1992

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HEICAL COIL SUSPENSION SPRINGS IN FINITE ELEMENT MODELS OF COMPRESSORS

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ABSTRACT

This paper discusses two approaches to modeling helical suspension springs for use in a dynamic finite element compressor model. The first approach represents the spring's stiffness characteristics with a simple spring element matrix. The appropriate spring constants are calculated using a separate finite element model of the spring. The spring modeling procedure is described and supplemented with classical solutions and experimental measurements. In addition, spring constants determined for a typical compressor suspension spring are presented and compared.

The second approach represents the spring with its own finite element mesh to include spring stiffness and mass effects in the compressor assembly model. This approach introduces spring surge which can significantly affect the compressor's modal properties. A major difficulty with the second approach is that incorporating a refined finite element mesh of each spring in the assembly model greatly expands the overall model size. To minimize this problem, we present procedures for making coarse-mesh spring models that maintain correct dynamic behavior over the frequency range of interest.

Finally, the dynamic characteristics of a compressor assembly model calculated with both spring representations is compared. The number of system eigenvalues approximately doubles in a given frequency range as a result of spring surge frequencies included by the meshed-spring models.

NOMENCLATURE

d  spring wire diameter
D  spring mean coil diameter
d.o.f.  degree of freedom
{d}  column matrix of finite element nodal displacements
E  modulus of elasticity
{f}  column matrix of finite element loads
g  gravitational constant
G  modulus of rigidity
I_{aa}  area moment of inertia about the a-a axis
J_{aa}  mass moment of inertia about the a-a axis
[k]  finite element stiffness matrix
K_a  spring constant corresponding to the a-direction
l  spring length
N  number of active spring coils
\rho  mass density
\gamma  specific weight

INTRODUCTION

Compressor suspension springs are key components in dynamic modeling and analysis of compressor assemblies. The springs support the compressor and are to provide vibration isolation between the compressor and the hermetic shell. This paper addresses two finite element method (FEM) approaches that may be taken to account for the springs and...
is a supplement to the paper "Dynamic Finite Element Modeling and Analysis of a Hermetic Reciprocating Compressor" [1].

A simple spring element or general stiffness element representation of the suspension springs will be presented first. In this approach, spring rates are found and entered into a matrix which characterizes the spring's static properties in the compressor assembly model. The second approach models the springs with their own finite element mesh in the compressor assembly model. This adds validity to the assembly model because spring surge, or the natural frequencies and normal mode shapes of the spring itself, will be included in the dynamic analysis. Both procedures are discussed and intermediate results are presented for a typical suspension spring. Finally, the dynamic characteristics for a compressor assembly model are presented for comparison of the approaches.

STIFFNESS FINITE ELEMENT REPRESENTATION OF HELICAL COIL SPRINGS

Simple Spring Finite Elements

In the first approach, the suspension springs are modeled by a single, two-node spring (or stiffness) finite element. Properties of the spring are characterized in an element stiffness matrix which describes the equilibrium relationship between nodal forces and nodal displacements. For linear elastic behavior, this relationship has the form

\[ [k][d]=[f] \]  

(1)

where \([k]\) is the element stiffness matrix, \([d]\) contains nodal degrees of freedom (d.o.f.), and \([f]\) contains the nodal forces. In a three dimensional problem, a 12x12 stiffness matrix is required to describe the six possible motions of each node.

To represent each spring in a compressor assembly model, a spring element is defined between a node on the compressor mechanism and a node in the housing. Then the appropriate spring rate coefficients are assigned to \([k]\). The principal translation and rotation terms are located along the diagonal. Off-diagonal terms of \([k]\) correspond to coupled force-displacement relationships between nodal d.o.f. These coupling terms should be included when their magnitudes are significantly large.

Depending on the specific finite element code chosen, entry of stiffness coefficients may be limited to the diagonal terms only. If this is the case, an equivalent linear beam element may be substituted for the spring element. The stiffness matrix of an isotropic beam element is defined by seven properties: cross-sectional area, a torsional constant, area moments of inertia, length, elastic modulus, and modulus of rigidity. These properties would be selected so that the beam-stiffness matrix approximates the desired spring-stiffness matrix.

Determining Spring Rates

In all, 21 coefficients are needed for the full stiffness matrix because of symmetry in \([k]\). The spring rates may be derived analytically, measured, or calculated from a separate finite element model of the spring. Among these methods FEM provides the full stiffness matrix, including the coupling terms, most efficiently.

