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A Computational Study of the Interaction Between Fluid Flow and Reed Valve Deformation

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ABSTRACT

The finite element method (FEM) is used to study the dynamic behaviour of a compressor valve of the half-annular reed type. The model solves for the flow field in the time-varying valve gap and simultaneously for the dynamic flexure of the reed. The predictions of the model are compared with the predictions of a standard model.

INTRODUCTION

For many compressors the moving element of the valve is a flexible element (the "reed") covering one or more ports. The reed is deflected by the aerodynamic forces and forms a gap the height of which varies in both space and time. The shape of the gap influences the flow field and its pressure distribution. The pressure distribution determines the force function driving and deforming the valve reed. Thus a strong conjunction exists between the deflection of the plate and the flow field in the gap. A feedback between the plate motion and the flow field is evident.

To calculate the plate deflection the pressure distribution is needed but equally for the pressure calculation the plate deflection is needed.

To solve the problem of valve dynamics two problems had to be analysed simultaneously. The first - the dynamics of a flexible reed and the second - the flow of viscous fluid in a gap of varying height. The first problem has been intensively investigated in recent years. Many numerical solutions have been presented [4,6,7,8,9]. The authors of these papers concentrated their attention mainly on the problem of stresses.

They used different element types and different methods of solution but one thing was common - the model of load. The problem of flow in the valve gap was avoided by assuming the gas force to be equal to the product of the port area and pressure drop across the valve.

From experiments [1,2,12] it is known that this assumption is of limited validity.

In this work the model of reed dynamics was connected interactively with the model of flow in the valve gap.

MODEL OF THE REED DYNAMICS

The discretisation of the flexible reed was performed by the use of the constant-moment plate-bending element. Triangular finite elements are well suited to deal with irregularly shaped boundaries and considerable effort has been devoted to their development for the purpose of solving plate-bending problems. One of the simplest and most efficient is the triangular element proposed by L.S.D. Morley [5]. In this element the normal deflection w is assumed to vary quadratically within the triangular element and is expressed in terms of the nodal deflections and the values of the slope $\partial w/\partial n$ at the mid-point of each side. The non-conformity of the inter-element displacements was ignored. Particulars of the stiffness matrix calculation can be found in [5].

The equation of reed motion in matrix form is as follows:

$$[K]\{\delta\}+[C]\frac{\partial}{\partial t}\{\delta\}+[M]\frac{\partial^2}{\partial t^2}\{\delta\}+\{F\}=0$$

where $[K]$ is the ∂t reed stiffness matrix
[C] the damping matrix
[M] the mass matrix
{F} the load vector
and {δ} the displacement vector

Taking the diagonal (lumped) mass matrix $[M_l]$ instead of the full matrix $[M]$ one can find a simple integration rule:

$$\{\delta\}^{n+1} = 2\{\delta\}^{n} - \{\delta\}^{n-1} \quad - \Delta t^2 \{\{F\} + [K]\{\delta\}^{n} + [C](\{\delta\}^{n} - \{\delta\}^{n-1})/\Delta t\} \quad [M_l]^{-1}$$

The proposed integration scheme was tested on the well-established case of the rectangular reed so that a direct comparison of the calculated period of free vibration with that obtained from an analytical solution, could be made.

MODEL OF FLOW IN THE VALVE GAP

From numerical analysis [10] it is known that in many cases the quasi-steady model of flow is adequate. Some models of flow in the gaps of valves with rigid moving elements are available [3,10] but can not be used for the case considered here.

For valves with flexible moving elements a relatively high ratio of the moving element area to the port area is common. The reed displacement is small. Consequently the flow field in the valve gap can be assumed to be two-dimensional.

The following assumptions have been used for the model of the flow in the gap:

a) the velocity component perpendicular to the gap is negligible,
b) derivatives of velocity components $\partial u/\partial z$ and $\partial v/\partial z$ (z-direction perpendicular to the gap) are dominant,
c) the variation of the gas temperature has a small effect,
d) the gas viscosity is constant.

