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Dynamic Behavior of a Free-Piston Stirling Engine Driven Heat Pump With Magnetic Coupling

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A new low-cost design for a free-piston Stirling engine driven heat pump is presented. The new approach uses a linear-acting magnetic coupling to transfer engine power across a hermetic shell. Compliance in the coupling affects the performance and stability of the system, especially with regard to matching the engine dynamics to the compressor load. This paper first describes test results that were obtained with the Mark I diaphragm-coupled heat pump. Engine efficiencies exceeded 25% based on fuel higher heating value and heat pump COP was greater than .95 at the 95° ambient, 3-ton point. Following the description of Mark I test results, the Mark II compressor design is presented.

INTRODUCTION

A residential-sized Stirling engine driven refrigerant compressor with attractive performance has been successfully developed at MTI. This work was performed under joint funding from the Gas Research Institute and DOE through a subcontract to Oak Ridge National Laboratory. Continuing design and development of a market prototype model, the Mark II, is progressing under contract to DOE. The Mark I heat-actuated heat pump (HAHP) includes an oil-filled transmission pressurized to 60 Bar. Chevron seal packs are used to seal two penetrations of the shafts that drive the compressor pistons, and two contoured steel diaphragms are used as flexible boundaries to contain the oil. Figure 1 shows a drawing of an advanced Mark I HAHP that would function essentially the same as the laboratory model. Reference 1 describes that machine in detail and also describes the basic principles of a free-piston Stirling engine.

Fig. 1 Mark I HAHP
The dynamic behavior of the Mark I HAHP is relatively easy to analyze, and laboratory experience has shown the machine to be extremely stable and controllable over a wide range of loads and compressor conditions (turn-down ratios exceeding 7-to-1 have been demonstrated). The dynamic model of the Mark I contains only two degrees-of-freedom, not including the external vibration absorbers and the displacer motor currents. These are: the displacer motion and the piston assembly motion (piston mass includes contributions from the diaphragms and from the transmission oil since these components move together as a single body). Both the displacer and piston are resonantly sprung using gas springs. A linear motor, incorporated in the displacer, provides stroke control of the system. The frequency of electric power supplied to this motor determines the operating frequency of the system. Although a small variation in frequency was initially used to obtain optimum tuning, final testing of the Mark I used fixed, 60-Hz frequency over the full range of compressor conditions. Power for the motor was obtained directly from the line using a Variac for voltage control.

The Mark I HAHP performed very well during extensive testing and development. Demonstration goals for system COP were achieved in both heating and cooling modes with engine efficiency exceeding 25% based on fuel higher heating value. A Mark I system package was also assembled and shipped to Lennox Industries in Carrollton, Texas. Verification tests and performance mapping were conducted at their site for 2-1/2 months with good reliability and little down time. A brief summary of the Lennox testing will be given later in this paper. Reference 2 presents the test results in detail and Reference 3 describes the history of development of this machine.

Following the Lennox testing and other development tests at MTI, a detailed assessment of manufacturing cost was begun with the help of engineers from John Deere and Co. Preliminary costing of major components had been done early in the program but overall manufacturing costs had never been accurately compiled. As a result of the recent study, manufacturing cost of the Mark I transmission/compressor was found to be very high. A major cost component that could not be greatly simplified was the oil management system, needed to collect, separate, and reinject transmission oil that leaks past the chevron seals. A new approach to the HAHP transmission was adopted to eliminate the need for an oil management system. This approach utilizes a linear magnetic coupling, designed to drive the compressor piston across a hermetic shell. This approach could not be cost effective using magnet materials available only three years ago, but certain grades of neodymium-iron-boron magnets are now projected to cost around $20/lb and the coupling cost will not be excessive. The design of the magnetic coupling is more fully described in a later section.

Magnetic couplings are not rigid connections. Coupling flexibility can be modeled to first order as a linear spring and damper with stiffness between 5 and 10,000 lb/in., depending on the design. Because of this flexibility, an additional degree-of-freedom is added to the system and the dynamic model now includes the displacer, the power piston, and the compressor piston. Dynamic analysis has shown that system operating points are altered because of coupling flexibility, but no problems with controllability or stability should arise.

