Final Report

PILE DESIGN BASED ON CONE PENETRATION TEST RESULTS

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Report

Title
Pile Designs Based on Cone Penetration Test Results

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Abstract
The bearing capacity of piles consists of both base resistance and side resistance. The side resistance of piles is in most cases fully mobilized well before the maximum base resistance is reached. As the side resistance is mobilized early in the loading process, the determination of pile base resistance is a key element of pile design.

Static cone penetration is well related to the pile loading process, since it is performed quasi-statically and resembles a scaled-down pile load test. In order to take advantage of the CPT for pile design, load-settlement curves of axially loaded piles bearing in sand were developed in terms of normalized base resistance \( q_b/q_c \) versus relative settlement \( s/B \). Although the limit state design concept for pile design has been used mostly with respect to either \( s/B = 5\% \) or \( s/B = 10\% \), the normalized load-settlement curves obtained in this study allow determination of pile base resistance at any relative settlement level within the \( 0 - 20\% \) range. The normalized base resistance for both non-displacement and displacement piles were addressed.

In order to obtain the pile base load-settlement relationship, a 3-D non-linear elastic-plastic constitutive model was used in finite element analyses. The 3-D non-linear elastic-plastic constitutive model takes advantage of the intrinsic and state soil variables that can be uniquely determined for a given soil type and condition. A series of calibration chamber tests were modeled and analyzed using the finite element approach with the 3-D non-linear elastic-plastic stress-strain model. The predicted load-settlement curves showed good agreement with measured load-settlement curves. Calibration chamber size effects were also investigated for different relative densities and boundary conditions using the finite element analysis.

The value of the normalized base resistance \( q_b/q_c \) was not a constant, varying as a function of the relative density, the confining stress, and the coefficient of lateral earth pressure at rest. The effect of relative density on the normalized base resistance \( q_b/q_c \) was most significant, while that of the confining stress at the pile base level was small. At higher relative densities, the value of \( q_b/q_c \) was smaller (\( q_b/q_c = 0.12 - 0.13 \) for \( D_h = 90\% \)) than at lower relative densities (\( q_b/q_c = 0.19 - 0.2 \) for \( D_h = 30\% \)). The values of the normalized base resistance \( q_b/q_c \) for displacement piles are higher than those for non-displacement piles, being typically in the \( 0.15 - 0.25 \) range for \( s/B = 5\% \) and in the \( 0.22 - 0.35 \) range for \( s/B = 10\% \).

The values of the normalized base resistance \( q_b/q_c \) for silty sands are in the \( 0.12 - 0.17 \) range, depending on the relative density and the confining stress at the pile base level. The confining stress is another important factor that influences the value of \( q_b/q_c \) for silty sands. For lower relative density, the value of \( q_b/q_c \) decreases as the pile length increases while that for higher relative density increases.

For effective use of CPT-based pile design methods in practice, the method proposed in this study and some other existing methods reviewed in this study were coded in a FORTRAN DLL with a window-based interface. This program can be used in practice to estimate pile load capacity for a variety of pile and soil conditions with relatively easy input and output of desired data.

Key Words
piles, sands, cone penetration test, bearing capacity, constitutive model, finite element analysis, limit states design, calibration chamber test.

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# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>TABLE OF CONTENTS</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>IMPLEMENTATION REPORT</td>
<td>x</td>
</tr>
<tr>
<td>IMPLEMENTATION REPORT</td>
<td>x</td>
</tr>
<tr>
<td>CHAPTER 1 INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>1.1 Background</td>
<td>1</td>
</tr>
<tr>
<td>1.2 Statement of Problem</td>
<td>2</td>
</tr>
<tr>
<td>1.3 Objective and Scope</td>
<td>3</td>
</tr>
<tr>
<td>1.4 Report Outline</td>
<td>4</td>
</tr>
<tr>
<td>CHAPTER 2 PILE DESIGN BASED ON IN-SITU TEST RESULTS</td>
<td>7</td>
</tr>
<tr>
<td>2.1 Introduction</td>
<td>7</td>
</tr>
<tr>
<td>2.2 Estimation of Pile Load Capacity Based on SPT Results</td>
<td>9</td>
</tr>
<tr>
<td>2.2.1 Meyerhof’s method</td>
<td>10</td>
</tr>
<tr>
<td>2.2.2 Aoki and Velloso’s method</td>
<td>11</td>
</tr>
<tr>
<td>2.2.3 Reese and O’Neill’s method</td>
<td>12</td>
</tr>
<tr>
<td>2.2.4 Briaud and Tucker’s method</td>
<td>14</td>
</tr>
<tr>
<td>2.2.5 Neely’s method</td>
<td>15</td>
</tr>
<tr>
<td>2.3 Estimation of Pile Load Capacity Based on CPT Results</td>
<td>17</td>
</tr>
<tr>
<td>2.3.1 The Dutch method</td>
<td>18</td>
</tr>
<tr>
<td>2.3.2 Schmertmann’s method</td>
<td>20</td>
</tr>
<tr>
<td>2.3.3 Aoki and Velloso’s method</td>
<td>22</td>
</tr>
<tr>
<td>2.3.4 LCPC method</td>
<td>22</td>
</tr>
<tr>
<td>2.4 Summary</td>
<td>27</td>
</tr>
<tr>
<td>CHAPTER 3 METHODS OF INTERPRETATION OF LOAD-SETTLEMENT CURVES</td>
<td>28</td>
</tr>
<tr>
<td>3.1 Introduction</td>
<td>28</td>
</tr>
<tr>
<td>3.2 Interpretation Methods</td>
<td>29</td>
</tr>
</tbody>
</table>
3.2.1 90% and 80% methods ............................................................... 29
3.2.2 Butler and Hoy’s method ........................................................... 31
3.2.3 Chin’s method ................................................................. 31
3.2.4 Davisson’s method ............................................................. 33
3.2.5 De Beer’s method .............................................................. 35
3.2.6 Permanent set method ........................................................ 35
3.3 Limit States Design ................................................................. 37
  3.3.1 Limit states design in Eurocode 7 ........................................... 37
  3.3.2 Limit states design for pile foundations .................................. 39
3.4 Tolerable Settlements for Buildings and Bridge Foundations ........... 43
  3.4.1 Buildings ........................................................................ 43
  3.4.2 Bridges .......................................................................... 47
3.5 Summary .............................................................................. 51

CHAPTER 4 MECHANICAL BEHAVIOR OF SAND .................................. 53

  4.1 Introduction ...................................................................... 53
  4.2 Stress Tensor and Invariants .................................................. 54
  4.3 Elastic Stress-Strain Relationship .......................................... 60
  4.4 Elastic Behavior of Soil ....................................................... 67
    4.4.1 Initial elastic modulus at small strain ................................ 67
    4.4.2 Hyperbolic stress-strain relationship ................................ 71
    4.4.3 Degradation of Elastic Modulus ...................................... 75
  4.5 Failure Criterion and Soil Plasticity ......................................... 76
    4.5.1 Failure criterion ............................................................ 76
    4.5.2 Flow rule and stress hardening ....................................... 79
    4.5.3 Soil dilatancy and critical state of sand ......................... 82
  4.6 Summary .......................................................................... 85

CHAPTER 5 3-D NON-LINEAR ELASTIC-PLASTIC STRESS-STRAIN MODEL 87

  5.1 Introduction ..................................................................... 87
  5.2 Intrinsic and State Soil Variables .......................................... 87
  5.3 Modified Hyperbolic Model for Non-linear Elasticity ................. 90
  5.4 Non-Linear Elastic Model for Three Dimensions ...................... 95
    5.4.1 Modified hyperbolic stress-strain relationship for three dimensions .. 95
    5.4.2 Variation of bulk modulus and Poisson’s ratio .................. 99
    5.4.3 Determination of the parameters f and g ......................... 101
  5.5 Plastic Stress-Strain Relationship for Three Dimensions ........... 118
    5.5.1 Drucker-Prager failure criterion ...................................... 118
    5.5.2 Non-linear failure surface and flow rule ......................... 120
    5.5.3 Incremental stress-strain relationship ............................... 121
  5.6 Summary ......................................................................... 126
CHAPTER 9  PILE DESIGN USING CPT RESULTS ........................................ 206

9.1  Introduction ................................................................................................. 206
9.2  Determination of Base and Shaft Resistance ............................................ 206
   9.2.1  Base resistance ..................................................................................... 206
   9.2.2  Shaft resistance ................................................................................... 209
   9.2.3  Factor of safety ................................................................................... 211
9.3  Use of SPT Blow Counts in CPT-based Method ....................................... 214
9.4  Program CONEPILE .................................................................................. 219
9.5  Summary ..................................................................................................... 223

CHAPTER 10  SUMMARY, CONCLUSIONS AND RECOMMENDATIONS .......... 224

10.1  Summary .................................................................................................... 224
10.2  Conclusions ............................................................................................... 226
10.3  Recommendations ..................................................................................... 228

LIST OF REFERENCES ..................................................................................... 229

APPENDIX ......................................................................................................... 242
### LIST OF TABLES

<table>
<thead>
<tr>
<th>Table</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.1</td>
<td>Values of $K$ and $\alpha$ for different soil types</td>
</tr>
<tr>
<td>2.2</td>
<td>Values of $F_1$ and $F_2$ for different pile types</td>
</tr>
<tr>
<td>2.3</td>
<td>Values of correlation factor $w$ for the Dutch method</td>
</tr>
<tr>
<td>2.4</td>
<td>Values of the factor $c_{sf}$ by Schmertmann (1978)</td>
</tr>
<tr>
<td>2.5</td>
<td>Values of $k_s$ for different soil and pile types</td>
</tr>
<tr>
<td>2.6</td>
<td>Values of $k_c$ for different soil and pile types</td>
</tr>
<tr>
<td>3.1</td>
<td>Relationship between angular distortion and total settlement (after Skepmton and MacDonald 1956)</td>
</tr>
<tr>
<td>3.2</td>
<td>Tolerable movement for buildings (after Eurocode 1)</td>
</tr>
<tr>
<td>3.3</td>
<td>Settlement criteria for bridges expressed in terms of settlement magnitude</td>
</tr>
<tr>
<td>3.4</td>
<td>Tolerable angular distortion for bridge by Moulton et al. (1985)</td>
</tr>
<tr>
<td>3.5</td>
<td>Data used by Moulton et al. (1985) to establish criteria for angular distortion</td>
</tr>
<tr>
<td>4.1</td>
<td>Relationship between different elastic modulus</td>
</tr>
<tr>
<td>4.2</td>
<td>Values of $C_g$, $e_g$, and $n_g$ for different sand type (after Salgado 1993, Salgado et al. 1999)</td>
</tr>
<tr>
<td>5.1</td>
<td>Basic properties of Ticino sand (after Ghionna et al. 1994)</td>
</tr>
<tr>
<td>5.2</td>
<td>Values of $f$ and $g$ from triaxial test results</td>
</tr>
<tr>
<td>5.3</td>
<td>Values of $f$ and $g$ for different relative densities</td>
</tr>
<tr>
<td>6.1</td>
<td>Soil and stress conditions in calibration chamber tests</td>
</tr>
<tr>
<td>6.2</td>
<td>Boundary conditions in calibration chamber tests</td>
</tr>
<tr>
<td>6.3</td>
<td>Size effect in calibration chamber test for BC1 condition</td>
</tr>
<tr>
<td>6.4</td>
<td>Size effect in calibration chamber test for BC2 condition</td>
</tr>
<tr>
<td>6.5</td>
<td>Size effect in calibration chamber test for BC3 condition</td>
</tr>
<tr>
<td>6.6</td>
<td>Size effect in calibration chamber test for BC4 condition</td>
</tr>
<tr>
<td>7.1</td>
<td>Basic soil properties used in finite element analysis</td>
</tr>
<tr>
<td>7.2</td>
<td>Pile geometry and soil conditions used in FEM analysis</td>
</tr>
<tr>
<td>7.3</td>
<td>Values of $q_b/q_c$ according to several authors</td>
</tr>
<tr>
<td>7.4</td>
<td>Values of $q_b/q_c$ at $s/B = 5%$ and $10%$</td>
</tr>
<tr>
<td>7.5</td>
<td>Base resistance ratio for displacement and non-displacement piles</td>
</tr>
<tr>
<td>7.6</td>
<td>Values of $q_b/q_c$ for displacement piles</td>
</tr>
<tr>
<td>7.7</td>
<td>Values of soil intrinsic parameters with different silt contents (after Salgado</td>
</tr>
</tbody>
</table>
et al. 1999, Bandini 1999). .................................................................................................................. 192
7.8 Values of friction angle $\phi_c$ at critical state and dilatancy parameters Q and R with different silt contents (after Salgado et al. 1999, Bandini 1999) ........ 192
7.9 Values of $f$ and $g$ used in finite element analyses for silty sands .................... 193
7.10 Values of $q_b/q_c$ for silty sands with different relative densities and pile lengths .................................................................................................................. 194
8.1 Values of $q_b/q_c$ from load tests on non-displacement and displacement piles .................................................................................................................. 199
9.1 Resistance modification factor $f_p$ and factor of safety for different field tests (after Canadian Geotechnical Society 1992) ......................... 212
9.2 Partial factor of safety for the base resistance ......................................................... 214
9.3 Correlation between CPT and SPT ........................................................................ 218
9.4 Correlation parameters for estimation of relative density ................................. 221
**LIST OF FIGURES**

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1</td>
<td>Research scope and process</td>
</tr>
<tr>
<td>2.1</td>
<td>Examples of methods for estimation of pile bearing capacity</td>
</tr>
<tr>
<td>2.2</td>
<td>Dutch method for determination of base resistance</td>
</tr>
<tr>
<td>2.3</td>
<td>Reduction factor in Schmertmann’s method (1978)</td>
</tr>
<tr>
<td>2.4</td>
<td>Equivalent cone resistance $q_{ca}$ for LCPC method</td>
</tr>
<tr>
<td>3.1</td>
<td>Definition of failure load in 90% criterion</td>
</tr>
<tr>
<td>3.2</td>
<td>Brinch Hansen’s 80% criterion</td>
</tr>
<tr>
<td>3.3</td>
<td>Definition of failure load in Butler and Hoy’s criterion</td>
</tr>
<tr>
<td>3.4</td>
<td>Chin’s criterion for definition of failure load</td>
</tr>
<tr>
<td>3.5</td>
<td>Definition of failure load in Davisson’s criterion</td>
</tr>
<tr>
<td>3.6</td>
<td>Definition of failure load in DeBeer’s criterion</td>
</tr>
<tr>
<td>3.7</td>
<td>Definition of failure load in permanent set criterion</td>
</tr>
<tr>
<td>3.8</td>
<td>Load levels at ultimate and serviceability limit states</td>
</tr>
<tr>
<td>3.9</td>
<td>Differential settlements for (a) smaller-diameter and (b) larger-diameter piles</td>
</tr>
<tr>
<td>3.10</td>
<td>Load-settlement curves with load versus $s_R$ (after Franke 1991)</td>
</tr>
<tr>
<td>3.11</td>
<td>Settlement criteria (after Wahls 1994)</td>
</tr>
<tr>
<td>3.12</td>
<td>Components of settlement and angular distortion in bridge for (a) uniform settlement, (b) uniform tilt or rotation, (c) nonuniform regular settlement, and (d) nonuniform irregular settlement (after Duncan and Tan 1991)</td>
</tr>
<tr>
<td>4.1</td>
<td>Nine components of stress tensor in a soil element</td>
</tr>
<tr>
<td>4.2</td>
<td>Definition of mechanical behavior of a body (after Chen and Han 1988)</td>
</tr>
<tr>
<td>4.3</td>
<td>Non-linear stress-strain behavior of soil</td>
</tr>
<tr>
<td>4.4</td>
<td>Seismic cone penetration test</td>
</tr>
<tr>
<td>4.5</td>
<td>Hyperbolic model (a) stress-strain curve and (b) linear representation</td>
</tr>
<tr>
<td>4.6</td>
<td>Stress states for elastic-plastic material</td>
</tr>
<tr>
<td>4.7</td>
<td>Stress-strain behavior for hardening, perfectly plastic and softening material</td>
</tr>
<tr>
<td>4.8</td>
<td>Different behavior of dense and loose sand (after Lambe and Whitmann 1986)</td>
</tr>
<tr>
<td>5.1</td>
<td>Secant modulus for non-linear stress-strain behavior</td>
</tr>
<tr>
<td>5.2</td>
<td>Modulus degradation relationship for normally consolidated sand</td>
</tr>
</tbody>
</table>
5.3 Modulus degradation curve for different values of f and g .......................... 91
5.4 Definition of $\tau_0$, $\tau$, and $\tau_{\text{max}}$ for (a) constant and (b) varying confinement. .................................................. 96
5.5 Modulus degradation curves for $D_R = 51.5\%$ and $\sigma_3 = 400$ kPa with $f = 0.97$ and $g = 0.18$ .......................................................... 104
5.6 Modulus degradation curves for $D_R = 48.8\%$ and $\sigma_3 = 200$ kPa with $f = 0.97$ and $g = 0.15$ .......................................................... 105
5.7 Modulus degradation curves for $D_R = 48.2\%$ and $\sigma_3 = 500$ kPa with $f = 0.97$ and $g = 0.18$ .......................................................... 106
5.8 Modulus degradation curves for $D_R = 50.8\%$ and $\sigma_3 = 110$ kPa with $f = 0.97$ and $g = 0.20$ .......................................................... 107
5.9 Modulus degradation curves for $D_R = 84.6\%$ and $\sigma_3 = 650$ kPa with $f = 0.93$ and $g = 0.20$ .......................................................... 108
5.10 Modulus degradation curves for $D_R = 82.3\%$ and $\sigma_3 = 100$ kPa with $f = 0.95$ and $g = 0.25$ .......................................................... 109
5.11 Modulus degradation curves for $D_R = 88.9\%$ and $\sigma_3 = 200$ kPa with $f = 0.95$ and $g = 0.20$ .......................................................... 110
5.12 Modulus degradation curves for $D_R = 91.1\%$ and $\sigma_3 = 150$ kPa with $f = 0.95$ and $g = 0.20$ .......................................................... 111
5.13 Modulus degradation curves for $D_R = 100\%$ and $\sigma_3 = 200$ kPa with $f = 0.95$ and $g = 0.25$ .......................................................... 112
5.14 Modulus degradation curves for $D_R = 100\%$ and $\sigma_3 = 400$ kPa with $f = 0.95$ and $g = 0.27$ .......................................................... 113
5.15 Modulus degradation curves for $D_R = 100\%$ and $\sigma_3 = 600$ kPa with $f = 0.94$ and $g = 0.32$ .......................................................... 114
5.16 Modulus degradation curves for $D_R = 100\%$ and $\sigma_3 = 800$ kPa with $f = 0.94$ and $g = 0.28$ .......................................................... 115
5.17 Modulus degradation curves for $D_R = 98.6\%$ and $\sigma_3 = 100$ kPa with $f = 0.94$ and $g = 0.20$ .......................................................... 116
5.18 Drucker-Prager failure surface (a) in $1_1-\sqrt{J_2}$ plane and (b) in principal Stress plane ................................................................. 119
5.19 Plastic strain in Drucker-Prager failure criterion with associated flow rule 122
5.20 Non-linear failure surface with non-associated flow rule .......................... 122
6.1 Plate load test in calibration chamber .................................................. 130
6.2 Types of boundary conditions in calibration chamber test ...................... 135
6.3 Finite element model for calibration chamber plate load test .................. 138
6.4 Deformed finite element mesh of calibration chamber plate load test with $D_R = 55.2\%$, $\sigma'_v = 62.0$ kPa, and $\sigma'_h = 24.4$ kPa at $s/B = 10\%$ ........ 141
6.5 Vertical stress distribution in calibration chamber plate load test with $D_R = 55.2\%$, $\sigma'_v = 62.0$ kPa, and $\sigma'_h = 24.4$ kPa at $s/B = 10\%$ ........ 142
6.6 Vertical displacement distribution in calibration chamber plate load test with $D_R = 55.2\%$, $\sigma'_v = 62.0$ kPa, and $\sigma'_h = 24.4$ kPa at $s/B = 10\%$ ........ 143
6.7 Horizontal displacement distribution in calibration chamber plate load test
with $D_R = 55.2\%$, $\sigma'_v = 62.0$ kPa, and $\sigma'_h = 24.4$ kPa at $s/B = 10\%$  \[144\]

6.8 Variation of shear modulus ................................................................. 145

6.9 Load-settlement curves for calibration chamber plate load tests  
(Test No. 300, 301, and 302) ................................................................. 146

6.10 Load-settlement curves for calibration chamber plate load tests  
(Test No. 303, 304, and 306) ................................................................. 147

6.11 Load-settlement curves for calibration chamber plate load tests  
(Test No. 307, 308, and 309) ................................................................. 148

6.12 Load-settlement curves for calibration chamber plate load tests  
(Test No. 310, 311, and 312) ................................................................. 149

6.13 Load-settlement curves for calibration chamber plate load tests  
(Test No. 313, 314, and 317) ................................................................. 150

6.14 Load-settlement curves for calibration chamber plate load tests  
(Test No. 321, 322, and 323) ................................................................. 151

6.15 Load-settlement curves for calibration chamber plate load tests  
(Test No. 324, 325, and 326) ................................................................. 152

6.16 Load-settlement curves for calibration chamber plate load tests  
(Test No. 327, 328, and 329) ................................................................. 153

6.17 Measured and predicted plate unit loads in calibration chamber tests  \[154\]

6.18 Comparison of pile base unit load with plate unit load in calibration  
chamber plate load tests ................................................................. 159

7.1 Load-settlement curves for pile load test at Georgia tech ................. 168

7.2 Different failure mechanisms for deep penetration .............................. 170

7.3 Slip pattern under cone penetrometer (after Salgado 1993) ................ 171

7.4 Stress pattern under cone penetrometer (after Salgado 1993) ............. 173

7.5 Finite element model for 5-m pile ..................................................... 176

7.6 Finite element model for 10-m pile ................................................... 177

7.7 Finite element model for 20-m pile ................................................... 178

7.8 Base load-settlement curves for (a) 5-m, (b) 10-m, and (c) 20-m piles  \[179\]

7.9 Normalized load-settlement curves for (a) 5-m, (b) 10-m, and  
(c) 20-m piles in terms of $q_b/q_c$ and $s/B$ ........................................ 182

7.10 Normalized base resistance $q_b/q_c$ with (a) mean effective stress ($\sigma'_m$)  
at the pile base level and (b) relative density ($D_R$) .............................. 186

7.11 Effect of $K_0$ on normalized base resistance $q_b/q_c$ ....................... 188

7.12 Values of $q_b/q_c$ for silty sand ..................................................... 195

8.1 Values of $q_b/q_c$ in calibration chamber plate tests for (a) $s/B = 5\%$ and  
(b) $s/B = 10\%$ ................................................................................ 203

9.1 Estimation of pile base resistance using different methods ............... 208

9.2 Estimation of pile shaft resistance using different methods .............. 210

9.3 CPT-SPT correlation with the mean grain size (after Robertson and  
Campanella 1983) ................................................................. 215

9.4 Cone resistance $q_c$ and SPT blow count $N$ with depth .................. 217

9.5 Estimation of the base resistance for a given soil condition .............. 222
IMPLEMENTATION REPORT

In the present study, in order to take advantage of the cone penetration test for pile design, load-settlement curves in terms of normalized base resistance \(q_b/q_c\) versus relative settlement \((s/B)\) where \(q_c\) = cone resistance, \(s\) = pile base settlement, \(B\) = pile diameter were developed. Although the limit state design concept for pile design has been used mostly with respect to either \(s/B = 5\%\) or \(s/B = 10\%\), the normalized load-settlement curves obtained in this study allow determination of pile base resistance for any relative settlement in the \(0 - 20\%\) range. This is important, as it permits consideration of specific project features, related to the superstructure or other components of the facility, by selecting a specific value of tolerable settlement for use in design.

The value of the normalized base resistance \(q_b/q_c\) is not a constant, varying as a function of the relative density, the confining stress, and the coefficient of lateral earth pressure at rest. The effect of relative density on the normalized base resistance is significant, while that of the confining stress at the pile base level is small. At higher relative densities, the value of \(q_b/q_c\) was smaller \((q_b/q_c = 0.12 - 0.13\) for \(D_R = 90\%)\) than at lower relative densities \((q_b/q_c = 0.19 - 0.2\) for \(D_R = 30\%\)). It is usually very difficult to obtain undisturbed granular soil samples. It is, therefore, recommended that the estimation of the relative density be made through reasonable correlations based on in-situ test results such as the cone penetration test.

The normalized base resistance \(q_b/q_c\) proposed in this study can also be used for displacement piles. The values of \(q_b/q_c\) were typically in the \(0.15 - 0.25\) range for \(s/B = 5\%\) and in the \(0.22 - 0.35\) range for \(s/B = 10\%\), depending on the value of relative density. CPT pile design methods can be used with the SPT blow count \(N\) for practical purposes, if a proper value of \(q_c/N\) is used for a given soil condition.

The evaluation of the relative settlement associated with the limit states design of piles should be done with consideration of the type, functionality, location, and
importance of the superstructure. The relative settlement s/B of piles leading to serviceability or ultimate limit states is usually in excess of 10%.

For implementation, use of the method in future INDOT piling projects is recommended. This may be done as part of an implementation project with extensive participation of INDOT design personnel. It is strongly recommended that a number of fully instrumented load tests on both driven piles and drilled shafts be performed along with the cone penetration test at representative sites in Indiana. The fully instrumented pile load test data would allow separate measurement of the base and the shaft resistance, and shaft resistance developed at the interface of the pile with different soil layers. This separation of load capacity in base and shaft capacities is essential for further validation of the design methods. It is also recommended that the selection of sites for pile load and cone penetration tests be such that sites with a variety of Indiana soil types be located. Results for such sites would be useful for further validation of the CPT-based pile design procedure for a wide range of soils.
CHAPTER 1 INTRODUCTION

1.1 Background

With the rapid growth of metropolitan areas, and fast industrialization resulting from the fast-paced economic globalization, there has been a need to build heavier and taller structures on marginal sites, where surface soils are weak and shallow foundations are usually not the best design solution. At the same time, advances in piling technology permit the installation of several types of piles, particularly non-displacement piles, at lower costs than was possible in the past. This in turn generates the motivation for further improvements in pile design capability. Additionally, there is a growing realization in the foundation engineering industry that certain types of deep foundation (such as large-diameter drilled shafts) are conservatively designed (Harrop-Williams 1989, Hirany and Kulhawy 1989, De Mello and Aoki 1993). In this context, advances in pile design methods can have significant economic impact and should be actively pursued.

Based on the method of installation, pile foundations are classified as either displacement or non-displacement piles. Driven piles are the most common type of displacement piles, and drilled shafts (bored piles) are the most common type of non-displacement piles. The load carrying capacity of both displacement and non-displacement piles consists of two components: base resistance and side resistance. The side resistance of piles is in most cases fully mobilized well before the maximum base resistance is reached (Franke 1993). After full mobilization of side resistance, any increment of axial load is transferred fully to the base. As the side resistance is
mobilized early in the loading process, the determination of base resistance is a key element in pile design.

1.2 Statement of Problem

Although friction piles are sometimes used, it is usually desirable to avoid relying solely on side resistance to develop the needed pile load capacity. This is done by placing the pile base on a bearing layer. Physically, what keeps a well designed pile from plunging when acted upon by an axial load is the base resistance developed in this layer, since the side resistance is fully mobilized early in the loading process. Plunging will take place only when the base unit load overcomes the limit base resistance \( q_{BL} \), which is dependent not only on the density but also on the lateral confinement imposed by the surrounding soil immediately beneath the pile base.