To determine spring rates with the FEM procedure, the spring is first modeled using many consecutive beam elements along the spring wire path as shown in figure 1. The spring geometry is readily developed using node and element copy procedures. Nodes are defined with uniform axial, radial, and angular position increments in a polar coordinate system. Then parabolic beam elements are defined having material and cross-section properties appropriate for the spring wire.
Next, spring rates are calculated from static solutions of the spring model. The procedure is to: 
1. restrain a node (set all 6 nodal d.o.f. equal to zero) at one spring end,
2. apply a unit deflection at the other spring end, for example, set \( \Theta_2 = 1.0 \) to calculate the torsional spring rate, and
3. zero the remaining 5 d.o.f. at the deflected end.

The reaction forces calculated at the deflected end correspond to the spring rates, including the coupling terms. Coefficients for the full spring stiffness matrix are found with six static solutions, one for each deflected d.o.f. Convergence of the finite element procedure should, of course, be assured by repeating the analysis with further discretization.

Deriving the spring rates is possible using simplified versions of beam theory or strain energy expressions in conjunction with Castigliano's second theorem \([2]\). This procedure was not followed through for each coefficient of the stiffness matrix, however, it is beneficial to find a few derived values for comparison to the FEM results during the spring model development. Basic design formulas for the principal-direction spring rates are widely available \([2,3,4]\) and are reportedly quite accurate when certain geometric configurations regarding the spring index and pitch angle are adhered to \([2]\).

For instance, in a helical spring having end conditions fixed as described above in the FEM procedure we have the following formulas.

\[
\text{Axial spring rate: } K_a = \frac{d^4G}{8D^3N} \quad (2)
\]

\[
\text{Lateral spring rate: } \frac{1}{K_L} = \left[ \frac{E(2G + E)}{6G} + D^2 \right] \frac{8ND}{d^4E} \quad (3)
\]

\[
\text{Torsional spring rate: } K_t = \frac{Ed^4}{64ND} \quad (4)
\]

Finally, spring rates may be measured. It is difficult, however, to measure spring rates other than those in the simple axial and lateral directions. Thus, obtaining the spring data necessary to complete the full stiffness matrix is not practical. As with the derived spring
rates, a few terms are beneficial to verify the FEM spring model.

Table 1 compares spring constants determined by the methods mentioned above. As shown by the variance of results, the particular spring chosen for this example presents as many difficulties as one may encounter. For example, some of the coils are wound with a tapered coil diameter, one end has several coils close-wound and is ground flat, the other end is closed and threaded into the compressor block. Analytically, it is important to estimate the number of active coils accurately. Also, the changing coil diameter is not accounted for in the formulation of the elementary solutions. The range of values shown in Table 1 from the elementary formulas correlate with the diameter range of the tapered sections. Experimentally, proper clamping of the ends and avoiding eccentric loads can make spring rate measurement difficult. All of these factors may lead to measurement and modeling differences.

### Table 1. Spring Rate Comparison

<table>
<thead>
<tr>
<th>Direction</th>
<th>Elementary</th>
<th>870 DOF FEM</th>
<th>TEST</th>
</tr>
</thead>
<tbody>
<tr>
<td>Axial(^1)</td>
<td>431-662 (75.5-116)</td>
<td>543 (95.1)</td>
<td>447 (78.3)</td>
</tr>
<tr>
<td>Lateral(^1)</td>
<td>352-441 (61.6-77.2)</td>
<td>386 (67.6)</td>
<td>410 (71.8)</td>
</tr>
<tr>
<td>Torsion(^2)</td>
<td>109-125 (12.3-14.1)</td>
<td>118 (13.3)</td>
<td>---</td>
</tr>
</tbody>
</table>

\(^1\) Translational units are lb/in (N/mm)
\(^2\) Rotational units are lb•in/rad (N•m/rad)

**FINITE ELEMENT MESH REPRESENTATION OF HELICAL COIL SPRINGS**

As modeling work progressed on the compressor studied in this investigation, the matrix spring representation was found to be limited. Dynamic characteristics calculated for the compressor assembly differed only moderately from those calculated for the empty housing. On the contrary, test data showed an approximate doubling of resonances in the compressor assembly. Numerous structural and fluid interactions among the assembly components, including spring surge, potentially contribute to modal density increases. Moreover, as reported by Hamilton [6], spring surge tends to "short circuit" the vibration isolation expected from the suspension springs.

