A Practical Method of Selecting Capillary Tubes for HCFC22 Alternatives

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A PRACTICAL METHOD OF SELECTING CAPILLARY TUBES
FOR HCFC22 ALTERNATIVES

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ABSTRACT

This paper presents a simple and practical method for sizing capillary tubes. An analytical equation is developed and a closed form of capillary tube length as a function of the mass flow rate is obtained. The present model, tested for HCFC22 and its alternatives R134a, R410a, R404a, R507a and R407c, is found to agree reasonably well with experimental data in open literature.

1. INTRODUCTION

Capillary tubes are extensively used on small refrigeration systems such as window air conditioning and household refrigerators. Capillary tubes are cheap, reliable and simple to install. But once the capillary length and diameter have been selected, the capillary tube can not adjust to load variation in contrast to other expansion devices such as thermostatic expansion valve. There is one point, called balance point, where the capillary tube is at its maximum efficiency. Next, the flow characteristic inside the tube is very complex. A number of research works have been carried out both in theoretical and experimental (Kuehl and Goldschmidt 1991). Computer models have been developed by many researchers to overcome the trial and error process in selecting the right length and diameter for a given set of operating conditions (Bittle and Pate 1996, Escanes et al. 1957, Li and Chen 1991). Even though, these models gave acceptable accuracy with experimental data, they are time consuming and they require computer skills. Empirical correlations are developed for simple predictions in the range of regressive data and can not arbitrarily extrapolated (Melo et al. 1999, Bansal and Rupasinghe 1996, Melo et al. 1997). Little analytical works have been done on capillary tubes. These approximate solutions can be useful tools for theoretical analysis and can provide quick engineering calculations for practicing engineers. Two major analytical works are presented. First, Cooper (1957) developed a simple method for capillary tube selection by noting that the mixture specific volume varied linearly along a fanno-pressure line on a semi-log plots. This method, however, was only tested for refrigerant R22. Second, Yilmaz and Unal (1996) developed a simple analytical equation for capillary tube design. In their work Yilmaz and Unal (1996) assumed an isenthalpic process and developed a relation between specific volume and pressure in the vapor mixture region, based on Clausius-Clapeyron equation. This work was further refined by Zhang and Ding (2001) who addressed some of the problems that have existed in the model. These problems include: choked flow condition was not considered and the reference point was not appropriately chosen. In the present study a simple analytical solution is developed for capillary tube sizing for pure as well as mixture refrigerants

2. BACKGROUND

As shown earlier, i.e., Cooper (1950) and Yilmaz-Unal (1996), the basic idea in developing analytical equations for capillary tube is to find a relation between pressure and specific volume in the vapor mixture
region. This relation is then used in the momentum equation in order to obtain a closed form for the capillary length. Cooper (1957) has developed a relation between \( P \) and \( \nu \) in the two-phase flow region by assuming an adiabatic process along a fanno line state path. This relation can be written as:

\[
\ln(\nu) = aP + b
\]  

(1)

Where \( a \) and \( b \) are constants. This model was only tested for refrigerant R22. Next, Yilmaz and Unal (1996) have assumed an isenthalpic process and by using the Clausius-Clapeyron equation, they have developed the following relation between \( P \) and \( \nu \) in the two-phase flow region:

\[
\nu^* = 1 + \beta(1 / P^* - 1)
\]  

(2)

Where, \( p^* = P / P_r \), \( \nu^* = \nu / \nu_r \). Subscript \( r \) is called the reference point, defined as the crossing of the isenthalpic line and the saturated liquid line in the pressure-enthalpy diagram. The slope, \( \beta \), was later redefined by Zhang and Ding (2004) for better correlation and it is equal to

\[
\beta = 1.63 \times 10^5 / P_r^{0.72}
\]  

(3)

In the present work, an accurate \( p - \nu \) relation is proposed for isenthalpic process in the vapor mixture region for pure as well as mixture refrigerants. This relation can be written as follow:

\[
\ln(\nu) = aP^3 + bP^2 + cP + d
\]  

(4)

