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BICYLINDRICAL COORDINATE FORMULATION FOR THE LEAKAGE FLOW THROUGH THE MINIMAL CLEARANCE IN A ROLLING PISTON COMPRESSOR


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ABSTRACT

The flow of pure lubricating oil through the minimal clearance has been calculated by solving the continuity and the full Navier - Stokes equations expressed in a bicylindrical coordinate system using the finite volume method. Precise oil flow measurements have been performed and an excellent agreement with the numerical data has been achieved. The flow of pure lubricating oil can be used to establish the upper limit for the real gas leakage through the minimal clearance and so it should be precisely calculated. The details of the experimental methods and the description of the experimental setup are presented.

NOMENCLATURE

- $E_p$: rolling piston eccentricity with respect to the cylinder center, [m].
- $h$: rolling piston height, [m].
- $h$: square root of the metric of the bicylindrical coordinate system, [m].
- $m$: oil mass flow rate through the minimal clearance, [kg/s].
- $p_l$: local pressure, [Pa].
- $p_d$: discharge side pressure, [Pa].
- $p_s$: suction side pressure, [Pa].
- $r_c$: cylinder radius, [m].
- $r_p$: rolling piston radius, [m].
- $T$: oil temperature, [$^\circ$C].
- $u$, $v$, $w$: velocity components in directions $\psi$, $\eta$ and $\zeta$, [m/s].
- $x$: distance from the piston to the cylinder wall, [m].
- $\psi$, $\eta$, $\zeta$: bicylindrical coordinates.
- $\alpha$: measuring angle.
- $\rho$: fluid density, [kg/m$^3$].
- $\beta$: arbitrary angle for the definition of the calculation domain.
- $\mu$: fluid absolute viscosity, [kg/m/s].
- $\delta_{\min}$: minimal clearance distance, [m].

INTRODUCTION

It is well known that the leakage through the minimal clearance in rolling piston type compressors constitutes the major contribution for the internal mass flow losses. It accounts for almost 70% of the total internal gas leakage.

Costa et al. [01] presented a one-dimensional model to evaluate
the mass flow rate of oil from the high pressure to the low pressure side of the compressor through the clearance between the roller and the cylinder wall. The flow of oil is responsible for the leakage of refrigerant through the minimal clearance. The validation of the 1-D model was performed using experimental results measured in a static test bench. The agreement between numerical and measured results was considered reasonably good for small values of the minimal clearance, despite the great difficulty in performing these experiments. First of all the test section with a predetermined minimum clearance (15-30μm) had to be carefully set up in order to avoid any mechanical deformation and other undesirable path leakages. The uncertainty involved in the evaluation of the minimal clearance was considered high, therefore several experimental runs had to be performed in order to reduce the dispersion of the measured values. Secondly, in order to reduce the viscosity of the lubricating oil being used in the static test experiment, heat was provided through resistance wires placed in the suction pipe and around the cylinder. In spite of using thermal insulation around the cylinder, it was rather laborious to exactly reproduce the thermal condition for all the experimental runs. A few degrees difference could substantially alter the test results as the oil viscosity was highly depended upon its temperature.

Associated with the previous discussion, more experiments are needed in order to overcome the detected deficiencies, and they should be performed as follows:

a) The experiments should be run at room temperature.
b) The oil pipeline should be provided with very fine oil filters in order to avoid the presence of particles in the test section.
c) A new pump kit should be used as a test section, after being checked against shape errors.
d) A more reliable method should be used to measure the minimal clearance before and after each test run, as it will be described in another section.
e) Higher values for the minimal clearance (40-80 μm) should be used.

The experimental results for the upper range of the minimal clearance showed poorer agreement with the 1-D model numerical results when compared to lower gaps. One of the reasons was related to the simplifying assumptions used in the 1-D model, as it does not consider the inertia terms within the Navier-Stokes equations. The inclusion of these terms in the model requires a more sophisticated numerical method for the solution of the governing equations. This has been done using a b bicylindrical coordinate system, needed by the geometry of the rolling piston compressor. The numerical method being used is the finite volume method developed by Patankar [22].