A number of different methods have been proposed to assess deep bearing capacity based on the load-settlement curve obtained from pile load tests (Brinch Hansen 1963; De Beer 1967; Chin 1970; Davisson 1972). One modern pile design philosophy is based on the limit state concept. A limit state is evaluated with respect to either a loss of functionality or collapse of the superstructure and/or foundation (Franke 1990; Salgado 1995). According to Franke (1991), drilled shafts must typically undergo settlements greater than 10% of the base diameter before a limit state is reached. In some design situations, however, settlements less than 10% of the pile diameter may cause the foundations or the supported structure to reach a limit state. Irrespective of the specific value of settlement leading to a limit state, it is clearly necessary that a methodology be available to calculate the settlements caused by a given load and vice-versa in order to design piles within the limit-state framework. As shaft resistance is mobilized early in the loading process, the determination of the load-settlement relationship for the pile base is a key element in pile design and is the focus of this study.
Pile design in sands have been mostly based on results of the SPT test, which is today widely recognized to have numerous limitations (Seed et al. 1985, Skempton 1986). A serious limitation is that its main measurement, the number of blows required to drive a sampler one foot into the ground, is obtained based on a dynamic process, which is not well related to the quasi-static pile loading process. The SPT blow counts will also vary, sometimes significantly, with the operator and operating procedures. A much better alternative for pile design is to base it on data from a cone penetration test (CPT). In this test, a cylindrical penetrometer with a conical point is pushed into the ground quasi-statically, and a number of measurements are made.

The CPT was invented in northern Europe precisely for the purpose of pile foundation design (the test can be seen as a scaled-down load test on a pile), and has since been increasingly used in Europe, and to a lesser extent in the Americas and Asia, for pile design and other purposes. Although there have been several proposed methods of pile design based on cone penetration test results, an example of which is the LCPC method (Bustamante and Gianeselli 1982), only recently has a method based on establishing the relationship between cone tip resistance and the load-settlement relationship of a pile been proposed (Ghionna et al. 1993, 1994; Salgado 1995). If such a relationship can be established reliably for a variety of soil conditions, significant economies in materials volume and pile installation charges will become possible with respect to current design methods.

1.3 **Objective and Scope**

The main objective of this research is to develop the methodology to determine pile load-carrying capacity based on the results of cone penetration test. The focus will be on the design of piles used to support typical transportation structures, with focus on the response of piles bearing on sandy soils to vertical loading.
In order to develop the load-settlement curves for a variety of soil conditions, 3-D finite element modeling is used. In general, the pile response to an external load is strongly non-linear and may involve large irreversible deformations. For more realistic modeling of soil behavior around axially loaded piles, a non-linear elastic-plastic stress-strain relationship will be used in the finite element analyses. The calculated load-settlement curves are normalized with the cone resistance $q_c$ and the pile diameter $B$ for the base resistance $q_b$ and the settlement $s$, respectively. The fully developed load-settlement curves in terms of $q_b/q_c$ versus $s/B$ can be used to determine the normalized pile base resistance $q_b/q_c$ for any settlement-based design criterion. The results of the analyses are compared with results from calibration chamber plate load tests and free field pile load tests. Figure 1.1 shows the general scope of this study.

1.4 Report Outline

This Report consists of ten chapters, including this introduction.

Chapter 2 reviews the pile design methods based on in-situ test results. The methods presented in Chapter 2 are based on the Standard Penetration Test (SPT) and Cone Penetration Test (CPT).

Chapter 3 describes the methodology for defining a "failure" load for a pile from its load-settlement curve. It also introduces the limit state design concept with main focus on ultimate and serviceability limit states that are important in geotechnical engineering. Tolerable settlements for different types of structures, including buildings and bridges are discussed as well.

Chapter 4 covers the conceptual framework for describing the mechanical behavior of soils, including stress tensors, invariants, linear and non-linear stress-strain relationships, and the concept of plasticity. Critical state, dilatancy and shear strength are all discussed.
Figure 1.1  Research scope and process
Chapter 5 presents the non-linear elastic-plastic soil model used in this study. The concepts of intrinsic and state soil variables are explained. Values of the parameters required for the non-linear soil model are presented based on experimental test results. The Drucker-Prager plastic model for the definition of post-failure behavior of the soil is described with incremental stress-strain relationship.

Chapter 6 presents the finite element modeling and analysis of the calibration chamber plate load tests. The analytical results are compared with the measured values of plate resistance in calibration chamber plate load tests. This chapter aims to verify the accuracy of plate resistance predictions and assess the existence of chamber size effects on plate resistance values.

Chapter 7 presents the determination of pile base resistance based on the normalized load-settlement curves fully developed for axially loaded piles bearing in sand for a variety of soil and stress conditions. The effects of relative density and the coefficient of lateral earth pressure on the pile base resistance are explained.

Chapter 8 presents case histories for the validation of the results obtained in this study. The case histories include both non-displacement and displacement piles.

Chapter 9 discusses pile design using CPT results in the light of the results of Chapters 1 - 8. In order to present a more complete discussion on the subjects, correlations between SPT and CPT are also addressed. A computer program developed for estimating pile load capacity in practice is briefly also introduced.

Chapter 10 presents the conclusions drawn from this study.
CHAPTER 2 PILE DESIGN BASED ON IN-SITU TEST RESULTS

2.1 Introduction

In general the application of in-situ tests to pile design is done through:

(1) Indirect Methods

or

(2) Direct Methods

Indirect methods require the evaluation of the soil characteristic parameters, such as the internal friction angle $\phi$ and the undrained shear strength $s_u$, from in-situ test results. This requires consideration of complicated boundary-value problems (Campanella et al. 1989). On the other hand, with direct methods, one can make use of the results from in-situ test measurements for the analysis and the design of foundations without the evaluation of any soil characteristic parameter. The application of direct methods to the analysis and the design of foundations is, however, usually based on empirical or semi-empirical relationships. Figure 2.1 shows some examples of the methods available for indirect and direct approaches in different applications.

Indirect methods for pile design include Vesic (1977), Coyle and Castello (1981), and $\beta$ method (Burland 1973) for cohesionless soil, and $s_u$ method (Bowles 1988), $\alpha$ method (Tomlinson 1971), $\beta$ method (Burland 1973), and $\lambda$ method (Vijayvergiya and Focht 1972) for cohesive soil. Most indirect pile design methods define the correlation
Estimation of Pile Bearing Capacity Using In-Situ Tests

- Indirect method
  - Sandy soil
  - Clayey soil
  - Rock
  - Base
    - $9_{su}$
  - Shaft
    - $\alpha$ method
    - $\beta$ method
    - $\lambda$ method

- Direct method
  - SPT
  - CPT
  - Base
    - Vesic
    - Coyle and Castello
  - Shaft
    - $\beta$ method
    - Coyle and Castello

Figure 2.1 Examples of methods for estimation of pile bearing capacity.
factor between the stress state and base or shaft resistance based on the soil-strength parameters.

Direct methods used for pile design have been mainly based on the standard penetration test (SPT) and the cone penetration test (CPT). Although the SPT has been used more extensively, it is widely recognized that it has a number of limitations (Seed et al. 1985, Skempton 1986). A serious limitation is that its main measurement (the SPT blow count) is not well related to the pile loading process. The SPT blow count can also vary depending on operation procedures. The CPT is a superior test for pile design purposes. In this test, a cylindrical penetrometer with a conical tip is pushed into the ground as if it were a scaled pile load test. In addition to the similarity between the pile loading and cone penetration testing mechanisms, the possibility of simultaneous measurement of shear wave velocities makes it possible to estimate elastic properties of subsurface soils, which may improve the quality of the design with more accurate in-situ soil properties.

The main focus of this study is the estimation of pile bearing resistance based on direct methods, the cone penetration test in particular. In this chapter, the existing methods for pile design using the SPT and CPT, will be reviewed.

2.2 Estimation of Pile Load Capacity Based on SPT Results

In most SPT methods, the pile load capacities are defined in terms of the SPT blow count \( N \) and the correlation parameters. These relationships are typically of the form (Bandini and Salgado 1998):

\[
q_b = n_b N_b
\]  
\[
q_s = \sum n_{si} N_{si}
\]

where \( q_b \) = base resistance
\( n_b = \text{factor to convert SPT blow count to base resistance} \)
\( N_b = \text{representative } N_{SPT} \text{ value at the pile base level} \)
\( q_s = \text{shaft resistance} \)
\( n_{si} = \text{factor to convert SPT blow count to shaft resistance} \)
\( N_{si} = \text{representative } N_{SPT} \text{ value along the pile shaft in layer } i. \)

For the computation of base resistance, it is recommended that the SPT N value should represent the condition near the pile base. Different ways to define the representative N value have been proposed. They will be discussed as the methods are presented.

2.2.1 Meyerhof's method

Meyerhof (1976, 1983) proposed the following expressions for the base resistance based on SPT results:

for sands and gravels
\[
q_b = 0.4N_{1.60} \frac{D}{B} P_a \leq 4.0N_{1.60}P_a
\]  
(2.3)

for nonplastic silts
\[
q_b = 0.4N_{1.60} \frac{D}{B} P_a \leq 3.0N_{1.60}P_a
\]  
(2.4)

where
\( q_b = \text{base resistance} \)
\( N_{1.60} = \text{SPT N value corrected for field procedures and overburden stress} \)
\( D = \text{pile embedment depth} \)
\( B = \text{pile diameter} \)
\( P_a = \text{reference stress} = 100 \text{ kPa} = 0.1 \text{ MPa} = 1 \text{ tsf} \).

The upper limits of base resistance given in (2.3) and (2.4) are always applied in case of \( D/B \geq 10 \) for sands and gravels, and \( D/B \geq 7.5 \) for nonplastic silts. For pile
diameters within the range of $0.5 < \frac{B}{B_R} < 2$, where $B_R = \text{reference length} = 1 \text{ m} = 100 \text{ cm} = 40 \text{ in.} = 3.28 \text{ ft.}$, $q_b$ is reduced using the factor $r_b$ as follows:

$$r_b = \left( \frac{B + 0.5B_R}{2B} \right)^n \leq 1$$

(2.5)

where $n = 1, 2, \text{ or } 3$ for loose, medium, or dense sand, respectively. Meyerhof (1976, 1983) also proposed the expression of shaft resistance for small- and large-displacement piles:

for small-displacement piles in cohesionless soil

$$q_s = \frac{P_o}{100} N_{60}$$

(2.6)

for large-displacement pile in cohesionless soil

$$q_s = \frac{P_o}{50} N_{60}$$

(2.7)

where $q_s = \text{shaft resistance}$

$N_{60} = \text{SPT N value corrected for field procedures only.}$

2.2.2 Aoki and Velloso's method

Aoki and Velloso (1975) proposed the following formula for different soil and pile types:

$$q_b = \frac{K}{F_1} N_b P_a$$

(2.8)
\[ q_{si} = \frac{\alpha K}{F_2} N_{si} P_a \]  

(2.9)

where

- \( K \) = empirical factor in function of soil type
- \( F_1 \) and \( F_2 \) = empirical factors in function of pile type
- \( \alpha \) = shaft resistance factor depending on soil type
- \( N_b \) = average of the three \( N_{SPT} \) values close to the pile base
- \( N_{si} \) = average of \( N_{SPT} \) values along the pile shaft in layer \( i \), excluding those used to calculate \( N_b \).
- \( P_a \) = reference stress = 100 kPa = 0.1 MPa = 1 tsf.

The values of \( K \), \( \alpha \), and \( F_1 \), \( F_2 \) are given in Table 2.1 and 2.2, respectively.

2.2.3 Reese and O’Neill’s method

Based on the observation of 41 loading tests, Reese and O’Neill (1989) proposed the following SPT-based relationship for the base resistance of drilled shafts embedded in sand:

\[ q_b = 0.6N \cdot P_a \leq 45P_a \]  

(2.10)

where \( q_b \) = base resistance; \( P_a \) = reference stress = 100 kPa = 0.1 MPa = 1 tsf. The limit value given in (2.10) was selected because no ultimate bearing pressure was observed beyond that value for any of the loading test results. In (2.10), the SPT \( N \) value should be mean uncorrected value within a distance of two times the base diameter (\( B_b \)) below the base of the drilled shaft.

In order to restrict the settlement of large-diameter shafts, they also suggested to use a reduced value of the base resistance (\( q_{br} \)) as follows:
Table 2.1 Values of K and α for different soil types.

<table>
<thead>
<tr>
<th>Type of Soil</th>
<th>K</th>
<th>α (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sand</td>
<td>10.0</td>
<td>1.4</td>
</tr>
<tr>
<td>Silty sand</td>
<td>8.0</td>
<td>2.0</td>
</tr>
<tr>
<td>Clayey silty sand</td>
<td>7.0</td>
<td>2.4</td>
</tr>
<tr>
<td>Clayey sand</td>
<td>6.0</td>
<td>3.0</td>
</tr>
<tr>
<td>Silty clayey sand</td>
<td>5.0</td>
<td>2.8</td>
</tr>
<tr>
<td>Silt</td>
<td>4.0</td>
<td>3.0</td>
</tr>
<tr>
<td>Sandy silt</td>
<td>5.5</td>
<td>2.2</td>
</tr>
<tr>
<td>Clayey sandy silt</td>
<td>4.5</td>
<td>2.8</td>
</tr>
<tr>
<td>Clayey silt</td>
<td>2.3</td>
<td>3.4</td>
</tr>
<tr>
<td>Sandy clayey silt</td>
<td>2.5</td>
<td>3.0</td>
</tr>
<tr>
<td>Clay</td>
<td>2.0</td>
<td>6.0</td>
</tr>
<tr>
<td>Sandy clay</td>
<td>3.5</td>
<td>2.4</td>
</tr>
<tr>
<td>Sandy silty clay</td>
<td>3.0</td>
<td>2.8</td>
</tr>
<tr>
<td>Silty clay</td>
<td>2.2</td>
<td>4.0</td>
</tr>
<tr>
<td>Silty sandy clay</td>
<td>3.3</td>
<td>3.0</td>
</tr>
</tbody>
</table>

Table 2.2 Values of F₁ and F₂ for different pile types.

<table>
<thead>
<tr>
<th>Type of Pile</th>
<th>F₁</th>
<th>F₂</th>
</tr>
</thead>
<tbody>
<tr>
<td>Franki Piles</td>
<td>2.50</td>
<td>5.0</td>
</tr>
<tr>
<td>Steel Piles</td>
<td>1.75</td>
<td>3.5</td>
</tr>
<tr>
<td>Precast Concrete Piles</td>
<td>1.75</td>
<td>3.5</td>
</tr>
<tr>
<td>Bored Piles</td>
<td>3.0 – 3.50</td>
<td>6.0 – 7.0</td>
</tr>
</tbody>
</table>
\[ q_{b,r} = 1.25 \left( \frac{B_R}{B_b} \right) \cdot q_b \quad \text{for } B_b \geq 1.25 \cdot B_R \quad (2.11) \]

where \( q_{b,r} \) = reduced base resistance; \( B_R \) = reference length = 1 m = 40 in. = 3.28 ft. According to Reese and O'Neill (1989), it is not recommended to use the above expressions for drilled shafts with a depth less than 15 ft (4.6 m) or a diameter less than 24 in (610 mm). For the shaft resistance of drilled shaft in sands, they recommend the use of the \( \beta \) method.

2.2.4 Briaud and Tucker's method

Based on the review and parametric study of 33 instrumented pile load tests, Briaud and Tucker (1984) developed a method for determining base (\( q_b \)) and shaft (\( q_s \)) resistance as a function of pile settlement (\( s \)). In this method, both \( q_b \) versus \( s \) and \( q_s \) versus \( s \) are modeled as hyperbolic curves. Residual stress resulting from rebounding after pile driving was also considered in that hyperbolic formula. The hyperbolic equations for both base and shaft resistance are given by:

\[ q_b = \frac{s}{1 + \frac{s}{K_p (q_{\text{max}} - q_{\text{res}})}} + q_{\text{res}} \quad (2.12) \]

\[ q_s = \frac{s}{1 + \frac{s}{K_s (q_{s,\text{max}} - q_{s,\text{res}})}} - q_{s,\text{res}} \quad (2.13) \]

where \[ K_p = 18684(N_p)^{0.0065} \left( \frac{P_a}{B_R} \right) \quad (2.14) \]
\[ q_{\text{max}} = 19.75(N_{pt})^{0.36} P_a \]  
\[ q_{\text{res}} = 5.57L\Omega P_a \]  
\[ \Omega = \frac{K_tP}{\sqrt{AE_p}} \]  
\[ K_t = 200(N_{\text{side}})^{0.27} \left( \frac{P_a}{B_R} \right) \]  
\[ q_{s,\text{max}} = 0.224(N_{\text{side}})^{0.29} P_a \]  
\[ q_{s,\text{res}} = q_{\text{res}} A_p / A_s < q_{s,\text{max}} \]

In (2.12) – (2.20), \( P_a \) = reference stress = 100 kPa = 0.1 MPa = 1tsf; \( B_R \) = reference length = 1 m ≈ 40 in.; \( L, P, E_p, \) and \( A \) are the pile length, perimeter, modulus and cross section area, respectively; \( A_p \) and \( A_s \) are the pile base and shaft areas; \( N_{pt} \) is the uncorrected average SPT blow count within the zone from 4 diameters above to 4 diameters below the pile base; and \( N_{\text{side}} \) is the uncorrected average SPT blow count within the layer where shaft resistance is considered.

2.2.5 Neely’s method

Neely (1990, 1991) suggested new empirical relationships between SPT N value and base resistance for expanded-base and auger-cast piles in sands. For expanded-base piles such as a Franki pile, he pointed out that the ultimate base resistance of twice the value for conventional driven piles, as suggested by Meyerhof (1956), would result in overestimated value of base resistance, based on the observation of load tests on 93 expanded-base piles. It was explained that the overestimated base resistance for expanded-base piles by Meyerhof’s (1956) suggestion is due to the ignorance of casing effect. The piles having uncased and compacted concrete shaft create very large
pressures between the shaft and surrounding soil and show greater load capacity than comparable piles having cased shaft.

According to Neely (1990), the ultimate base resistance of expanded-base piles in sands can be given as:

\[ q_b = 0.28 \cdot N \frac{D}{D_b} P_a \leq 2.8 \cdot N \cdot P_a \]  \hspace{1cm} (2.21)

where

- \( q_b \) = base resistance
- \( D \) = embedment depth of the maximum cross section of the base resistance as the sum of the driven length and on-half the base diameter.
- \( D_b \) = diameter of expanded base
- \( N \) = SPT N value
- \( P_a \) = reference stress = 100 kPa = 0.1 MPa = 1 tsf.

The limit value of 2.8N applies whenever the ratio of \( D/D_b \) is greater than 10. For augered, cast-in-place (auger-cast) piles, the base resistance was suggested as follows (Neely 1991):

\[ q_b = 1.9N \cdot P_a \]  \hspace{1cm} (2.22)

The auger-cast piles are different from the conventional drilled shafts in terms of the installation process. The auger-cast piles are installed supporting the side of augered hole by the soil-filled auger without use of temporary casing or bentonite slurry.

From the comparison with the results of loading test on auger-cast piles, it was shown that (2.22) results reasonable agreement with the measured value of base resistance of auger-cast piles. It was also observed that the ratio \( q_b/N \) increases as mean grain size \( (D_{50}) \) increases, and decreases as fines content increases (Neely 1989).
2.3 Estimation of Pile Load Capacity Based on CPT Results

Similarly to what is done in the case of the SPT, the determination of pile load capacity based on CPT results can be expressed as:

\[ q_b = c_b q_c \]  \hspace{1cm} (2.23)

\[ q_s = \sum c_{si} q_{ci} \]  \hspace{1cm} (2.24)

where
- \( q_b \) = base resistance
- \( c_b \) = empirical parameter to convert \( q_c \) to base resistance
- \( q_c \) = cone resistance at the pile base level
- \( q_s \) = shaft resistance
- \( c_{si} \) = empirical parameter to convert \( q_c \) to shaft resistance
- \( q_{ci} \) = representative cone resistance for layer \( i \)

Values of \( c_b \) and \( c_{si} \) have been proposed mostly based on empirical correlations developed between pile load test results and CPT results. Because different authors proposed different values of \( c_b \) and \( c_{si} \), the use of such parameters should be applied under conditions similar to those under which they were determined (Bandini and Salgado 1998). Although most expressions were based on cone resistance \( q_c \), some authors (Price and Wardle 1982, Schmertmann 1978) suggested the use of cone sleeve friction \( f_s \) for the estimation of shaft resistance with the following general expression:

\[ q_s = c_{sf} f_{si} \]  \hspace{1cm} (2.25)

where \( c_{sf} \) is a empirical parameter to convert cone sleeve friction to shaft resistance and \( f_{si} \) is a representative cone sleeve friction for layer \( i \). In this section, some of the methods for the determination of pile load capacity using CPT results are described.
2.3.1 The Dutch method

In the Dutch method (DeRuiter and Beringen 1979), pile base resistance in cohesionless soil is computed from the average cone resistance $q_c$ between the depth of 8B above and 4B below a pile base, where B is the pile diameter. As can be seen in Figure 2.2, the average cone resistance $q_{c1}$ for the layer below the pile base is determined along the path 'abcd', in which 'x' is selected so as to minimize $q_{c1}$. Similarly, the average cone resistance $q_{c2}$ for the layer above the pile base is calculated along the path 'efgh'. The base resistance $q_b$ is then obtained from the average of $q_{c1}$ and $q_{c2}$ as follows:

$$q_b = \frac{w(q_{c1} + q_{c2})}{2} \leq 150 \text{ Pa}$$

where

$q_b$ = base resistance

$w$ = correlation factor

$q_{c1}$ = average cone resistance for the layer below pile base

$q_{c2}$ = average cone resistance for the layer above pile base

$P_a$ = reference stress = 100 kPa = 0.1 MPa = 1 tsf

The values of correlation factor $w$ for several soil conditions are given in Table 2.3.

<table>
<thead>
<tr>
<th>Soil Condition</th>
<th>Values of w</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sand with OCR = 1</td>
<td>1.0</td>
</tr>
<tr>
<td>Very gravelly coarse sand; sand with OCR = 2 to 4</td>
<td>0.67</td>
</tr>
<tr>
<td>Fine gravel; sand with OCR = 6 to 10</td>
<td>0.50</td>
</tr>
</tbody>
</table>
Figure 2.2 Dutch method for determination of base resistance.
2.3.2 Schmertmann’s method

For the estimation of pile base resistance in stiff cohesive soil, Schmertmann (1978) proposed the use of an average cone resistance with multiplying the reducing factor shown in Figure 2.3. The average cone resistance is calculated within a depth between 8B above and 0.7B to 4B below a pile base in the same way as in the Dutch method described previously. He also recommended reducing the base resistance that is obtained from the Dutch method by 60% in case of using mechanical cone for a cohesive soil.

For shaft resistance in sand, the following values of the shaft resistance factor $c_s$ of (2.24) were proposed for different pile types:

\[
\begin{align*}
    c_s &= 0.008 & \text{for open-end steel tube piles} \\
    c_s &= 0.012 & \text{for precast concrete and steel displacement piles} \\
    c_s &= 0.018 & \text{for vibro and cast-in-place displacement piles with steel} \\
    & & \text{driving tube removed, and timber piles}
\end{align*}
\]

According to Schmertmann (1978), the cone sleeve friction $f_s$ can also be used to estimate shaft resistance in cohesive soil. The values of $c_{sf}$ of (2.25) that relates cone sleeve friction to shaft resistance are given by Table 2.4 for displacement piles.

<table>
<thead>
<tr>
<th>$f_s/P_a$</th>
<th>Values of $c_{sf}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Steel Piles</td>
</tr>
<tr>
<td>0.25</td>
<td>0.97</td>
</tr>
<tr>
<td>0.50</td>
<td>0.70</td>
</tr>
<tr>
<td>0.75</td>
<td>0.48</td>
</tr>
<tr>
<td>0.88</td>
<td>0.40</td>
</tr>
<tr>
<td>1.00</td>
<td>0.36</td>
</tr>
<tr>
<td>1.50</td>
<td>0.27</td>
</tr>
<tr>
<td>2.00</td>
<td>0.20</td>
</tr>
</tbody>
</table>

* $P_a$ = reference stress = 100 kPa = 0.1 MPa = 1 tsf.
Figure 2.3 Reduction factor in Schmertmann’s method (1978).
2.3.3 Aoki and Velloso’s method

Based on the load tests and CPT results, Aoki and Velloso (1975) proposed the following relationship for both shaft and base resistance in terms of cone resistance $q_c$:

$$q_b = \frac{1}{F_1} q_c \quad (2.27)$$

$$q_s = \frac{\alpha}{F_2} q_c \quad (2.28)$$

where $\alpha$, $F_1$ and $F_2$ are the same empirical parameters as shown in Table 2.1 and 2.2.

2.3.4 LCPC method

From a number of load tests and CPT results for several pile and soil types, Bustamante and Gianeselli (1982) presented a pile design method using factors related to both pile and soil types. The method presented by them is often referred to as the LCPC method. The basic formula for the LCPC method can be written as:

$$q_b = k_c q_{ca} \quad (2.29)$$

$$q_s = \frac{1}{k_s} q_c \quad (2.30)$$

where

- $k_c =$ base resistance factor;
- $q_{ca} =$ equivalent cone resistance at pile base level;
- $k_s =$ shaft resistance factor;
- $q_c =$ representative cone resistance for the corresponding layer
The values of \( k_c \) and \( k_s \) depend on the nature of soil and its degree of compaction as well as the pile installation method. Tables 2.5 and 2.6 show the values of \( k_s \) and \( k_c \) with different soil and pile types, respectively. According to Bustamante and Gianeselli (1982), the values of \( k_c \) for driven piles cannot be directly applied to H-piles and tubular piles with an open base without proper investigation of full-scale load tests.

The equivalent cone resistance \( q_{ca} \) used in (2.29) represents an arithmetical mean of the cone resistance measured along the distance equal to 1.5B above and below the pile base, where B = pile diameter. The procedure for determining \( q_{ca} \) consists of the following steps (see also Figure 2.4):

1. The curve of the cone resistance \( q_c \) is smoothened in order to eliminate local irregularities of the raw curve.
2. Beginning with the smoothened curve, the mean cone resistance \( q_{cm} \) of smoothened resistance between the distance equal to 1.5B above and below pile base is obtained.
3. The equivalent cone resistance \( q_{ca} \) is calculated as the average after clipping the smoothened curve at 0.7\( q_{cm} \) to 1.3\( q_{cm} \). This clipping is carried out for the values higher than 1.3 \( q_{cm} \) below the pile base, and the values higher than 1.3 \( q_{cm} \) and lower than 0.7\( q_{cm} \) above the pile base.

In the LCPC method, separate factors of safety are applied to the shaft and base resistance. A factor of safety equal to 2 for shaft resistance and 3 for base resistance were considered, so that the carrying load is given by:

\[
Q_w = \frac{Q_L^i}{2} + \frac{Q_L^b}{3} \tag{2.31}
\]

where

- \( Q_w \) = allowable load
- \( Q_L^i \) = limit shaft load
- \( Q_L^b \) = limit base load
Table 2.5  Values of $k_s$ for different soil and pile types.

<table>
<thead>
<tr>
<th>Nature of Soil</th>
<th>$q_c / P_a$</th>
<th>Value of $k_s$</th>
<th>Maximum $q_s / P_a$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>IA</td>
<td>IB</td>
</tr>
<tr>
<td>Soft clay and mud</td>
<td>&lt;10</td>
<td>30</td>
<td>30</td>
</tr>
<tr>
<td>Moderately compact clay</td>
<td>10 to 50</td>
<td>40</td>
<td>80</td>
</tr>
<tr>
<td>Silt and loose sand</td>
<td>≤ 50</td>
<td>60</td>
<td>150</td>
</tr>
<tr>
<td>Compact to stiff clay and compact chalk</td>
<td>&gt; 50</td>
<td>60</td>
<td>120</td>
</tr>
<tr>
<td>Soft chalk</td>
<td>≤ 50</td>
<td>100</td>
<td>120</td>
</tr>
<tr>
<td>Moderately compact sand and gravel</td>
<td>50 to 120</td>
<td>100</td>
<td>200</td>
</tr>
<tr>
<td>Weathered to fragmented chalk</td>
<td>&gt; 50</td>
<td>60</td>
<td>80</td>
</tr>
<tr>
<td>Compact to very compact sand and gravel</td>
<td>&gt; 120</td>
<td>150</td>
<td>300</td>
</tr>
</tbody>
</table>

- $P_a$ = reference stress = 100 kPa = 0.1 MPa = 1 tsf
- Type IA: Plain bored piles, mud bored piles, hollow auger bored piles, cast screwed piles, piers, barrettes, and micropiles installed with low injection pressure.
- Type IB: Bored piles with steel casing and driven cast piles.
- Type IIA: Driven or jacked precast piles and prestressed concrete piles.
- Type IIB: Driven or jacked steel piles.
- Type IIIA: Driven grouted piles and driven rammed piles.
- Type IIIB: High pressure grouted piles with diameter greater than 250 mm and micropiles installed with high injection pressure.
Table 2.6  Values of $k_c$ for different soil and pile types.