The validity of these assumptions holds only for valves with a high ratio of reed area to port area, small reed deflections and small pressure drops.

The following equations (continuity and momentum) describe the model of flow of viscous fluid in a narrow gap.

$$\frac{\partial \rho u}{\partial x} + \frac{\partial \rho v}{\partial y} + \frac{\partial \rho w}{\partial z} = 0,$$

$$\frac{\partial p}{\partial x} = \mu \frac{\partial^2 u}{\partial z^2}$$

$$\frac{\partial p}{\partial y} = \mu \frac{\partial^2 u}{\partial z^2}$$

$$0 = \mu \frac{\partial^2 u}{\partial y^2}$$

$$p = (R\ T)\,\rho$$

Because p is independent of z

$$u = -\frac{1}{2\mu}\ \frac{\partial p}{\partial x}\ f(z)$$

$$v = -\frac{1}{2\mu}\ \frac{\partial p}{\partial y}\ f(z)$$

$$w = W \quad z/H$$

with f(z) = z(H-z)

where $H(x,y)$ is a local height of the gap and W is the reed velocity.

After substitution into the continuity equation we have

$$-\frac{1}{2\mu}\left[\frac{\partial}{\partial x}\left(p\,\frac{\partial p}{\partial x}f(z)\right)+\frac{\partial}{\partial y}\left(p\,\frac{\partial p}{\partial y}f(z)\right)\right] \;+\; W\;\frac{\partial}{\partial z}\left(p\,\frac{z}{H}\right)=0$$

Integrating the first term with respect to z we have

$$\int_0^H \frac{\partial}{\partial x}\left(p\,\frac{\partial p}{\partial x}f(z)\right)dz=\frac{\partial}{\partial x}\left[\left(p\,\frac{\partial p,}{\partial x}\right)\int_0^H f(z)\,dz\right] \;=\; \frac{1}{6}\frac{\partial}{\partial x}H^3 p\,\frac{\partial p}{\partial x}=\frac{1}{12}\frac{\partial}{\partial x}H^3\,\frac{\partial p^2}{\partial x}$$

In a similar way we can integrate the second term.

And the last term

$$\int_0^H W\,\frac{\partial}{\partial z}\left(p\,\frac{z}{H}\right)dz=\int_0^H\frac{\partial}{\partial z}\left(W\,p\,\frac{z}{H}\right)dz-\int_0^H\left(p\,\frac{z}{H}\right)\frac{\partial W}{\partial z}\,dz$$

Unfortunately, $W(x,y,t)$ = the reed velocity in z direction is not constant.

But taking the assumption

$$\frac{\partial W}{\partial z}=\frac{\partial^2 H}{\partial z\,\partial t}=\frac{\partial}{\partial t}\frac{\partial}{\partial z}$$

(slope of the reed) $\equiv 0$ we have

$$\frac{\partial}{\partial x}H^3(x,y)\,\frac{\partial p^2}{\partial x}+\frac{\partial}{\partial y}H^3(x,y)\,\frac{\partial p^2}{\partial y}=24\,\mu\,W\,p$$

The flow field in the valve gap is described by Poisson's equation. Because the reed velocity is small in comparison with the velocity in the gap we can make the last simplification, $W = 0$, and have

$$\frac{\partial}{\partial x}H^3(x,y)\,\frac{\partial p^2}{\partial x}+\frac{\partial}{\partial y}H^3(x,y)\,\frac{\partial p^2}{\partial y}=0$$

If the geometry of the valve gap is known, the pressure distribution can be obtained from a solution of the Laplace equation using proper boundary conditions. They are: a known distribution of p_s^2 at the boundary corresponding to the inlet cross-section and p_{*}^2 in the outlet cross section.

The assumption made here is that the viscosity is independent of the pressure distribution.

Since the moving element is of relatively complicated geometry a finite element method [13,14] has been used to achieve a solution.

