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Dynamic Behavior of Variable Displacement Wobble Plate Compressor for Automotive Air Conditioners

by

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ABSTRACT

The present study examines the dynamic behavior of a variable displacement wobble plate compressor which makes it possible to control the cooling capacity continuously. The continuous cooling capacity control is achieved by the complicated motion of the piston, the piston rod, the wobble plate, the rotating journal and the drive shaft. In order to analyze the complicated motion of each moving element, the inertial forces and moments of the moving elements are calculated first. Subsequently, the constraint force at each pair of compressor elements and the frictional forces given by Coulomb's law of friction are calculated. The unknown frictional coefficient was determined on the basis of experimental data on the cylinder pressure and the driving power. It can be concluded that the inertial and frictional forces have a significant effect upon the feasibility to control the cooling capacity, and upon the quiet operation and durability of compressor parts at high operating speeds.

INTRODUCTION

Variable displacement wobble plate compressors have brought on a revolution in automobile air conditioning since commercial production [see Ref. 1]. The refrigerant flow rate discharged from the compressor is controlled by continuously changing the piston displacement, thus resulting in wide-range continuous control of the cooling capacity and in high efficiency. The variable displacement wobble plate compressors are now widely considered to be the best choice for automobile air conditioning with its stringent requirements for comfort and reduced energy consumption.

The continuous control of the cooling capacity is achieved, however, by the complicated motion of many compressor elements, such as the piston, the piston rod, the wobble plate, the rotating journal and the drive shaft. Subsequently, questions concerning the quiet operation and durability of these parts at high operating speeds arise. The complicated motion of variable displacement wobble plate compressors and the dynamic balancing of the rotating shaft have been treated in several previous studies [2, 3] from the kinematics viewpoint. However, the inertial forces and moments of the moving elements and also the frictional force at each pair of compressor elements have not been taken into account. As the operating speed of the compressor increases, the effects of the inertial and frictional forces become increasingly significant.

The present study examines the dynamic behavior of a 6-cylinder variable displacement wobble plate compressor. The complicated motion of each moving element of the compressor is precisely analyzed, from both the kinematics and dynamics viewpoints, and thus the inertial forces and moments of the moving elements and the frictional forces acting on the element pairs are calculated. Based on the calculated results, the effects of the inertial and frictional forces upon the feasibility to control the cooling capacity, and upon the quiet operation and durability of compressor parts at high operating speeds, are examined. Furthermore, the power losses due to friction are examined and design criteria for quiet operation and high speed durability of compressor parts are obtained.
The vertical cross-sectional view of a 6-cylinder variable displacement wobble plate compressor is shown in Fig. 1. The compression mechanism is composed of many compressor elements, such as the drive shaft, the drive plate, the rotating journal, the wobble plate, the piston rods and the pistons. The rotating journal driven by the drive plate rotates but the wobble plate on the rotating journal undergoes a wobbling motion, since its rotation around the drive shaft is constrained by an anti-rotation mechanism with five degrees of freedom. Thus, the piston connected to the wobble plate through the double-ended ball piston rod reciprocates in the cylinder. The variable piston displacement is achieved by the kinematics of the rotating journal. The rotating journal can rotate about the pivot pin fixed on the shaft sleeve, but its nutation angle and position are controlled by a slide pin which is located in the guide slot. As the nutation angle decreases, the shaft sleeve slides up along the drive shaft to keep the piston head clearance at a constant value for any nutation angle, and thus the piston displacement is decreased. It is a matter of course that the nutation angle of the rotating journal is determined by the equilibrium of the moments around the pivot pin, which act on the rotating journal. The major factor which determines the nutation angle is the gas force acting on each piston. Therefore, the piston displacement can be controlled by adjusting the gas pressure in the crankcase. A pressure control valve is attached between the discharge chamber and the crankcase. At full load, the pressure control valve closes and the crankcase pressure becomes equal to the suction pressure. Thus, the maximum nutation angle of the rotating journal is maintained. At partial load, the pressure control valve opens and the crankcase pressure is increased. Thus, the nutation angle is decreased. In this way, the pressure control valve regulates the crankcase pressure, resulting in the continuous control of the piston displacement. Thus, the wide-range continuous control of the cooling capacity as well as high efficiency can be obtained.