<table>
<thead>
<tr>
<th>Nature of Soil</th>
<th>$q_c / P_a$</th>
<th>Value of $k_c$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Group I</td>
</tr>
<tr>
<td>Soft clay and mud</td>
<td>&lt; 10</td>
<td>0.40</td>
</tr>
<tr>
<td>Moderately compact clay</td>
<td>10 to 50</td>
<td>0.35</td>
</tr>
<tr>
<td>Silt and loose sand</td>
<td>$\leq$ 50</td>
<td>0.40</td>
</tr>
<tr>
<td>Compact to stiff clay and compact silt</td>
<td>$&gt; 50$</td>
<td>0.45</td>
</tr>
<tr>
<td>Soft chalk</td>
<td>$\leq$ 50</td>
<td>0.20</td>
</tr>
<tr>
<td>Moderately compact sand and gravel</td>
<td>50 to 120</td>
<td>0.40</td>
</tr>
<tr>
<td>Weathered to fragmented chalk</td>
<td>$&gt; 50$</td>
<td>0.20</td>
</tr>
<tr>
<td>Compact to very compact sand and gravel</td>
<td>120</td>
<td>0.30</td>
</tr>
</tbody>
</table>

- $P_a =$ reference stress $= 100$ kPa $= 0.1$ MPa $= 1$ tsf.
- Group I: Plain bored piles, cased bored piles, mud bored piles, hollow auger bored piles, piers, barrettes, micropiles installed with low injection pressure.
- Group II: Driven cast-in-place piles and piles in Type IIA, IIB, IIIA, and IIIB of Table 2.5.
Figure 2.4 Equivalent cone resistance $q_{ca}$ for LCPC method.
2.4 **Summary**

Pile design methods using in-situ test results can be classified in two categories, indirect and direct methods. Indirect methods require the evaluation of the soil characteristic parameters in the estimation of pile bearing capacity. On the other hand, the direct methods utilize the in-situ test results directly in the estimation of pile bearing capacity.

Direct methods have been based mainly on the standard penetration test (SPT) and the cone penetration test (CPT). In SPT methods, most proposed expressions relate the pile bearing capacity to the SPT blow count N and correlation parameters. These methods include Meyerhof (1976, 1983), Aoki-Velloso (1975), Reese and O’Neil (1989), and Neely (1990, 1991), while Briaud and Tucker (1984) presented a hyperbolic formula for the base and shaft resistance as a function of pile settlement with SPT N value in the evaluation of equation parameters. It should be noticed that, since every method has been developed under different conditions, including soil and pile type, the consideration of such factors must be taken into account for the selection of pile design methods.

The cone penetration test is regarded as a better alternative to the SPT because it reflects well the vertical pile loading mechanism. The widely used CPT methods include the Dutch method (DeRuiter and Beringen 1979), Schmertmann (1978), Aoki-Velloso (1975), and the LCPC method (Bustamante and Gianeselli 1982). Most CPT methods relate the base and shaft resistance to the cone penetration resistance q_c using empirical parameters. The empirical parameters relating pile resistance to q_c are given as a function of soil and pile type. The LCPC method (Bustamante and Gianeselli 1982) provides relatively detailed information regarding soil and pile types. Some authors propose the use of cone sleeve friction f_s for the estimation of shaft resistance, while others propose that it be done on the cone penetration resistance q_c.
CHAPTER 3 METHODS OF INTERPRETATION OF LOAD-SETTLEMENT CURVES

3.1 Introduction

The pile load-settlement curve obtained from a load test provides an important indication of pile load-carrying capacity. In general, however, there is no unique criterion that can clearly define a "failure load" or "bearing capacity" of a pile based on a load-settlement curve. Although several methods for evaluating the "failure" load of a pile have been proposed, they produce a very wide range of results (Horvitz et al. 1981). The approach selected for interpreting a load-settlement curve should account for the characteristics of the load-settlement curve and the soil condition.

For geotechnical structures to perform "properly", it is necessary that the structure satisfy certain fundamental requirements. Whenever a geotechnical structure or a part thereof fails to satisfy a performance criterion, it is said to have reached a "limit state". Potentially, the number of limit state events is infinite. It is therefore necessary that the consideration of limit state events be reduced to a relatively small number of critical events in order to achieve a balance between the needs for safety and economy in design (Bolton 1989).

In this chapter, some of the methods proposed for interpretation of pile load-settlement curves will be reviewed. Additionally two important types of limit states in geotechnical engineering and tolerable settlement for different types of structures are discussed.
3.2 Interpretation Methods

3.2.1 90% and 80% methods

The 90% method was proposed by Brinch Hansen (1963). In the 90% method, the failure point in the load-settlement curve is defined as the load that causes a settlement twice as large as that caused by 90% of that same load (see Figure 3.1). The main goal of this method is to define the failure load as the point from which significant change in the rate of displacement to load increment occurs.

For the application to both the quick and slow maintained pile load tests, Brinch Hansen also suggested 80% method. In this method, the failure load is defined as the load that produces four times the strain caused by 80% of the same load. The failure load \( Q_f \) and corresponding settlement \( s_f \) in the 80% method are defined based on the hyperbolic relationship of a transformed load-settlement curve. As shown in Figure 3.2, the load-settlement curve is plotted in terms of \( \sqrt{s/Q} \) versus \( s \), where \( s = \) settlement and \( Q = \) load. From the relationships between \( 0.8Q_f \) versus \( 0.25s_f \) and \( Q_f \) versus \( s_f \) through a hyperbolic equation in Figure 3.2, the failure load \( Q_f \) and corresponding settlement \( s_f \) can be obtained as:

\[
Q_f = \frac{1}{2\sqrt{C_1C_2}} \tag{3.1}
\]

\[
s_f = \frac{C_2}{C_1} \tag{3.2}
\]

in which \( C_1 \) and \( C_2 \) are the slope and the intersection point of the curve in Figure 3.2, respectively.
Figure 3.1 Definition of failure load in 90% method.

Figure 3.2 Brinch Hansen’s 80% method.
3.2.2 Butler and Hoy’s method

Butler and Hoy (1977) considered the failure load as the load that is the intersection point of two lines tangent to the load-settlement curve at different points. One tangent line is the initial straight line that can be thought of as an elastic compression line. The other line is tangent to a point having a slope of \(0.00125B_R/Q_r\) in the load-settlement curve, where \(B_R = \text{reference length} = 1 \text{ m} \approx 40 \text{ in.} \approx 3.28 \text{ ft}\) and \(Q_r = \text{reference load} = 1 \text{ ton} = 9.8 \text{ kN}\.\)

Usually, the rebound portion of the load-settlement curve is more or less parallel to the true elastic line. Based on this observation, Fellenius (1980) suggested the use of a rebound line as an elastic compression line instead of an initial straight line in Figure 3.3 for determining a “failure” load.

3.2.3 Chin’s method

Based on the assumption that the pile load-settlement curve is approximately hyperbolic, Chin (1970) proposed the following (Figure 3.4):

1. The load-settlement curve is drawn in terms of \(s/Q\) versus \(s\);
2. The failure load \((Q_f)\) or ultimate load \((Q_{ult})\) is defined as \(Q_f = 1/C_1\).

Chin’s failure method is applicable to both the quick and slow maintained load tests. It may, however, provide an unrealistic “failure load” if a constant time increment is not used in the pile load test. Extrapolation for hyperbolic load-settlement curve also requires that the load test be extended sufficiently far.
Figure 3.3 Definition of failure load in Butler and Hoy’s method.

Figure 3.4 Chin’s method for definition of failure load.
3.2.4 Davisson’s method

In Davisson’s method (Davisson 1972), the “failure” load is defined as the load leading to a deformation equal to the summation of the pile elastic compression and a deformation equal to a percentage of the pile diameter. This relationship is given by:

\[ s_f = \frac{QL}{AE} + 0.00381B_R + \frac{1}{3.05} \cdot \frac{B}{B_R} \]  

(3.3)

where
- \( s_f \) = settlement at failure condition
- \( Q \) = applied load
- \( L \) = pile length
- \( E \) = Young’s modulus of pile
- \( A \) = cross sectional area of pile
- \( B_R \) = reference length = 1 m = 40 in. ≈ 3.28 ft
- \( B \) = pile diameter.

As can be seen in Figure 3.5, the elastic compression line of the pile can be obtained from the elastic deformation equation of a column which is given by a equation of \( s_{\text{elastic}} = \frac{QL}{AE} \).

Since this method is, in general, regarded conservative, it appears to work best with data from quick maintained load tests. Due to the dynamic effect, loads obtained from the quick maintained load tests tend to be higher than loads obtained from the slow maintained load tests, sometimes significantly so in clayey material (Fellenius 1975). It may, therefore, lead to overly conservative results when applied to data from the slow maintained load tests resulting in considerable underestimation of a pile failure load (Coduto 1994).
Figure 3.5 Definition of failure load in Davisson’s method.
3.2.5 De Beer’s method

De Beer (1967) defined the “failure load” as the load corresponding to the point of maximum curvature on the load-settlement curve. In this method, the load-settlement curve is plotted using log-log scale as shown in Figure 3.6. The failure load is then determined as the load corresponding to the point at which two straight lines intersect. This method was, however, originally proposed for the slow maintained pile load test.

3.2.6 Permanent set method

In the permanent set method (Horvitz et al. 1981), failure is defined by a certain specified amount of permanent deformation that occurs after full load removal. To determine the failure load with this method, the value of the permanent settlement should be predefined prior to performing a load test.

As shown in Figure 3.7, the failure load corresponding to a specified permanent settlement can be determined by conducting the load-rebound for each applied load. The permanent settlement appearing as a result of unloading is generally associated with plastic soil deformation. This method does not provide one “failure” load, as “failure” in this case depends on what level of permanent settlement the user associates with “failure”.
Figure 3.6 Definition of failure load in De Beer’s method.

Figure 3.7 Definition of failure load in permanent set method.
3.3 Limit States Design

In general, there are two types of limit states in civil engineering design: ultimate and serviceability limit states (Ovesen and Orr 1991). An ultimate limit state is reached when loss of static equilibrium, severe structural damage, or rupture of critical components of the structure occurs. On the other hand, a serviceability limit state is associated with loss of functionality of the structure, typically related to settlement, deformation, utility, appearance, and comfort. Figure 3.8 illustrates the characteristic difference of the load levels between ultimate and serviceability limit states on the load-deformation curve.

In practice, it is usually difficult to determine which type of limit states governs design. It is, therefore, required that both the ultimate and serviceability limit states be investigated.

3.3.1 Limit states design in Eurocode 7

The Eurocodes were established for common use of design codes in European communities. Design criteria in the Eurocodes are based on the limit states concept. Geotechnical design is addressed in Eurocode 7 (1993). Three categories of design problems are defined in order to establish minimum requirements for the extent and quality of geotechnical investigation and calculations as well as construction control checks (Franke 1990, Ovesen and Orr 1991).

The factors taken into consideration for determination of the geotechnical categories in each particular design situation are as follows:

(1) Nature and size of the structure;

(2) Conditions related to the location of the structures (neighboring structures, utilities, vegetation, etc.);
Figure 3.8 Load levels at ultimate and serviceability limit states.
(3) Ground conditions;
(4) Groundwater conditions;
(5) Regional seismicity;
(6) Influence of the environment (hydrology, surface water, subsidence, seasonal changes of moisture, etc.)

Geotechnical Category 1

This includes small and relative simple structures for which it is possible to ensure that the performance criteria will be satisfied on the basis of experience and qualitative geotechnical investigations with no risk for property and life.

Geotechnical Category 2

This category includes structures for which quantitative geotechnical data and analysis are necessary to ensure that the performance criteria will be satisfied, but for which conventional procedures of design and construction may be used. These necessitate the involvement of qualified engineers with relevant experience.

Geotechnical Category 3

This category includes very large or unusual structures, structures involving abnormal risks or unusual or exceptionally difficult ground or loading conditions, or structures in highly seismic areas.

3.3.2 Limit states design for pile foundations

The limit states for a pile foundation under axial loading condition, according to Eurocode 7, are defined as follows (Salgado 1995):
(IA) Bearing capacity failure of single pile, which may correspond to either
(IA-1) a condition of large settlement for unchanged load on the pile;
(IA-2) the crushing or other damage to the pile element itself;
(IB) Collapse or severe damage to the superstructure due to foundation movement;
(II) Loss of functionality or serviceability of the superstructure due to displacement of
the foundations;
(III) Overall stability failure, consisting of the development of a failure mechanism
involving the pile foundation or a part thereof.

Limit state (III) is a possibility for the case where the structure is located close to a
topographic discontinuity area such as a slope, river and retaining wall. In most pile
design situations under axial loading condition, either limit state (II) or limit state (IB)
governs design. This is so because either the serviceability or the stability of the
supported structure would be jeopardized before a given pile developed a classical
bearing capacity failure. The settlement required for the limit state (IB) is usually greater
than that for limit state (II).

It is necessary to establish tolerable settlements $s_{IB}$ and $s_{II}$ according to limit state IB
and II (Franke 1991, 1993). The tolerable settlements $s_{IB}$ and $s_{II}$ can be related to the
corresponding differential settlements $\Delta s_{IB}$ and $\Delta s_{II}$. The angular distortion ($\beta$), defined
as the ratio of differential settlement between two adjacent columns to the distance
between them, is often used to determine tolerable differential settlements.

Figure 3.9 shows the settlements at the foundation level with smaller- and larger-
diameter piles, caused by the superstructure. As can be seen in Figure 3.9, $a_s$ and $a_l$
represent the distances between two adjacent piles, and $\Delta s_{s,\text{max}}$ and $\Delta s_{l,\text{max}}$ are the
maximum differential settlements for the smaller- and larger-diameter piles, respectively.

It must be realized that the distance $a_l$ is generally greater than $a_s$ because larger-
diameter piles are used to carry heavier axial loads with larger spans. The maximum
angular distortion $\beta_{\text{max}}$ for each case can be defined as:
Figure 3.9  Differential settlements for (a) smaller-diameter and (b) larger-diameter piles.
for smaller-diameter piles

$$\beta_{\text{max}} = \frac{\Delta s_{\text{s, max}}}{a_s}$$  \hspace{1cm} (3.4)

for larger-diameter piles

$$\beta_{\text{max}} = \frac{\Delta s_{\text{l, max}}}{a_l}$$  \hspace{1cm} (3.5)

Irrespective of pile size, the maximum angular distortion $\beta_{\text{max}}$ given in (3.4) and (3.5) cannot be larger than the value of the tolerable angular distortion. This implies that the tolerable differential settlement $\Delta s$ usually increases with the span. The total tolerable settlement accordingly, also increases with span. Given that the pile diameters also increase with span, a common way to define limit states for piles is to establish a value of tolerable relative settlement $s_R$ as a ratio of the pile settlement $s$ to the pile diameter $D$. According to Franke (1991), the relative settlement required to cause either loss of functionality or collapse of the superstructure is larger than $s_R = 0.1$. As shown in Figure 3.10, by the time $s_R$ reaches 0.1, the shaft resistance has already been fully mobilized. This implies that the evaluation of the base resistance is a key element in the limit states design of piles.

Figure 3.10  Load-settlement curves with load versus $s_R$ (after Franke 1991).
3.4 Tolerable Settlements for Buildings and Bridge Foundations

3.4.1 Buildings

Tolerable settlements for buildings have been extensively studied by several authors (Skempton and MacDonald 1956, Polshin and Tokar 1957, Burland and Wroth 1974, Wahls 1981). Skempton and MacDonald (1956) presented the values of tolerable movement for building structures based on the observed settlements and damages of 98 buildings. These values of tolerable movement for building structures are still widely accepted as a satisfactory criterion. Because most of the observed damage appeared to be related to distortional deformation, the angular distortion ($\beta$) was selected as the critical index of settlement. From the field data, a limit value of angular distortion equal to $\beta = 1/300$ was suggested for the condition of cracking in panel walls. The corresponding differential settlement for a typical span equal to 20 ft is about 3/4 in. The limit value of angular distortion that caused structural damage in frames was observed to be $\beta = 1/150$.

Based on the examination of the case histories, Skempton and MacDonald (1956) also suggested the correlation between the maximum angular distortion and total settlement. As would be expected, the angular distortion for a given maximum settlement was smaller for a raft than for isolated foundations. It was observed as well that the angular distortion for a given maximum settlement was smaller for a clay than for a sand. These relationships can be given by (3.6) – (3.9) with the numerical factors suggested by Skempton and MacDonald (1956):

\[
\begin{align*}
\rho_{\text{max}} &= 25B_R \cdot \beta_{\text{max}} & \text{for clay with isolated foundations} & (3.6) \\
\rho_{\text{max}} &= 15B_R \cdot \beta_{\text{max}} & \text{for sand with isolated foundations} & (3.7) \\
\rho_{\text{max}} &= 31.25B_R \cdot \beta_{\text{max}} & \text{for clay with raft foundations} & (3.8) \\
\rho_{\text{max}} &= 18.75B_R \cdot \beta_{\text{max}} & \text{for sand with isolated foundations} & (3.9)
\end{align*}
\]
in which \( \rho_{\text{max}} \) = the maximum settlement; \( \beta_{\text{max}} \) = the maximum angular distortion; and \( B_R \) = reference length = 1 m = 40 in. Table 3.1 summarizes the limit values of total settlement, and the correlation factor \( R \) representing the ratio of angular distortion to the total settlement suggested by Skempton and MacDonald (1956). The values of Table 3.1 were based on the tolerable angular distortion of \( \beta = 1/300 \) for the different types of foundations and soils.

Polshin and Tokar (1957) proposed separate tolerable settlement criteria for framed structures and load bearing walls. For framed structures, the limit values of angular distortion are similar to those by Skempton and MacDonald (1956), ranging from 1/500 to 1/200. For load bearing walls, the limit values of angular distortion were suggested in terms of deflection ratio \( \Delta/L \), where \( \Delta \) = maximum relative settlement from two reference points and \( L \) = distance between them, depending on the length to height ratio. Figure 3.11 shows the common settlement criteria including the deflection ratio.

More recently, guidelines for tolerable settlement can be found in Eurocodes. The tolerable movement criteria in the Eurocodes are very similar to those by Skempton and MacDonald (1956) and Polshin and Tokar (1957). Table 3.2 shows the values of tolerable settlements in Eurocode 1 (1993). It can be seen that the values of tolerable angular distortion in the Eurocode 1 appear to be more restrictive than those by Skempton and Macdonald (1956) and Polshin and Tokar (1957).

Table 3.1 Relationship between angular distortion and total settlement
(after Skempton and MacDonald 1956).

<table>
<thead>
<tr>
<th></th>
<th>Isolated foundation</th>
<th>Raft foundation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Clay</td>
<td>( R )</td>
<td>1/25</td>
</tr>
<tr>
<td></td>
<td>( \rho_{\text{max}} )</td>
<td>0.075·( B_R ) (^a)</td>
</tr>
<tr>
<td>Sand</td>
<td>( R )</td>
<td>1/15</td>
</tr>
<tr>
<td></td>
<td>( \rho_{\text{max}} )</td>
<td>0.05·( B_R ) (^a)</td>
</tr>
</tbody>
</table>

\(^a\)\( B_R \) = reference length = 1 m = 40 in. = 3.28 ft.
Table 3.2  Tolerable movement for buildings (after Eurocode 1).

<table>
<thead>
<tr>
<th></th>
<th>Total settlement</th>
<th>Differential settlement between adjacent columns</th>
<th>Angular distortion</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>• Isolated foundations</td>
<td>• Open frame</td>
<td>1/500</td>
</tr>
<tr>
<td></td>
<td>• Raft foundations</td>
<td>• Frames with flexible cladding or finishing</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Frames with rigid cladding or finishing</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td><strong>Isolated foundations</strong></td>
<td><strong>Open frame</strong></td>
</tr>
<tr>
<td></td>
<td></td>
<td>25 mm</td>
<td><strong>Frames with flexible cladding or finishing</strong></td>
</tr>
<tr>
<td></td>
<td></td>
<td>50 mm</td>
<td><strong>Frames with rigid cladding or finishing</strong></td>
</tr>
</tbody>
</table>
\( \Delta s_{AB} \) = differential settlement between A and B
\( \beta_{AB} \) = angular distortion between A and B
\( \rho_{max} \) = total settlement
\( L \) = distance between two reference points (A and E)
\( \omega \) = tilt = rigid body rotation
\( \Delta \) = relative deflection
\( \Delta/L \) = maximum displacement from a straight line connecting two reference points (A and E)

Figure 3.11 Settlement criteria (after Wahls 1994).
3.4.2 Bridges

According to Walkinshaw (1978), the tolerability of the movement should be assessed qualitatively by the agency responsible for each bridge using the following definition: “Movement is not tolerable if damage requires costly maintenance and/or repairs and a more expensive construction to avoid this would have been preferable”. Although said of bridges, this is a concept of general applicability.

As shown in Figure 3.12, the settlement of a bridge can be divided into three components: uniform settlement, tilt (or rotation), and nonuniform settlement. Uniform settlement represents a condition where all foundation elements settle by the same amount. Although it may cause such problems as drainage and clearance at the end of the bridge, it does not cause significant distortion of the bridge superstructure (Stermac 1978, Yokel 1990, Wahls 1990). Uniform tilt or rotation corresponds to a uniform angular distortion. This pattern of settlement is possible only for single-span bridges or bridges with very stiff superstructures. This can also cause some problems with the approach slab, with drainage, and with clearance, while distortion effects are largely absent in the superstructure.

Nonuniform settlement can be categorized by two representative types: regular nonuniform [see Figure 3.12 (c)] and irregular [see Figure 3.12 (d)] nonuniform settlement. If the same amount of total settlement is considered, irregular settlement will cause more significant distortion of the superstructure than regular settlement, mainly due to the greater differential settlement (Xanthakos 1995). Foundation design must control the differential settlement and angular distortion not to exceed the limit state.

Limit values of tolerable settlements for bridge have been proposed by several investigators (Walkinshaw 1978, Bozozuk 1978, Grover 1978, Wahls 1990). No single measure of settlement or distortion, however, can be regarded as a sole indicator of bridge damage, due to the complexity of settlement patterns. Table 3.3 shows criteria of tolerable bridge settlements in terms of the magnitude of the settlement. Although the
Angular distortion $= \frac{\delta}{s}$

Figure 3.12 Components of settlement and angular distortion in bridge for (a) uniform settlement, (b) uniform tilt or rotation, (c) nonuniform regular settlement, and (d) nonuniform irregular settlement (after Duncan and Tan 1991).
values and degree of damage were suggested by different authors, it appears that they are reasonably consistent.

As shown in Table 3.3, the smallest value of approximately 50 mm was suggested by Bozozuk (1978) as not harmful. The upper limit value of approximately 100 mm was suggested as the settlement that may cause some damage, yet remaining tolerable. Later findings by the Federal Highway Administration (FHWA) (Moulton et al. 1985, Moulton 1986) turned out to be reasonably consistent with the tolerable values given in Table 3.3. They found that the movement could be regarded as tolerable for 90% of the cases for which the vertical movement was less than 100 mm and the horizontal movement was less than 50 mm, based on the measurements of bridge movement for 439 abutment and 269 piers.

According to Moulton et al. (1985), the angular distortion gives a rational basis for establishing tolerable movement magnitude for bridges. The values of tolerable angular distortion for bridges, suggested by Moulton et al. (1985), are shown in Table 3.4. The criteria given in Table 3.4 were based on the observation for 56 simple span and 119 continuous span bridges.

Table 3.3 Settlement criteria for bridges expressed in terms of settlement magnitude.

<table>
<thead>
<tr>
<th>Settlement Magnitude (mm)</th>
<th>Basis for recommendation</th>
<th>Recommended by</th>
</tr>
</thead>
<tbody>
<tr>
<td>51</td>
<td>Not harmful</td>
<td>Bozozuk (1978)</td>
</tr>
<tr>
<td>63</td>
<td>Ride quality</td>
<td>Walkinshaw (1978)</td>
</tr>
<tr>
<td>&gt; 63</td>
<td>Structural distress</td>
<td>Walkinshaw (1978)</td>
</tr>
<tr>
<td>102</td>
<td>Ride quality and structural distress</td>
<td>Grover (1978)</td>
</tr>
<tr>
<td>102</td>
<td>Harmful but tolerable</td>
<td>Bozozuk (1978)</td>
</tr>
<tr>
<td>&gt; 102</td>
<td>Usually intolerable</td>
<td>Wahls (1990)</td>
</tr>
</tbody>
</table>
The details considered in Table 3.4 are summarized in Table 3.5. Although most bridges are less complex than buildings, the values shown in Table 3.4 are just slightly higher than those for buildings. The criteria for the tolerable angular distortion in AASHTO (1994) were also established based on the recommendations of Moulton et al. (1985).

Duncan and Tan (1991), however, indicated that the tolerable angular distortion for single-span bridges, equal to $\beta = 1/200$, may be overly conservative based on the review of data used by Moulton et al. (1985). They recommended the use of the tolerable angular distortion for single-span bridge as $\beta = 1/125$.

<table>
<thead>
<tr>
<th>Angular distortion</th>
<th>Basis for recommendation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/250</td>
<td>Tolerable for multi-span bridge</td>
</tr>
<tr>
<td>1/200</td>
<td>Tolerable for single-span bridge</td>
</tr>
</tbody>
</table>

Table 3.5 Data used by Moulton et al. (1985) to establish criteria for angular distortion.

<table>
<thead>
<tr>
<th>Angular distortion</th>
<th>% of 199 continuous bridges for which this amount of angular distortion was considered to be tolerable</th>
<th>% of 56 single-span bridges for which this amount of angular distortion was considered to be tolerable</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.000 - 0.001</td>
<td>100%</td>
<td>98% (100%)</td>
</tr>
<tr>
<td>0.001 - 0.002</td>
<td>97%</td>
<td>98% (100%)</td>
</tr>
<tr>
<td>0.002 - 0.003</td>
<td>97%</td>
<td>98% (100%)</td>
</tr>
<tr>
<td>0.003 - 0.004</td>
<td>96%</td>
<td>98% (100%)</td>
</tr>
<tr>
<td>0.004 - 0.005</td>
<td>92%</td>
<td>98% (100%)</td>
</tr>
<tr>
<td>0.005 - 0.006</td>
<td>88%</td>
<td>96% (98%)</td>
</tr>
<tr>
<td>0.006 - 0.008</td>
<td>85%</td>
<td>93% (95%)</td>
</tr>
</tbody>
</table>
3.5 **Summary**

The concept of a "failure load" of an axially loaded pile is potentially controversial. Most criteria proposed to define a failure load based on pile load-settlement curves associate failure with relatively dramatic changes in settlement increment for a given load increment. These methods include the 90% criteria method (Brinch Hansen 1963), the Butler and Hoy method (1977), the Davisson method (1972) and the permanent set method (Horvitz et al. 1981). The 80% method (Brinch Hansen 1963), Chin's method (1970), and DeBeer's method (1967), on the other hand, define the failure conditions from the transformed load-settlement curve using either hyperbolic relationship or log-log scale. Because different methods may produce widely different values of failure load, the selection of the method for defining the failure condition should account for the characteristic shape of load-settlement curve and soil condition.

The limit state concept has been proposed as a modern design approach, in which the adequate technical quality of foundations and superstructures is considered. Whenever a geotechnical structure or part of a geotechnical structure fails to satisfy one of its performance criteria, it is said to have reached a "limit state". In general, there are two limit states in civil engineering design: ultimate and serviceability limit states. A serviceability limit state is reached when a structure loses functionality, while an ultimate limit state is reached when there is a loss of static equilibrium or severe structural damage. Franke (1991) suggested that if relative settlement $s_R$ is kept below a limit value of 10% neither loss of functionality nor collapse of the superstructure will take place.