Where, \( a, b, c, d \) are evaluated for each isenthalpic line in the mixture region. These constants are determined as follow: first, the pressure and enthalpy are found at the crossing point between the isenthalpic line and the saturated liquid line, as shown in figure 1. Then the vapor quality, specific volume are calculated for different pressure nodes along the isenthalpic line. Next, The relation between specific volumes and pressures are found using a simple MATLAB function called polyfit. The proposed cubic fit function relation, tested for a wide pressure ranges and for different types of pure and mixture refrigerants, gives an excellent \( p - \nu \) relation and more accurate than that of Yilmaz and Unal (1996). For example, figures 2 and 3 show data comparison between the present relation and that of Yilmaz-Unal (1996) for R22 and R410a, respectively. As one can see the proposed relation nearly match the actual data with a percentage error less than 6%, while the Yilmaz-Unal (1996) equation deviates considerably at lower pressure values as can be seen in figures 4 and 5, respectively. Next, the Yilmaz-Unal equation does not well represent the \( p - \nu \) relation for R407C, a zeotropic refrigerant mixture in which the liquid saturation pressure differs considerably from the vapor saturation pressure.

\[\text{Figure 1: Schematic of the throttling process}\]
Figure 2: R22 $P - \nu$ Relation for isenthalpic process for $Pr = 19.427$ bar 
(Coefficient constants: $a = -0.6501$, $b = 2.4976$, $c = -4.8180$, $d = -2.3143$)

Figure 3: R410a $P - \nu$ Relation for isenthalpic process for $Pr = 30.65$ bar 
(Coefficient constants: $a = -0.1418$, $b = 0.9143$, $c = -2.8875$, $d = -2.5061$)

Figure 4: Deviation of specific volume for R22 at $Pr = 19.427$ bar
Figure 5: Deviation of specific volume for R410a at $P_r = 30.65\text{bar}$

Figure 6 shows the difference between the actual $P - v$ relation and the relation generated by the Yilmaz-Unal (1996) equation; and an error of 50% was found for this particular inlet condition, as shown in figure 7. This poor representation of R407C can be explained by the fact the reference pressure $P_r$ used in equations 2 and 3, respectively, is taken as the pressure of the saturation liquid and did not account for the changes between the liquid saturation pressure and that of saturation vapor.

Figure 6: R407C $P - v$ Relation for isenthalpic process for $P_r = 17.37\text{bar}$
(Coefficient constants: $a=-1.2254$, $b=3.8843$, $c=-6.0499$, $d=-1.7879$)
The flow in capillary tube can be treated as homogenous equilibrium flow (Kuehl and Goldschmidt 1991). In this work the effect of underpressure and pressure drop due to area contraction at the capillary tube inlet are not considered. The governing equations can be written as follow:

\[
\frac{dG}{dL} = 0 \quad (5)
\]

\[
d(L + \frac{1}{2} G^2 v^2) = 0 \quad (6)
\]

\[-dp = G^2 dv + \frac{f}{2D} G^2 vdL \quad (7)\]

The process in capillary tube is assumed to be an isenthalpic process, which is a good approximation for adiabatic flow (Kuehl and Goldschmidt 1991). Capillary tube can be divided into a subcooled and a two-phase flow region. The length of the subcooled region can be obtained by integrating the momentum equation from capillary pressure inlet \( (p_{in}) \) to the pressure point \( (p_r) \), defined in this work as the pressure at the crossing point between the saturated liquid line and the isenthalpic line in the pressure-enthalpy diagram. This can be written as:

\[
L_{liq} = \frac{2D(p_{in} - p_r)}{f_{in} G^2 v_{in}} \quad (8)
\]

In the two phase region, Eq. (4) can be used to relate pressure and specific volume. Substituting Eq. (4) into Eq. (7) and taking integral from \( p_r \) to outlet pressure \( p_{out} \), one can obtain the length of the two-phase flow region as flow

\[
L_{lp} = \frac{2D}{f_{lp} G^2} \left[ \frac{\exp(-F(p_{out}))}{F'(p_{out})} - \frac{\exp(-F(p_r))}{F'(p_r)} \right] - \frac{2D}{f_{lp}} \left[ F(p_{out}) - F(p_r) \right] \quad (9)
\]

Where, \( F \) and \( F' \) are the cubic function fit and its derivative defined by:

\[
F(p) = aP^3 + bP^2 + cP + d \quad (10)
\]

and,

\[
F' = \frac{dF}{dP} = 3aP^2 + 2bP + c \quad (11)
\]

Figure 7: Deviation of specific volume for R407C at Pr = 17.37bar
The total length of the capillary tube can then be written as:

\[
L = \frac{2D(p_{in} - p_{r})}{f_{in}G^2v_{in}} + \frac{2D}{f_{ip}G^2} \left[ \frac{\exp(-F(p_{out}))}{F'(p_{out})} - \frac{\exp(-F(p_{r}))}{F'(p_{r})} \right] - \frac{2D}{f_{ip}} \left[ (F(p_{out}) - F(p_{r})) \right] \tag{12}
\]

Eq. (10) is an explicit approximate solution of the capillary tube length when the flow is not choked. As one can this equation is easy to use for capillary tube sizing.