It is desirable to include the spring modes in the compressor models if spring resonances occur in the frequency range of interest. The full spectrum of spring resonances may be determined from an eigensolution of the independent spring models of the previous section. Alternatively, a quick calculation of the spring surge frequency can be made with elementary spring design formulas. For the undamped helical spring with fixed ends, the natural frequency, in Hz, for vibration along the spring length is

\[
f_i = \frac{i}{2} \sqrt{\frac{Kg}{W_s}} \quad i = 1, 2, 3, \ldots \tag{5a}
\]

where \(K\) is the axial spring rate as determined from eq. 2, \(g\) is the gravitational constant 386 in/sec\(^2\) (9.81 m/sec\(^2\)), and \(W_s\) is the spring weight which is found from

\[
W_s = \frac{\pi^2 d^2 DNY}{4} \tag{5b}
\]

where \(\gamma\) is the spring material specific weight. The fundamental axial frequency is found from eq. 5a using \(i = 1\), the second harmonic is found using \(i = 2\), etc. This equation shows the first axial surge mode of the example spring to occur between 567 and 755 Hz.
Equation (5a) will generally overestimate the surge frequency because the derivation considers translational motion only, rotational inertia of the spring wire is ignored. It is nonetheless a good approximation to determine if spring surge is a concern in a given frequency range.

An internal torsional surge relation can be found for helical springs by adapting the fixed-fixed torsional mode solution of a continuous rod. Substituting $\pi ND$ for the length of the rod, one obtains,

$$f_{int} = \frac{i}{2\pi ND} \sqrt{\frac{Gg}{\gamma}}$$

(6)

However, these modes usually begin at much higher frequencies. Equation (6) yields the first internal torsion surge of the example spring to be between 3770 and 4350 Hz.

Lateral surge frequency relations have also been derived. Wahl [7] reported an analysis by Haring in 1949. He related the lateral surges to the axial surge frequencies, a length-to-diameter ratio, and an optional ratio of compression to free length. For $I/D$ of 3 or less, the lateral surge frequencies are about 1.0 to 1.5 times higher than axial surges. The spring modes may be included in the compressor assembly model by adding the independent spring finite element models, described in the previous section, in place of the simple spring elements. Note that simply including the refined spring models is logical but will unnecessarily expand the assembly model size drastically. For this reason, it is advantageous to develop coarse-mesh spring models which maintain dynamic accuracy for use in the compressor model.

Reduced Degree of Freedom Spring Model Development

Figure 2 shows a coarse-mesh spring model which has only four linear beam elements per coil, and is comparable to the refined model of figure 1. This d.o.f. reduction results in significant static stiffening. Through a combination of material geometric volume conservation and model tuning, the coarse-mesh model can, however, be coaxed to agree with the fine-mesh model.

Figure 2. Coarse-Mesh FEM Spring Model having 150 d.o.f.

Preserving volume of the structure upon discretization is recognized as good modeling practice. For the spring model application, the coil diameter of the coarse model is increased so that the combined length of four elements is equivalent to the true coil circumference. However, this modification alone does not resolve over-stiffening by the coarse mesh.
Consequently, natural frequency predictions will be elevated, for example, frequencies found with the reduced springs considered here ranged from (0 to 30%) high. Spring rates are compared in Table 2.

Table 2: Reduced-Spring Rate Comparison

<table>
<thead>
<tr>
<th>Direction</th>
<th>870 DOF FEM</th>
<th>150 DOF FEM</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Axial¹</td>
<td>543 (95.1)</td>
<td>701 (12.3)</td>
<td>+29%</td>
</tr>
<tr>
<td>Lateral¹</td>
<td>386 (67.6)</td>
<td>405 (70.9)</td>
<td>+4.9%</td>
</tr>
<tr>
<td>Torsion²</td>
<td>118 (13.3)</td>
<td>118 (13.3)</td>
<td>0%</td>
</tr>
<tr>
<td>Bending²</td>
<td>312 (35.2)</td>
<td>323 (36.5)</td>
<td>+3.5%</td>
</tr>
</tbody>
</table>

¹Translational units are lb/in (N/mm)
²Rotational units are lb•in/rad (N•m/rad)