This paper will briefly review the test results obtained with the Mark I HAHP, both at Lennox and MTI, and then present the main features of the Mark II design. Emphasis will be given to the magnetic coupling. Finally, dynamic behavior of the Mark II HAHP will be described.

**MARK I TEST RESULTS**

A Mark I "system package" was prepared at MTI and shipped to the Lennox Engineering Center in March, 1987. The unit was installed in an environmental test cell and connected to their existing 5-ton calorimeter loop. Figure 2 shows the Lennox installation. Testing at the Lennox facility involved over 140 hours of operating time and provided performance data at 72 test points ranging over equivalent ambient temperature conditions from 0 to 105°F.
The purpose of this effort was fourfold: verification testing, qualification tests, parametric-mapping tests, and testing at varying cooler temperatures. The verification test point was run periodically to ensure that the unit was performing consistently and the data was repeatable. Lennox instrumentation was used for all critical readings and the data agreed very well with performance data that had been measured earlier for this unit. A more complete description of this effort is given in References 2 and 3, along with extensive plots of performance data. Two figures are given here. Figure 3 summarizes the heating and cooling COP's (without accounting for electrical parasitics such as fans, displacer motor, coolant pump, etc.) and Figure 4 shows the repeatability of the unit at the verification point over the 70 day duration of tests.

Testing and development of the Mark I continued until February of this year. Improvements such as adding a lightly contacting split-ring seal to the displacer, running the machine at fixed 60-Hz frequency, and experimenting with PTFE-based bearing materials were tried. Performance of the Mark I continued to improve above that obtained in the Lennox testing. It will be useful to compare the final measured performance at the 95°F ambient to goals that were set in 1981. Table I provides this comparison.

<table>
<thead>
<tr>
<th>PARAMETER</th>
<th>FPSE/HP TARGETS</th>
<th>MEASURED DATA</th>
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<tr>
<td>Date</td>
<td>1981 Scan 32, 1-6-88</td>
<td>95, 95</td>
</tr>
<tr>
<td>Ambient Temperature (°F)</td>
<td>95</td>
<td>95</td>
</tr>
<tr>
<td>Coolant Temperature (°F)</td>
<td>80</td>
<td>85</td>
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<tr>
<td>Capacity (RT)</td>
<td>3.0</td>
<td>3.0</td>
</tr>
<tr>
<td>Displacer Motor Power (watts)</td>
<td>&lt;500</td>
<td>612</td>
</tr>
<tr>
<td>Engine Efficiency (%)</td>
<td>27.5</td>
<td>26.3</td>
</tr>
<tr>
<td>Hydraulic Trans Effic (%)</td>
<td>82.7</td>
<td>83.7</td>
</tr>
<tr>
<td>Compressor Isentropic Efficiency (%)</td>
<td>83.2</td>
<td>81.5</td>
</tr>
<tr>
<td>Lower End COP</td>
<td>3.63</td>
<td>3.78</td>
</tr>
<tr>
<td>HP Thermal COP</td>
<td>1.00</td>
<td>0.96</td>
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</table>
The laboratory reliability of the Mark I was more than adequate and its performance equaled the demonstration goals set to establish marketability. The manufacturing cost, however, was found to be excessive. Prime centers of high cost in the Mark I compressor were identified as: size and weight of the transmission housing, relatively expensive contoured steel diaphragms, and the complexity of the oil management system. Extensive value engineering of the Mark I compressor would have resulted in significant cost reduction, but it was felt that the cost target of approximately $250.00 to $300.00 would be impossible to attain unless drastic changes were adopted.

Numerous new compressor configurations were conceptualized and evaluated. The final Mark II concept is a refinement and simplification of the Mark I. Cost reduction and performance improvement acted as drivers throughout.

The Mark II design, shown in Figure 5, includes the following departures and improvements from the Mark I:

1. Single-piston, single-acting compressor arrangement. Piston and displacer are mounted in line so the vibration absorbers can provide perfect balance.