The value of tolerable settlement associated with serviceability and ultimate limit states may differ depending on the type, functionality and importance of a given structure. For building structures, values of tolerable settlement have been proposed mainly based on the results proposed by Skempton and MacDonald (1956) and Polshin and Tokar (1957). The results suggested by these two authors are relatively consistent, showing that the angular distortion varies from 1/150 to 1/500.
For bridge structures, several authors proposed limit values for tolerable settlement. No single measure of settlement or angular distortion, however, can be regarded as a sole indicator of bridge damage due to the complex settlement patterns. Bozozuk (1978) suggested the limit value of settlement equal to approximately 100 mm as a tolerable settlement for bridges. This value is similar to that proposed by Federal Highway Administration (Moulton 1986).
CHAPTER 4 MECHANICAL BEHAVIOR OF SAND

4.1 Introduction

The simplest way to describe mechanical behavior of a soil may be linear elasticity. In linear elasticity, the stress-strain relationship is represented by linearity in the absence of the definition of a failure condition. Although linear elasticity is simple, hence convenient to use, it is usually not suitable for typical stress-strain ranges in geotechnical problems. Linear elastic behavior can be observed in soil from the initially unstrained condition up to strains of the order of \(10^{-5}\). After this strain range, the soil usually shows highly non-linear behavior.

In order to represent the non-linear stress-strain behavior of a soil before the failure condition takes place, several soil models have been proposed (Kondner 1963, Duncan and Chang 1970, Hardin and Drnevich 1972, Fahey and Carter 1993, Tatsuoka et al. 1993). The hyperbolic types of soil models are among the most popular soil models for representing the non-linear behavior of a soil. These types of soil models are relatively simple to use, and are reasonably accurate. Fundamentally, the hyperbolic types of soil models are based on the quasi-linear elastic model assuming piece wise linear behavior for each increment of stress and strain.

Stability problems are concerned mostly with the portion of the stress-strain curve where large strains result from modest stress increases. These large deformations are usually associated with plastic response. Description of the plastic condition requires two basic concepts, i.e. definition of a failure criterion and flow rule. A failure criterion defines a stress state in a soil that leads to a failure condition. After the stress state
satisfies a failure criterion, the stress-strain relationship can no longer be defined by either linear elasticity or non-linear elasticity. The flow rule defines the relationship between stress and strain in the plastic state through the plastic potential function. If the plastic potential function is the same as the failure criterion, the flow rule is referred to as associated. Otherwise, it is referred to as a non-associated flow rule. After satisfying the failure criterion, the soil may harden, soften, or remain without any change in stress.

This chapter will start by describing the basic concept of the stress tensor. The fundamental ways to describe the stress-strain relationship of soil, including linear elastic, non-linear elastic and plastic responses are then discussed. Indicial notation, originally adopted by Einstein, will be used in the mathematical treatment of these concepts.

4.2 Stress Tensor and Invariants

As shown in Figure 4.1, there are nine components of stress in a soil element in a Cartesian coordinate system. The stress tensor is a set of these nine components of stress and is denoted by \( \sigma_{ij} \). The stress tensor \( \sigma_{ij} \) can be expressed in matrix notation as follows:

\[
\sigma_{ij} = \begin{bmatrix}
\sigma_{11} & \sigma_{12} & \sigma_{13} \\
\sigma_{21} & \sigma_{22} & \sigma_{23} \\
\sigma_{31} & \sigma_{32} & \sigma_{33}
\end{bmatrix}
\]  

(4.1)

Imposing moment equilibrium on the stress tensor \( \sigma_{ij} \) shown in (4.1), it follows that it is symmetric:

\[
\sigma_{ij} = \sigma_{ji}
\]  

(4.2)

or

\[
\sigma_{12} = \sigma_{21}
\]
Figure 4.1 Nine components of stress tensor in a soil element.
\[ \sigma_{13} = \sigma_{31} \]
\[ \sigma_{23} = \sigma_{32} \]  
(4.3)

The stress tensor \( \sigma_{ij} \) of (4.1) can also be written, using other symbols, as:

\[
\sigma_y = \begin{bmatrix}
\sigma_{11} & \sigma_{12} & \sigma_{13} \\
\sigma_{21} & \sigma_{22} & \sigma_{23} \\
\sigma_{31} & \sigma_{32} & \sigma_{33}
\end{bmatrix} = \begin{bmatrix}
\sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\
\sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\
\sigma_{zx} & \sigma_{zy} & \sigma_{zz}
\end{bmatrix} = \begin{bmatrix}
\sigma_x & \tau_{xy} & \tau_{xz} \\
\tau_{yx} & \sigma_y & \tau_{yz} \\
\tau_{zx} & \tau_{zy} & \sigma_z
\end{bmatrix}
\]  
(4.4)

The diagonal stress components of (4.4) are normal to the planes \( xz, xy, yz \) while the other stress terms are tangential to these planes and are referred to as shear stresses. The stress tensor given by (4.4) is a second-order symmetric tensor. According to the characteristics of a tensor, stresses have the invariant property. The invariants of a tensor represent the quantities that are constant irrespective of the rotation of the coordinate axes. The invariants of a tensor can be obtained from the characteristic equation of the square matrix given by:

\[ \sigma^3 - I_1 \sigma^2 + I_2 \sigma - I_3 = 0 \]  
(4.5)

From the characteristic equation of (4.5) and the stress tensor matrix of (4.4), the quantities of \( I_1, I_2, \) and \( I_3 \) are obtained as follows:

\[ I_1 = \sigma_{ii} = \sigma_{11} + \sigma_{22} + \sigma_{33} = \sigma_x + \sigma_y + \sigma_z \]  
(4.6)

\[ I_2 = \begin{vmatrix}
\sigma_{11} & \sigma_{12} \\
\sigma_{12} & \sigma_{22}
\end{vmatrix} + \begin{vmatrix}
\sigma_{22} & \sigma_{23} \\
\sigma_{23} & \sigma_{33}
\end{vmatrix} + \begin{vmatrix}
\sigma_{11} & \sigma_{13} \\
\sigma_{13} & \sigma_{33}
\end{vmatrix}
\]

\[ = \begin{vmatrix}
\sigma_x & \tau_{yz} \\
\tau_{zy} & \sigma_z
\end{vmatrix} + \begin{vmatrix}
\sigma_y & \tau_{xz} \\
\tau_{zx} & \sigma_x
\end{vmatrix} + \begin{vmatrix}
\sigma_z & \tau_{xy} \\
\tau_{yx} & \sigma_y
\end{vmatrix} \]  
(4.7)
\[ I_3 = \text{determinant of } \sigma_{ij} = |\sigma_{ij}| \quad (4.8) \]

The invariants of the stress tensor shown in (4.6) – (4.8) are often expressed using different formulation:

\[ \bar{I}_1 = I_1 = \sigma_{ii} = \sigma_{11} + \sigma_{22} + \sigma_{33} \quad (4.9) \]
\[ \bar{I}_2 = \frac{1}{2} \sigma_{ij} \sigma_{ij} = \frac{1}{2} (I_1^2 - 2I_2) \quad (4.10) \]
\[ \bar{I}_3 = \frac{1}{3} \sigma_{ki} \sigma_{km} \sigma_{mi} = \frac{1}{3} (I_1^3 - 3I_1I_2 + 3I_3) \quad (4.11) \]

where \( \bar{I}_1, \bar{I}_2, \) and \( \bar{I}_3 \) are the first, second, and third invariants of the stress tensor, respectively while some authors refer \( I_1, I_2, \) and \( I_3 \) to as the first, second, and third invariants of the stress tensor. Another important property of the second-order symmetric tensor is the existence of principal directions related to invariants. For a given stress tensor, this implies that a set of planes, for which only the normal stresses are non-zero, can be found. The directions of these normal stresses acting on such planes are referred to as the principal directions, and the corresponding normal stresses are the principal stresses. In principal planes, the shear stresses are always equal to zero. Using the principal stresses, the stress tensor of (4.1) can be rewritten as:

\[ \sigma_{ij} = \begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{bmatrix} \quad (4.12) \]

where \( \sigma_1, \sigma_2, \) and \( \sigma_3 \) are the principal stresses. In general, the largest principal stress is referred to as the major principal stress while the smallest is called the minor principal stress. The third principal stress is referred to as the intermediate principal stress. The
principal stresses can be determined from the characteristic equation of the stress tensor given in (4.5). Three real roots of the equation correspond to the major, minor, and intermediate principal stresses.

It is possible to decompose the stress tensor of (4.4) into two parts; one called the spherical or the hydrostatic stress tensor, and the other one called the deviatoric stress tensor. In matrix form, the hydrostatic tensor is given by:

$$ p\delta_{ij} = \begin{bmatrix} p & 0 & 0 \\ 0 & p & 0 \\ 0 & 0 & p \end{bmatrix} $$

(4.13)

where

$$ p = \frac{1}{3} \sigma_{kk} = \frac{1}{3} (\sigma_{11} + \sigma_{22} + \sigma_{33}) = \frac{1}{3} I_1 $$

(4.14)

and

$$ \delta_{ij} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} $$

(4.15)

The deviatoric stress tensor $S_{ij}$ is defined by subtracting the hydrostatic stress tensor from the original stress tensor of (4.4). Then the deviatoric stress tensor is written as:

$$ S_{ij} = \sigma_{ij} - p\delta_{ij} $$

(4.16)

with the corresponding matrix form given as:

$$ S = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix} = \begin{bmatrix} \sigma_{11} - p & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} - p & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} - p \end{bmatrix} $$

(4.17)
The deviatoric stress tensor represents physically the shear deviatoric state of stress excluding the hydrostatic state of stress. Because the deviatoric stress tensor is also a second-order symmetric tensor, the invariants of the deviatoric stress tensor can be obtained from the characteristic equation. It should be noticed that the first invariant of the deviatoric stress tensor is zero, as:

\[ S_i = S_{11} + S_{22} + S_{33} \]

\[ = \sigma_{ii} - \frac{1}{3} \sigma_{kk} \delta_{ii} \]

\[ = \sigma_{ii} - \sigma_{mm} \]

\[ = 0 \] (4.18)

The second invariant \( J_2 \) of the deviatoric stress tensor is given by:

\[ J_2 = \frac{1}{2} S_{ij} S_{ij} \]

\[ = \frac{1}{2} \left( S_{11}^2 + S_{12}^2 + S_{13}^2 + S_{21}^2 + S_{22}^2 + S_{23}^2 + S_{31}^2 + S_{32}^2 + S_{33}^2 \right) \]

\[ = \frac{1}{2} \left[ (\sigma_{11} - p)^2 + (\sigma_{22} - p)^2 + (\sigma_{33} - p)^2 + 2S_{12}^2 + 2S_{23}^2 + 2S_{31}^2 \right] \] (4.19)

Since \( S_{12} = \sigma_{12}, S_{23} = \sigma_{23}, \) and \( S_{13} = \sigma_{13}, \) (4.19) can be rewritten as:

\[ J_2 = \frac{1}{2} \left( \sigma_{11}^2 + \sigma_{22}^2 + \sigma_{33}^2 + 2\sigma_{12}^2 + 2\sigma_{23}^2 + 2\sigma_{13}^2 - 3p^2 \right) \] (4.20)

or

\[ J_2 = \frac{1}{6} \left[ (\sigma_{11} - \sigma_{22})^2 + (\sigma_{22} - \sigma_{33})^2 + (\sigma_{11} - \sigma_{33})^2 \right] + \sigma_{12}^2 + \sigma_{23}^2 + \sigma_{13}^2 \] (4.21)

Using the principal stresses,
The third invariant of the deviatoric stress tensor can be expressed as:

\[
J_3 = \frac{1}{3} S_{ij} S_{jm} S_{mi} = \frac{1}{3} (\sigma_{ij} - \frac{\bar{I}_1}{3} \delta_{ij})(\sigma_{jm} - \frac{\bar{I}_1}{3} \delta_{jm})(\sigma_{mi} - \frac{\bar{I}_1}{3} \delta_{mi}) = \bar{I}_3 - \frac{2}{3} \bar{I}_1 \bar{I}_2 + \frac{2}{27} \bar{I}_1^3
\]  

(4.23)

where \(\bar{I}_1, \bar{I}_2,\) and \(\bar{I}_3\) are the first, second and third invariants of the stress tensor.

4.3 Elastic Stress-Strain Relationship

The complete definition of mechanical behavior of a body requires three basic relationships: equilibrium condition, compatibility condition and stress-strain relationship. Figure 4.2 illustrates how these relationships are connected with each other. The equilibrium condition defines the relationship between the internal stresses \(\sigma\) and the external forces including surface tractions \(T\) and body forces \(F\). The compatibility condition defines the relationship between the displacement \(u\) and the strain \(\varepsilon\). The stress-strain relationship is also called constitutive equation because it reflects the internal constitution of the material.

The linear relationship between stress \(\sigma_{ij}\) and strain \(\varepsilon_{kl}\) is referred to as linear elasticity and can be defined by the generalized Hooke's law as follows:
Figure 4.2 Definition of mechanical behavior of a body (after Chen and Han 1988).
where \( i, j, k, \) and \( l = 1, 2, \) and 3; and \( C_{ijkl} \) is the fourth-order elastic moduli tensor. Eq. (4.24) implies that the strain generated by the stress is always recoverable for the condition of eliminating or decreasing the stress. Since both \( \sigma_{ij} \) and \( \varepsilon_{kl} \) are the symmetric tensors, \( C_{ijkl} \) is also symmetric. Using a number of modulus symmetries the number of independent constants can be reduced to 21. The final expression for \( C_{ijkl} \) of (4.24) is then written in matrix form as:

\[
\begin{bmatrix}
\sigma_{11} \\
\sigma_{22} \\
\sigma_{33} \\
\sigma_{12} \\
\sigma_{23} \\
\sigma_{13}
\end{bmatrix} = \begin{bmatrix}
C_{1111} & C_{1122} & C_{1133} & C_{1112} & C_{1123} & C_{1133} \\
C_{2211} & C_{2222} & C_{2233} & C_{2212} & C_{2223} & C_{2233} \\
C_{3311} & C_{3322} & C_{3333} & C_{3312} & C_{3323} & C_{3333} \\
C_{1211} & C_{1222} & C_{1233} & C_{1212} & C_{1223} & C_{1233} \\
C_{2311} & C_{2322} & C_{2333} & C_{2312} & C_{2323} & C_{2333} \\
C_{1311} & C_{1322} & C_{1333} & C_{1312} & C_{1323} & C_{1333}
\end{bmatrix} \begin{bmatrix}
\varepsilon_{11} \\
\varepsilon_{22} \\
\varepsilon_{33} \\
\gamma_{12} \\
\gamma_{23} \\
\gamma_{13}
\end{bmatrix}
\]

(4.25)

where \( \varepsilon_{11}, \varepsilon_{22}, \) and \( \varepsilon_{33} = \) axial strains; \( \gamma_{12}, \gamma_{23}, \) and \( \gamma_{13} = \) shear strains. For an isotropic material, the general form of the fourth-order tensor \( C_{ijkl} \) can be given by:

\[
C_{ijkl} = \lambda \delta_{ij} \delta_{kl} + \mu \delta_{ik} \delta_{jl} + \alpha \delta_{il} \delta_{jk}
\]

(4.26)

where \( \lambda, \mu, \) and \( \alpha \) are scalar constants. Because \( C_{ijkl} \) is a symmetric tensor, the following relationship should be satisfied:

\[
C_{ijkl} = C_{jikl}
\]

(4.27)

This leads to

\[
\lambda \delta_{ij} \delta_{kl} + \mu \delta_{ik} \delta_{jl} + \alpha \delta_{il} \delta_{jk} = \lambda \delta_{ij} \delta_{kl} + \mu \delta_{jk} \delta_{il} + \alpha \delta_{jl} \delta_{ik}
\]

(4.28)
From (4.27) and (4.28), it can be seen that $\mu = \alpha$. Then we are left with:

$$C_{ijkl} = \lambda \delta_{ij} \delta_{kl} + \mu (\delta_{ik} \delta_{jl} + \delta_{il} \delta_{jk}) \quad (4.29)$$

Taking (4.29) into (4.24), we get:

$$\sigma_{ij} = \lambda \delta_{ij} \epsilon_{kl} + \mu (\delta_{ik} \delta_{jl} + \delta_{il} \delta_{jk}) \epsilon_{kl} \quad (4.30)$$
or

$$\sigma_{ij} = \lambda \delta_{ij} \epsilon_{ik} + 2 \mu \epsilon_{ij} \quad (4.31)$$

The two independent material constants $\mu$ and $\lambda$ in (4.31) are referred to as Lame's constants. Following Hooke's law for the three dimensional and isotropic element, the stress-strain relationships can be expressed as follows:

$$\epsilon_{11} = \frac{1}{E} [\sigma_{11} - \nu (\sigma_{22} + \sigma_{33})] \quad (4.32)$$

$$\epsilon_{22} = \frac{1}{E} [\sigma_{22} - \nu (\sigma_{11} + \sigma_{33})] \quad (4.33)$$

$$\epsilon_{33} = \frac{1}{E} [\sigma_{33} - \nu (\sigma_{11} + \sigma_{22})] \quad (4.34)$$

$$\gamma_{12} = \frac{1}{G} \sigma_{12} \quad (4.35)$$

$$\gamma_{23} = \frac{1}{G} \sigma_{23} \quad (4.36)$$

$$\gamma_{13} = \frac{1}{G} \sigma_{13} \quad (4.37)$$

where $E$ and $\nu =$ Young's modulus and Poisson's ratio; $G =$ elastic shear modulus. In (4.35) - (4.37), $\sigma_{12}$, $\sigma_{23}$ and $\sigma_{13}$ represent the shear stresses. Using indicial notation, (4.32) - (4.37) can be rewritten as:
\[ \varepsilon_{ij} = \frac{1 + \nu}{E} \sigma_{ij} - \frac{\nu}{E} \sigma_{kk} \delta_{ij} \] 

(4.38)

or

\[ \sigma_{ij} = \frac{E}{1 + \nu} \varepsilon_{ij} - \frac{\nu E}{(1 + \nu)(1 - 2\nu)} \varepsilon_{kk} \delta_{ij} \] 

(4.39)

Comparing (4.39) and (4.31), Lame's constants \( \mu \) and \( \lambda \) can be obtained as:

\[ \mu = G = \frac{E}{2(1 + \nu)} \] 

(4.40)

\[ \lambda = \frac{\nu E}{(1 + \nu)(1 - 2\nu)} \] 

(4.41)

There is another elastic parameter, called the bulk modulus \( K \), which defines the relationship between the hydrostatic stress and the volumetric strain. For the condition of hydrostatic compression, (4.31) is expressed as:

\[ \sigma_{kk} = 3\lambda \varepsilon_{kk} + 2\mu \varepsilon_{kk} \] 

(4.42)

From the relationship of \( p = \sigma_{11} = \sigma_{22} = \sigma_{33} = \frac{\sigma_{kk}}{3} \),

\[ p = (\lambda + \frac{2}{3} \mu) \varepsilon_{kk} \] 

(4.43)

where \( \varepsilon_{kk} = \varepsilon_{11} + \varepsilon_{22} + \varepsilon_{33} = \) volumetric strain. Then the bulk modulus \( K \) is defined as:

\[ K = \frac{p}{\varepsilon_{kk}} = \lambda + \frac{2}{3} \mu \] 

(4.44)
Using (4.40) and (4.41), the bulk modulus $K$ is rewritten as:

$$K = \frac{E}{3(1 - 2\nu)} \tag{4.45}$$

Table 4.1 summarizes the relationships between the different elastic parameters described so far. Using these elastic parameters, the elastic moduli tensor $C_{ijkl}$ shown in (4.25) can be written in a matrix form as:

$$[C] = \frac{E}{(1 + \nu)(1 - 2\nu)} \begin{bmatrix}
(1-\nu) & \nu & \nu & 0 & 0 & 0 \\
\nu & (1-\nu) & \nu & 0 & 0 & 0 \\
\nu & \nu & (1-\nu) & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{(1-2\nu)}{2} & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{(1-2\nu)}{2} & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{(1-2\nu)}{2}
\end{bmatrix} \tag{4.46}$$

or

$$[C] = \begin{bmatrix}
\frac{K + \frac{4}{3}G}{3} & \frac{K - \frac{2}{3}G}{3} & \frac{K - \frac{2}{3}G}{3} & 0 & 0 & 0 \\
\frac{K - \frac{2}{3}G}{3} & \frac{K + \frac{4}{3}G}{3} & \frac{K - \frac{2}{3}G}{3} & 0 & 0 & 0 \\
\frac{K - \frac{2}{3}G}{3} & \frac{K - \frac{2}{3}G}{3} & \frac{K + \frac{4}{3}G}{3} & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{G}{3} & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{G}{3} & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{G}{3}
\end{bmatrix} \tag{4.47}$$
Table 4.1 Relationship between different elastic modulus.

<table>
<thead>
<tr>
<th>Constant</th>
<th>Basic Pair</th>
<th>( \lambda, \mu = G )</th>
<th>( E, \nu )</th>
<th>( K, \mu )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \lambda )</td>
<td>( \lambda )</td>
<td>( \frac{\nu E}{(1+\nu)(1-2\nu)} )</td>
<td>( \frac{3K-2\mu}{3} )</td>
<td></td>
</tr>
<tr>
<td>( \mu )</td>
<td>( \mu, G )</td>
<td>( \frac{E}{2(1+\nu)} )</td>
<td>( \mu )</td>
<td></td>
</tr>
<tr>
<td>( K )</td>
<td>( \frac{3\lambda + 2\mu}{3} )</td>
<td>( \frac{E}{3(1-2\nu)} )</td>
<td>( K )</td>
<td></td>
</tr>
<tr>
<td>( E )</td>
<td>( \frac{\mu(3\lambda + 2\mu)}{\lambda + \mu} )</td>
<td>( E )</td>
<td>( \frac{9K\mu}{3K + \mu} )</td>
<td></td>
</tr>
<tr>
<td>( \nu )</td>
<td>( \frac{\lambda}{2(\lambda + \mu)} )</td>
<td>( \nu )</td>
<td>( \frac{3K-2\mu}{6K + 2\mu} )</td>
<td></td>
</tr>
</tbody>
</table>
4.4 Elastic Behavior of Soil

4.4.1 Initial elastic modulus at small strain

It is well known that the stress-strain behavior of soil is highly non-linear. The non-linear soil response is in part associated with strains that cannot be fully recovered upon unloading. There is, however, a certain strain range within which the soil behaves as a linear elastic material and the stress-strain relationship is fully recoverable. As can be seen in Figure 4.3, this linear-elastic strain range is usually very narrow with an upper limit of $10^{-5}$ for sands. The elastic modulus for this strain range is referred to as the initial elastic modulus at small strains. It has been observed that the initial shear modulus $G_0$ is a constant, for a given soil condition, regardless of the nature of the loading type, i.e., whether the loading is monotonic or cyclic (Shibuya et al. 1992).

There are a number of ways to evaluate the initial shear modulus for a given soil. Those include in-situ tests, laboratory tests, and empirical equations (Janbu 1963, Hardin and Richart 1963, Yu and Richart 1984, Baldi et al 1989, Viggiani and Atkinson 1995, Salgado et al. 1997c). In most field and laboratory tests, the initial shear modulus is determined by measuring the shear wave velocity based on the following relationship:

$$G_0 = \rho (V_s)^2$$

(4.48)

where $G_0 =$ initial shear modulus; $\rho =$ mass density of the medium through which the shear wave is transmitted; and $V_s =$ shear wave velocity. The cross-hole test, the spectral analysis of surface wave test (SASW) and the seismic cone penetration test are the typical examples of in-situ tests for evaluating initial shear modulus. The seismic cone penetration test is regarded as one of the most economical and rapid in-situ tests for obtaining the initial shear modulus. The test is performed in a similar manner to a down-hole test using the wave generated at the surface (Robertson et al. 1985). Figure 4.4 shows the schematic illustration of the seismic cone penetration test.
Figure 4.3  Non-linear stress-strain behavior of soil.
Figure 4.4  Seismic cone penetration test.
For laboratory testing, the resonant column test and the bender element test are often used. The bender element test has been developed relatively recently, but has been used increasingly since. The test can be easily performed using the same soil sample as in a conventional triaxial test, with the wave generator and receiver (called bender elements) attached at the end caps of a triaxial sample. Because the strains generated by the shear wave do not disturb the initial soil condition, the conventional triaxial test can be performed immediately after the bender element test for the initially assumed soil condition.

The empirical equations for the initial shear modulus are generally expressed as a form of:

\[
\frac{G_o}{P_a} = CF(e) \left( \frac{p'}{P_a} \right)^n
\]  

(4.49)

where \( G_o \) = initial shear modulus; \( P_a \) = reference pressure used for normalization; \( C \) = non-dimensional material constant; \( F(e) \) = function of the void ratio; \( n \) = material constant; \( p' \) = mean effective stress in the same units as \( P_a \). Eq (4.49) indicates the dependence of the initial shear modulus on the degree of compactness of the soil and the magnitude of the confinement. One of the commonly used empirical equations for the initial shear modulus of sand is the one suggested by Hardin and Black (1966) and written as follows:

\[
G_o = C_g \left( \frac{e_g - e_o}{1 + e_o} \right)^2 \left( \frac{P_a}{\sigma_m'} \right)^{(1-n_g)} \]  

(4.50)

where \( C_g \), \( e_g \) and \( n_g \) = material constants that depend only on the nature of the soil; \( e_o \) = initial void ratio; \( P_a \) = reference pressure = 100 kPa = 1 kgf/cm\(^2\); and \( \sigma_m' \) = initial mean effective stress in the same unit as \( P_a \). In the original work by Hardin and Black (1966), the values of \( C_g \), \( e_g \), and \( n_g \) were suggested for round grains (Ottawa sand) and angular
grains (crushed Quartz). After Hardin and Black (1966), several authors presented the values of $C_g$, $e_g$, and $n_g$ for other kinds of sands. Table 4.2 shows the values of $C_g$, $n_g$, and $e_g$ for different sands.

4.4.2 Hyperbolic stress-strain relationship

As discussed in the previous section, the stress-strain behavior of a soil shows highly non-linear relationship from the early stage of loading. The hyperbolic family of soil models has been widely used to represent such non-linear behavior of a soil over a wide range of strains. Since Kondner (1963) first proposed the original hyperbolic equation for the stress-strain relationship, several modifications have been suggested (Duncan and Chang 1970, Hardin and Drnevich 1972, Fahey and Carter 1993, Purzin and Burland 1996). The hyperbolic soil-models are based on the quasi-linear elastic stress-strain relationship assuming piece-wise linear behavior with respect to a stress and strain level.

The conventional hyperbolic equation for stress-strain curve by Kondner (1963) is written for triaxial or plane-strain condition as:

$$\sigma_1 - \sigma_3 = \frac{\varepsilon}{a + b\varepsilon}$$  \hspace{1cm} (4.51)

where $\sigma_1$ and $\sigma_3$ are the major and minor principal stresses; $\varepsilon$ is the axial strain; and $a$ and $b$ are the material constants that characterize the feature of stress-strain curve. As can be seen in Figure 4.5, the constants $a$ and $b$ in the conventional hyperbolic equation by Kondner (1963) correspond to the value of the reciprocals of the initial elastic modulus $E_o$, and of the asymptotic value $(\sigma_1 - \sigma_3)_{ult}$ of deviatoric stress in the hyperbolic stress-strain curve.
Table 4.2 Values of $C_g$, $e_g$, and $n_g$ for different sand type
(after Salgado 1993, Salgado et al. 1999)

<table>
<thead>
<tr>
<th>Sand type</th>
<th>$C_g$</th>
<th>$e_g$</th>
<th>$n_g$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ottawa</td>
<td>612</td>
<td>2.17</td>
<td>0.44</td>
</tr>
<tr>
<td>Ticino</td>
<td>647</td>
<td>2.27</td>
<td>0.43</td>
</tr>
<tr>
<td>Toyoura</td>
<td>900</td>
<td>2.17</td>
<td>0.40</td>
</tr>
<tr>
<td>Hokksund</td>
<td>942</td>
<td>1.96</td>
<td>0.46</td>
</tr>
<tr>
<td>Monterey No.0</td>
<td>326</td>
<td>2.97</td>
<td>0.50</td>
</tr>
</tbody>
</table>
Figure 4.5  Hyperbolic model (a) stress-strain curve and (b) linear representation.
From (4.51) and Figure 4.5, it is easily seen that an infinite amount of strain is required for the stress-strain curve to reach the ultimate stress level \((\sigma_1 - \sigma_3)_{\text{ult}}\). This also implies that the ultimate stress \((\sigma_1 - \sigma_3)_{\text{ult}}\) in the conventional hyperbolic stress-strain relationship is greater than the actual soil strength, which is taken as the maximum stress at failure.

