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ANALYSIS OF THE APPLIED FORCES IN TWIN-SCREW REFRIGERANT COMPRESSORS

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ABSTRACT

An improved method to determine the axial and radial forces applied to the helical rotors of a refrigerant compressor is presented. The axial force consists of two parts: the force applied to the end surfaces and the force applied to the helical profile surface. The radial force also consists of two parts: the force applied to the non-contact segments and the force applied to the contact segments. These forces were calculated basing on the computer numerical calculation and the computer graphical plotting. Results can be used for bearing design and for rigidity calculation of screw compressor rotors.

1. INTRODUCTION

Rotary twin-screw compressors are widely used in the refrigeration machine of medium and large capacity. Some new constructions and profiles such as the SRM-D profile, the NEW HITATCH profile etc. can improve compressors with higher efficiency and better reliability.

The design procedure for a twin screw refrigerant compressor is more complicate than other type displacement compressors. While using computer calculation and plotting, it can be made easier, more accurate and time saving. The computation and plotting procedures include the generation of the following items: the profiles, the pressure distribution curves, the rotor's meshed curves and the rotor's contact curves etc.. These computer output data and geometric graph were to be saved to disc files as the initial values for force analyzing.

2. THE CALCULATION OF AXIAL FORCE

The axial force applied to the rotor by gas is the axial component of forces applied to the rotor surface.

2.1 The Axial force Applied to the End Surface of Rotor

It is assumed that the gas pressure applied to the end surface of rotor on the suction side is the suction pressure $p_s$. As on the discharge side, the mean pressure of suction pressure and discharge pressure is applied to a half the area of end surface, while the discharge pressure is applied to the other half. Therefore, the sum of axial forces applied to each rotor on both ends is as follows: (Fig. 1)

$$Q_d = \frac{1}{2} (F_a - F_b) (P_m - P_e)$$  \hspace{1cm} (1)
where \( F_a = \frac{1}{2} D^2 - mf_a \) ; \( F_b = \frac{1}{2} d^2 \)
\( m \) = number of lobes
\( D \) = outside diameter of rotor
\( d \) = inner diameter of rotor
\( f_a \) = the trough area between lobes

2.2 The Axial Forces Acting on the Profile Surface

For any inter lobe volume, it can be divided into two regions: the contact line region and non-contact line region as shown in fig. 2. The trough region between section I-I and section II-II is the contact region, and the region outside of section I-I and II-II are the non-contact regions.

Pressure in the non-contact region is uniform everywhere. The axial projected area of the front lobe surface is equal to the back lobe surface but the axial forces acting on them are in opposite directions. Therefore, the resultant force is zero, only the axial force in the contact region need to calculate.

2.2.1 The Axial Force at the Complete Contact Line Region

2.2.1.1 Male Rotor (Fig. 2a)

For the number i th inter lobe volume, the pressures on both sides of the contact line are \( p_i \) and \( p_e \) respectively. The projected area of the lobe area enclosed by contact line 1-5-4-2-1' is represented by the symbol \( f_i \). The axial projected area of area 1-h-c-1' is the inter lobe area \( f_{hi} \). The axial component forces applied to these two areas are \( p_i \) \( f_i \) and \( p_e \) \( f_{hi} \) respectively. For the same reason, \( s_2 \) is the axial projected area of area 2-2'-4-2', \( p_i \) \( s_2 \) is the applied axial force; \( s_3 \) is the axial projected area of area 1'-5-5'-2'-2' and \( -p_i \) \( s_3 \) is the applied axial force. Because

\[
\sum (s_1 + s_2 + s_3 + s_4) = f_i + f_{hi} + s_2 + s_3
\]

therefore, the sum axial force applied to this trough section is as follows:

\[
Q_{ax} = A P_i ( f_i + f_{hi} )
\]

where \( A P_i = p_i - p_e \).

If the number of inter lobe volumes that have complete contact lines is \( n \), then the sum of axial forces is as follows:

\[
Q_{ax} = \sum Q_{ax} = (f_i + f_{hi}) A P_i
\]

2.2.1.2 Female Rotor (Fig. 2b)

In the section between I and II, the lobe surface is divided into four regions, \( s_1, s_2, s_3, s_4 \) by the contact line and the helical line along the trough bottom. Those areas are the axial projected areas of the lobe surface areas enclosed by 1-2-2'-5-1.1-5-5'-1'-1, 5-4-3-5'-5 and 5-2'-3'-3-4-5. The axial forces applied to the above four areas are \( p_i s_1 \), \( p_i s_2 \), \( -p_i s_3 \) and \( -p_i s_4 \) respectively.