2. Inexpensive diaphragms to seal the oil and yet allow volumetric displacement.
3. Use of tuned vibration absorbers mounted on torque bars.

4. Magnetic coupling to eliminate the need for an oil management system.

A fluid-filled transmission is retained to assure longevity. The fluid is hermetically sealed and sufficient flexibility is provided in the gas spring diaphragm to allow for thermal expansion. The transmission fluid will likely be a water-glycol mixture with added lubricity enhancers. This choice was due to the low thermal expansion coefficient associated with water-based fluids.

As indicated in Figure 5, an inexpensive, convoluted steel diaphragm, approximately 9 inches in diameter, is used to separate the transmission fluid from the helium working gas of the engine. Centering stiffness will be satisfactory if material thickness is approximately 14 to 16 mils. A molded elastomeric part, only 5 inches in diameter, will be used as the gas spring diaphragm. The shape indicated in the figure is only a schematic representation of the current design.

The key component of the Mark II design, both in terms of cost and in terms of its influence on system behavior is the magnetic coupling. The remainder of the paper will focus on the design and analysis of this element.

Fig. 5 Mark IIIB Lower End Design

*A "dry-compressor" Mark II design has also been developed that eliminates the oil-filled transmission and the need for diaphragms. This approach will be adopted only if on-going research indicates that piston rings and dry sliding bearings can deliver the required life. Wear-couple testing is being conducted at MTI and suitable designs for long-life piston rings are also being developed.
Magnetic Coupling Design

The basic function of most magnetic couplings is to allow motion or power to be transmitted across a hermetic shell. In the case of the Mark II heat pump, one side of the shell is exposed to the transmission fluid at mean pressure equal to 60 bar (the Stirling engine working fluid has this mean pressure) while the other side of the shell is exposed to the refrigerant mean pressure, varying from 161 to 96.5 psia depending on the operating point. Figure 6 shows a schematic of the main components in a linear magnetic coupling.

Several configurations of the magnetic coupling have been under study for the Mark II compressor; efficiency of magnet utilization and ease of fabrication are the primary drivers to consider. Figure 7 shows two alternative magnet arrangements that have proven interesting. The arrangement shown in Figure 7b may be especially useful since the magnet rings have been eliminated from one member of the coupling. Therefore, its fabrication cost will be greatly reduced. Extensive use of finite element magnetic codes will aid in establishing the final design. Several bench-top couplings have already been assembled and tested, and a full-scale trial coupling is currently in fabrication.

![Fig. 6 Schematic of Magnetic Coupling](image)

![Fig. 7a Radially Poled Magnets](image)

![Fig. 7b Magnets on One Member](image)

The amplitude of the coupling force varies greatly depending on the compressor operating point. At the 95°, 3-ton point, this force is 440 lbf if the compressor piston stroke is .62 inches, frequency is 60 Hz, and the piston mass is chosen so that the equivalent spring from the compressor is perfectly balanced by the piston's inertia force. (This assumes that mechanical piston power is 2900 W at the 3-ton point.) It can be readily appreciated that the design of an inexpensive magnetic coupling, strong enough to carry this load, is not trivial. The use of longer piston stroke will reduce the coupling force requirement, but additional transmission power losses will be incurred because of the higher velocities. Coupling efficiency is expected to be very high. Analysis has shown that with proper construction techniques, coupling loss will be about 50 W at the 95° point. Most of the loss occurs as eddy current heating in the hermetic shell.
Coupling Flexibility and Engine Dynamics

As was previously stated, analysis of magnetic coupling designs indicated that coupling stiffness on the order of 5 to 10,000 lb/in could be attained. With this flexibility included, the lower end dynamics can be represented by a linear, two degree-of-freedom system, as shown in Figure 8. The additional degrees-of-freedom are: displacer motion, the motor coil current, and the motor eddy currents. Compressor piston stroke is determined prior to the engine dynamic analysis and, therefore, only four state variables need to be found. Since the system is assumed to behave harmonically, closed form solutions for the four remaining degrees-of-freedom can be obtained and these are programmed in a "function" subprogram that is iterated with the engine's thermodynamic model. This is done automatically until specified convergence criteria are met.