The main objective of this work is to present the validated numerical results of the leakage through the minimal clearance in the form of dimensionless groups according to the variables involved in the problem. The experimental procedures and the experimental test rig are concisely described.

PROBLEM FORMULATION

The geometry of the pressure driven flow through the minimal clearance of the rolling piston compressor has to be described by a bicylindrical coordinate system (ψ, η, z), as shown in Fig. 1. The oil flows circumferentially from the high pressure to the low pressure side while the rolling piston is kept still.

The hydrodynamic problem is governed by the continuity (1) and Navier-Stokes (2)-(4) equations written in bicylindrical coordinates for laminar, incompressible, isothermal and steady conditions.
Fig. 1 - Geometry of the problem

\[ 1/h^2 [\partial(\partial_p u)/\partial x + \partial(\partial_p v)/\partial y + \partial(\partial_p w)/\partial z] = 0 \]  \hspace{1cm} (1)

\[ 1/h^2 [\partial(\partial_p u)/\partial x + \partial(\partial_p v)/\partial y + \partial(\partial_p w)/\partial z] = \]

\[ -1/h \partial p/\partial x + \mu/h^2 [\partial^2 u/\partial x^2 + \partial^2 u/\partial y^2 + h^2 \partial^2 u/\partial z^2] + \]

\[ + \mu/h^2 (2/h \partial u/\partial y + 2/h \partial u/\partial z - 2/h \partial u/\partial x) - \]

\[ - u/h (\partial^2 h/\partial x^2 + \partial^2 h/\partial y^2) - \rho/h^2 [u \partial u/\partial x - v \partial u/\partial y] \] \hspace{1cm} (2)

\[ 1/h^2 [\partial(\partial_p v)/\partial y + \partial(\partial_p v)/\partial y + \partial(\partial_p w)/\partial z] = \]

\[ -1/h \partial p/\partial y + \mu/h^2 [\partial^2 v/\partial x^2 + \partial^2 v/\partial y^2 + h^2 \partial^2 v/\partial z^2] + \]

\[ + \mu/h^2 (2/h \partial v/\partial x + 2/h \partial v/\partial z - 2/h \partial v/\partial y) - \]

\[ - v/h (\partial^2 h/\partial x^2 + \partial^2 h/\partial y^2) - \rho/h^2 [u \partial v/\partial y - u \partial v/\partial y] \] \hspace{1cm} (3)

\[ 1/h^2 [\partial(\partial_p w)/\partial z + \partial(\partial_p v)/\partial y + \partial(\partial_p w)/\partial z] = \]

\[ - \partial p/\partial z + \mu/h^2 [\partial^2 w/\partial x^2 + \partial^2 w/\partial y^2 + h^2 \partial^2 w/\partial z^2] \] \hspace{1cm} (4)

where \( u, v, \) and \( w \) are the velocity components in the \( x, y, \) and \( z \) directions, respectively, \( p \) is the pressure, \( \rho \) is the fluid density, \( \mu \) is the absolute viscosity and \( h \) is the square root of the metrics, or Lam's coefficient \((03)\), related to the bicylindrical coordinate system, given by
where $a$ is the geometric parameter of the coordinate system.