4. FRICTION FACTOR AND VISCOSITY MODELS

Friction factor is an important variable in capillary tube calculation. Different correlations have been suggested for the friction factors to best represent experimental data. In general the friction factor can be evaluated as follow:

\[
f = c_1 \frac{\text{Re}^{C_2}}{\mu} = c_1 \left( \frac{GD}{\mu} \right)^{C_3} \tag{13}
\]

Where \(C_1\) and \(C_2\) are empirical constants. Table 1 summarizes different friction factor correlations found in literature. Next, there are three suggested models for the viscosity, \(\mu\), in the two-phase flow region. These models are presented in table 2. Friction factor with the viscosity are considered as factors of uncertainties in predicting capillary tube lengths or mass flow rates. Jung et al. (1999) have shown that different capillary tube lengths can be obtained by using the same friction factor correlation but with different viscosity models for one set of operating conditions. In the present study the stocker (1982) friction factor model is employed along with the McAdams (1942) viscosity model. The average friction factor along is taken along the capillary tube.

<table>
<thead>
<tr>
<th>Model</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stocker (1982)</td>
<td>(f = 0.33 \frac{1}{R_e^{0.25}})</td>
</tr>
<tr>
<td>Modified Blasius (1982)</td>
<td>(f = 0.3 \frac{1}{R_e^{0.25}})</td>
</tr>
<tr>
<td>Goldestein (1981)</td>
<td>(f = 0.02)</td>
</tr>
<tr>
<td>Erth (1970)</td>
<td>(f = 0.31 \frac{1}{R_e^{0.25}} \exp \left( \frac{1 - x^{0.25}}{2.4} \right))</td>
</tr>
<tr>
<td>Pate (1950)</td>
<td>(f = 3.49 \frac{1}{R_e^{0.47}})</td>
</tr>
<tr>
<td>Hopkins (1950)</td>
<td>(f = 0.217 \frac{1}{R_e^{0.2}})</td>
</tr>
<tr>
<td>Bittle and Pate (1966)</td>
<td>(f = 0.23 \frac{1}{R_e^{0.216}})</td>
</tr>
</tbody>
</table>
Table 2: Two-phase dynamic viscosity models found in literature

<table>
<thead>
<tr>
<th>Model</th>
<th>Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cicchitti (1960)</td>
<td>$\mu_{2\phi} = (1-x)\mu_f + x\mu_g$</td>
</tr>
<tr>
<td>McAdams (1942)</td>
<td>$\mu_{2\phi} = \frac{1-x}{\mu_f} + \frac{x}{\mu_g}$</td>
</tr>
<tr>
<td>Duckler (1964)</td>
<td>$\mu_{2\phi} = \frac{\nu_f (1-x) + x \nu_m}{\nu_m}$</td>
</tr>
</tbody>
</table>

5. CHOKE FLOW CONDITION

Choked flow occurs when further decrease in outlet pressure does not result in an increase in the mass flow rate, which reaches a plateau at certain exit pressure. In the present work the choked flow condition is taken at the exit plane. This condition can be expressed by the following equation (Zang and Ding 2004):

$$\frac{dL}{dp_{out}} \leq 0$$ (14)

This equation implies that further decrease in pressure below the critical pressure does not increase the capillary tube length. At the critical pressure condition, the following equation is satisfied:

$$\frac{dL}{dp_{out}} = 0$$ (15)

The derivative of equation (10) can be taken with respect to $P_{out}$ and solve for the critical pressure. This will lead to iterative solution for $P_{out}$. However, a simple straightforward equation, developed by Zhang and Ding (2004), is used in the present study to determine the critical pressure. This equation can be written as:

$$p_c = \sqrt{p_r \nu_f \beta G}$$ (16)

6. RESULTS AND DISCUSSIONS

The flow chart used to compute the capillary length is shown in figure 8; and as one can see the procedure is straightforward and less complicated. The mass flow rate can also be determined by an iterative process for a given capillary length. The mass flow determined by the present analytical model are compared with other data in open literature. Deviation of measured mass flow rates from prediction are shown in figures 9 and 10, respectively. Figure 9 shows that the majority of data are within -15% to +10% for both refrigerants R134a and R22. Data of R22 and R134a are also compared with those of Yilmaz and Unal (1996). These are presented in table 3. The comparison of the present study results with those of Melo's (1994) and Shultz’s (1987) gives a better correlation than that of Yilmaz-Unal (1996) results. On the other hand, the Wijaya (1992) experimental data is better correlated with the Yilmaz-Unal (1996) method. Figure 10 shows deviation of mass flow rate for mixture refrigerants. The majority of data lie within the range of -15 to +10%. Zang and Ding (2004) have used the Yilmaz-Unal model for pure and refrigerant mixtures and they found that the predicted data lie in the range of ±15%.