For the purpose of better fitting the hyperbolic relationship to a real soil stress-strain curve, Duncan and Chang (1970) modified the hyperbolic equation by introducing the material constant \(R_f\) into the equation. The factor \(R_f\) is referred to as the failure ratio, relating the ultimate deviatoric stress \((\sigma_1 - \sigma_3)_{\text{ult}}\) of the original hyperbolic stress-strain curve to the actual deviatoric stress of soil at failure \((\sigma_1 - \sigma_3)_f\):

\[
(\sigma_1 - \sigma_3)_f = R_f (\sigma_1 - \sigma_3)_{\text{ult}}
\]

(4.52)

The modified hyperbolic equation is then written as:

\[
(\sigma_1 - \sigma_3) = \frac{\varepsilon}{\frac{1}{E_o} + \frac{\varepsilon \cdot R_f}{(\sigma_1 - \sigma_3)_f}}
\]

(4.53)

where the deviatoric stress at failure \((\sigma_1 - \sigma_3)_f\) is determined from the Mohr-Coulomb failure criterion:

\[
(\sigma_1 - \sigma_3)_f = \frac{2c \cdot \cos \phi + 2\sigma_3 \cdot \sin \phi}{1 - \sin \phi}
\]

(4.54)

where \(c\) and \(\phi\) are the Mohr-Coulomb shear strength parameters. If the factor \(R_f\) is less than 1, the asymptotic value \((\sigma_1 - \sigma_3)_{\text{ult}}\) is \(1/R_f\) times greater than the actual deviatoric stress \((\sigma_1 - \sigma_3)_f\) at failure. The hyperbolic model describes the non-linearity of the stress-
strain relationship without recourse to plasticity concepts. As a result, it can be readily incorporated into numerical analyses using an incremental elastic implementation.

4.4.3 Degradation of Elastic Modulus

The hyperbolic equation given by (4.53) can be rewritten so as to be expressed in terms of shear stress and strain:

\[
\tau = \frac{\gamma}{1 + \frac{\gamma \cdot R_f}{G_o \cdot \tau_{\max}}} \tag{4.55}
\]

where \(G_o\) = initial shear modulus in the very small strain range; and \(\tau_{\max}\) = maximum shear stress at failure. Using the relationship \(\gamma = \tau / G\), (4.55) can be written as:

\[
\tau = \frac{\tau}{G} = \frac{1}{1 + \frac{\tau \cdot R_f}{G_o \cdot \tau_{\max} \cdot G}} \tag{4.56}
\]

Thus,

\[
\frac{G}{G_o} = 1 - R_f \frac{\tau}{\tau_{\max}} \tag{4.57}
\]

where \(G = \) secant shear modulus. Eq. (4.57) describes the degradation of the shear modulus from its initial maximum value \(G_o\) according to the magnitude of shear stress. Duncan and Chang (1970) implicitly described the degradation ratio \(G/G_o\) of elastic stiffness as varying linearly with the stress level \(\tau/\tau_{\max}\) according to (4.57). The degraded
magnitude of the elastic modulus at failure is determined by the value of $R_f$ in (4.57). According to Duncan and Chang (1970), the value of $R_f$ typically lies between 0.75 and 1.0.

4.5 Failure Criterion and Soil Plasticity

4.5.1 Failure criterion

Although the non-linear behavior of soil starts from the very early stages of loading, there is a condition for which relatively dramatic changes of stress, for a given strain increment, can be observed. When the soil is in this condition, it is said to have “failed” and behaves as a plastic material. The failure criterion is used to define the limit stress state at which the material exhibits plastic behavior. In most classical plastic theories, the material is considered as elastic below the failure surface which is defined by a failure criterion. Once the material has reached a failure condition and started exhibiting plastic behavior, Hooke's law, given by (4.24), is no longer valid.

For a given temperature, the failure criterion $F$ can be expressed as a function of stress components as follows:

$$F = F(\sigma_y) = F(\sigma_{11}, \sigma_{22}, \sigma_{33}, \sigma_{12}, \sigma_{23}, \sigma_{13})$$

(4.58)

where $F =$ failure criterion; $\sigma_{ij} =$ six stress components. If the soil is assumed to be isotropic, the soil properties are the same as all directions, thus the material constants are not affected by coordinate transformations. It follows that the failure criterion can be given in terms of principal stresses or stress invariants, for isotropic conditions, as follows:
where \( \sigma_1, \sigma_2, \) and \( \sigma_3 = \) major, intermediate, and minor principal stresses; \( I_1 = \) the first invariant of stress tensor; and \( J_2 \) and \( J_3 = \) the second and third invariants of deviatoric stress tensor. For certain materials, experimental observations show that the influence of hydrostatic pressure on failure of the material is minimal. Based on these observations, the failure criterion is expressed for these materials in a more simplified form as follows:

\[
F = F(I_1, J_2, J_3) \tag{4.60}
\]

In some elastic-plastic models in soil mechanics, a soil remains in the elastic range before the failure criterion is reached and deforms under a constant shear stress in the plastic range. For such materials, referred to as perfectly plastic, the following holds:

\[
F < 0 \quad \text{for the elastic range} \tag{4.62}
\]

and

\[
F = 0 \quad \text{at failure and beyond} \tag{4.63}
\]

For a perfectly plastic material, the condition \( F > 0 \) is physically impossible. Figure 4.6 illustrates the elastic and plastic stress states for the elastic-plastic material. In the figure \( \sigma_{ij} \) and \( d\sigma_{ij} \) represent the current stress and the stress increment at failure, respectively. If the stress state takes place inside the failure surface given by (4.59) or (4.60), it corresponds to the elastic condition of (4.62). As shown in Figure 4.6, when the stress state remains on the failure surface, this is referred to as the loading condition. On the other hand, the case where the stress state drops below the failure surface is called unloading condition.
Figure 4.6 Stress states for elastic-perfectly plastic material.
Considering a small stress increment $d\sigma_{ij}$ from the current stress state $\sigma_{ij}$, the conditions for loading and unloading are given by:

$$f(\sigma_{ij}) = 0 \quad \text{and} \quad df = \frac{\partial f}{\partial \sigma_{ij}} d\sigma_{ij} = 0$$  \hspace{1cm} (4.64)

for loading and

$$f(\sigma_{ij}) = 0 \quad \text{and} \quad df = \frac{\partial f}{\partial \sigma_{ij}} d\sigma_{ij} < 0$$  \hspace{1cm} (4.65)

for unloading.

### 4.5.2 Flow rule and stress hardening

When the stress state of an elastic-plastic material reaches a failure surface, the material starts exhibiting plastic behavior. The total strain increment tensor at plastic state can be given by the sum of the elastic and plastic strain increment:

$$d\varepsilon_{ij} = d\varepsilon_{ij}^e + d\varepsilon_{ij}^p$$  \hspace{1cm} (4.66)

where $d\varepsilon_{ij} = \text{total strain increment;} \ d\varepsilon_{ij}^e$ and $d\varepsilon_{ij}^p = \text{elastic and plastic strain increment.}$

The flow rule defines the relationship between the plastic strain and the current stress state. Based on the flow rule, the magnitude and direction of the plastic strain increment can be determined. The flow rule is given by:

$$d\varepsilon_{ij}^p = \lambda \frac{\partial \Omega}{\partial \sigma_{ij}}$$  \hspace{1cm} (4.67)
where $\lambda$ is a positive scalar factor of proportionality and $\Omega$ is a plastic potential function. When the plastic potential function $\Omega$ is the same as the failure criterion $F$, the flow rule is called an associated flow rule. Otherwise it is referred to as a non-associated flow rule. Eq. (4.67) also indicates that the direction of the plastic strain vector $d\varepsilon_{ij}^p$ is normal to the plastic potential surface defined by a plastic potential function $\Omega$ in the stress space. This condition is referred to as the normality rule.

For the perfectly plastic material, the work done by the stress increment $d\sigma_{ij}$ and strain increment $d\varepsilon_{ij}$ in the plastic state should be equal to zero since no further increment of stress, after the failure condition is reached, is possible. Some materials, however, show positive amount of work done by additional stress and strain increments. This condition is referred to as work hardening or a stable material condition. According to Drucker, the following two conditions should be satisfied for the hardening material:

1. During the application of the added set of forces, the work done by the external force on the changes in displacements it produces is positive.
2. Over the cycle of application and removal of the added set of forces, the new work performed by the external agency on the change in displacements it produces is non-negative.

The first condition can be given by:

$$d\sigma_{ij} \cdot d\varepsilon_{ij} > 0 \quad (4.68)$$

or

$$d\sigma_{ij} (d\varepsilon_{ij}^e + d\varepsilon_{ij}^p) > 0 \quad (4.69)$$

The second condition implies that:

$$d\sigma_{ij} \cdot d\varepsilon_{ij}^p > 0 \quad (4.70)$$
Figure 4.7 Stress-strain behavior for hardening, perfectly plastic and softening material.
If the work done by additional stress and strain increment is negative, the material is referred to as work softening or unstable. Figure 4.7 shows the conditions for the work hardening, perfectly plastic, and work softening materials.

4.5.3 Soil dilatancy and critical state of sand

An important property that characterizes the state of cohesionless granular materials is the relative density. The relative density is defined as the degree of compactness of the soil with respect to its most and least dense states. The relative density $D_R$ is written as:

$$D_R = \frac{e_{\text{max}} - e_o}{e_{\text{max}} - e_{\text{min}}} \times 100\% = \frac{\gamma_d_{\text{max}} - \gamma_d_{\text{min}}}{\gamma_d} \times 100\%$$  \hspace{1cm} (4.71)

where $D_R = $ relative density; $e_{\text{max}}$ and $e_{\text{min}} = $ maximum and minimum void ratios; $e_o = $ initial void ratio; $\gamma_d_{\text{max}}$ and $\gamma_d_{\text{min}} = $ maximum and minimum dry unit weight; $\gamma_d = $ initial dry unit weight. Relative densities equal to $D_R = 100\%$ and $0\%$ represent the densest and loosest conditions of sand, respectively.

The stress-strain behavior and volumetric changes of sand during shearing differ significantly depending on the level of relative density. As can be seen in Figure 4.8, the looser sand shows a stress-strain curve with gradually decreasing curvature until failure, showing hardening behavior. On the other hand, the denser sand has a stress-strain curve with a clear peak.

With respect to the volumetric strain of dense sand, it is usually observed that negative volumetric strains, representing increases of volume, occur following the contractive behavior initially observed. This phenomenon observed in dense sand is referred to as dilatancy. For the same amount of confinement, consequently, the denser the sand, the greater the shear strength, and the higher the peak friction angle. The other
Figure 4.8 Different behavior of dense and loose sand (after Lambe and Whitman 1986).
factor that affects the dilatancy of sand is the confinement. It has been observed that lower confinement produces higher dilatancy in sand.

There is a condition far beyond the peak point of the stress-strain curve, for which no volume change is observed. This stage is referred to as the critical state. As shown in Figure 4.8, the volumetric strain $\varepsilon_{\text{vol}}$, deviatoric stress $\sigma'_1 - \sigma'_3$ and void ratio $e$ at the critical state remain constant. The friction angle at this stage is regarded as a material property that depends only on the nature of the sand, and not on either the initial density or confining stress.

The friction angle at the critical state of sand is given by:

$$\sin \phi_c = \frac{\sigma'_1 - \sigma'_3}{\sigma'_1 + \sigma'_3}$$

(4.72)

where $\phi_c =$ friction angle at the critical state; and $\sigma'_1$ and $\sigma'_3 =$ major and minor principal stresses at the critical state.

In order to quantify the dilatancy of sand, Bolton (1986) proposed the following relationship based on the experimental test results:

$$\phi_p = \phi_c + 0.8 \psi_p$$

(4.73)

in which $\phi_p =$ peak friction angle; $\phi_c =$ friction angle at the critical state; and $\psi_p =$ dilatancy angle. The dilatancy angle $\psi_p$ is given by:

$$\psi_p = 6.25 I_R$$

(4.74)

for plane-strain conditions and

$$\psi_p = 3.75 I_R$$

(4.75)

for triaxial conditions.
The dilatancy index $I_R$ in (4.74) and (4.75) is given by:

$$I_R = I_D \left[ Q + \ln \left( \frac{P_a}{100P_p} \right) \right] - 1$$

(4.76)

where $I_D = \text{relative density as a number between 0 and 1}$; $P_a = \text{reference stress} = 100\text{kPa} = 1\text{kgf/cm}^2$; $P'_p = \text{mean effective stress at peak strength in the same units as } P_a$; and $Q = \text{material constant approximately equal to 10 for clean silica sand}$. From (4.73) through (4.76), it is seen that the dilatancy angle $\psi_p$ depends on both relative density and confinement. The higher the relative density, the higher the dilatancy angle, whereas the higher the confinement, the lower the dialtancy angle. For practical purposes, Bolton (1986) limits the values of the dilatancy angle at:

$$\phi_p - \phi_c < 20^\circ \quad \text{for plane-strain conditions}$$

and

$$\phi_p - \phi_c < 12^\circ \quad \text{for triaxial conditions}$$

(4.77)  (4.78)

4.6 **Summary**

In this chapter, the mechanical behavior of soil was discussed. The linear elastic relationship between stress $\sigma_{ij}$ and strain $\epsilon_{kl}$ can be defined by the generalized Hooke’s law as following equation:

$$\sigma_{ij} = C_{ijkl} \epsilon_{kl}$$

where the elastic moduli tensor $C_{ijkl}$ is expressed in terms of two elastic constants, either the bulk modulus $K$ and shear modulus $G$ or Young’s modulus $E$ and Poisson’s ratio $v$. 
Hyperbolic stress-strain models have been used to represent the non-linear behavior beyond the very small strains for which soil behaves as a linear elastic material. Hyperbolic soil models are based on quasi-linear elasticity and can be expressed in terms of the degraded elastic modulus as a function of the stress or strain level. The degraded elastic modulus can be obtained from the ratio of the current stress level to the maximum stress level and the initial elastic modulus, which is given by the confinement and the relative density for sands.

At large strains, soil exhibits plastic behavior. For the description of plastic behavior of soil, two conditions are required: a failure criterion and a flow rule. The failure criterion defines the stress limit under which soil remains as an elastic material. After this stress limit, soil no longer follows the elastic stress-strain relationship. The flow rule represents the relationship between the stress and strain rate in the plastic range. The magnitude and direction of the plastic strain increment can be determined based on the flow rule.

The peak friction angle of sands can be expressed in terms of the friction angle at the critical state and the dilatancy angle. The friction angle at the critical state is a constant for a given sand, and depends only on the nature of the sands. The dilatancy angle, on the other hand, is a function of confinement and relative density. Bolton (1986) proposed the following relationship in order to quantify the dilatancy of sand:

\[ \phi_p = \phi_c + 0.8\psi_p \]

where \( \phi_p \) = peak friction angle; \( \phi_c \) = friction angle at critical state; and \( \psi_p \) = dilatancy angle determined as a function of density and confining stress. As a result, the failure surface of sands, which is determined by the peak friction angle, is regarded as a non-linear surface.
CHAPTER 5 3-D NON-LINEAR ELASTIC-PLASTIC STRESS-STRAIN MODEL

5.1 Introduction

The non-linear elastic models mentioned in the previous chapter were developed on the basis of a plane-strain condition. Vertical loading of either a pile in the free field or a circular plate within a calibration chamber is an axi-symmetric problem, thus requiring three-dimensional modeling. In order to obtain more realistic results for such problems, therefore, the full description of the stress-strain relationship including non-linear elastic and plastic behavior in three dimensions is necessary.

In this chapter, we discuss first the characteristics of the intrinsic and state soil variables that will be used in the stress-strain model. The full non-linear elastic-plastic soil model for three dimensions is then presented.

5.2 Intrinsic and State Soil Variables

Soil variables used for the description of soil behavior can be classified as either intrinsic or state variables. Intrinsic soil variables do not change with soil state and are only a function of soil particle, mineralogy, shape, and size distribution (Been et al. 1991, Salgado et al. 1997a). This implies that the intrinsic soil variables for a given soil can be uniquely determined irrespective of the stress state, history or initial condition. These variables include the friction angle at the critical state (φ_c), specific gravity (G_s), and
maximum and minimum void ratios \((e_{\text{max}}, e_{\text{min}})\). The parameters \(C_g, e_g\), and \(n_g\) used in (4.50) for the initial shear modulus are also regarded as intrinsic soil variables.

Soil state variables, on the other hand, are determined by the soil state. The soil state represents the physical condition under which the soil exists. The initial void ratio \((e_0)\) or relative density \((D_R)\), and in-situ vertical and horizontal stresses \((\sigma'_v, \sigma'_h)\) are the most important state variables of sands, and control the behavior of the sand (Been et al. 1986). In contrast to the intrinsic variables, the determination of state variables (for example by using laboratory tests) requires undisturbed soil samples. This presents significant difficulties for the determination of soil state variables, particularly for cohesionless soils. The recent use of in-situ testing methods, such as the cone penetration test and the pressuremeter test, however, offers more effective, indirect ways to estimate soil state variables under in-situ conditions.

A useful soil model should be able to relate the stress-strain behavior to soil intrinsic and state variables. With the rapid growth of computing power, the use of more complex soil models have come to be practical for more realistic analyses of geotechnical problems. In the applications of such soil models, the most critical factor for obtaining accurate results may be the reasonably accurate determination of soil and model parameters. The procedures to determine these parameters in general require significant experimental efforts. The more complicated the soil models are, the more laborious the determination of soil and model parameters.

As a simplified approach, the secant modulus is sometimes used to represent the non-linear behavior of soil before a failure condition (Figure 5.1). However, the non-linearity that soil shows is quite complex, varying significantly with the initial density level, stress state, and displacement level of interest. Consequently, the selection of the value of the secant modulus depends on the soil condition and the characteristics of the geotechnical structure being analyzed. It also implies that there is no single value of secant modulus that will produce acceptable results for all possible initial conditions.

It is possible to back-estimate the secant modulus using experimental results as a function of displacement level. This approach, however, is not a fundamental solution,
Figure 5.1  Secant modulus for non-linear stress-strain behavior.
and may result in an inaccurate relationship between modulus, density level and stress state due to the limited number of experimental results. The use of the intrinsic and state soil variables in a non-linear elastic-plastic soil model combined with suitable numerical techniques allows a more fundamental solution to geotechnical problems.

5.3 Modified Hyperbolic Model for Non-Linear Elasticity

The conventional hyperbolic equation of (4.57) implies that the degradation of the elastic modulus with respect to stress level is linear. However, the measured degradation curves of real soils under static or quasi-static loading can be quite different from linear. Figure 5.2 shows measured modulus degradation curves of normally consolidated sand and the degradation line by the conventional hyperbolic equation plotted together. The figure was plotted in terms of normalized shear modulus \( G/G_0 \) and normalized shear stress \( \tau/\tau_{\text{max}} \) where \( G_0 \) and \( \tau_{\text{max}} \) represent the initial shear modulus and the maximum shear stress at failure, respectively. As can be seen, the modulus degradation curve for cyclic loading shows good agreement with the conventional hyperbolic equation. On the other hand, the modulus degradation curve for monotonic loading reveals significantly different response from that of the conventional hyperbolic equation. The measured modulus degradation for monotonic loading from the initial shear modulus \( G_0 \) is initially quite rapid. The rate of the degradation then drops as the normalized shear stress increases.

In order to account for the characteristics of the modulus degradation observed in real soils, Fahey and Carter (1993) proposed a modification of the conventional hyperbolic model. The modification consists of the introduction of a parameter \( g \) into (4.57):

\[
\frac{G_s}{G_0} = 1 - f\left(\frac{\tau}{\tau_{\text{max}}}\right)^g
\] 
(5.1)
Figure 5.2  Modulus degradation relationship for normally consolidated sand (after Teachavorasinskun et al. 1991).
The parameter \( f \) in (5.1) has the same role as \( R_f \) in Duncan and Chang’s hyperbolic equation of (4.53) and (4.55). The parameter \( g \) determines the shape of the degradation curve in terms of the stress level. If \( f \) and \( g \) are both set equal to 1, (5.1) becomes the original hyperbolic equation by Kondner (1963). If \( f \) is equal to 0, (5.1) represent the linear elastic relationship with a constant value of shear modulus irrespective of stress level as:

\[
\frac{G}{G_0} = 1 \quad \text{or} \quad G = G_o
\]  

(5.2)

If \( g \) is set to be equal to 1 with a certain value of \( f \), the Duncan and Chang hyperbolic relationship given by (4.57) results. Figure 5.3 shows typical modulus degradation curves for different values of \( f \) and \( g \). For a typical normally consolidated sand, Fahey and Carter (1993) suggested that the value of \( f = 0.98 \) and \( g = 0.25 \) lead to reasonable agreement with observed modulus degradation curve.

In numerical analysis of soil behavior using non-linear stress-strain models, a tangent modulus with successive incremental procedure rather than a secant modulus with successive iterative procedure is often used (Duncan and Chang 1970, Desai and Christian 1977). According to Duncan and Chang (1970), the advantage of the incremental procedure is that the initial stress state can readily be taken into account, which is very important in geotechnical problems. The accuracy of the incremental procedure can significantly be improved by combining the iterative method for each increment. The tangent shear modulus \( G_t \) can be obtained from the differentiation of (5.1). Using the relationship \( \tau = G \cdot \gamma \), (5.1) is rewritten as:

\[
\frac{\tau}{G_o \gamma} = 1 - f \left( \frac{\tau}{\tau_{\max}} \right)^g
\]  

(5.3)

or

\[
\frac{1}{\gamma} = G_o \tau^{-1} - G_o f \left( \frac{1}{\tau_{\max}} \right)^g (\tau)^{g-1}
\]  

(5.4)
Figure 5.3  Modulus degradation curve for different values of \( f \) and \( g \).
Replacing $1/\gamma$ by $\xi$:

$$\gamma = \xi^{-1} \quad (5.5)$$

Then,

$$\frac{d\gamma}{d\tau} = -\xi^{-2} \frac{d\xi}{d\tau}$$

$$= -\xi^{-2}[-G_o \tau^{-2} - G_o f (g - 1)(\frac{1}{\tau_{\text{max}}})^g (\tau)^{g-2}]$$

$$= \xi^{-2}[G_o \tau^{-2} + G_o f (g - 1)(\frac{1}{\tau_{\text{max}}})^g (\tau)^{g-2}]$$

$$= \frac{G_o \tau^{-2}[1 - f(1-g)(\frac{\tau}{\tau_{\text{max}}})^g]}{[G_o \tau^{-1} - G_o f(\frac{1}{\tau_{\text{max}}})^g (\tau)^{g-1}]^2} \quad (5.6)$$

Thus,

$$\frac{d\gamma}{d\tau} = \frac{1 - f(1-g)(\frac{\tau}{\tau_{\text{max}}})^g}{G_o[1 - f(\frac{\tau}{\tau_{\text{max}}})^g]^2} \quad (5.7)$$

Because $G_i = \frac{d\tau}{d\gamma}$, (5.7) becomes:

$$\frac{1}{G_i} = \frac{G_o}{G_o(G_o-G)^2} \quad (5.8)$$

or

$$\frac{G_i}{G_o} = \frac{(\frac{G_o}{G})^2}{1 - f(1-g)(\frac{\tau}{\tau_{\text{max}}})^g} \quad (5.9)$$
Using (5.9), the tangent modulus $G_t$ corresponding to the current stress level $\tau/\tau_{\text{max}}$ and the current secant shear modulus $G$ can be obtained.

5.4 Non-Linear Elastic Model for Three Dimensions

5.4.1 Modified hyperbolic stress-strain relationship for three dimensions

Application of the hyperbolic model to the analysis of the loading of a natural soil deposit requires the resolution of two issues. Firstly, there is the issue of how to define what shear stresses to use in the stress-strain model. In naturally deposited soil, the stress state is anisotropic with an existing initial shear stress which is determined by a coefficient of lateral earth pressure at-rest, $K_0 = \sigma'_h/\sigma'_v$. Most laboratory studies on stress-strain response, however, have been done on isotropically consolidated samples. In order for the analysis to be more realistic, the initial shear stress existing in a natural soil deposit should be taken into account. A second issue is related to the extension of the non-linear elastic models, which have been based on common laboratory tests and are two-dimensional, to three dimensions.

In the modified hyperbolic model by Fahey and Carter (1993), the degradation of shear modulus is expressed in terms of shear stress level $\tau/\tau_{\text{max}}$ with the model parameters $f$ and $g$. To take account of the initial shear stress due to initial stress anisotropy in the hyperbolic model, the following formulation should be used instead of (5.1):

$$\frac{G_t}{G_o} = 1 - f \left( \frac{\tau - \tau_o}{\tau_{\text{max}} - \tau_o} \right)^g$$  \hspace{1cm} (5.10)

where $\tau_o = \text{initial shear stress due to the initial K}_0 \text{ condition}$. Figure 5.4 illustrates how $\tau_o$, $\tau$ and $\tau_{\text{max}}$ were defined in the present study. In Figure 5.4 (a), it should be noticed
Figure 5.4 Definition of $\tau_0$, $\tau$ and $\tau_{\text{max}}$ for (a) constant and (b) varying confinement.
that $\tau_o$, $\tau$ and $\tau_{\text{max}}$ are all defined at the same confining stress as represented by $(\sigma'_1 + \sigma'_3)/2$. The stress path AB is only possible in a simple shear test with initially anisotropic stress condition. The stress path CF in Figure 5.4 (b), on the other hand, is more representative of the axial loading of a foundation for soils with an initial $K_0$ condition, where failure is reached as a result of increases in both shear and confining stresses. For the stress path CF, the maximum shear stress $\tau_{\text{max}}$ varies along the line DF, depending on the current shear stress on line CF. As an illustration, for point E in Figure 5.4 (b) representing the current shear stress $\tau$, point G (obtained by vertically projecting point E onto the $K_f$ line) represents the maximum shear stress corresponding to the current shear stress $\tau$ and confining stress $\sigma'_m$.

Full description of the stress state under various types of external loading requires a specification of the intermediate principal stress as well as the major and minor principal stresses. Three-dimensional stress conditions may also be represented by the use of stress invariants. The first invariant $I_1$ of the stress tensor given by (4.9) is a measure of confining stress, and the square root of the second invariant $J_2$ of the deviatoric stress tensor given by (4.22) is a measure of shear stress in three dimensions. Using the stress invariants, the hyperbolic relationship of (5.10) may now be rewritten for three dimensions as:

$$\frac{G_s}{G_o} = 1 - f\left(\frac{\sqrt{J_2} - \sqrt{J_{2o}}}{\sqrt{J_{2\text{max}}} - \sqrt{J_{2o}}}\right)^8$$  \hspace{1cm} (5.11)$$

where $\sqrt{J_2}$, $\sqrt{J_{2o}}$ and $\sqrt{J_{2\text{max}}}$ are the 3-D equivalents of $\tau_o$, $\tau$ and $\tau_{\text{max}}$ in (5.10). It is necessary for $J_{2\text{max}}$ to be defined using a three-dimensional failure criterion. The Drucker-Prager failure criterion was selected in this study for that purpose. The Drucker-Prager failure criterion is given by:

$$F = \sqrt{J_2} - \alpha d_1 - \kappa = 0$$  \hspace{1cm} (5.12)$$
where $\alpha$ and $\kappa$ are related to the Mohr-Coulomb strength parameters $c$ and $\phi$ through:

\[
\alpha = \frac{2\sin \phi}{\sqrt{3}(3 - \sin \phi)} \quad (5.13)
\]

and

\[
\kappa = \frac{6c \cdot \cos \phi}{\sqrt{3}(3 - \sin \phi)} \quad (5.14)
\]

In (5.13) and (5.14), $\phi$ and $c$ represent the friction angle and cohesion, respectively. For sands, $c$ and hence $\kappa = 0$. From (5.12), $J_{2\text{max}}$ for sands at a given confinement would be obtained as:

\[
J_{2\text{max}} = \alpha^2 I_1^2 \quad (5.15)
\]

As a result, the 3-D stress plane of $I_1$ versus $\sqrt{J_2}$ is used, instead of the 2-D stress plane of $\sigma$ versus $\tau$, to obtain the stress level associated with (5.11) in a manner similar to that of Figure 5.4.