Since \( s_1 + s_2 = s_3 + s_4 \), the resultant force of above four forces is as follows:

\[
Q_{ax} = (p_i - p_e) (f_{ax} - f) = (p_i - p_e) (s_3 - s_3)
\]

therefore, the sum axial applied to the completed contact line is as follows:

\[
Q_{ax} = (f_{ax} - f) A P_i
\]
2.2.2 The Axial force in the Incomplete Contact Line Region

2.2.2.1 Male Rotor

Usually, the incomplete contact lines are at the first and the last lobes. The applied pressures in those regions are \( p \) and \( p_x \) respectively.

The shape of discharge port is determined by the discharge angle \( \alpha \); the axial projected area of the incomplete contact line region \( e-4-5 \) consists of \( s_1, s_2 \) and \( s_3 \).

The projected areas of lobe surfaces enclosed by \( a-e-4-5-a \), \( 4-2'-b-e-4' \) and \( 2'-5'-f-b-2' \) are \( s_1, s_2 \) and \( s_3 \) respectively. (Fig. 3, 4).

Then the axial forces applied to the above three areas are \( p_x s_1, p_x s_2, \) and \( p_x s_3 \) respectively. Therefore, the resultant of force is as follows (i.e. the axial applied force in the last inter lobe volume, see Fig. 3 and Fig. 4)

\[
Q_{p1} = p_x s_1 + p_x s_2 - p_x s_3 = (p_x - p) s_1 \tag{7}
\]

Similarly if graphs, (Fig. 5 and Fig. 6), are plotted for suction end, then the axial applied force for suction of lobe trough with incomplete contact line 1-2-3-f can be obtained.

Let \( s_4 \) represent the projected area of lobe surface enclosed by 1-2-3-f-e-1', and the axial applied force on it : \( p, s_4 + s_5 \); the projected area of 2-2'-f-2 and the axial applied force on it: \( p_x s_5 \); the projected area of 1-5-4-2'-2-1 and the applied force on it: \( p_0 s_6 \).

The axial applied force in the first inter lobe volume is as follows:

\[
Q_{31} = p, s_4 + p, s_5 - p_0 s_5 + p_0 s_6 + p_0 f_4 \tag{3}
\]

where \( s_4 = s_5 + s_5 + f_4 \), therefore,

\[
Q_{31} = (p, -p_0) (f_4 + s_4)
\]

The sum of axial force applied to the incomplete contact line region is \( Q_{31} + Q_{31} \).

2.2.2.2 Female Rotor

Longitudinal and projections (Fig. 7, 8) of suction end and discharge end for female rotor are plotted first. Segments of incomplete contact line in the first and the last lobe troughs are projected to the corresponding suction and discharge ends and projections. Thus we obtain projected areas of lobe surfaces with different pressures acting on. The procedures are similar to those for male rotor.

The axial forces \( Q_{31}, Q_{32} \) applied to the first inter lobe surface and to the last inter lobe surface are as follows:

\[
Q_{32} = (p_x - p_x)(f_4 - f - s_7) \tag{9}
\]

\[
Q_{32} = (p_x - p_0)(-s_7) \tag{10}
\]

where \( s_7 \) is the projected area of the area 3-h-g-3

\( s_8 \) = the projected area of the area m-n-3-m

3. THE CALCULATION OF THE RADIAL FORCE

Radial force is the radial component of the force applied by gas to the rotor. It can be divided into two parts: the radial force applied to the non-contact line lobe segment and the contact line lobe segment.
3.1 Radial Force Applied to the Non-contact Line Lobe Segment

Assume that the gas pressure in the ith inter lobe volume is \( P_i \), and take a unit length along axis, the lobe camber \( a-b-c-d-d'-c-b-a \), is intercepted. (Fig.9). The radial force applied to this surface is as follows:

\[
q = (P_i - P) a \\
\alpha = 2\sin(\pi / m)
\]

where \( m \) is the number of lobes.

The \( X \) and \( Y \) components of radial force \( q \) are as follows:

\[
q_x = (P_i - P) a \cos \theta \\
q_y = (P_i - P) a \sin \theta
\]

Since \( \cos \theta = X / R, \sin \theta = Y / R \), (here \( X, Y \) are the coordinate of the point where the force act at.). Therefore,

\[
q_x = -a P / 2 X \sin(\pi / m) \\
q_y = -a P / 2 Y \sin(\pi / m)
\]

Because the acting point of \( q \) is at the middle of arc AD, so, the variation of \( X \) and \( Y \) along axial direction are as follows:

\[
X = R \cos \left( \tau + \varphi \right) \\
Y = R \sin \left( \tau + \varphi \right)
\]

where \( \varphi \) = the initial angle
\( \tau \) = the twist angle
\( R \) = outside radius of rotor

It is evident that the locus of acting point of applied force along rotor axis is a helical line which is called load line. The projection of \( q \) on the coordinate plane of \( YOZ \) and \( XOZ \) can be plotted when initial angle \( \varphi \), number of lobes \( m \) and difference of pressures \( \Delta P \) are given. Those projections represent the distribution of \( q_x \) and \( q_y \) along the rotor axis. Fig.10 shows the \( q_x \) and \( q_y \) of the male rotor.