The boundary conditions needed for the specification of the problem are given by Eqs. (5).

\begin{align}
  u = v = w &= 0 \quad \text{for } 0 \leq z \leq H; \quad \eta = \eta_1; \quad \beta \leq \psi \leq 2\pi - \beta \\
  u = v = w &= 0 \quad \text{for } 0 \leq z < H; \quad \eta = \eta_2; \quad \beta \leq \psi \leq 2\pi - \beta \\
  u = v = w &= 0 \quad \text{for } z = 0; \quad \eta_1 \leq \eta \leq \eta_2; \quad \beta \leq \psi \leq 2\pi - \beta \\
  u = v = w &= 0 \quad \text{for } z = H; \quad \eta_1 \leq \eta \leq \eta_3; \quad \beta \leq \psi \leq 2\pi - \beta \\
  u = \frac{\partial p}{\partial n}, \quad v = w &= 0 \quad \text{for } 0 \leq z \leq H; \quad \eta_1 \leq \eta \leq \eta_2; \quad \psi = 2\pi - \beta
\end{align}

where $\beta$ is a small angle, in order to take into account the presence of the blade ($\beta \sim 5^\circ$) and $A_{in}$ is the flow entrance area into the calculation domain.

**NUMERICAL SOLUTION**

The differential equations (1)-(4) governing the laminar flow field and the associated boundary conditions (6) are discretized using the finite volume method and the algebraic equations are solved using the methodology developed by Patankar [02]. The detailed discretization of all the terms appearing in the differential equations is presented by Gasche [04].

The final mesh used to generate the results to be presented here has 1232 nodal points, being 28 points in $\psi$ - direction, 11 points in $\eta$ - direction and 4 points in $z$ - direction. Finer meshes have been tested without showing greater improvement of the results.

The validation of the numerical model is performed through comparison with the experimental results.

**EXPERIMENTAL SETUP AND PROCEDURES**

The experimental setup employed to generate the data used to validate the numerical model is shown in Fig. 2 and 3. The pumping kit of a rolling piston type compressor is kept stationary with all clearances hermetically sealed, except the minimal clearance. An auxiliary reciprocating hermetic compressor is used to generate the pressure difference between two graduated reservoirs filled with pure lubricating oil. The pressure in both chambers are measured with two HBM absolute pressure transducers carefully mounted in order to avoid any air trapped in the oil pipeline. The oil flow rate is measured with a Micromotion Coriolis type flow meter with an uncertainty of 0.4% of the read value. Copper-Constantan thermocouples are used to monitor the oil temperature at both chambers, as the experimental is performed at room temperature (25-30°C). All data are acquired by a Yokogawa 3087 Hybrid Recorder in intervals of one minute during the whole test run.

The measurement of the minimal clearance is performed by a Carl Zeiss Measuring Microscope, model ZKM 01-250D with an amplification range of 100 X. The center of the roller of the pump kit is placed exactly in the center of the microscope rotating table and the distances between the roller and the cylinder are measured at $-30^\circ$, $-20^\circ$, $0^\circ$, $20^\circ$, $30^\circ$, being taken two values in each point in the ascending and descending directions of the angle values. The $0^\circ$ angle
corresponds to the position of the minimal clearance. With the measured distances at other angles, the roller eccentricity can be determined by Eq. (7).

\[
R_c^2 = E_c \sqrt{R_c^2 - (x+R_p)^2 \sin^2 \alpha + (x+R_p) \left(R_c^2 - E_c^2 \sin^2 \alpha\right)}
\]

where \( \alpha \) is the angular position, \( x \) is the measured distance between the walls of the roller and the cylinder, \( R_p \) is the piston radius, \( R_c \) is the cylinder radius and \( E_c \) is the eccentricity, according to Fig. 1. The calculated minimal clearance for each angle is then determined.

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**Fig. 2 - Schematic view of the experimental setup**

**Fig. 3 - General view of the experimental setup**
The average of all ten values furnishes the minimal clearance reported in the experiments. The measurement of the minimal clearance is performed before and after each series of experiments.

Table 1 shows the results obtained for \( R_2 = 21.3225 \text{ mm}, \ R_p = 17.662 \text{ mm}, \ H = 11.0 \text{ mm} \) with the Texaco X-10 lubricating oil. For the range of temperatures used in the tests, the absolute viscosity and the oil density are given by Eqs. (8) and (9), respectively,

\[
\mu \text{ [kg/m/s]} = 0.6761 T^{-1.204}
\]

(8)

\[
\rho \text{ [kg/m}^3\text{]} = 867.5 - 0.67 T
\]

(9)

for \( T \) given in [°C].