Considering the stress path CH in Figure 5.4 (b), the shear modulus should increase due to the increase of confinement, while the magnitude of shear stress is kept constant. In order to account for the influence of confinement on shear modulus during loading, (5.11) is modified as follows:

\[
\frac{G_s}{G_o} = [1 - f(f \left( \frac{\sqrt{J_2} - \sqrt{J_{2_0}}}{\sqrt{J_{2\text{max}}} - \sqrt{J_{2_0}}} \right)_{s} (\frac{I_1}{I_{1o}})^{n_g}]
\]

(5.16)

where $I_1$ and $I_{1o}$ are the first invariants of the stress tensor at the current and initial states. The parameter $n_g$ is the same as appears in (4.50). In both equations, $n_g$ represents the dependence of shear modulus on confinement. Use of (5.16), rather than (5.11), permits that the degradation of shear modulus be properly expressed in terms of both shear stress
99

and confining stress levels. Eq. (5.16) reverts to (5.11) for the simple shear case in which
$I_1 = I_{1o}$ at any stress level.

5.4.2 Variation of bulk modulus and Poisson's ratio

As discussed in chapter 4, the stress-strain response of an elastic material is
described by two constants; the bulk modulus $K$ and the shear modulus $G$ are often used.
The elastic stress-strain relationship using the bulk modulus $K$ and the shear modulus $G$
can be written in matrix form as follows:

$$\begin{bmatrix}
\sigma_{11} \\
\sigma_{22} \\
\sigma_{33} \\
\sigma_{12} \\
\sigma_{23} \\
\sigma_{13}
\end{bmatrix} = \begin{bmatrix}
K + \frac{4G}{3} & K - \frac{2G}{3} & K - \frac{2G}{3} & 0 & 0 & 0 \\
K - \frac{2G}{3} & K + \frac{4G}{3} & K - \frac{2G}{3} & 0 & 0 & 0 \\
K - \frac{2G}{3} & K - \frac{2G}{3} & K + \frac{4G}{3} & 0 & 0 & 0 \\
0 & 0 & 0 & G & 0 & 0 \\
0 & 0 & 0 & 0 & G & 0 \\
0 & 0 & 0 & 0 & 0 & G
\end{bmatrix} \begin{bmatrix}
\varepsilon_{11} \\
\varepsilon_{22} \\
\varepsilon_{33} \\
\gamma_{12} \\
\gamma_{23} \\
\gamma_{13}
\end{bmatrix}$$

(5.17)

As the stress state changes in a non-linear elastic model, the elastic parameters $K$
and $G$ of (5.17) also change. Thus, the complete description of a non-linear elastic
relationship requires proper representation of variations in both shear modulus $G$ and bulk
modulus $K$. As described earlier, the shear modulus is given by the non-linear stress-
strain relationship as a function of current stress state. The magnitude of the bulk
modulus depends mainly on the magnitude of the confining stress (Naylor et al. 1981).
The K-G model is one of the ways for accounting for the nonlinear elastic characteristics
before yield (Naylor et al. 1981, Salgado 1993). The basic considerations of the K-G
model are:

1. The magnitude of the shear modulus increases with increasing confining stress.
2. The magnitude of the shear modulus decreases with increasing shear stress.
3. The magnitude of the bulk modulus increases with increasing confining stress.

The first two considerations have already been included in the development of the non-linear elastic relationship of (5.16).

Based on the discussion on the K-G model by Naylor et al. (1981), the tangent bulk modulus \( K_t \) can be represented by the following equation:

\[
K_t = D_s \cdot (\sigma'_m)^n (P_a)^{(1-n_k)}
\]

(5.18)

where \( P_a \) = reference stress = 100 kPa = 1 kgf/cm\(^2\); \( \sigma'_m \) = mean effective stress in the same units as \( P_a \); \( D_s \) = material constant that can be calculated from the initial values of bulk modulus and confining stress; and \( n_k \) can be taken as 0.5 with reasonable accuracy. The values of the initial bulk modulus may be obtained from the initial shear modulus \( G_0 \) and the initial Poisson’s ratio \( v_o \), which can, in most cases, be taken in the 0.1 - 0.15 range.

The expression of Poisson’s ratio \( v \) in terms of the current loading state can be obtained from the relationships of Table 4.1. Based on these relationships, the ratio of the tangent Young’s modulus \( E_t \) to the initial Young’s modulus \( E_o \) can be written as either:

\[
\frac{E_t}{E_o} = \frac{G_t(1+v)}{G_o(1+v_o)}
\]

(5.19)

or

\[
\frac{E_t}{E_o} = \frac{K_t(1-2v)}{K_i(1-2v_o)}
\]

(5.20)

in which \( v_o \) = initial Poisson’s ratio; \( K_i, K_t, G_o, \) and \( G_t \) = initial and current bulk and shear moduli, respectively. From (5.19) and (5.20), the Poisson’s ratio \( v \) at the current stress state can be given by:
\[
\nu_i = \frac{K_i(1+\nu_o) - G_i(1-2\nu_o)}{2K_i(1+\nu_o) + G_i(1-2\nu_o)}
\] (5.21)

In (5.21), it is seen that the Poisson's ratio approaches 0.5 as the shear modulus approaches zero, as expected.

5.4.3 Determination of the parameters f and g

The parameters f and g in (5.16) determine the characteristics of degradation of the elastic modulus. In order to evaluate the values of f and g, a set of triaxial test results performed by Giuseppe (1991) and Vecchia (1991) for Ticino sand was analyzed. Ticino sand has been studied extensively (Salgado 1993, Bellotti et al. 1996) and involved in a number of laboratory plate load and cone penetration tests (Ghionna et al. 1994, Salgado et al. 1997a). The properties of Ticino sand are shown in Table 5.1.

When the modulus degradation relationship is to be determined from triaxial test results, it should be pointed out that the soil specimen under triaxial loading is subjected to continuously increasing confinement. The conventional triaxial test is performed by applying first an isotropic cell pressure \(\sigma'_3\), then a deviatoric axial stress, which is increased (or decreased) until failure. If confinement is defined through the mean effective stress \(\sigma'_m = (\sigma'_1+2\sigma'_3)/3\) rather than \(\sigma'_3\), a triaxial soil specimen can be considered to undergo changes in both confinement and shear stress. As a result, it is not possible to define a single value of maximum shear stress for a triaxial loading condition, due to continuously varying confinement, as discussed in the previous section. Instead, it is possible to identify a different value of maximum shear stress corresponding to each value of current confinement \(\sigma'_m\), as illustrated in Figure 5.4(b).

The use of Young's modulus is more suitable for obtaining the modulus degradation relationship from triaxial tests. Young's modulus in a triaxial test is calculated from the
Table 5.1  Basic properties of Ticino sand (after Ghionna et al. 1994).

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D_{10}$ (mm)</td>
<td>0.36</td>
</tr>
<tr>
<td>$D_{50}$ (mm)</td>
<td>0.54</td>
</tr>
<tr>
<td>Specific gravity ($G_s$)</td>
<td>2.623</td>
</tr>
<tr>
<td>Coefficient of uniformity ($U$)</td>
<td>1.5</td>
</tr>
<tr>
<td>Friction angle at critical state ($\phi_c$)</td>
<td>34.8°</td>
</tr>
<tr>
<td>$e_{\text{max}}$</td>
<td>0.922</td>
</tr>
<tr>
<td>$e_{\text{min}}$</td>
<td>0.573</td>
</tr>
<tr>
<td>$\gamma_{\text{max}}$ (kN/m$^3$)</td>
<td>16.68</td>
</tr>
<tr>
<td>$\gamma_{\text{min}}$ (kN/m$^3$)</td>
<td>13.65</td>
</tr>
<tr>
<td>$C_g$</td>
<td>647</td>
</tr>
<tr>
<td>$n_g$</td>
<td>0.44</td>
</tr>
<tr>
<td>$e_g$</td>
<td>2.27</td>
</tr>
</tbody>
</table>
applied axial stress \((\sigma'_1 - \sigma'_3)\) and the axial strain \(\varepsilon_{\text{axial}}\) with the lateral stress \(\sigma'_3\) remaining constant. The stress level can be determined from the applied axial stress \((\sigma'_1 - \sigma'_3)\) normalized with respect to the unique value of maximum axial stress at failure, \((\sigma'_1 - \sigma'_3)_f\).

Figures 5.5 through 5.17 show the measured and calculated modulus degradation curves obtained from triaxial tests on samples with two different relative density levels (i.e. medium dense sand with relative density equal to approximately 50%, and dense sand with relative density equal to or higher than 90%). As can be seen in the figures, the degradation curves were plotted in terms of normalized Young’s modulus \(E_s/E_o\) versus deviatoric axial stress level \((\sigma'_1 - \sigma'_3)/(\sigma'_1 - \sigma'_3)_f\), where \(E_s\) and \(E_o\) represent the secant and initial Young’s modulus respectively. The initial Young’s modulus \(E_o\) was calculated from the initial shear modulus given by \((4.50)\) and the initial Poisson’s ratio at small strain taken as 0.15.

The values of \(f\) and \(g\) for the medium dense sand shown in Figures 5.5 – 5.8 were found to be in the 0.96 - 0.97 and 0.15 - 0.20 range, respectively. Those for the medium dense sand were in the 0.93 – 0.95 and 0.2 - 0.32 range, respectively. Table 5.2 summarizes the initial elastic modulus and the values of \(f\) and \(g\) for the triaxial soil samples used in Figures 5.5 - 5.17. From Figures 5.5 - 5.17 and Table 5.2, it is observed that the values of \(f\) and \(g\) vary according to the relative density level. As the relative density increases, the value of \(f\) decreases while the value of \(g\) increases. This result indicates that the ratio of the elastic modulus at failure to its initial value is higher for denser than for looser sand, and the rate of degradation of elastic modulus is higher for looser than for denser sand. The results for both cases are in agreement with intuition.

Because the triaxial tests by Giuseppe (1991) and Vecchia (1991) were performed for only two relative density levels, the values of \(f\) and \(g\) shown in Table 5.2 cannot be directly applied to other relative density levels. In order to determine the parameters \(f\) and \(g\) for other relative density levels, the values of \(f\) and \(g\) for \(D_R = 30\%\) and \(70\%\) were extrapolated and interpolated, respectively, from those corresponding to \(D_R = 50\%\) and \(90\%\) which were obtained based on the measured modulus degradation relationship.
Figure 5.5  Modulus degradation curves for $D_R = 51.5\%$ and $\sigma_3 = 400$ kPa with $f = 0.97$ and $g = 0.18$. 
Figure 5.6  Modulus degradation curves for $D_R = 48.8\%$ and $\sigma_3 = 200$ kPa with $f = 0.97$ and $g = 0.15$. 
Figure 5.7  Modulus degradation curves for $D_R = 48.2\%$ and $\sigma_3 = 500$ kPa with $f = 0.97$ and $g = 0.18$. 
Figure 5.8  Modulus degradation curves for $D_R = 50.8\%$ and $\sigma_3 = 110$ kPa with $f = 0.97$ and $g = 0.20$. 
Figure 5.9  Modulus degradation curves for $D_R = 84.6\%$ and $\sigma_3 = 650 \text{ kPa}$ with $f = 0.93$ and $g = 0.20$. 
Figure 5.10  Modulus degradation curves for $D_R = 82.3\%$ and $\sigma_3 = 100$ kPa with $f = 0.95$ and $g = 0.25$. 
Figure 5.11  Modulus degradation curves for $D_R = 88.9\%$ and $\sigma_3 = 200 \text{ kPa}$ with $f = 0.95$ and $g = 0.20$. 
Figure 5.12  Modulus degradation curves for $D_R = 91.1\%$ and $\sigma_3 = 150$ kPa with $f = 0.95$ and $g = 0.20$. 
Figure 5.13  Modulus degradation curves for $D_R = 100\%$ and $\sigma_3 = 200$ kPa with $f = 0.95$ and $g = 0.25$. 
Figure 5.14  Modulus degradation curves for $D_R = 100\%$ and $\sigma_3 = 400$ kPa with $f = 0.95$ and $g = 0.27$. 
Figure 5.15  Modulus degradation curves for $D_R = 100\%$ and $\sigma_3 = 600$ kPa with $f = 0.94$ and $g = 0.32$. 
Figure 5.16  Modulus degradation curves for $D_R = 100\%$ and $\sigma_3 = 800$ kPa with $f = 0.94$ and $g = 0.28$. 
Figure 5.17  Modulus degradation curves for $D_R = 98.6\%$ and $\sigma_3 = 100$ kPa with $f = 0.94$ and $g = 0.20.$
Table 5.3 shows the values of $f$ and $g$ for $D_R = 30\%, 50\%, 70\%$ and $90\%$. These values will be used for the analysis of calibration chamber plate load tests and pile load tests in the next chapters.

**Table 5.2  Values of $f$ and $g$ from triaxial test results.**

<table>
<thead>
<tr>
<th>$D_R$ (%)</th>
<th>$\sigma'_3$ (kPa)</th>
<th>$E_0$ (MPa)</th>
<th>$f$</th>
<th>$g$</th>
</tr>
</thead>
<tbody>
<tr>
<td>51.5</td>
<td>400</td>
<td>367.1</td>
<td>0.97</td>
<td>0.18</td>
</tr>
<tr>
<td>48.8</td>
<td>200</td>
<td>266.0</td>
<td>0.97</td>
<td>0.15</td>
</tr>
<tr>
<td>48.2</td>
<td>500</td>
<td>396.6</td>
<td>0.97</td>
<td>0.18</td>
</tr>
<tr>
<td>50.8</td>
<td>110</td>
<td>207.2</td>
<td>0.97</td>
<td>0.20</td>
</tr>
<tr>
<td>84.6</td>
<td>650</td>
<td>563.6</td>
<td>0.93</td>
<td>0.20</td>
</tr>
<tr>
<td>82.3</td>
<td>100</td>
<td>243.7</td>
<td>0.95</td>
<td>0.25</td>
</tr>
<tr>
<td>88.9</td>
<td>200</td>
<td>344.2</td>
<td>0.95</td>
<td>0.20</td>
</tr>
<tr>
<td>91.1</td>
<td>150</td>
<td>307.8</td>
<td>0.95</td>
<td>0.20</td>
</tr>
<tr>
<td>100</td>
<td>200</td>
<td>369.6</td>
<td>0.95</td>
<td>0.25</td>
</tr>
<tr>
<td>100</td>
<td>400</td>
<td>501.4</td>
<td>0.95</td>
<td>0.27</td>
</tr>
<tr>
<td>100</td>
<td>600</td>
<td>599.3</td>
<td>0.94</td>
<td>0.32</td>
</tr>
<tr>
<td>100</td>
<td>800</td>
<td>680.2</td>
<td>0.94</td>
<td>0.28</td>
</tr>
<tr>
<td>98.6</td>
<td>100</td>
<td>270.5</td>
<td>0.94</td>
<td>0.20</td>
</tr>
</tbody>
</table>

**Table 5.3  Values of $f$ and $g$ for different relative densities.**

<table>
<thead>
<tr>
<th>$D_R$ (%)</th>
<th>$f$</th>
<th>$g$</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>0.98</td>
<td>0.17</td>
</tr>
<tr>
<td>50</td>
<td>0.97</td>
<td>0.20</td>
</tr>
<tr>
<td>70</td>
<td>0.96</td>
<td>0.23</td>
</tr>
<tr>
<td>90</td>
<td>0.95</td>
<td>0.26</td>
</tr>
</tbody>
</table>
5.5 Plastic Stress-Strain Relationship for Three Dimensions

5.5.1 Drucker-Prager failure criterion

In geotechnical engineering, the Mohr-Coulomb failure criterion has been often used to describe plastic soil behavior. The basic assumption in the Mohr-Coulomb failure criterion is the plane-strain condition assuming two-dimensional stress state in which only the major and minor principal stresses are considered. It cannot, therefore, be directly applied to the analysis of three-dimensional problems in which the major and minor principal stresses as well as the intermediate principal stress should be taken into account. In order to describe failure and post-failure soil response for the three-dimensional stress state, the Drucker-Prager plastic model was adopted in this study.

As can be seen in (5.12), the Drucker-Prager failure criterion is expressed in terms of the stress invariants $J_2$ and $I_1$ given by (4.22) and (4.9). When the stress state reaches the failure surface in the Drucker-Prager plastic model, it should satisfy the condition of:

$$F = \sqrt{J_2} - (\alpha I_1 + \kappa) = 0$$  (5.22)

where the parameters $\alpha$ and $\kappa$ are given by (5.13) and (5.14) in terms of the Mohr-Coulomb strength parameter $c$ and $\phi$ obtained from the triaxial conditions. For plane-strain conditions, the parameters $\alpha$ and $\kappa$ are expressed as follows:

$$\alpha = \frac{\tan \phi}{(9 + 12 \tan^2 \phi)^{1/2}}$$  (5.23)

$$\kappa = \frac{3c}{(9 + 12 \tan^2 \phi)^{1/2}}$$  (5.24)
Figure 5.18 Drucker-Prager failure surface (a) in $I_1-\sqrt{J_2}$ plane and (b) in principal stress plane.
Since the stress invariants shown in (5.22) include all components of the three principal stresses, (5.22) can be used for the description of a failure condition under the three-dimensional stress states.

Figure 5.18 shows the failure surface defined by the Drucker-Prager failure criterion in both $I_1$-$\sqrt{J_2}$ plane and principal stress plane. As shown in Figure 5.18, the Drucker-Prager failure surface appears as a straight line in $I_1$-$\sqrt{J_2}$ plane, and a smooth circle in $\pi$-plane. As a result, the Drucker-Prager failure criterion is readily incorporated into a numerical procedure while the Mohr-Coulomb cannot readily be used in numerical computation due to the corners of the hexagon in Figure 5.18 (b).

5.5.2 Non-linear failure surface and flow rule

The failure surface given by the original Drucker-Prager failure criterion is defined as a straight line as shown in Figure 5.18 (a) with the Drucker-Prager friction parameter $\alpha$. As discussed in chapter 4, the peak friction angle $\phi_p$ for sand is not constant, varying with relative density and confining stress. This relationship was expressed in (4.73) – (4.76). Because the $\alpha$ parameter in the Drucker-Prager failure criterion is obtained from the peak friction angle using (5.13), the envelope of the Drucker-Prager failure surface is also non-linear. As a result, the failure surface becomes steeper as the level of confining stress decreases.

It has been widely recognized that the original Drucker-Prager plastic model with an associated flow rule always causes a large negative volumetric increment, i.e., excessive dilational behavior (Desai and Siriwarden 1984, Chen and Baladi 1985). This is illustrated in Figure 5.19. The associated flow rule requires that the plastic strain increment vector $\mathbf{d}\epsilon_{ij}^p$ be perpendicular to the failure surface, such as at point A in Figure 5.19. The plastic strain increment vector $\mathbf{d}\epsilon_{ij}^p$ can be decomposed into the vertical ($\mathbf{d}\epsilon_{ij}^{p_v}$) and horizontal ($\mathbf{d}\epsilon_{ij}^{p_h}$) components as:
\[ d\varepsilon_{ij}^p = d\varepsilon_{ij}^{ps} + d\varepsilon_{ij}^{pv} \]  \hspace{1cm} (5.25)

The vertical component \(d\varepsilon_{ij}^{ps}\) represents the plastic shear-strain. The horizontal component \(d\varepsilon_{ij}^{pv}\) represents the plastic volumetric-strain that has a negative direction for an associated flow rule. This indicates that the plastic flow in the original Drucker-Prager plastic model is always accompanied by an increase in volume.

In order to suppress unrealistic dilation in the plastic state, a non-associated flow rule with the von Mises plastic potential function was adopted (Borja et al. 1989). The von Mises plastic potential function can be given by:

\[ \Omega = \sqrt{J_2} - \kappa \]  \hspace{1cm} (5.26)

where \(\Omega\) is the von Mises plastic potential function and \(J_2\) is the second invariant of the deviatoric stress tensor. Figure 5.20 shows the non-linear failure surface used in this study and plastic strain increment with the non-associated flow rule.

5.5.3 Incremental stress-strain relationship

The most common approach for applying plasticity theory to numerical analysis is the incremental method calculating the tangent stiffness for a plastic condition. According to the concept of perfect plasticity theory, the increment of plastic strain cannot be uniquely determined from the current stress state \(\sigma_{ij}\) and stress increment \(d\sigma_{ij}\). The stress increment \(d\sigma_{ij}\), however, can be obtained from a current stress \(\sigma_{ij}\) and a given plastic strain increment \(d\varepsilon_{ij}^p\). This relationship is refereed to as the consistency condition, which forces the stress state to remain on the failure surface, and given by:

\[ dF = \frac{\partial F}{\partial \sigma_{ij}} d\sigma_{ij} = 0 \]  \hspace{1cm} (5.27)
Figure 5.19 Plastic strain in Drucker-Prager failure criterion with associated flow rule.

Figure 5.20 Non-linear failure surface with non-associated flow rule.
The consistency condition of (5.27) has already been discussed in Chapter 4 and is one of the conditions required for perfectly plastic behavior. From the flow rule given by (4.66) and (4.67), the stress increment $d\sigma_{ij}$ with Hooke’s law can be written as:

$$d\sigma_{ij} = C_{ijkl} (d\varepsilon_{kl} - d\varepsilon_{kl}^p) \quad (5.28)$$

Thus,

$$d\sigma_{ij} = C_{ijkl} d\varepsilon_{kl} - \lambda C_{ijkl} \frac{\partial \Omega}{\partial \sigma_{kl}} \quad (5.29)$$

in which $C_{ijkl} = $ elastic moduli matrix; $d\varepsilon_{kl}$ and $d\varepsilon_{kl}^p = $ total and plastic strain increment; and $\Omega = $ Von-Mises plastic potential function. Plugging (5.29) into (5.27), $\lambda$ is obtained as:

$$\lambda = \frac{\frac{\partial F}{\partial \sigma_{ij}} C_{ijkl} d\varepsilon_{kl}}{\frac{\partial F}{\partial \sigma_{rs}} C_{rs} \frac{\partial \Omega}{\partial \sigma_{tu}}} \quad (5.30)$$

Equation (5.30) indicates that, for a given material with a failure surface $F$ and strain increment $d\varepsilon_{ij}$, the factor $\lambda$ can be uniquely determined through (5.30). Substituting (5.30) into (5.29), the incremental stress-strain relationship is expressed as a form of:

$$d\sigma_{ij} = [C_{ijkl} - \frac{C_{ijkl}}{C_{ijkl}} \frac{\partial F}{\partial \sigma_{rs}} \frac{\partial \Omega}{\partial \sigma_{tu}}] d\varepsilon_{kl} \quad (5.31)$$

All indicies in (5.31) are based on index notation as used in Chapter 4. The stress-strain relationship in a plastic state can then be defined using (5.31). The coefficient tensor of (5.31) represents the elastic-plastic tensor of tangent modulus:
\[ C_{ijkl}^{ep} = C_{ijkl} - \frac{C_{i j m n} \frac{\partial \Omega}{\partial \sigma_{m n}} \frac{\partial F}{\partial \sigma_{p q}} C_{p q k l}}{\frac{\partial F}{\partial \sigma_{r s}} C_{r s t u} \frac{\partial \Omega}{\partial \sigma_{t u}}} \]  

(5.32)

Since the function \( \Omega \) and \( F \) are expressed in terms of the stress invariants, the following relationships can be obtained:

\[ \frac{\partial \Omega}{\partial \sigma_{m n}} = \frac{\partial \Omega}{\partial I_1} \frac{\partial I_1}{\partial \sigma_{m n}} + \frac{\partial \Omega}{\partial I_2} \frac{\partial I_2}{\partial \sigma_{m n}} + \frac{\partial \Omega}{\partial I_3} \frac{\partial I_3}{\partial \sigma_{m n}} \]  

(5.33)

and

\[ \frac{\partial F}{\partial \sigma_{p q}} = \frac{\partial F}{\partial I_1} \frac{\partial I_1}{\partial \sigma_{p q}} + \frac{\partial F}{\partial I_2} \frac{\partial I_2}{\partial \sigma_{p q}} + \frac{\partial F}{\partial I_3} \frac{\partial I_3}{\partial \sigma_{p q}} \]  

(5.34)

in which \( I_1 \) = the first invariant of stress tensor; and \( J_2 \) and \( J_3 \) = the second and third invariants of the deviatoric stress tensor. In (5.33) and (5.34), the derivatives of the stress invariants are written as:

\[ \frac{\partial I_1}{\partial \sigma_{i j}} = \delta_{i j} \]  

(5.35)

\[ \frac{\partial J_2}{\partial \sigma_{i j}} = S_{i j} \]  

(5.36)

\[ \frac{\partial J_3}{\partial \sigma_{i j}} = S_{i k} S_{k j} - \frac{2}{3} J_2 \delta_{i j} \]  

(5.37)
where \( \delta_{ij} = \) Kronecker delta; and \( S_{ij} = \) deviatoric stress tensor. From (5.22) and (5.26), the derivatives of the functions \( \Omega \) and \( F \) with the stress invariants, which appear in (5.33) and (5.34), can be obtained as follows:

\[
\frac{\partial \Omega}{\partial I_1} = 0
\]

(5.38)

\[
\frac{\partial \Omega}{\partial J_2} = \frac{1}{2\sqrt{J_2}}
\]

(5.39)

\[
\frac{\partial \Omega}{\partial J_3} = 0
\]

(5.40)

and

\[
\frac{\partial F}{\partial I_1} = -\alpha
\]

(5.41)

\[
\frac{\partial F}{\partial J_2} = \frac{1}{2\sqrt{J_2}}
\]

(5.42)

\[
\frac{\partial F}{\partial J_3} = 0
\]

(5.43)

Then, (5.33) and (5.34) are rewritten as:

\[
\frac{\partial \Omega}{\partial \sigma_{mn}} = \frac{1}{2\sqrt{J_2}} S_{mn}
\]

(5.44)

and

\[
\frac{\partial F}{\partial \sigma_{pq}} = \frac{1}{2\sqrt{J_2}} S_{pq} - \alpha \delta_{pq}
\]

(5.45)

Now the elastic-plastic tangent modulus matrix of (5.32) can be given more explicitly by:
\[ C_{ijkl}^{ep} = C_{ijkl} - \frac{C_{ijmn} \left( \frac{1}{2\sqrt{J_2}} S_{mn} \right) \left( \frac{1}{2\sqrt{J_2}} S_{pq} - \alpha \delta_{pq} \right) C_{pql}}{\left( \frac{1}{2\sqrt{J_2}} S_{rs} - \alpha \delta_{rs} \right) C_{rstu} \left( \frac{1}{2\sqrt{J_2}} S_{tu} \right)} \]  \tag{5.46}

As a result, the incremental stress-strain relationship for the plastic state is obtained as:

\[ d\sigma_{ij} = C_{ijkl}^{ep} d\varepsilon_{kl} \]  \tag{5.47}

And the plastic strain component can be expressed as:

\[ d\varepsilon_{ij}^{p} = \lambda \left( \frac{S_{ij}}{2\sqrt{J_2}} - \alpha \delta_{ij} \right) \]  \tag{5.48}

in which \( \lambda \) is given by (5.30).

5.6 Summary

The conventional hyperbolic stress-strain relationship is based on plane-strain analyses which may be used for two-dimensional stress states. Vertical loading of either a pile in the free field or a circular plate within a calibration chamber is an axi-symmetric problem. Although the analysis of such problem is much simpler than that of ordinary three-dimensional problems, it still requires the definition of stress states in three dimensions, including vertical, radial, and tangential stresses. In that sense, an axi-symmetric problem is certainly different from the plane-strain condition, which is suitable to treatment using two-dimensional stress-strain relationships and failure criteria.