Before the dynamic behavior of the variable displacement compressor can be examined, the kinematics of each moving element must first be analyzed. For this purpose, the rectangular coordinates and variables shown in Fig. 2 are introduced. The static coordinates are represented by x, y and z, where the z axis coincides with the drive shaft center. The \( x_m \) axis represents the pivot pin center of the shaft sleeve. Therefore, the \( x_m-y_m \) coordinates rotate with the drive shaft. The rotating angle of the drive shaft is represented by \( \theta \). When rotating the \( x_m-y_m-z \) coordinates around the \( x_m \)-axis by the nutation angle \( \alpha \), the \( x_m-y_{\alpha-2\alpha} \) coordinates fixed on the rotating journal are obtained. For example, the point \( P_0 \) on the x-y plane moves to the point \( P \) on the \( x_m-y_m \) plane. The point \( P \) has to move to the point \( S \), since the rotation of the wobble plate is controlled by the anti-rotation mechanism.

The piston rod TS is shown in Fig. 3. The position of a cylinder is represented by the radius \( r_p \) and the angle \( \Theta_0 \) from the x-axis. The subscript \( n \) takes the integers from 1

![Fig. 1 Mechanism of variable displacement compressor](image-url)
The position of the piston rod center $S_n$ on the wobble plate side can be analyzed on the basis of the coordinate transformations shown in Fig. 2, resulting in the following x-y-z coordinates:

$$
\begin{bmatrix}
    x_{S_n} \\
    y_{S_n} \\
    z_{S_n}
\end{bmatrix} = A(\theta) \begin{bmatrix}
    r_n \cos(\Theta_n - \theta + \delta) \\
    r_n \sin(\Theta_n - \theta + \delta) \cos \alpha - h_n \sin \alpha \\
    r_n \sin(\Theta_n - \theta + \delta) \sin \alpha + h_n \cos \alpha
\end{bmatrix}
$$

where $r_n$ and $h_n$ represent the radius from the origin $O_0$ and the height from the $x_m$-$y_m$ plane, respectively, of the piston rod center $S_n$. $A(\theta)$ is a coordinate transformation matrix and $\delta$ is a constraint angle correcting the rotation of the wobble plate controlled by the anti-rotation mechanism, given by the following expressions:

$$
A(\theta) = \begin{bmatrix}
    \cos \alpha & -\sin \alpha & 0 \\
    \sin \alpha & \cos \alpha & 0 \\
    0 & 0 & 1
\end{bmatrix}
$$

$$
\delta = \tan^{-1} \left[ \frac{\tan(\Theta_B - \theta)}{r_n \cos(\Theta_B - \theta)} + \frac{h_n \tan \alpha}{r_n \cos(\Theta_B - \theta)} \right] - \Theta_B + \theta
$$

The piston rod center $T_n$ on the piston side is represented by the following x-y-z coordinates:

$$
\begin{align*}
    x_{T_n} &= r_n \cos \Theta_n, \\
    y_{T_n} &= r_n \sin \Theta_n, \\
    z_{T_n} &= z_{S_n} + l_p \cos \phi_n
\end{align*}
$$

where $\phi_n$ is the rotation angle of the piston rod from the $z$ axis and is given by the following expression:

$$
\phi_n = \sin^{-1} \left[ \sqrt{\left((x_{S_n} \cdot x_{T_n})^2 + (y_{S_n} \cdot y_{T_n})^2\right)} \right]^{0.5} / l_p
$$

**INERTIAL FORCES AND MOMENTS**

The inertial forces and moments of the moving elements are derived as follows.