<u>Program</u>

A specialized program including routines for calculating stiffness and mass matrices and integration routines for the prediction of reed deflections and the solution of the Laplace equation describing the pressure distribution in the valve gap has been developed.

Use was made of symmetry so that only a half of the reed was considered.

The calculation starts from an initial reed deflection so that the temporary shape of the gap determines the boundary conditions for the flow problem. One of the routines solves the flow field problem and describes the pressure distribution. This pressure distribution is then used for calculating the pressure load and the new deflected form. In this step by step way the problem is solved.

Boundary Conditions and Initial Conditions

The geometry of the valve under consideration is shown in Fig. 1.

The reed thickness used in the calculation was 0.5 mm. The motion of the reed was limited by the stopper located 1.5 mm above it.

To reduce the number of terms influencing the problem, the pressure in the valve port was assumed to be constant and equal to 1.22 [bar]. The ambient pressure has a value of 1.0 [bar]. The initial condition for deflection is taken to be zero and the pressure is assumed to be constant over the port area and zero elsewhere.

The grid used for reed discretisation is shown in Fig. 2. Attention was applied to node no. 33 (centre of the port) and node no. 63 (reed mid-point).

In Fig. 3 the grid used in the flow calculation is presented.

To test the method the free oscillation of the reed was calculated and the period compared with a known solution [7]. The agreement was good, 60 Hz as against 58 Hz.

DISCUSSION

To reveal differences between the new model proposed here and a standard model, both calculations were performed. The same data have been used, the only difference being the load assumption. In the first calculation the pressure distribution (an example is shown in Fig. 5) was calculated at every time step and in the second the pressure was constant.

The instantaneous deflection of the reed, the pressure distribution in the gap, and the tangential stress are shown in graphical form. For example in Fig. 4 the reed deflection at time = 0.55 ms is presented (the vertical scale is 9 times greater than the horizontal) and in Fig. 5 the corresponding pressure distribution is shown; Fig. 6 shows the temporary τ_{xy} stress distribution.

When the predictions of standard model (not shown) are compared with those of the new model differences in the shape of the gap are noticeable, as are small differences in the pressure distributions and large differences in the velocity fields.

The time history of some reed node displacements is presented in Figs. 7 and 8 in which the new and standard models are compared. In both cases the reed reaches the stopper.

In all time steps maximum values of stresses were found. Variations of these values are presented in the following figures.

Figs. 9, 10, 11 and 12 show a comparison of stresses (σ_x, σ_y, τ_{xy} and σ_{red}). The value of σ_{red} was calculated using the following formula:

$$\sigma_{red} = \sqrt{\sigma_x^2 - \sigma_x\sigma_y + \sigma_y^2 + 3\tau_{xy}^2}$$

It is worthy of note that in the analyzed case the tangential stresses σ_{xy} resulting from the twisting of the reed are crucial.

CONCLUSIONS

The new model of valve dynamics presented here is relatively simple and has limitations. The model of flow is valid for small displacements and a high ratio of reed area to port area. With the constant viscosity assumption, velocities in regions of high displacement are overestimated in relation to velocities in regions of small displacement. However it is a first attempt by the author to produce a solution in which the motion of the reed and its aerodynamic load is coupled with the flow phenomena in the valve gap. The model of flow with variable fluid viscosity and a model of a fully 3-D viscous fluid flow are under development.

The results presented show that a commonly used standard model of load underestimates the load, acceleration, deflection and stresses. Experimental verification of these predictions is under consideration.