Next, model tuning is employed to improve dynamic correlation with the refined models. The best tuning parameter was found to be mass density which results in a nearly uniform shift of natural frequencies. In addition, the modified mass density desired, $\rho_{\text{modified}}$, necessary for correlation of the reduced d.o.f. spring models may be calculated directly from

$$
\frac{\rho_{\text{modified}}}{\rho_{\text{reduced}}} = \left(\frac{f_{\text{reduced}}}{f_{\text{refined}}}\right)^2
$$

where $\rho_{\text{reduced}}$ and $f_{\text{reduced}}$ are found in an initial eigensolution of the reduced model, and $f_{\text{refined}}$ is the target frequency found by the refined model. It was necessary to increase the mass density of the spring of this study by a factor of about 1.3 to reach agreement with the refined model.

Table 3 compares the first ten spring modes found by the refined and reduced side spring models. All mode shapes are correlated and the corresponding frequencies are within 5.3 percent difference. Spring rates of the reduced model, however, do not improve with the density correction. This artificial stiffening is expected to affect only the low-frequency compressor-suspension modes of the assembly model.

Table 3. Reduced Spring Normal Mode Comparison
The natural frequencies are sensitive to other parameters but tuning the reduced model by other avenues were not successful. At first the torsional constant, J, was changed in an effort to correlate natural frequencies. The effect proved to be localized, depending on the mode shape. For example, reducing J to match the first axial mode has no effect on a radially expansive mode and has inconsistent effects on the lateral modes. The localized effects result in a non-uniform reduction of natural frequencies and consequently a change in the numerical order of mode extraction. The jumbled modes are difficult to correlate. Modifications to area moments of inertia, I, cause local effects similar to those found with torsional constant modifications. In addition, relations between I, J and the natural frequency are not readily available, so iterations are necessary to arrive at the appropriate modified values.

Elastic modulus is another potential parameter for tuning the coarse-mesh model but was not considered here. Tuning with the elastic modulus may prove to be more successful than tuning with mass density because it will improve stiffness agreement as well. In addition, natural frequencies are related to $\sqrt{E}$, so the modified value may be solved for directly.

**SPRINGS IN THE COMPRESSOR ASSEMBLY DYNAMIC FINITE ELEMENT MODEL**

In this study, spring surge proved to be an important factor in the compressor assembly's dynamic properties. Table 4 compares the normal modes of a compressor having the simple matrix springs to a compressor model having the coarse-mesh springs. The significance of surge frequencies on the compressor is discussed more thoroughly by Kelly and Knight [1], but the spring representation differences are of interest here. For the frequency range shown, spring surge frequencies account for nearly one-half of the assembly resonances. The meshed-spring representation is necessary to find these modes. Presently, the coarse-mesh spring is somewhat stiffer than the more refined mesh model. This is evident in slightly elevated compressor suspension frequencies.
Table 4. Compressor Assembly Normal Modes