\[ P_c \sin \omega t \]

\[ \begin{align*}
\mathbf{K}_p &= \mathbf{K}_{lower \ gas} + \mathbf{K}_{mechanical \ spring} \\
\mathbf{C}_p &= \mathbf{C}_{lower \ gas} + \mathbf{C}_{coil \ spring} \\
\mathbf{C}_{cpl} &= 46.8 \ \text{N-sec/m} \\
\end{align*} \]

\[ \mathbf{m}_p = 6.68 \ \text{kg} \]

\[ \mathbf{m}_{cp} = 3.85 \ \text{kg} \]

\[ \mathbf{K}_{cpl} = 1.75 \times 10^6 \ \text{N/m} \]

The object of the engine matching analysis is to determine the behavior of the engine at the desired compressor operating points, specified by the compressor piston stroke, and the power and equivalent spring rate associated with that stroke. Compressor analysis is performed prior to the engine analysis so that these values are known. MTI's CYLINDER code, a time-stepping, pv analysis code including valve dynamics and plenum pressure variation, allows accurate prediction of the required stroke, power, and equivalent spring rate at the four key operating points: 95°, 80°, 47°, and 17° ambients.

The engine matching analysis was performed for three values of coupling stiffness: \( K_{cpl} \) equal to 10,000, 5,000, and 2,500 lbf/in. Compressor piston stroke was fixed at the same specified values regardless of coupling stiffness. Tables II, III, and IV give the engine matching results along with a description of the nomenclature used.
### Table II Matching Results for $K_{cpl} = 10,000$ lb/in. (1.75 kN/mm)

<table>
<thead>
<tr>
<th>Ambient</th>
<th>$\dot{m}$ (lb/hr)</th>
<th>$X_c$ (mm)</th>
<th>$X_p$ (mm)</th>
<th>$\alpha_p$</th>
<th>$\delta_{cpl}$ (mm)</th>
<th>$\phi_{cpl}$ (°)</th>
<th>$\phi_D$ (°)</th>
<th>$P_{motor}$ (W)</th>
<th>$P_{diaph}$ (W)</th>
<th>$n_{PV}$</th>
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<tbody>
<tr>
<td>90°</td>
<td>572</td>
<td>15.75</td>
<td>18.49</td>
<td>1.8</td>
<td>1.16</td>
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<td>17.96</td>
<td>56.5°</td>
<td>248</td>
<td>393</td>
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<tr>
<td>80°</td>
<td>327</td>
<td>9.14</td>
<td>9.18</td>
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<td>1.02</td>
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<td>60°</td>
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<td>10.29</td>
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<td>3.6</td>
<td>2.23</td>
<td>593</td>
<td>6.98</td>
<td>61.7°</td>
<td>-7.9</td>
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<tr>
<td>17°</td>
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<td>14.22</td>
<td>106.3°</td>
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### Table III Results for $K_{cpl} = 5000$ lb/in. (.875 kN/mm)

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<th>Ambient</th>
<th>$\dot{m}$ (lb/hr)</th>
<th>$X_c$ (mm)</th>
<th>$X_p$ (mm)</th>
<th>$\alpha_p$</th>
<th>$\delta_{cpl}$ (mm)</th>
<th>$\phi_{cpl}$ (°)</th>
<th>$\phi_D$ (°)</th>
<th>$P_{motor}$ (W)</th>
<th>$P_{diaph}$ (W)</th>
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<tbody>
<tr>
<td>90°</td>
<td>572</td>
<td>15.75</td>
<td>18.49</td>
<td>1.8</td>
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<td>2030</td>
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<td>3582</td>
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<tr>
<td>80°</td>
<td>267</td>
<td>9.14</td>
<td>9.18</td>
<td>1.8</td>
<td>1.52</td>
<td>897</td>
<td>7.65</td>
<td>60.5°</td>
<td>6.5</td>
<td>863</td>
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<tr>
<td>60°</td>
<td>116</td>
<td>10.29</td>
<td>10.67</td>
<td>3.6</td>
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<td>593</td>
<td>6.98</td>
<td>30.7°</td>
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<td>17°</td>
<td>343</td>
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<td>2008</td>
<td>13.68</td>
<td>162.0°</td>
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### Table IV Results for $K_{cpl} = 2500$ lb/in. (.4375 kN/mm)