**Piston Rod**

Assuming a point in the piston rod represented by the $(x_n, y_n, z_n)$ coordinates, the inertial moments $M_{pzn}$ around the $z_p$ axis and $M_{\Theta n}$ around the orthogonal axis of the piston rod are given by the following integral expressions:

$$
\begin{align*}
    M_{pzn} &= \int (x_n \cdot y_n - (x_n \cdot x_{T_n}) y_n) \ dm \\
    M_{\Theta n} &= \int r (x_n \cos \theta_n \cos \phi_n + y_n \sin \theta_n \cos \phi_n + z_n \sin \theta_n) \ dm
\end{align*}
$$

where $dm$ is a small mass in the piston rod, $r$ the distance from the piston rod center $T_n$, and $\theta_n$ the rotation angle of the piston rod from the $x_p$ axis. The integral sign represents a definite integral for the whole piston rod. $\theta_n$ is given by the following expression:

$$
\theta_n = \tan^{-1} \left( \frac{y_{S_n} - y_{T_n}}{x_{S_n} - x_{T_n}} \right)
$$
Wobble Plate

Assuming a point in the wobble plate represented by the \((x,y,z)\) coordinate, the inertial forces in the \(x\), \(y\) and \(z\) directions are given by the following integral expressions, respectively:

\[
F_{wx} = -\int x \, dm, \quad F_{wy} = -\int y \, dm, \quad F_{wz} = -\int z \, dm
\]

The inertial moments around the \(x\), \(y\) and \(z\) axes are given by the following integral expressions, respectively:

\[
M_{wx} = \int (\dot{y} \cdot z - y \cdot \ddot{z}) \, dm, \quad M_{wy} = \int (\dot{z} \cdot x - z \cdot \ddot{x}) \, dm, \quad M_{wz} = \int (\dot{x} \cdot y - x \cdot \ddot{y}) \, dm
\]

Rotating Journal

Assuming a point in the rotating journal represented by the \((x,y,z)\) coordinates, the inertial forces and moments of the rotating journal are given by the same integral form of the expressions in (8) and (9):

\[
F_{Tz} = -\int \dot{x} \, dm, \quad F_{Ty} = -\int \dot{y} \, dm, \quad F_{Tz} = -\int \dot{z} \, dm
\]

\[
M_{Tz} = \int (\dot{y} \cdot z - y \cdot \ddot{z}) \, dm, \quad M_{Ty} = \int (\dot{z} \cdot x - z \cdot \ddot{x}) \, dm, \quad M_{Tz} = \int (\dot{x} \cdot y - x \cdot \ddot{y}) \, dm
\]

EQUATIONS OF MOTION

The equations of motion of the moving elements are derived from the equilibrium of forces and moments acting on each moving element.

Piston and Piston Rod

Fig. 4 shows the forces acting on the piston. The reaction forces from the piston rod are represented by \(T_x, T_y\) and \(T_z\), and the resultant gas force by \(P_n\). The reaction forces \(T_x\) and \(T_y\) balance with the reaction forces from the cylinder. Assuming Coulomb's law of friction, the frictional force \(f_{pn}\) between the piston and the cylinder wall is represented by the following expression:

\[
f_{pn} = \mu_p \left( T_{xn}^2 + T_{yn}^2 \right)^{0.5}
\]

The equation of motion of the piston is given by the following expression:

\[- m_p \ddot{z}_{Tn} - P_n - s g a (z_{Tn}) f_{pn} + T_{zn} = 0\]

Fig. 5 shows the forces and moments acting on the piston rod. The reaction forces from the piston are represented by \(T_x\) and \(T_y\), and those from the wobble plate by \(S_{xn}, S_{yn}\) and \(S_{zn}\). The frictional moments represented by \(f_{Tn}\) and \(f_{Sn}\) arise at the ball joints \(T\) and \(S\), respectively. The inertial moments \(M_{pzn}\) and \(M_{\theta zn}\) are given by (6). When these forces and moments are at equilibrium, the 5 equations of motion with respect to \(x_p, y_p, z_p, \theta_n\) and \(\phi_n\) can be derived.