3.1.1 Male Rotor

Fig. 11 shows the projection of the first inter lobe load helical line on the \( XOY \) and \( YOZ \) planes.

The second, third etc. inter lobe load helical line can be obtained by the same procedure, and the radial force can be obtained by the following equation:

\[
q = \sqrt{q_x^2 + q_y^2}
\]

3.1.2 Female Rotor

The load helical line of female rotor is similar to the male rotor.

3.2 The calculation of radial force in the contact line segment.

The method of calculation is similar to that of non-contact line segment. For example, contact line segment and position of initial section of load helical line \( q_L \) are assigned to ith inter lobe volume of a male rotor.

The right side of initial section of \( q_L \) is taken into account as non-contact region where pressure \( P_i \) is applied. Since pressure \( P_i \) is applied to the area \( 1-5-k-3-1 \), therefore, to calculate the radial force \( q_L \) in this region a force of \( (P_i - P) s_{1-c} \) should be deducted, where \( s_{1-c} \) is the area \( 1-5-k-3-1 \). On the other hand, since the initial section has been assigned, area \( J-5'-4-3-J' \), that is \( s_{5-5'} \), is included in the high pressure region; therefore, to calculate \( q_L \), a force \( (P_i - P) s_{5-5'} \) should be added.
For the same reason, the projected areas $s_{1-2}$ and $s'_{1-2}$ which are the areas $1-5-k-3-1$ and $J-5'-4-3-J$ on the XOY coordinate plane can be found. To calculate $q_{xy}$ in this region, forces $(p_i - p_s)s_{1-2}$ and $(p_i - p_s)s'_{1-2}$ should be deducted or added.

This method is also suitable for female rotor calculation.

For the first and last inter lobe, since contact line are incomplete there, area $s$ and $s'$ are should be calculated individually. While for the inter lobe of complete contact line region, area $s$ is the same as $s'$.

Let
\[ N_x = (p_i - p_s) s \]
\[ N_y = (p_i - p_s) s' \]
where $s$ and $s'$ can be positive or negative.

Using symbols $n_x$ and $n_y$ as the unit length of $N_x$ and $N_y$. $s$ and $s'$ can be changed at the original position and $n_x(z)$, $n_y(z)$ can be obtained. (Fig. 12)

4. CALCULATION RESULTS

The axial and radial force of a prototype twin-rotor screw refrigerant was calculated. The results are listed as follows:

4.1 The axial force acting on the end surface of rotor

Male rotor: $Q = 1784.31$ N
Female rotor: $Q = 988.43$ N

4.2 The axial force acting on the profile surface

Male rotor: $Q = 3588$ N
Female rotor: $Q = 54.3$ N

4.3 The distribution of radial force of male and female rotors along the rotor axis

Fig. 12 shows the distribution of radial force of male rotor. Fig. 13 shows the distribution of radial force of female rotor.

5. CONCLUSIONS

1. This improved method for screw rotor forces calculation has the features of rapid, flexible, accurate and visualized.

2. Some simplifications are made in this force calculation, results are satisfied for engineering design.

3. This computer-assisted technique can be used for different rotor profiles, refrigerants and different ratings.

6. REFERENCES

Fig. 1 The profile of male and female rotors

Fig. 2 The contact line  a) Male rotor  b) Female rotor

Fig. 3 A part of contact line on the last lobe

Fig. 4 A part of contact line on the first lobe
Fig. 5 The profile of discharge side of male rotor

Fig. 6 The profile of suction side of male rotor

Fig. 7 The contact line of male rotor

Fig. 8 The profile of female rotor
a) Suction side
b) Discharge side
Fig. 9 The radial force of the male rotor

Fig. 10 The locus of acting point of $q_x$ and $q_y$ along rotor axis

a) $\Sigma q_x$

b) $\Sigma q_y$
Fig. 12 The horizontal component and the vertical component of the radial force of male rotor

a) $\Sigma q_x$, b) $\Sigma q_y$
Fig. 13 The horizontal component and the vertical component of the radial force of female rotor a) $\Sigma q_x$ b) $\Sigma q_y$