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<tr>
<th>Ambient</th>
<th>$\dot{m}$ (lb/hr)</th>
<th>$X_c$ (mm)</th>
<th>$X_p$ (mm)</th>
<th>$\alpha_p$</th>
<th>$\delta_{cpl}$ (mm)</th>
<th>$\phi_{cpl}$ (°)</th>
<th>$\phi_D$ (°)</th>
<th>$P_{motor}$ (W)</th>
<th>$P_{diaph}$ (W)</th>
<th>$n_{PV}$</th>
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<tbody>
<tr>
<td>90°</td>
<td>572</td>
<td>15.75</td>
<td>18.49</td>
<td>1.8</td>
<td>2.83</td>
<td>2030</td>
<td>17.23</td>
<td>48.5°</td>
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<td>1.8</td>
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<td>16.08</td>
<td>98.7°</td>
<td>526</td>
<td>2721</td>
</tr>
</tbody>
</table>

**Temperature**
- $\dot{m}$ - refrigerant flow rate
- $X_c$ - compressor piston stroke
- $X_p$ - power piston stroke
- $\alpha_p$ - load angle of $X_p$ with respect to $X_c$
- $\delta_{cpl}$ - amplitude of coupling deflection
- $P_{cpl}$ - amplitude of coupling force
- $\phi_D$ - displacer stroke
- $\phi_{cpl}$ - displacer phase angle relative to $X_p$
- $P_{diaph}$ - power delivered to transmission diaphragm
- $n_{PV}$ - engine efficiency excluding combuster
- $P_{motor}$ - electrical power to displacer motor
- $P_{motor}$ - electrical power to displacer motor

As can be seen from Table II, the maximum amplitude of coupling deflection is only 1.16 mm when the coupling stiffness is 10,000 lb/in. This amplitude is small in comparison to the compressor piston amplitude of 7.875 mm and the engine behavior is very similar to that obtained for a rigid coupling. The range of displacer motor power is from 392 W at the 17° ambient to -19.2 W at the 80° point.* The goal for maximum motor power is 350 W, which can probably be achieved through refinement of the design.

Table III shows that system behavior is still well controlled when coupling stiffness is reduced to half the previous value. The rather high level of displacer motor power can be reduced significantly if negative motor power is achieved at the 80° point. The results in Table IV indicate that problems may be encountered if coupling stiffness is reduced another factor of two, to 2,500 lb/in. Coupling deflection is now a large percentage of piston stroke, especially at the 17° ambient. Also, the large difference in motor power between the 47° and 17° points indicates that maximum motor power cannot be greatly reduced from the 350 W level.

*As much as 50 to 100 W of displacer generated electricity can be fed back to the line if a properly designed Triac is used. However, high levels of negative displacer motor power will cause low engine efficiency at the corresponding operating points.
System stability was verified at all the critical operating points in Tables II and III. Nonlinear, dynamic analysis of the five degree-of-freedom was performed using a time stepping code that includes an ideal compressor model to represent the load. The code uses numerical integration of the equations of motion because a harmonic solution is no longer possible when the nonlinear compressor model is included. The forcing function in the system is simply the sinusoidal voltage supplied to the displacer motor. To analyze system stability, the operating points in the tables were first established, and then perturbations to the system were shown to die out within several cycles, indicating stability.

CONCLUSIONS

Significant progress has been made in the design of a second generation heat-actuated heat pump that will be manufacturable in large quantities. The preliminary design described here offers performance improvement and low cost manufacture relative to the Mark I. Two areas of component development, the magnetic coupling and the elastomeric diaphragm, need to be addressed and the concepts proven. Assuming satisfactory resolution of these issues, the Mark II design can be considered suitable for market prototype machines. Final design and development should commence as soon as funding permits.

ACKNOWLEDGEMENTS

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REFERENCES