In this chapter, the non-linear elastic-plastic soil model for three dimensions was presented. This soil model takes advantage of the intrinsic and state soil variables that
can be uniquely determined for a given soil type and condition. For the description of soil behavior before a failure condition is reached, the three-dimensional non-linear elastic model was developed based on the modified hyperbolic stress-strain relationship by Fahey and Carter (1993). This non-linear elastic model represents changes of elastic parameters (the shear modulus $G$ and the bulk modulus $K$) according to the stress level. The stress invariants were used to represent the three-dimensional stress state. The expressions for the variation of shear modulus and bulk modulus according to the stress level are given by:

$$\frac{G}{G_o} = [1 - f \left( \frac{J_2 - J_{2o}}{J_{2 \text{max}} - J_{2o}} \right)^g ] (\frac{I_1}{I_{1o}})^{\gamma},$$

and

$$K = D_s \cdot (\sigma_m')^\alpha \cdot (P_a)^{(1-\alpha)}$$

The parameters $f$ and $g$ in (5.49) were determined from triaxial test results. From the measured modulus degradation curves, it is observed that the values of $f$ and $g$ vary according to the relative density level. As the relative density increases, the value of $f$ decreases while the value of $g$ increases.

In order to describe failure and post-failure soil response for the three-dimensional stress state, the Drucker-Prager plastic model with non-associate flow rule was adopted in this study. The Drucker-Prager failure criterion is given by:

$$F = \sqrt{J_2} - (\alpha I_1 + \kappa) = 0$$

where the parameters $\alpha$ and $\kappa$ are related to the Mohr-Coulomb strength parameter $c$ (which is zero for cohesionless soils) and $\phi$ obtained from the triaxial conditions. Because the peak friction angle for sand is not constant due to the dilatancy, the friction
parameter $\alpha$ was defined in terms of the dilatancy friction angle and the friction angle at critical state.
CHAPTER 6 NUMERICAL ANALYSIS AND EXPERIMENTAL INVESTIGATION OF CALIBRATION CHAMBER TESTS

6.1 Introduction

Calibration chamber tests have been used to investigate both the load-settlement response of the base of non-displacement piles and cone penetration resistance under a variety of conditions (Parkin 1991, Ghionna et al. 1994). Calibration chamber tests can be performed at any desired values of relative density and vertical and horizontal stresses under controlled conditions.

Based on the 3-D non-linear elastic-plastic stress-strain model presented in the previous chapter, calibration chamber tests are modeled and analyzed using the finite element approach. The analytical results will be compared with the measured values of plate resistance in calibration chamber plate load tests. The objective of this chapter is to verify the accuracy of the model predictions for plate resistance and assess calibration chamber size effects on plate resistance values.

6.2 Calibration Chamber Plate Load Tests

6.2.1 Description of test and experimental procedures

A calibration chamber plate load test can be used to simulate the axial loading of a non-displacement pile. In such a test, a cylindrical sand specimen is carefully prepared,
Figure 6.1  Plate load test in calibration chamber.
consolidated to a desired stress state, and tested (Salgado et al. 1998a,b, Lee and Salgado 1999a). In order to simulate the load-settlement response of the base of a non-displacement pile under a variety of conditions, plate load tests were performed within a flexible calibration chamber (Figure 6.1) with a diameter equal to 1.2 m and height equal to 1.5 m. The series of 30 tests was previously described by Ghionna et al. (1994), where details regarding the experimental procedures are discussed at length.

The test samples were formed by pluviation, which was interrupted when the surface of the sample reached mid-height, so that a rigid circular plate having a diameter of 104 mm could be positioned on top of the sample. After the plate connected to an inner rod was positioned, an outer casing with the same diameter as the plate was positioned above the plate and fixed to the calibration chamber. Sample pluviation was then continued outside the outer casing. A sample prepared in this manner simulates the conditions near the base of a non-displacement pile. Since the loading rod is separated from the soil by the outer casing, there is no side resistance along the push rod and the vertical load-displacement response of the plate simulates the load-displacement response of the base of a non-displacement pile.

After preparation is completed, the sample is consolidated under $K_o$-conditions. The relative density of each sample was controlled by the intensity of the sand flow during pluviation, and was accurately determined at the end of each test when the sample was disassembled. Both dense and medium-dense samples were tested. The test results are presented later.

6.2.2 Test material and boundary conditions for calibration chamber plate load tests

The sand used in the calibration chamber plate load tests was Ticino sand, a silica sand, whose properties are shown in Table 5.1. The relative densities used in the tests are divided typically into two different levels, medium dense and dense. The medium dense and dense samples represent the relative-density levels equal to around $D_R = 50\%$
and \( D_R = 90\% \), respectively. Both normally- and over-consolidated conditions were used in the tests. For over-consolidated soil samples, the over-consolidation ratio (OCR) were within the 2.73 – 7.61 range. The desired confining stress levels were established by applying separately the vertical and lateral boundary stresses on the sample surfaces. This allows different \( K_0 \) values. The vertical boundary stresses were in the 62 – 513 kPa range while the lateral boundary stresses were in the 25 – 235 kPa range. Accordingly, the value of \( K_0 \) used in the tests were in the 0.34 – 0.97 range. Table 6.1 shows the soil and stress conditions of the test samples used in calibration chamber plate load tests.

In calibration chamber tests, four different types of boundary conditions can be used. These boundary conditions include BC1, BC2, BC3, and BC4 conditions according to the types of boundary conditions imposed on the lateral, top and bottom chamber surfaces. Table 6.2 and Figure 6.2 illustrate the types of boundary conditions used in the calibration chamber plate load tests and how these boundary conditions differ from each other. Based on the lateral boundary conditions that has indeed significant influence on the plate resistance or cone penetration resistance, the boundary conditions can be classified into two categories, constant-stress and fixed boundary conditions. The constant-stress boundary conditions include BC1 and BC4 conditions while BC2 and BC3 conditions are categorized as the fixed boundary conditions.

It should be noticed that none of these boundary conditions perfectly reproduces the boundary conditions corresponding to real field situations. This is so because the calibration chamber used in a test has a limited size. The difference of results between calibration chamber tests and free-field tests would not exist if calibration chambers with infinite sizes were used. The other limitation is that BC2 and BC3 conditions are not as useful as BC1 and BC4 conditions (Salgado et al. 1998b). This is because achieving a true no-displacement condition for the lateral sample boundary is very difficult due to the characteristics of flexible chambers. The 30 calibration chamber plate load test by Ghionna et al. (1994) include 26 tests under BC1 condition, 1 test under BC2 conditions, 2 tests under BC3 conditions, and 1 test under BC4 conditions. The boundary conditions used in each calibration chamber test are shown in Table 6.1.
Table 6.1  Soil and stress conditions in calibration chamber tests.

<table>
<thead>
<tr>
<th>Test No.</th>
<th>Boundary Condition</th>
<th>D_R (%)</th>
<th>(\sigma'_v) (kPa)</th>
<th>(\sigma'_h) (kPa)</th>
<th>K_0</th>
<th>OCR</th>
</tr>
</thead>
<tbody>
<tr>
<td>300</td>
<td>BC1</td>
<td>51.0</td>
<td>115.0</td>
<td>51.0</td>
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<tr>
<td>301</td>
<td>BC1</td>
<td>92.0</td>
<td>115.0</td>
<td>40.0</td>
<td>0.347</td>
<td>1.00</td>
</tr>
<tr>
<td>302</td>
<td>BC1</td>
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<td>113.0</td>
<td>51.0</td>
<td>0.451</td>
<td>2.73</td>
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<td>313.0</td>
<td>140.0</td>
<td>0.447</td>
<td>1.00</td>
</tr>
<tr>
<td>304</td>
<td>BC1</td>
<td>55.2</td>
<td>214.0</td>
<td>92.0</td>
<td>0.429</td>
<td>1.00</td>
</tr>
<tr>
<td>305</td>
<td>BC1</td>
<td>58.4</td>
<td>512.0</td>
<td>223.0</td>
<td>0.435</td>
<td>1.00</td>
</tr>
<tr>
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<td>BC1</td>
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<td>116.0</td>
<td>76.0</td>
<td>0.655</td>
<td>2.70</td>
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<td>123.0</td>
<td>0.391</td>
<td>1.00</td>
</tr>
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<td>216.0</td>
<td>85.0</td>
<td>0.393</td>
<td>1.00</td>
</tr>
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<td>115.0</td>
<td>77.0</td>
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<td>0.378</td>
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<td>0.458</td>
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<td>62.0</td>
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<td>216.0</td>
<td>92.0</td>
<td>0.425</td>
<td>1.00</td>
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<td>53.0</td>
<td>0.815</td>
<td>6.34</td>
</tr>
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<td>0.793</td>
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<td>321</td>
<td>BC1</td>
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<td>1.00</td>
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<td>322</td>
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<td>314.0</td>
<td>129.0</td>
<td>0.410</td>
<td>1.00</td>
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<td>323</td>
<td>BC1</td>
<td>92.0</td>
<td>66.0</td>
<td>64.0</td>
<td>0.969</td>
<td>6.27</td>
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<td>BC1</td>
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<td>27.0</td>
<td>0.409</td>
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<td>BC3</td>
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<td>47.0</td>
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<td>65.0</td>
<td>26.0</td>
<td>0.400</td>
<td>1.00</td>
</tr>
<tr>
<td>328</td>
<td>BC4</td>
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<td>65.0</td>
<td>26.0</td>
<td>0.400</td>
<td>1.00</td>
</tr>
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<td>90.6</td>
<td>65.0</td>
<td>26.0</td>
<td>0.400</td>
<td>1.00</td>
</tr>
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</table>
Table 6.2  Boundary conditions in calibration chamber tests.

<table>
<thead>
<tr>
<th>Boundary Condition</th>
<th>Lateral Boundary Condition</th>
<th>Top/Bottom Boundary Condition</th>
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</thead>
<tbody>
<tr>
<td>BC1</td>
<td>Constant stress ($\sigma_h = \text{constant}$)</td>
<td>Constant stress ($\sigma_v = \text{constant}$)</td>
</tr>
<tr>
<td>BC2</td>
<td>No displacement ($u_h = 0^a$)</td>
<td>No displacement ($u_v = 0^b$)</td>
</tr>
<tr>
<td>BC3</td>
<td>No displacement ($u_h = 0$)</td>
<td>Constant stress ($\sigma_v = \text{constant}$)</td>
</tr>
<tr>
<td>BC4</td>
<td>Constant stress ($\sigma_h = \text{constant}$)</td>
<td>No displacement ($u_v = 0$)</td>
</tr>
</tbody>
</table>

$^a u_h$ represents horizontal displacement

$^b u_v$ represents vertical displacement
Figure 6.2 Types of boundary conditions in calibration chamber test.
6.3 **Numerical Modeling of Plate Load Tests in Calibration Chambers**

6.3.1 **Program ABAQUS**

Finite-element analysis of calibration chamber plate load tests or pile load tests in the free field requires an accurate model of the stress-strain relationship that can represent the complicated behavior of the soil around the rigid plate or pile base. The description of a stress-strain relationship for soils, therefore, must take into account the non-linear, stress-dependent stress-strain response before failure as well as the post-failure soil behavior with non-linear strength envelope.

The commercial finite-element program ABAQUS (Hibbitt, Karlsson & Sorensen, Inc., Pawtucket, R.I.) was used to model both the calibration chamber plate load tests and the axial loading of non-displacement piles. The program ABAQUS has been used to analyze many engineering problems. It provides a set of material models available for geotechnical problems and several types of elements. The analysis procedure is divided in two parts (Lee et al. 1999):

1. interpretation of model information;
2. implementation of history data.

Model information includes element types, material definitions, and boundary conditions, which are required in the analysis. The history data consist of analysis type and any control parameter necessary for a non-linear solution procedure. In most geotechnical problems, the initial condition is defined as a geostatic equilibrium state.

Instead of using the optional material model provided by the original program, a subroutine was written for the 3-D non-linear stress-strain relationship described in the previous chapter.
6.3.2 Finite element modeling of plate load test

The load-displacement curve for each calibration chamber plate load test was predicted numerically using the finite element method with the non-linear elastic plastic model discussed earlier. Figure 6.3 shows a typical finite element mesh used to model the plate load tests in calibration chamber. This finite element mesh was constructed using the same dimensions as those of the real calibration chamber having a diameter equal to 1.2 m and height equal to 1.5 m. The elements were eight-noded axisymmetric elements, necessary for modeling the three-dimensional loading conditions, with four internal integration points. In ABAQUS, the finite element model with axi-symmetric elements can be plotted using any visual angle desired. The finite element mesh shown in Figure 6.3 was plotted with a visual angle equal to 180° for better visualization.

The finite element model for the calibration chamber plate load test shown in Figure 6.3 consists of two different element groups, i.e. soil and casing elements. The soil elements were modeled using the 3-D non-linear stress-strain relationship written in a specific subroutine. Because the steel casing is relatively rigid compared to the soil, the casing elements were modeled as a linear elastic material with very high stiffness. The steel casing in the finite element model was set to be fixed allowing no displacement throughout the analysis, as that was how the actual tests were performed.

Since the axial load is applied only on the circular rigid-plate located at the middle of the calibration chamber, no significant friction between the soil and casing is expected to occur. However, in order to simulate the calibration chamber plate load test more realistically, thin-layer interface elements were used between the soil and casing elements. The thin-layer interface elements have zero initial thickness, and allow the relative movement of the soil and casing, i.e. slippage. The interface elements follow a Coulomb friction mechanism, where slippage takes place when the tangential stress exceeds a critical shear stress defined by a friction angle and normal stress acting on the surface between the casing and soil.
Figure 6.3  Finite element model for calibration chamber plate load test.
As discussed in the previous section, four different types of boundary conditions were used in the calibration chamber plate load tests. In the numerical modeling, the boundary conditions imposed on the finite element mesh were those actually imposed on the samples in the actual tests.

The values of the parameters \( f \) and \( g \), which define the degradation of the elastic modulus, were \( f = 0.97 \) and \( g = 0.20 \) for \( D_R = 50\% \) level, and \( f = 0.95 \) and \( g = 0.26 \) for \( D_R = 90\% \) level, selected with basis on the values given in Table 5.3. The only other soil parameters required in the analyses were the intrinsic and state variables, which are given in Tables 5.1 and 6.1 (\( C_g, n_g, e_g, \phi_c, e_{\text{max}}, e_{\text{min}}, e_o, \sigma'_v, \sigma'_h \)).

After defining all the required geometry information and material properties, the analysis is performed first by checking the initial geostatic equilibrium condition for a given boundary condition and stress state.

6.3.3 Predicted and measured plate resistance

Figures 6.4 - 6.7 show the graphical results of the numerical analyses for the calibration chamber plate load tests. All these results were obtained at the relative settlement (defined as the ratio of the vertical settlement to the plate or pile base diameter) equal to \( s/B = 10\% \). The displacement of the deformed finite element mesh shown in Figure 6.4 was exaggerated by a magnification factor equal to 5 for better visualization.

Figures 6.5 - 6.7 represent the vertical stress, vertical displacement, and lateral displacement distribution, respectively. From the distribution of stress and displacement, it can be seen that significant stress concentration and shear stress increases are observed near the plate edge. Consequently, it is also expected that the most significant reduction of shear modulus would occur near the plate edge.

Figure 6.8 shows the variation of secant shear modulus with horizontal and vertical distances from the plate for three different settlement levels. As can be seen in Figure
6.8 (a), the reduction of secant shear modulus at the level of the plate base is most severe near the plate edge due to high shear stresses there. The shear modulus of the soil immediately below the center of the plate undergoes rather slow degradation, as the stress state is dominated by the increase of confinement rather than the increase of shear stress. Such a slow reduction of shear modulus underneath the plate is in agreement with the observation of the formation of an "elastic core" beneath the base of axially loaded piles (BCP 1971; Salgado et al. 1997a). Figure 6.8 (b) shows the degradation of the secant shear modulus with depth along the plate axis. At the early stages of loading (e.g. curve 1), the shear modulus initially decreases with depth down to a depth of around 10 cm (equal to the plate diameter), and then increases again. For higher loads, similar reductions can be seen down to the depth of 10 cm below the plate, but this time no increase of shear modulus is observed below that depth, due to the expansion of the shear zones (curve 2, 3). Similar observations of modulus reduction with depth from the analyses of footings were noted by Fahey et al. (1994).

Figures 6.9 – 6.16 show the measured and predicted load-settlement curves of the calibration chamber plate load tests. The curves extend up to a settlement of approximately 10 mm, corresponding to a relative settlement level equal to \( s/B = 10\% \). The predicted load-settlement curves obtained by the finite element analyses compare favorably with the measured responses in most of the cases.

Figure 6.17 shows measured versus predicted plate unit load at the relative settlement levels equal to \( s/B = 5 \) and \( 10\% \) for all calibration chamber tests. Overall agreement is very satisfactory, showing a maximum error of about 20\%. There is a slight underestimation of plate resistance for dense sand. This may be due to differences between the dilational response of sand under triaxial loading conditions (from which \( f \) and \( g \) were obtained) and plate load test loading conditions.
Figure 6.4  Deformed finite element mesh of calibration chamber plate load test with $D_R = 55.2\%$, $\sigma'_v = 62.0$ kPa, and $\sigma'_h = 24.4$ kPa at $s/B = 10\%$. 

Figure 6.5  Vertical stress distribution in calibration chamber plate load test with $D_R = 55.2\%$, $\sigma'_v = 62.0$ kPa, and $\sigma'_h = 24.4$ kPa at $s/B = 10\%$. 
Figure 6.6  Vertical displacement distribution in calibration chamber plate load test with $D_R = 55.2\%$, $\sigma'_v = 62.0$ kPa, and $\sigma'_h = 24.4$ kPa at $s/B = 10\%$. 
Figure 6.7  Horizontal displacement distribution in calibration chamber plate load test with $D_R = 55.2\%$, $\sigma'_v = 62.0$ kPa, and $\sigma'_h = 24.4$ kPa at $s/B = 10\%$. 
Figure 6.8 Variation of shear modulus.
Figure 6.9 Load-settlement curves for calibration chamber plate load tests (Test No. 300, 301 and 302).
Figure 6.10 Load-settlement curves for calibration chamber plate load tests (Test No. 303, 304, and 306).
Figure 6.11 Load-settlement curves for calibration chamber plate load tests (Test No. 307, 308 and 309).
Figure 6.12 Load-settlement curves for calibration chamber plate load tests (Test No. 310, 311 and 312).
Figure 6.13 Load-settlement curves for calibration chamber plate load tests (Test No. 313, 314 and 317).
Figure 6.14 Load-settlement curves for calibration chamber plate load tests (Test No. 321, 322 and 323).
Figure 6.15 Load-settlement curves for calibration chamber plate load tests (Test No. 324, 325 and 326).
Figure 6.16 Load-settlement curves for calibration chamber plate load tests (Test No. 327, 328 and 329).
Figure 6.17 Measured and predicted plate unit loads in calibration chamber tests.
6.4 Calibration Chamber Size Effects on Plate Load Test Results

6.4.1 Definition of size effect

Because of their finite size, calibration chambers do not perfectly reproduce free-field conditions. As a result, measurements made in a calibration chamber sand specimen are different from what would be observed in the field for the same relative density and stress state. These difficulties tend to be more pronounced, the smaller the chamber; hence the term size effect.

Calibration chamber size effects have been extensively studied in connection with cone penetration testing, where large deformations take place (e.g., Schnaid and Houlsby 1991; Mayne and Kulhawy 1991; Salgado et al. 1998a), but not with respect to plate load testing. It has been determined that size effects in calibration chambers are more significant in dense sands than in loose sands. In plate load testing, the displacements induced in the sand specimen are much less than those caused by cone penetration. On the other hand, the plate diameter used in the study (100 mm) is much larger than typical cone diameters (most commonly 35.7 mm).

For the results of calibration chamber tests to be effectively applied to pile design, the possible size effect should be properly addressed. If size effects are well understood, calibration chambers can be used to experimentally assess pile base resistance under controlled conditions.

6.4.2 Investigation of size effects for different boundary conditions

In order to investigate size effects in calibration chamber plate load tests, numerical analyses of both full-scale non-displacement piles and calibration chamber plate tests were performed. For studying the loading of non-displacement piles, three 60-cm diameter piles with lengths L = 5 m, 10 m and 20 m were analyzed. The corresponding
ratios \( L/B \) of the length \( L \) to the diameter \( B \) were equal to approximately 8, 16 and 33, respectively. The piles were positioned within a granular soil deposit with assumed values of \( D_R = 30, 50, 70 \) and \( 90\% \). The initial vertical and lateral effective stresses at the pile base level for each pile length are as follows:

1. \( \sigma'_v = 100 \) kPa and \( \sigma'_h = 43 \) kPa for the 5-m pile
2. \( \sigma'_v = 200 \) kPa and \( \sigma'_h = 86 \) kPa for the 10-m pile
3. \( \sigma'_v = 400 \) kPa and \( \sigma'_h = 172 \) kPa for the 20-m pile.

All of these states correspond to a normally consolidated condition. Plate load tests in a calibration chamber were also simulated numerically for these same values of densities and stresses. Comparison of the base unit load in the full-scale pile load tests with plate unit load in the calibration chamber tests for the same relative settlement level provides the basis for conclusions regarding size effects. The sand adopted was Ticino sand, whose properties were discussed previously.

The comparison of pile load tests and chamber tests were made for all boundary conditions that were defined in Table 6.2 and Figure 6.2. Tables 6.3 - 6.6 and Fig. 6.18 show pile base unit loads versus plate unit loads in calibration chamber tests with BC1, BC2, BC3, and BC4 conditions for the relative settlement \( s/B \) equal to 10\%. The size effect in Tables 6.3 - 6.6 was defined as a ratio of the calibration chamber plate resistance to the free-field pile base resistance.

In Figure 6.18, the different points for the same pile length represent the results for relative densities \( D_R = 30\%, 50\%, 70\% \), and \( 90\% \). As can be seen in Figure 6.18, the results for BC4 conditions were similar to those for BC1 conditions, and the results for BC3 conditions were similar to those for BC2. This illustrates that the lateral boundary condition has an overwhelming influence on plate unit load measured in a calibration chamber.
Table 6.3  Size effect in calibration chamber test for BC1 condition.

<table>
<thead>
<tr>
<th>Pile Load Test</th>
<th>Calibration Chamber Test</th>
<th>Size Effect (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pile length (m)</td>
<td>D_R (%)</td>
<td>q_b (kPa) (s/B = 10%)</td>
</tr>
<tr>
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<td>1516</td>
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<tr>
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<td>50</td>
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Table 6.4  Size effect in calibration chamber test for BC2 condition.

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<th>Size Effect (%)</th>
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<td>D_R (%)</td>
<td>q_b (kPa) (s/B = 10%)</td>
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<tr>
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Table 6.5 Size effect in calibration chamber test for BC3 condition.

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<th>Size Effect (%)</th>
</tr>
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<td>D_R (%)</td>
<td>q_b (kPa) (s/B = 10%)</td>
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<tr>
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Table 6.6 Size effect in calibration chamber test for BC4 condition.

<table>
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<th>Pile Load Test</th>
<th>Calibration Chamber Test</th>
<th>Size Effect (%)</th>
</tr>
</thead>
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<td>Pile length (m)</td>
<td>D_R (%)</td>
<td>q_b (kPa) (s/B = 10%)</td>
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<tr>
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Figure 6.18 Comparison of pile base unit load with plate unit load in calibration chamber plate load tests.
Figure 6.18  Comparison of pile base unit load with plate unit load in calibration chamber plate load tests (continued).
Differences between pile base unit load and plate unit load were small for all relative density levels. It is observed, however, that the plate unit loads under BC1 conditions were smaller than the pile base unit loads, and more substantially so in the case of short piles, for which confinement is low at the level of the pile base. The differences between pile base unit load and plate unit load were around 14 to 15% for $L = 5$ m, 6 to 9% for $L = 10$ m, and 1 to 3% for $L = 20$ m.

The results for BC2 conditions, as shown in Figure 6.18 (b), differ from those for BC1 conditions in that the plate unit loads from the calibration chamber plate load tests were found to be greater than pile base unit load values under corresponding conditions. The differences were around 1 to 3% for $L = 5$ m, 3 to 9% for $L = 10$ m, and 7 to 15% for $L = 20$ m. This indicates that the differences are more significant for long piles, for which confinement is high at the pile base level.

6.5 Summary

Calibration chamber plate load tests have been used to investigate the base load-settlement relationship of non-displacement piles. In this chapter, the calibration chamber plate load tests were analyzed through the finite element method using the three-dimensional non-linear elastic-plastic model presented in the previous chapter. A series of calibration chamber tests performed by Ghionna et al. (1994) were modeled and compared with the results of finite element analyses. The predicted load-settlement curves showed good agreement with measured load-settlement curves. The comparison between computed and measured plate unit loads for the relative settlement of $s/B = 5$ and 10% was also made for both dense ($D_R = 90\%$) and medium dense ($D_R = 50\%$) sand. Predicted plate unit loads were in good agreement with measured results, showing relative errors not larger than about 20% at $s/B = 10\%$.

Calibration chamber size effects, resulting from the finite size of the chamber, were also investigated for different relative densities and boundary conditions using finite
element analysis. The comparison was made between pile base unit load for piles loaded under field conditions and plate unit loads from the calibration chamber tests. The piles were modeled with three different pile lengths and four different relative densities. Plate unit loads in calibration chamber tests tend to be lower (for BC1, BC4) and higher (for BC2, BC3) than pile base unit loads. The confining stress level at the pile base level also influences size effect. The calibration chamber size effects under BC1 were more pronounced at low confinement, corresponding to shorter piles, while size effects under BC2 were more pronounced at high confinement, corresponding to longer piles. The magnitude of size effect is small, validating the use of chamber tests to simulate pile base loading. In keeping with the recommendation of Salgado et al. (1998b) regarding CPT testing in calibration chambers, it is recommended here that BC1 or BC4 be used in calibration chamber plate load testing. If this is done, our numerical results show that, for practical purposes, no correction is needed to the measured plate unit load in estimating pile base unit load, unless very short piles are being simulated.
CHAPTER 7  DETERMINATION OF PILE BASE RESISTANCE

7.1  Introduction

The standard penetration test (SPT) and the cone penetration test (CPT) are the two most popular methods for pile design using in-situ test results (Bandini and Salgado 1998). While the process followed to obtain SPT blow counts is not well related to the quasi-static pile loading process, that of the static cone penetration is better related to the pile loading process. The test is performed quasi-statically and resembles a scaled-down pile load test. According to many authors (e.g. De Beer 1984; Franke 1989,1993; Ghionna et al. 1993, 1994; Jamiolkowski and Lancellotta 1988; Fioravante et al. 1995), cone penetration resistance may be used as a proxy for limit base resistance in piles.

In this chapter, the base load-settlement curves of axially loaded piles bearing in sand are obtained for different stresses and densities using the finite element analysis with a non-linear elastic-plastic stress-strain model. Cone resistance $q_c$ is determined for the same soil conditions from the penetration resistance analysis of Salgado et al. (1997a) using the program CONPOINT (Salgado 1993; Salgado et al. 1997a, b; Salgado et al. 1998a); $q_c$ values determined in this manner are used to normalize the pile base load-settlement curves. The fully developed load-settlement curves in terms of $q_b/q_c$ versus relative settlement $s/B$ can be used to determine the normalized pile base resistance $q_b/q_c$ for any settlement-based design criterion.
7.2 Methods for Investigating Load-Settlement Response

If a pile were continuously pushed down into a homogenous granular soil mass, it would eventually reach a condition of penetration at a constant load in the same way as a cone penetrometer does in a cone penetration test (CPT). This is a condition of limited interest in pile design, and nearly always impossible to establish with conventional load test procedures and equipment, unless the pile is relatively small in diameter and length, it is not bearing in a very strong soil layer, and the engineer specifically requires a plunging load test. Typical load-settlement curves for piles embedded in sand show gradually increasing curvature rather than a clear peak load.