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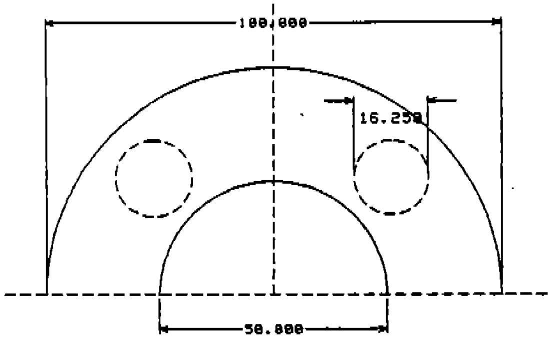


FIGURE 1

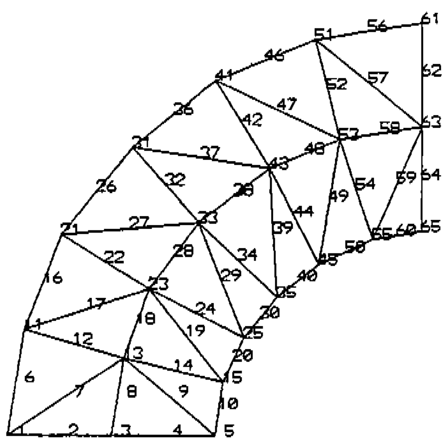
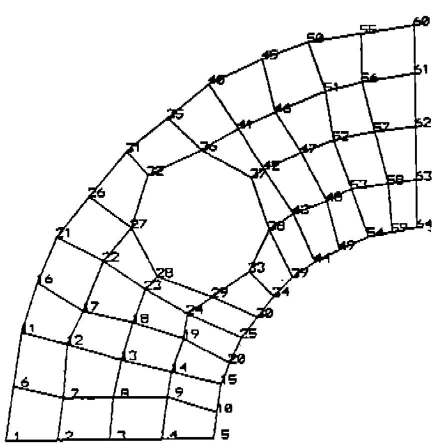


FIGURE 2



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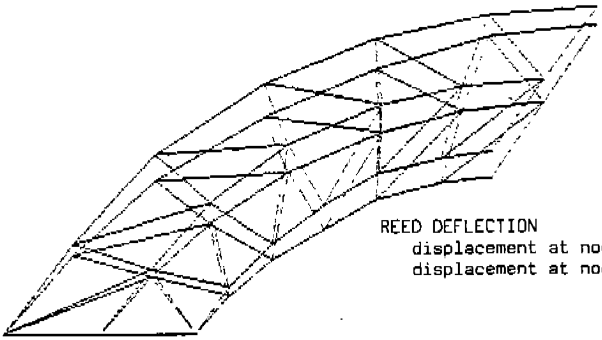
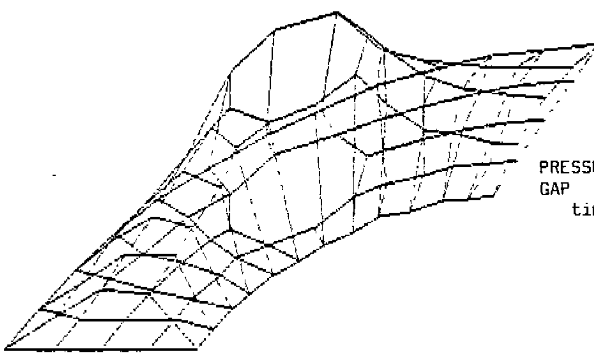


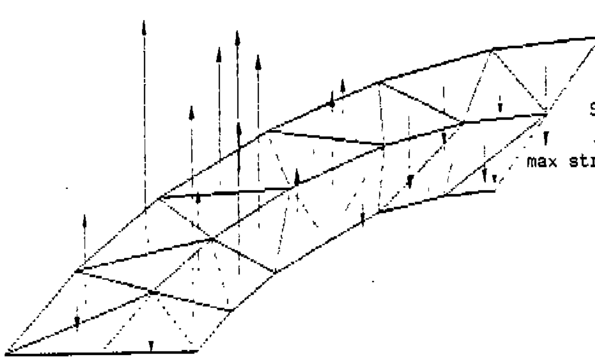
REED DEFLECTION
displacement at node 33 = 0.446 mm
displacement at node 63 = 0.365 mm