Wobble Plate

Fig. 6 shows the forces acting on the wobble plate. The reaction forces from the piston rod are represented by \(S_{xan}, S_{yan}\) and \(S_{zan}\), and the reaction forces and moments
from the rotating journal by $F_{Qx}$, $F_{Qy}$, $F_{Qz}$, $M_{Qx}$ and $M_{Qy}$, respectively. The inertial forces and moments are represented by $F_{Wx}$, $F_{Wy}$, $F_{Wz}$, $M_{Wx}$, $M_{Wy}$ and $M_{Wz}$, and can be derived by applying the coordinate transformations to (8) and (9). The point B represents the joint of the wobble plate and the anti-rotation mechanism. The frictional forces from the anti-rotation mechanism are represented by $f_{Qx}$ in the $x$ direction and $f_{Qy}$ in the $y$ direction. The frictional moment from the thrust bearing is represented by $f_Q$ and that from the radial bearing by $f_R$. When these forces and moments are at equilibrium, the 6 equations of motion in the $x, y, z$ directions and around each axis can be derived.

**Rotating Journal**

Fig. 7 shows the forces and moments acting on the rotating journal. The nutation of the rotating journal is controlled by the slide pin (G) and the shaft sleeve pin ($x$ axis). The reaction force and frictional force at the point G from the drive shaft are represented by $F_G$ and $f_G$, respectively. The reaction forces and moments from the wobble plate are represented by $F_{Qx}$, $F_{Qy}$, $F_{Qz}$, $M_{Qx}$, $M_{Qy}$ and those from the drive shaft by $F_{Cxa}$, $F_{Cya}$, $M_{Cxa}$, $M_{Cya}$ and $M_{Cza}$. The inertial forces and moments are represented by $F_{Wx}$, $F_{Wy}$, $F_{Wz}$, $M_{Wx}$, $M_{Wy}$ and $M_{Wza}$, and can be derived by applying the coordinate transformations to (10) and (11). $f_C$ is the frictional force between the shaft sleeve and the drive shaft, and $F_s$ the reaction force of the spring as it presses the shaft sleeve down. When these forces and moments are at equilibrium, the 6 equations of motion in the $x, y, z$ directions and around each axis can be derived. For instance, the equilibrium equation of the moment around the $x_m$ axis is given by the following expression:

$$M_{Qx} + M_{Qy} + (F_{Qy} - f_{Qy}) + (F_{Qy} + f_{Qy}) = 0$$

where the moment $M_{Cxa}$ is introduced when the nutation angle reaches a maximum value.

**Drive Shaft**

Fig. 8 shows the forces and moments acting on the drive shaft. The drive shaft is supported by the two radial bearings denoted by E and F. The reaction forces from the bearing E are represented by $E_{xm}$ and $E_{ym}$, and those from the bearing F by $F_{xm}$ and $F_{ym}$. The frictional moments from the two radial bearings are represented by $f_E$ and $f_F$, respectively. The reaction force and frictional moment from the thrust bearing are represented by $f_{th}$ and $f_{th}$, respectively. Therefore, one can obtain the equilibrium equations of the forces in the $x, y, z$ directions and the moments around the $x, y, z$ axes exerted on the drive shaft. When these forces and moments are at equilibrium, the equations of motion of the drive shaft can be derived. The rotation of the drive shaft is subject to the following equation of motion:

$$T_r = I_c \ddot{\theta} - M_{Cya} \sin \alpha + M_{Cza} \cos \alpha + f_E + f_F + f_{th}$$

where $T_r$ represents the driving torque and $I_c$ the moment of inertia of the drive shaft.
CALCULATED RESULTS

In order to evaluate the total dynamic behavior of the variable displacement compressor, the equation of motion (15) of the rotating drive shaft must first be calculated. However, from the basic standpoint of examining the effects of the inertial forces upon the reaction forces, the reaction moments and the frictional forces, they were calculated for various constant operating speeds of the drive shaft. For given data concerning the mechanical constants, the dimensions and the operating parameters, the variables representing the motion of the moving elements were calculated first, and subsequently, the time-differential terms of the variables were calculated. Thus, the inertial forces and the moments of inertial of the moving elements were calculated. By using the calculated results, the reaction forces and moments acting on the compressor elements were calculated, and finally, the driving torque and the gas pressure necessary for cooling capacity control were examined. The unknown frictional coefficient was determined on the basis of experimental data on the cylinder pressure and the driving power.