<table>
<thead>
<tr>
<th>Mode</th>
<th>[k] Springs Freq., Hz</th>
<th>Meshed-Springs Freq., Hz</th>
<th>Mode Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-6</td>
<td>0</td>
<td>0</td>
<td>Rigid Body</td>
</tr>
<tr>
<td>7</td>
<td>8.6</td>
<td>18.4</td>
<td>compressor suspension modes</td>
</tr>
<tr>
<td>8</td>
<td>13.3</td>
<td>20.1</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>14.8</td>
<td>22.2</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>22.2</td>
<td>31.9</td>
<td>compressor suspension modes</td>
</tr>
<tr>
<td>11</td>
<td>26.2</td>
<td>33.6</td>
<td>compressor suspension modes</td>
</tr>
<tr>
<td>12</td>
<td>32.3</td>
<td>35.7</td>
<td>compressor suspension modes</td>
</tr>
<tr>
<td>13</td>
<td>99.4</td>
<td>99.4</td>
<td>shockloop</td>
</tr>
<tr>
<td>14</td>
<td>163</td>
<td>163</td>
<td>shockloop</td>
</tr>
<tr>
<td>15</td>
<td>208</td>
<td>208</td>
<td>shockloop</td>
</tr>
<tr>
<td>16</td>
<td>---</td>
<td>347</td>
<td>top spring</td>
</tr>
<tr>
<td>17</td>
<td>---</td>
<td>369</td>
<td>top spring</td>
</tr>
<tr>
<td>18</td>
<td>492</td>
<td>493</td>
<td>shockloop</td>
</tr>
<tr>
<td>19</td>
<td>---</td>
<td>497</td>
<td>top spring</td>
</tr>
<tr>
<td>20</td>
<td>---</td>
<td>504</td>
<td>top spring</td>
</tr>
<tr>
<td>21</td>
<td>535</td>
<td>536</td>
<td>shockloop</td>
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<tr>
<td>22</td>
<td>---</td>
<td>627</td>
<td>top spring</td>
</tr>
<tr>
<td>23</td>
<td>--- 646</td>
<td>652</td>
<td>side spring</td>
</tr>
<tr>
<td>24</td>
<td>---</td>
<td>652</td>
<td>side spring</td>
</tr>
<tr>
<td>25</td>
<td>662</td>
<td>662</td>
<td>modified housing mode M8(3,1)</td>
</tr>
<tr>
<td>26</td>
<td>--- 685</td>
<td>690</td>
<td>top spring</td>
</tr>
<tr>
<td>27</td>
<td>---</td>
<td>710</td>
<td>modified housing mode M9(3,1)</td>
</tr>
<tr>
<td>28</td>
<td>726</td>
<td>719</td>
<td>top spring</td>
</tr>
<tr>
<td>29</td>
<td>--- 731</td>
<td>760</td>
<td>side springs, housing M9(3,1), M10(2,1)</td>
</tr>
<tr>
<td>30</td>
<td>---</td>
<td>760</td>
<td>side springs</td>
</tr>
<tr>
<td>31</td>
<td>--- 777</td>
<td>790</td>
<td>side springs</td>
</tr>
<tr>
<td>32</td>
<td>---</td>
<td>808</td>
<td>all springs, housing M10(2,1)</td>
</tr>
<tr>
<td>33</td>
<td>--- 810</td>
<td>854</td>
<td>shockloop</td>
</tr>
<tr>
<td>34</td>
<td>---</td>
<td>854</td>
<td>modified housing mode M11(4,1)</td>
</tr>
<tr>
<td>35</td>
<td>---</td>
<td>909</td>
<td>top spring</td>
</tr>
<tr>
<td>36</td>
<td>--- 937</td>
<td>968</td>
<td>top spring, housing M12(top), M13(4,1)</td>
</tr>
<tr>
<td>37</td>
<td>---</td>
<td>968</td>
<td>top spring, housing M12(top)</td>
</tr>
<tr>
<td>38</td>
<td>---</td>
<td>977</td>
<td>modified housing mode M12(top)</td>
</tr>
<tr>
<td>39</td>
<td>--- 971</td>
<td>985</td>
<td>modified housing mode M13(4,1)</td>
</tr>
<tr>
<td>40</td>
<td>---</td>
<td>1020</td>
<td>shockloop</td>
</tr>
<tr>
<td>41</td>
<td>---</td>
<td>1020</td>
<td>modified housing mode M14(2,1,top)</td>
</tr>
<tr>
<td>42</td>
<td>---</td>
<td>1266</td>
<td>shockloop</td>
</tr>
<tr>
<td>43</td>
<td>---</td>
<td>1266</td>
<td>top spring, housing mode M15(5,1)</td>
</tr>
<tr>
<td>44</td>
<td>---</td>
<td>1291</td>
<td>modified housing mode M15(5,1)</td>
</tr>
</tbody>
</table>

1Identified housing deflection experimentally
2Housing deflection is described by an (n,m) type descriptor, where n corresponds to the circumferential pattern and m corresponds to the axial pattern.
CONCLUSIONS

One approach to represent helical suspension springs in a compressor assembly model is by a simple spring or stiffness element. For this method, an independent finite element spring model can efficiently and accurately calculate all of the necessary spring stiffness coefficients, including the coupling terms. In addition, measurements and elementary spring relations are beneficial to this procedure.

If spring surge exists in the frequency range of interest, the springs should be represented with their own finite element mesh in the assembly. The spring surge characteristics may be calculated with the independent spring model. Alternately, simple axial and torsional surge frequencies may be quickly approximated from the elementary relations presented.

ACKNOWLEDGMENTS

The authors wish to thank Bristol Compressors, Inc. and the Virginia Center for Innovative Technology for their sponsorship. We also thank Dr. L. D. Mitchell, Dr. R. G. Mitchiner, and Mr. David Gilliam for their efforts and contributions to this research.

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