FIGURE 4



PRESSURE DISTRIBUTION IN GAP
time = 0.550 ms

FIGURE 5



SIGMA XY
max stress=180 x 10^6 N/m^2

FIGURE 6

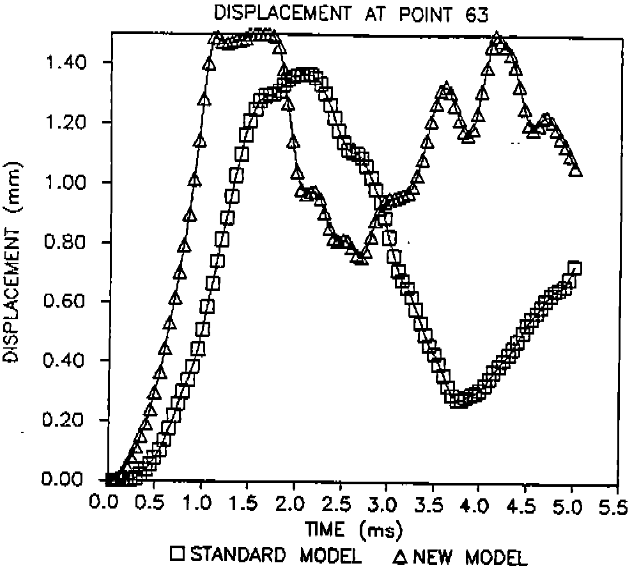


FIGURE 7

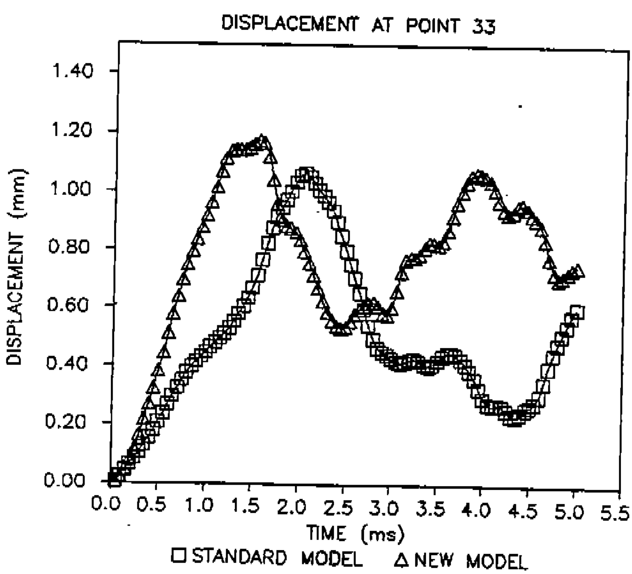