Motion of the Piston Rod

Fig. 9 shows the calculated results for the constraint angle $\delta$ given by (3). The angle $\delta$ fluctuates twice for one revolution of the drive shaft. This means that any point on the wobble plate follows two elliptical orbits in each revolution. The calculated results for the rotation angles $\phi_1$ and $\phi_2$ (see Figs. 3 and 5) of the 6 piston rods are shown in Figs. 10 and 11, respectively. Their motions are very complicated, and, moreover, each motion is different. As a result of the complicated motion of the piston rod, the moments of inertial ($M_{pza}$ and $M_{pza}$) of the piston rod, shown in Figs. 12 and 13, arise. Depending upon the position of the piston rod, the moments of inertial of the piston rods are fairly different in amplitude and mode.
Reaction Force on the Piston Rod

The reaction force $F_r$ on the piston rod is shown in Fig. 14. The solid line represents the reaction force and the dotted line the reaction force component due to the gas force. Therefore, the difference between the solid and dotted lines suggests the magnitude of the effect of the inertial and frictional forces upon the reaction force. Due to the inertial effects, a negative reaction force appears during the suction process. As the operating speed of the drive shaft increases, the magnitude of this negative reaction force increases. It can be concluded that alternating compression and tensile forces act on the piston rod and thus durability will be severely compromised at the piston-rod ball joints, especially at high operating speeds.

Constraint Force on the Anti-rotation Mechanism

The reaction force $B_y$ acting on the anti-rotation mechanism is shown by the solid line in Fig. 15. In order to examine the effects of the inertial and frictional forces upon the reaction force, the reaction force component due to the gas force is shown by the dotted line, the component due to the frictional and gas forces by the broken line and the component due to the inertial and gas forces by the chain line. The broken line represents a step curve, due to a rapid change in direction of the frictional forces. By comparing the calculated results in this way, it can be seen that the effects of the inertial and frictional forces upon the resultant reaction force at the anti-rotation mechanism is fairly large and that the gas force has only a slight effect.

Driving Torque

The calculated results of the driving torque $T_r$ given by (15) are shown by the solid line in Fig. 16. In order to examine the effects of the inertial and frictional forces, the driving torque components are shown in the same way as in Fig. 15. This figure also shows that the effects of the inertial and frictional forces upon the driving torque are great.

Control Pressure

In order to control the piston displacement for various operating speeds of the compressor, the gas pressure difference (control pressure) between the crankcase and the suction chamber can be used for regulating the crankcase pressure. The control pressure can be determined from (14). This means that the control pressure depends not only upon the gas force but upon the inertial and frictional forces. It is interesting enough to examine the effects of the inertial and frictional forces upon the control pressure. Here, the effects of the inertial forces of the moving elements were calculated, as shown in Figs. 17a and b. It can be concluded from these figures that as the cooling capacity or the operating speed increases, the inertial forces of the moving elements have a greater effect upon the feasibility of controlling the cooling capacity. However, the impact of these inertial effects upon the control pressure can be decreased by the careful design of each moving element.