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<th>Minimal Clearance [µm]</th>
<th>Average Temperature [°C]</th>
<th>Discharge Pressure [kPa]</th>
<th>Suction Pressure [kPa]</th>
<th>Mass Flow Rate [g/s]</th>
<th>Volumetric Flow Rate [mm³/s]</th>
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The agreement between the three-dimensional numerical results and the experimental results for the same mass flow rate is less than 10% for the highest minimal clearance being tested.

The main variables involved in the leakage through the minimal clearance phenomenon are the pressure difference across the gap \( (p_d - p_s) \), the dimension of the minimum clearance \( \delta_{min} \), the fluid density \( \rho \), the absolute viscosity \( \mu \), the height of the roller \( H \) and the mass flow rate \( m \).
Fig. 4 - Validation of the numerical model

Therefore, the mass flow rate can be expressed in a functional relationship by

\[ m = f((p_d - p_e), \delta_{\text{min}}, \rho, \mu, H) \]  \hspace{1cm} (10)

The following three dimensionless groups can be formed with those variables:

\[ G((m/\mu \delta_{\text{min}}), (p_d - p_e) \rho \delta_{\text{min}}^2 / \mu^2, H/\delta_{\text{min}}) = 0 \]  \hspace{1cm} (11)

Fig. 5 shows the numerical results obtained for the oil leakage using those dimensionless groups. Of course, for other pump dimensions which are not included in the dimensional analysis, the results, here obtained, should be carefully used.

Fig. 5 - Dimensional analysis of the flow through the minimal clearance
From a parametric analysis, one can see that the dimensionless oil leakage varies with the dimensionless pressure difference to the first power and with the dimensionless roller width to the power 0.504. Therefore, for the pump mechanism under analysis, Eq. (11) yields

\[ \left( \frac{a}{\mu \delta_{\text{min}}} \right) = 0.0162 \left( \frac{(p_d - p_s) \rho \delta^2_{\text{min}}}{\mu^2} \right) \left( \frac{H}{\delta_{\text{min}}} \right)^{0.504} \]  

Eq. (12) identifies that the leakage through the minimal clearance varies directly with \( \delta_{\text{min}}^2 \) and inversely with the kinematic viscosity \( \mu/\rho \).

The pressure variation along the minimal clearance, calculated using the bicylindrical coordinate formulation, shows a very steep gradient when the oil flows from the high pressure to the low pressure side of the compressor, as it can be observed in Fig. 6, for a minimal clearance of 14 \( \mu \)m. The same trend was observed with the one-dimensional model as reported by Costa et alii. [01].

The equation which converts the bicylindrical coordinate \( \psi \) into the cylindrical angle \( \theta \) is given by

\[ \theta = \tan^{-1} \left[ \sin \psi \right. \left. \sin \psi / \left( \cos \psi \cos \psi - 1 \right) \right] \]  

![Fig. 6 - Pressure profile along the flow](image)

It is really important to know the pressure and the velocity fields in order to calculate the leakage through the minimal clearance when dissolved refrigerant is flowing with the lubricating oil. It is expected that the flow of oil, when fluid refrigerant is dissolved in it, should be smaller than the values reported here due to the growth of the refrigerant bubbles when submitted to a very high favorable pressure gradient.

**CONCLUSIONS**

This paper reports the results of an experimentally validated bicylindrical formulation used to calculate the pure lubricating oil flow transported through the minimal clearance in a rolling piston type compressor. Experimental results have been carefully performed and are fully reported. The experimental procedures and setup are concisely described.
Use has been made of dimensional analysis in order to reduce the numerical data and an equation for the oil leakage through the minimal clearance is proposed.

BIBLIOGRAPHIC REFERENCES