The present model can be used also to investigate the effect of varying mass flow rates, inner diameters and subcool temperatures on the performance of capillary tubes. For example, figure11 shows the mass flow rates of R22 as a function of capillary tube length for various diameters. As expected, the mass flow rates decreases with an increase in capillary tube length for the same diameter tubes. For the same tube length,
mass flow rate increases as the inner diameters increases. The present data agree well with ASHRAE (2002) data except at smaller length where deviation error increases with an increase of the capillary diameter.

![Flow chart for sizing capillary Tube](image)

Figure 8: Flow chart for sizing capillary Tube

![Comparison of predicted mass flow rates with data measured by other researchers.](image)

Figure 9: Comparison of predicted mass flow rates with data measured by other researchers.
Table 3: Comparison of the mass flow rates between the present model, Yilmaz-Unal model and experimentally obtained data.

<table>
<thead>
<tr>
<th>Literature</th>
<th>Present Model</th>
<th>Yilmaz-Unal Model (1996)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wijaya [26], R134a</td>
<td>-17.92% -- +7.5%</td>
<td>-12.53% -- +6.51%</td>
</tr>
<tr>
<td>Melo et al. [24], R134a</td>
<td>-3.30% -- +12.52%</td>
<td>4.01% -- +19.17%</td>
</tr>
<tr>
<td>Dirik et al. [25], R134a</td>
<td>-11.87% -- -4.24 %</td>
<td>N.A</td>
</tr>
<tr>
<td>Schultz [27], R22</td>
<td>-10.40% -- -9.4%</td>
<td>-12.54% -- -9.50%</td>
</tr>
<tr>
<td>Melo et al. [22], R22</td>
<td>-10.91% -- +8.49%</td>
<td>N.A</td>
</tr>
<tr>
<td>Kim &amp; Kim [23], R22</td>
<td>-12.86% -- -3.77%</td>
<td>N.A</td>
</tr>
</tbody>
</table>

Figure 10: Comparison of predicted mass flow rates with data measured by other researchers.

Figure 11: Variation of refrigerant mass flow rate with capillary tube length for R22 ($P_C = 18$ bar and $\Delta T_{sub} = 5.0^\circ C$)
7. CONCLUSIONS

A simple analytical model for the design of capillary tubes for is presented. The present model is tested for HCFC22 and its alternatives R134a, R410a, R404a, R507a and R407c. The prediction of mass flow rates is within \( -15\% \) to \(+10\%\) of the experimental data reported in literature. The deviation of the present study can be considered satisfactory on account of the simplicity of the present analytical method.

NOMENCLATURE

\[ a, b, c, d \] constants defined in Eq. (4)
\[ C_1, C_2 \] empirical constants defined in Eq. (11)
\[ D \] inner diameter (m)
\[ F \] third polynomial function
\[ f \] friction factor
\[ G \] mass flux (kg s\(^{-1}\) m\(^{-2}\))
\[ H \] enthalpy (J kg\(^{-1}\))
\[ L \] capillary tube length (m)
\[ M \] mass flow rate (kg s\(^{-1}\))
\[ P \] pressure (Pa)
\[ R_e \] Reynolds number
\[ T \] temperature (°C)
\[ V \] specific Volume (m\(^3\) kg\(^{-1}\))
\[ X \] quality

Greek letters
\[ \beta \] slope of Eq. (2)
\[ \mu \] viscosity (kg m\(^{-1}\) s\(^{-1}\))

Superscripts and Subscripts

\[ cr \] critical or choked flow condition
\[ f \] saturated liquid
\[ g \] saturated vapor
\[ in \] inlet
\[ liq \] liquid phase
\[ out \] outlet
\[ r \] crossing point between saturation liquid line and isenthalpic line in the pressure-enthalpy diagram
\[ tp \] two-phase region

REFERENCES


Cichitti, A., Lombardi, C., Silvestri, M., Soldaini, G., Zavattarelli, R., 1960, Two-phase cooling

International Refrigeration and Air Conditioning Conference at Purdue, July 17-20, 2006