Fig. 14 Reaction force on the piston rod

Fig. 15 Reaction force on the anti-rotation mechanism

Fig. 16 Drive torque
CONCLUSIONS

This study examined the dynamic behavior of a 6-cylinder variable displacement wobble plate compressor. The complicated motion of each moving element of the compressor was precisely analyzed. By taking into account the inertial forces and moments of the moving elements, the equation of motion of each element is derived, and the motions of the moving elements, and the forces and moments acting on the compressor elements are calculated. The conclusions can be summarized as follows:

1) When the wobble plate is constrained by the anti-rotation mechanism, each of the six piston rods has a complicated motion. As a result, the inertial forces and moments become complicated. Moreover, as the direction of the relative motion at each pair of elements changes, the direction of the frictional force changes rapidly. Mainly as a result of these two factors, the constraint forces at each element and the driving torque also fluctuate complicated manner of a step. This phenomenon causes the control pressure for the cooling capacity to be unstable;

2) As the cooling capacity or the operating speed increases, the inertial effects of the moving elements upon the constraint forces become increasingly important. Especially the inertial forces of the piston and the piston rod during high speed operation have a significant effect upon the feasibility to control the cooling capacity, upon the increase in driving power and the driving variation;

3) By analyzing the dynamic behavior of each moving element, design criteria were obtained for quiet operation and durability of the compressor parts at high operating speeds.

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REFERENCES

**Appendix: Nomenclature**

- \( l_0 \): piston rod length
- \( B_y \): reaction force on anti-rotation mechanism
- \( F_{x_m}, F_{y_m}, F_{x_n}, F_{y_n} \): constraint forces on drive shaft
- \( f_{B_x}, f_{B_y} \): frictional forces on anti-rotation mechanism
- \( f_c \): frictional force on rotating journal and shaft
- \( f_R, f_P \): frictional moments at radial bearings of drive shaft
- \( f_G \): frictional force on slide pin
- \( f_p \): frictional force on piston
- \( f_Q, f_R \): frictional moments at thrust and radial bearings between wobble plate rotating journal
- \( f_{th} \): frictional moment at thrust bearing of drive shaft
- \( f_{T_n}, f_{S_n} \): frictional moments on piston rod joints
- \( F_0 \): spring force
- \( F_{C_{za}}, F_{C_{ya}}, M_{C_{ya}}, M_{C_{za}} \): reaction forces and moments on rotating journal and drive shaft
- \( F_C \): reaction force on slide pin
- \( F_{Q_{za}}, F_{Q_{ya}}, F_{Q_{za}}, M_{Q_{za}}, M_{Q_{ya}} \): reaction forces and moments on wobble plate and rotating journal
- \( F_{th} \): reaction force on drive shaft from thrust bearing
- \( F_{T_x}, F_{T_y}, F_{T_z}, M_{T_x}, M_{T_z} \): inertial forces and moments of rotating journal
- \( F_{W_x}, F_{W_y}, F_{W_z}, M_{W_y}, M_{W_z} \): inertial forces and moments of wobble plate
- \( I_c \): moment of inertia of drive shaft
- \( m_p \): mass of piston
- \( m_{pr} \): mass of piston rod
- \( M_{P_{zh}}, M_{theta} \): inertial moments piston rod
- \( n \): cylinder number
- \( P_n \): gas force on piston
- \( r_p \): arranged radius of piston center
- \( r_s, h_s \): arranged radius and height of wobble plate/piston rod joint
- \( r_b, h_b \): arranged radius and height of anti-rotation mechanism center
- \( f_o, h_o \): arranged radius and height of slide pin center
- \( S_{x_n}, S_{y_n}, S_{z_n} \): reaction forces on piston rod and wobble plate
- \( T_{x}, T_{y}, T_{z} \): reaction forces on piston
- \( x, y, z \): orthogonal coordinate
- \( x_m, y_m, z \): rotating orthogonal coordinate
- \( x_p, y_p, z_p \): orthogonal coordinate fixed on each cylinder
- \( \alpha \): nutation angle of rotating journal
- \( \delta \): constraint angle of wobble plate
- \( \theta \): rotating angle of drive shaft
- \( \theta_n \): angular position of each cylinder bore center
- \( \theta_B \): angular position of anti-rotation mechanism
- \( \mu_p \): frictional coefficient at piston
- \( \phi_n \): rotating angle of piston rod from \( z_p \) axis