FIGURE 8

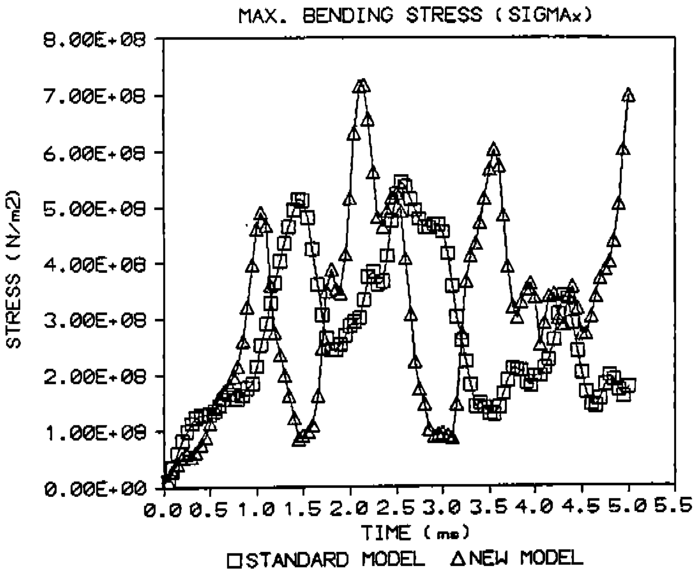


FIGURE 9

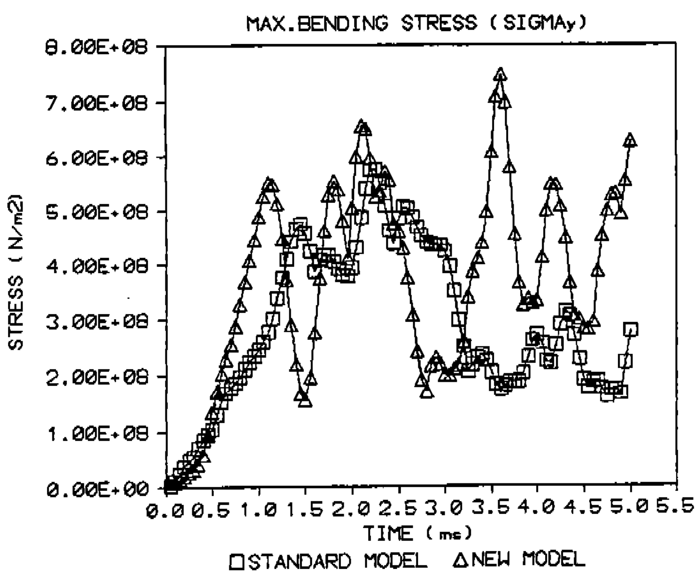


FIGURE 10

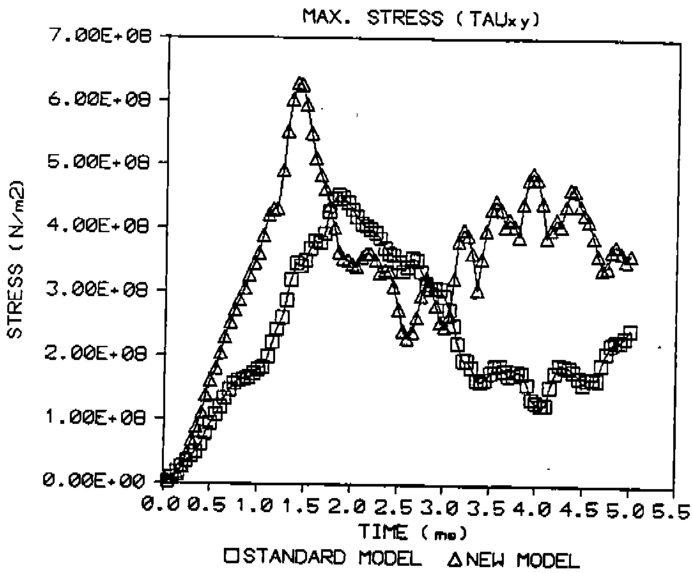


FIGURE 11

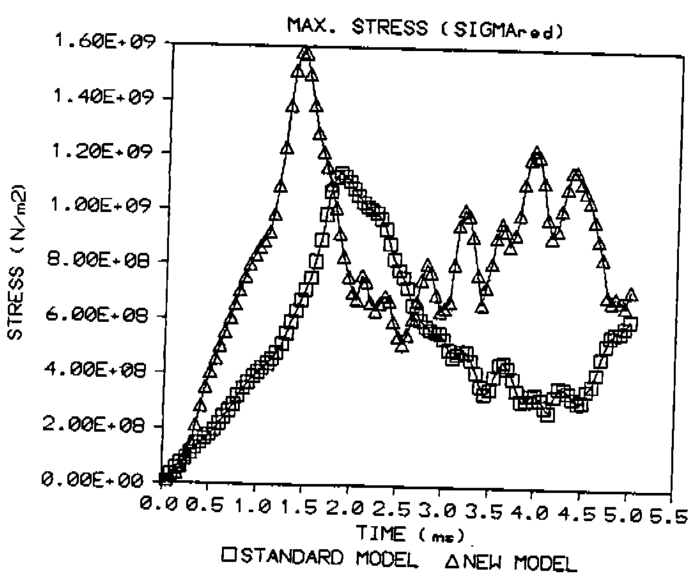


FIGURE 12