Three approaches are possible to obtain the vertical load-settlement relationship for a pile. Those include full-scale pile load tests, calibration chamber tests and numerical modeling. Full-scale pile load tests are the best option to investigate the load-settlement relationship for a specific site and pile, but cannot typically be used to obtain an accurate correlation between base resistance and the soil state. Only fully instrumented load tests with significant efforts to characterize the soil around the pile could potentially be used for that purpose. Difficulties that would still need to be addressed include the fact that relative density and lateral stress are not known in the field, and that typical natural granular deposits tend to be variable (Lee and Salgado 1999b).

Calibration chamber testing and numerical analysis are more flexible than pile load testing. A variety of stress states, densities and boundary conditions can be considered. Calibration chamber plate load tests, as discussed in the previous chapter, have been used to investigate the load-settlement response of non-displacement piles (Ghionna et al. 1994, Lee and Salgado 1999a). A number of numerical techniques have been identified for the same purpose (Desai and Christian 1977, Lee et al. 1989, Poulos 1989). The finite element method is among the most popular, as it allows modeling of complicated non-linear soil behavior and various interface conditions, with different geometries and soil conditions (Lee and Salgado 1999c). A key element in a finite element analysis is the use of a relevant constitutive model, which should model the strongly non-linear soil behavior. In the present study, the non-linear elastic-plastic stress-strain presented in
chapter 5 is used in a finite element analysis to determine the load-settlement response of vertically loaded piles.

7.3 Finite Element Modeling of Pile Load Test

The commercial finite element program ABAQUS was used to model vertically loaded piles. The finite element modeling of pile load tests for obtaining load-settlement curves was done in the same way as used in the investigation of size effects in calibration chamber tests. Instead of using one of the material models available in the program, a subroutine was written for the non-linear elastic-plastic model described previously. Because the pile stiffness is very large compared to soil stiffness, the pile was assumed made of linear elastic material throughout the analysis. Eight-noded axisymmetric elements with four internal integration points were used to model both the soil and the pile. Thin layer interface elements with zero initial thickness, allowing slippage, were used between the pile and the soil. The necessity of interface elements in the analysis of axially loaded piles is discussed in Trochanis et al. (1991).

The analyses of the calibration chamber plate load tests in the previous chapter showed the validity of the proposed finite element analysis of the pile base load-settlement response in sand. In order to further assess the performance of the proposed finite element analysis, a pile load test performed by the Georgia Institute of Technology (Mayne and Harris 1993) was modeled, and the numerical and experimental results were compared. The test site has a layer of residual, silty silica sand extending down to 15.8 – 19.5 m underlain by partially-weathered rock down to 20 – 24.8 m and then sound bedrock. Grain size distribution analysis showed the soil to be composed of about 70% sand, with the clay fraction under 10%. A series of laboratory tests on the soil samples collected at several depths were performed to obtain basic soil properties. Table 7.1 shows the soil property profile with depth and layers used in the analysis.
Table 7.1 Basic soil properties used in finite element analysis.

<table>
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<tr>
<th>Layer No.</th>
<th>Depth (m)</th>
<th>$\phi'$</th>
<th>$K_0$</th>
<th>$\sigma'_{vo}$ (kPa)</th>
<th>$e_0$</th>
<th>$G_0$ (kPa)</th>
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<td>18.2</td>
<td>0.70</td>
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<tr>
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</tbody>
</table>
The bottom boundary of the finite element mesh was located at a depth of 21.93 m from the surface, at which the bedrock was encountered. The last layer of the mesh from 18.28 m to 21.93 m was the partially weathered rock layer, which was assumed to behave as an elastic material. Values of initial shear modulus $G_o$ for each soil layer were calculated based on Hardin and Black's equation for angular sand. According to the Hardin and Black equation and the observation made by Salgado et al. (1999) regarding changes in stiffness as a function of fines content, the values adopted for $C_e$, $e_e$ and $n_e$ were $C_e = 214$, $e_e = 2.97$ and $n_e = 0.5$.

The values of the model parameters $f$ and $g$ of the non-linear elastic plastic model were taken as 0.98 and 0.05 respectively, based on the observed non-linear elastic properties of silty sand (Salgado et al. 1999). The test drilled shaft had a diameter equal to 76 cm and a length equal to 16.8 m. Measured and predicted base load-settlement curves were plotted together in Figure 7.1. Overall, agreement is observed to be satisfactory.

7.4 Cone Penetration Resistance from Cavity Expansion Analysis

A number of methods have been proposed analytically and experimentally to correlate the cone penetration resistance $q_c$ to stress state and soil conditions (Terzaghi 1943, Vesic 1972, Durgunoglu and Mitchell 1975, Baligh 1985, Yu and Houlby 1991, Salgado 1993, Salgado et al. 1997a). For undrained clay and fully drained sand, those are generally classified into (Yu and Mitchell 1998):

1. Bearing capacity theory
2. Cavity expansion theory
3. Steady state deformation
4. Incremental finite-element analysis
5. Calibration chamber testing.
Figure 7.1 Load-settlement curves for pile load test at Georgia Tech.
A useful penetration resistance theory should be able to relate values of cone penetration resistance $q_c$ to the intrinsic and state soil variables (Salgado et al. 1997a). The cavity expansion solution by Salgado (1993) is one of the well-validated theories for obtaining the cone penetration resistance $q_c$, and takes advantage of utilizing the intrinsic and state soil variables. According to Salgado et al. (1997a), for clean, uncremented soils, the cone penetration resistance $q_c$ can be expressed as follows:

$$q_c = q_c(D_R, \sigma'_v, \sigma'_h)$$

(7.1)

in which $q_c = $ function containing intrinsic variables; $D_R = $ relative density of sand; and $\sigma'_v$ and $\sigma'_h = $ in-situ vertical and horizontal effective stress. When a penetrometer is pushed into soil, it creates and expands a cylindrical cavity that has an initial radius equal to zero. The cavity expansion analysis by Salgado (1993) allows the calculation of cone penetration resistance $q_c$ based on the cavity expansion resistance required to form such a cylindrical cavity.

Figure 7.2 shows different types of assumed failure mechanisms proposed for deep penetration. It should be noticed that the mechanism shown in Figure 7.2 (a), (b) and (d) are theoretically impossible for soils with the usual value of the coefficient of lateral earth pressure ratio $K_0$ that lies in the 0.3 – 1.0 range. This is so because those mechanisms violate the path of least work. In other words, the slip lines should be directed towards the side since the values of lateral stresses are smaller for these values of $K_0$ than that of vertical stresses. The mechanism of Figure 7.2 (c) is kinematically possible.

Based on the observation of the displacement field under the pile base (BCP 1971), Salgado (1993) proposed the slip pattern shown in Figure 7.3 considering the axisymmetric condition of cone penetration. In Figure 7.3, $\sigma^p_i$ represents horizontal principal stress on the slip line under cone penetrometer and is related to the cavity expansion pressure $p_L$. 
Figure 7.2 Different failure mechanisms for deep penetration.
Figure 7.3  Slip pattern under cone penetrometer (after Salgado 1993).
The calculation of cavity expansion pressure $p_L$ requires the numerical sequence considering plastic, non-linear elastic and elastic stress region around the cavity. The relationship between cavity expansion pressure $p_L$ and cone penetration resistance $q_c$ was established based on stress rotation analysis. Following Bolton (1979), the major principal stresses in two different zones shown in Figure 7.4 are related for the rotation angle $\Delta \psi$ as:

$$\sigma_1^A = \sigma_1^B e^{2\Delta \psi \tan \phi}$$  \hspace{1cm} (7.2)

in which $\sigma_1^A$ and $\sigma_1^B$ = major principal stresses in zones A and B in Figure 7.4; $\Delta \psi$ = rotation angle between different principal stresses in zones A and B; and $\phi$ = friction angle. For the stresses in Figure 7.3, (7.2) can be written with $\Delta \psi = \pi/2$ as:

$$\sigma_1^Q = \sigma_1^P e^{\pi \tan \phi_T}$$  \hspace{1cm} (7.3)

where $\sigma_1^P$ is related to the cavity expansion pressure $p_L$; $\sigma_1^Q$ is related to $q_c$; and $\phi_T$ is a representative friction angle in the transition zone $T$ in Figure 7.3. Integrating $\sigma_1^Q$ with respect to the projected area of cone tip, the cone penetration resistance $q_c$ can be determined numerically for a given stress state and soil conditions. More details regarding the procedure to compute $q_c$ using cavity expansion analysis can be found in Salgado et al. (1997a).

The cone penetration resistance $q_c$ obtained from this procedure will be used to normalize the pile base resistance using the program CONPOINT containing the procedure for cavity expansion solution (Salgado et al. 1997a, b, Salgado et al. 1998a, b).
Figure 7.4 Stress rotation between different zones (after Salgado 1993).
7.5 Determination of Base Resistance for Non-Displacement Piles

7.5.1 Load-settlement response for various soil conditions

The pile base load-settlement response for various soil and stress conditions can be obtained by analyzing axially loaded piles with different pile lengths and relative densities. The pile dimensions and soil conditions used for obtaining load-settlement curves in this section are the same as used in the investigation of size effect in calibration chamber test in Chapter 6. Three pile lengths of 5 m, 10 m, and 20 m with a diameter equal to 60 cm were used. Different pile lengths imply different confining stress levels at the pile base. For an assumed unit weight $= 20 \text{kN/m}^3$, initial vertical stresses at the pile base level (corresponding to 5 m, 10 m and 20 m) are $\sigma'_{vo} = 100 \text{kPa}$, 200 kPa and 400 kPa, respectively.

All the piles were positioned within a granular soil deposit assumed as normally consolidated Ticino sand with $K_o = 0.43$. A $K_o$ value of 0.43 was selected because it is near the center of the typical range of 0.39 to 0.47 observed for $K_o$ in sands. Ticino sand was selected because it has been studied extensively (Salgado 1993, Bellotti et al. 1996) and has been used in hundreds of calibration chamber plate load and cone penetration tests (Ghionna et al. 1994, Salgado et al. 1997a). Values of relative density $D_r$ used in calculations were 30%, 50%, 70% and 90% for each of the pile lengths assumed. The basic pile geometry and soil conditions are illustrated in Table 7.2. In Table 7.2, $\sigma'_v$ and $\sigma'_h$ represent the in-situ vertical and horizontal effective stresses at the pile base level, respectively.

The finite element meshes for each pile length are shown in Figures 7.5 - 7.7. The bottom boundaries of the meshes were located at a depth larger than two times the corresponding pile length measured from the ground surface. The widths of the meshes were equal to or larger than the pile lengths. Finite element analyses performed separately with infinite elements at the lateral boundary showed that the mesh dimensions used in this study are sufficiently large to eliminate geometric boundary effects.
Figure 7.8 shows a set of pile base load-settlement curves obtained from finite element analyses done for the different pile lengths and relative densities. The curves in Figure 7.8 extend up to a base settlement equal to 12 cm, corresponding to a relative settlement $s/B$ of 20%. It is observed that the load-settlement curves for higher relative densities show stiffer responses than those for lower relative densities under the same confinement level or pile length. The initial shear modulus of soil with higher relative density is also higher, and the rate of modulus degradation is lower.

The differences of pile base unit load between $D_r = 30\%$ and $90\%$ appear more pronounced as the confinement level or pile length increases. For a 5-m pile, the difference was about 2.2 MPa at a settlement equal to 6 cm, which corresponds to $s/B = 10\%$. The difference for a 20-m pile was about 3.0 MPa at the same relative settlement.

Table 7.2. Pile geometry and soil conditions used in FEM analyses.

<table>
<thead>
<tr>
<th>Pile Length (m)</th>
<th>Relative density (%)</th>
<th>Type of Sand</th>
<th>$\sigma'_v$ (kPa)</th>
<th>$\sigma'_h$ (kPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>30</td>
<td>Ticino</td>
<td>100</td>
<td>43</td>
</tr>
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<td></td>
<td>50</td>
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<td>100</td>
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<td></td>
<td>70</td>
<td>Ticino</td>
<td>100</td>
<td>43</td>
</tr>
<tr>
<td></td>
<td>90</td>
<td>Ticino</td>
<td>100</td>
<td>43</td>
</tr>
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<td>10</td>
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<td>Ticino</td>
<td>200</td>
<td>86</td>
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<td>Ticino</td>
<td>200</td>
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<tr>
<td></td>
<td>90</td>
<td>Ticino</td>
<td>400</td>
<td>172</td>
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</table>
Figure 7.5  Finite element model for 5-m pile.
Figure 7.6 Finite element model for 10-m pile.
Figure 7.7  Finite element model for 20-m pile.
Figure 7.8  Base load-settlement curves for (a) 5-m, (b) 10-m, and (c) 20-m piles.
Figure 7.8  Base load-settlement curves for (a) 5-m, (b) 10-m, and (c) 20-m piles (continued).
7.5.2 Normalized base resistance for non-displacement piles

In order to relate pile base resistance to cone penetration resistance, the load-settlement curves obtained previously were normalized as follows:

1. the base resistance $q_b$ at a given $D_R$ and stress state was divided by the cone resistance $q_c$ for the same soil conditions;
2. the settlement was divided by the pile diameter $B$.

The cone resistance $q_c$ was determined using the penetration resistance analysis of Salgado et al. (1997a), contained in the program CONPOINT (Salgado 1993; Salgado et al. 1997a, b; Salgado et al. 1998a, b). Figure 7.9 shows the fully developed load-settlement curves in terms of $q_b/q_c$ and $s/B$ for each of the pile lengths and relative densities.

A common design approach for non-displacement piles is to stipulate that so long as the load is less than the load required for the pile settlement to reach a certain percentage of the shaft diameter $B$, serviceability and ultimate limit states are not reached. A modest safety factor can also be used. Franke (1993), for example, proposed that the critical load corresponds to $s/B = 0.1$, while Reese and O’Neill (1988) define a critical load corresponding to $s/B = 0.05$. The British code for pile design is based on the $s/B = 0.1$ criterion. It may be stipulated in a given project, based on structural or architectural requirements, that some other values of $s/B$ not be exceeded. Table 7.3 shows different values of $q_b/q_c$ recommended by several authors.

Table 7.4 shows the values of $q_b/q_c$ at $s/B = 5\%$ and $10\%$ for different relative densities and pile lengths obtained using the finite element and penetration resistance analyses. It can be seen that values of $q_b/q_c$ fall within the $0.07 - 0.13$ range for $s/B = 5\%$ and the $0.12 - 0.21$ range for $s/B = 10\%$. 
Figure 7.9 Normalized load-settlement curves for (a) 5-m, (b) 10-m, and (c) 20-m piles in terms of $q_l/q_c$ and $s/B$. 
Figure 7.9 Normalized load-settlement curves for (a) 5-m, (b) 10-m, and (c) 20-m piles in terms of $q_b/q_c$ and $s/B$ (continued).
Table 7.3 Values of \( q_b/q_c \) according to several authors.

<table>
<thead>
<tr>
<th></th>
<th>( q_b/q_c )</th>
<th>( q_b/q_c )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>((s/B = 5%))</td>
<td>((s/B = 10%))</td>
</tr>
<tr>
<td>German Specification</td>
<td>N/A</td>
<td>0.2</td>
</tr>
<tr>
<td>(DIN 4014, Franke 1993)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Franke (1989)</td>
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</tr>
<tr>
<td>Jamiolkowski and Lancellotta (1988)</td>
<td>0.2</td>
<td>N/A</td>
</tr>
<tr>
<td>(for ( B &lt; 60 ) cm)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ghionna et al. (1994)</td>
<td>0.09 ± 0.02</td>
<td>0.13 ± 0.02</td>
</tr>
<tr>
<td>Salgado (1995)</td>
<td>N/A</td>
<td>0.15</td>
</tr>
</tbody>
</table>

Table 7.4 Values of \( q_b/q_c \) at \( s/B = 5\% \) and 10\%.

<table>
<thead>
<tr>
<th>Pile Length (m)</th>
<th>( D_R ) (%)</th>
<th>( q_b ) ((s/B = 5%)) (kPa)</th>
<th>( q_b ) ((s/B = 10%)) (kPa)</th>
<th>( q_c ) (kPa)</th>
<th>( q_b/q_c ) ((s/B = 5%))</th>
<th>( q_b/q_c ) ((s/B = 10%))</th>
</tr>
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<tbody>
<tr>
<td>5</td>
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<td>939</td>
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<tr>
<td></td>
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<td>1303</td>
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<tr>
<td></td>
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<td>2789</td>
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<td></td>
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<td>3630</td>
<td>30121</td>
<td>0.07</td>
<td>0.12</td>
</tr>
<tr>
<td>10</td>
<td>30</td>
<td>1343</td>
<td>2158</td>
<td>10922</td>
<td>0.12</td>
<td>0.20</td>
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<tr>
<td></td>
<td>50</td>
<td>1817</td>
<td>2915</td>
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<td>0.10</td>
<td>0.16</td>
</tr>
<tr>
<td></td>
<td>70</td>
<td>2409</td>
<td>3871</td>
<td>26644</td>
<td>0.09</td>
<td>0.14</td>
</tr>
<tr>
<td></td>
<td>90</td>
<td>3054</td>
<td>4970</td>
<td>38816</td>
<td>0.08</td>
<td>0.13</td>
</tr>
<tr>
<td>20</td>
<td>30</td>
<td>1933</td>
<td>3106</td>
<td>16716</td>
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<td>0.19</td>
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<td>2590</td>
<td>4158</td>
<td>25694</td>
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<td>0.16</td>
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<td></td>
<td>70</td>
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<td>5401</td>
<td>36718</td>
<td>0.09</td>
<td>0.15</td>
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<tr>
<td></td>
<td>90</td>
<td>4289</td>
<td>6845</td>
<td>50524</td>
<td>0.08</td>
<td>0.13</td>
</tr>
</tbody>
</table>
Figure 7.10 illustrates the influence of pile length (i.e., confinement at pile base level) and relative density on the normalized base resistance $q_b/q_c$. The values of $q_b/q_c$ in Figure 7.10 correspond to the relative settlement level of $s/B = 10\%$. From Figure 7.10 (a), it is observed that the effect of pile length on $q_b/q_c$ is not significant. This is because the pile-base resistance $q_b$ and the cone penetration resistance $q_c$ depend on initial confining stress in a similar way. Only a slight decrease of $q_b/q_c$ can be observed for loose sand as the pile length increases. For dense sand, pile length has essentially no influence on $q_b/q_c$ for the range of lengths investigated.

As can be seen in Figure 7.10(b), the influence of relative density on the normalized base resistance is substantial. The normalized base resistance $q_b/q_c$ decreases as the relative density increases. The value of $q_b/q_c$ at $s/B = 10\%$ is 0.19 to 0.2 for $D_R = 30\%$, whereas it is 0.12 to 0.13 for $D_R = 90\%$. These results indicate that larger settlements are required for soils with higher relative densities to reach a base resistance equal to a set percentage of cone penetration resistance $q_c$.

The results also offer some insight into why most pile design methods that calculate $q_b$ by multiplying $q_c$ by a certain constant (e.g., 0.2 for 10\% relative settlement, according to Franke 1989) also place an upper limit, which is usually taken as a value in the 4.5 - 5 MPa range, on possible values of $q_b$. When piles are embedded in very dense sand layers, the results of the present analysis indicate that, as an example, $q_b = 0.2 \times q_c$ [the value proposed by Franke (1989) irrespective of relative density] would be too high. Following the results given in Table 7.4, the value of $q_b/q_c = 0.12$ would be more appropriate. Placing a limit on $q_b$ (of say 5 MPa) serves a purpose in that case. However, if the $q_b/q_c$ values of Table 7.4 and Figure 7.9 are used, there may not be a need for setting an upper limit on $q_b$. 
Figure 7.10 Normalized base resistance $q_b/q_c$ with (a) mean effective stress ($\sigma'_m$) at the pile base level and (b) relative density ($D_R$).
7.5.3 The effect of initial stress ratio $K_0$

Normally consolidated sand deposits have initial values of the coefficient of lateral earth pressure at rest ($K_0$) in the 0.39 – 0.5 range. Overconsolidated sand deposits have $K_0$ values typically higher than that of normally consolidated deposits. In order to investigate the effects of $K_0$ on normalized base resistance $q_b/q_c$, three $K_0$ values (0.4, 0.7 and 1.0) were assumed in a series of finite element analyses. The value of $K_0 = 1.0$, corresponding to an isotropic stress condition, may be regarded, for practical purposes, as the upper limit on $K_0$, observed for highly overconsolidated sand deposits.

Figure 7.11 represents the effects of $K_0$ on the normalized base resistance $q_b/q_c$. The value of normalized base resistance $q_b/q_c$ in Figure 7.11 were determined for the relative settlement equal to $s/B = 10\%$. The curves shown in Figure 7.11 were plotted for different relative densities ($D_R = 30\%, 50\%, 70\%$ and $90\%$) and three pile lengths ($L = 5\, m$, $10\, m$ and $20\, m$).

It is observed that $q_b/q_c$ tends to decrease as the initial $K_0$ increases, although deviation from this trend can be seen in the case for the highest relative density of $D_R = 90\%$. This trend was most obvious for the lowest relative density. For $D_R = 30\%$ and $L = 5\, m$, the difference between the normalized base resistance $q_b/q_c$ at $K_0 = 0.4$ and at $K_0 = 1.0$ was equal to approximately 0.05. No difference in the normalized base resistance $q_b/q_c$ was found at $D_R = 90\%$ for $K_0 = 0.4$ to 1.0. Comparing the three cases ($L = 5, 10$ and $20\, m$) in Figure 7.11, it can also be observed that the confinement at the pile base level, which is determined by the pile length, does not have as much influence as the relative density on the relationship between $q_b/q_c$ and $K_0$.

These results suggest that, when the soil is loose and has a high $K_0$ value, the values of normalized base resistance $q_b/q_c$ given in Table 7.4 and Figure 7.9 need to be modified considering the variations in the values of $q_b/q_c$ shown in Figure 7.11. For very dense soil, no such modification of the value of $q_b/q_c$ is necessary.
Figure 7.11  Effect of $K_0$ on normalized base resistance $q_b/q_c$. 

--- $D_R = 30\%$  
--- $D_R = 50\%$  
--- $D_R = 70\%$  
--- $D_R = 90\%$
7.6 Determination of Base Resistance for Displacement Piles

Values and analyses presented previously were developed for non-displacement piles. The limit base resistance $q_{bl}$, as mentioned earlier, is mobilized at very large settlement levels and is conceptually identical for both non-displacement and displacement piles (De Beer 1984, 1988, Ghionna et al. 1993, Salgado et al. 1997a). The normalized base resistance values of Figure 7.9 for non-displacement piles, however, cannot be directly applied to displacement piles, because of the very different load-settlement response of displacement and non-displacement piles for low to moderate settlement levels.

De Beer (1984, 1988) has shown that, under the same conditions, the loads carried by a displacement pile and a geometrically identical non-displacement pile differ significantly for values of relative settlement $s/B$ of interest in pile design. Non-displacement piles settle more than displacement piles for the same applied load. This is mainly due to the different installation processes for these two types of piles. The installation of displacement piles usually causes considerable densification of the soil around the pile. In terms of base resistance, this process could be seen as a preloading of the soil in the immediate neighborhood of the pile base, hence the stiffer response when compared with non-displacement piles. The difference between the loads carried by the two types of pile for the same settlement level becomes less pronounced as settlement increases. De Beer’s observations were later confirmed by other authors (e.g., Jamiołkowski and Lancellotta 1988, Ghionna et al. 1993).

Table 7.5 shows the typical ratio of base resistance of displacement piles to base resistance of non-displacement piles for different values of relative settlement but same initial soil conditions. As can be seen, the ratio of $q_b$ of a non-displacement pile to $q_b$ of a geometrically identical displacement pile is much smaller than 1 at small relative settlements, but approaches 1 as the settlement approaches infinity. A simple approach that may be used to determine $q_b/q_c$ values as a function of $s/B$ for displacement piles is the application of the ratios of Table 7.5 to the results of Table 7.4 for non-displacement
piles. Such an approach can provide useful design criteria, but is naturally subject to more uncertainties than the values proposed for non-displacement piles in Table 7.4. Table 7.6 provides the normalized base resistance \( q_b/q_c \) for displacement piles obtained using this approach.

### Table 7.5 Base resistance ratio for displacement and non-displacement piles.

<table>
<thead>
<tr>
<th>Relative Settlement (s/B)</th>
<th>( q_{b,ND}^a ) / ( q_{b,D}^b )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.5%</td>
<td>0.482</td>
</tr>
<tr>
<td>5%</td>
<td>0.517</td>
</tr>
<tr>
<td>10%</td>
<td>0.587</td>
</tr>
<tr>
<td>25%</td>
<td>0.715</td>
</tr>
<tr>
<td>( \rightarrow \infty )</td>
<td>( \rightarrow 1.0 )</td>
</tr>
</tbody>
</table>

\( ^a q_{b,ND} = \) base resistance for non-displacement pile
\( ^b q_{b,D} = \) base resistance for displacement pile

### Table 7.6 Values of \( q_b/q_c \) for displacement piles.

<table>
<thead>
<tr>
<th>Pile Length (m)</th>
<th>( D_R ) (%)</th>
<th>( q_b/q_c ) (s/B = 5%)</th>
<th>( q_b/q_c ) (s/B = 10%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>30</td>
<td>0.25</td>
<td>0.35</td>
</tr>
<tr>
<td></td>
<td>50</td>
<td>0.19</td>
<td>0.29</td>
</tr>
<tr>
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<td>70</td>
<td>0.17</td>
<td>0.24</td>
</tr>
<tr>
<td></td>
<td>90</td>
<td>0.14</td>
<td>0.20</td>
</tr>
<tr>
<td>10</td>
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</tr>
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<td>0.22</td>
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<td>0.26</td>
</tr>
<tr>
<td></td>
<td>90</td>
<td>0.15</td>
<td>0.22</td>
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</tbody>
</table>
7.7 Normalized Base Resistance for Silty Sands

Natural sand deposits often contain a certain amount of fines. These sand deposits are also bearing layers for piles, if the fines contents of soils are not high and soils are sufficiently strong to support axial loads. In order to investigate the normalized base resistance \( q_b/q_c \) for soils containing fines, the load-settlement response of piles in silty sands was analyzed.

The mechanical properties of silty sands were extensively studied by Salgado et al. (1999) and Bandini (1999). They investigated the behavior of silty sands focusing on stiffness and strength characteristics. The material used was Ottawa sand with silt contents equal to 5\%, 10\%, 15\%, and 20\% in weight. Tables 7.7 and 7.8 show the soil intrinsic parameters of silty sands with different fines contents. The parameters shown in the tables are the same as used in (4.50) and (4.76) for the initial shear modulus and dilatancy angle of sand, respectively. As can be seen in Table 7.7, the parameter \( C_g \) decreases as the silt content increases, while the exponent parameter \( n_g \) for the confining stress increases. On the other hand, as shown in Table 7.8, the higher the silt content, the higher the friction angle \( \phi_c \) at critical state. This also resulted in higher peak friction angles with increasing silt content. The dilatancy parameters \( Q \) and \( R \) for silty sands in Table 7.8 were obtained from the best-fit regression of test results, while the original Bolton (1986) correlation was based on fixing \( R = 1 \).

The finite element analysis of pile load tests in silty sands was done using the parameters given in Tables 7.7 and 7.8. Since it is usually desirable for piles to be placed into granular soil deposit with low fines content, silt contents equal to 5\% and 10\% were considered in the finite element analyses. The values of the parameters \( f \) and \( g \) for the non-linear elastic-plastic stress-strain model were determined based on the stress-strain relationship of silty sands by Bandini (1999). Table 7.9 shows the values of the parameters \( f \) and \( g \) for different relative densities and silt contents.
Table 7.7  Values of soil intrinsic parameters with different silt contents
(after Salgado et al. 1999, Bandini 1999).

<table>
<thead>
<tr>
<th>Silt content (%)</th>
<th>$C_g$</th>
<th>$e_g$</th>
<th>$n_g$</th>
<th>$e_{\text{min}}$</th>
<th>$e_{\text{max}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>612</td>
<td>2.17</td>
<td>0.439</td>
<td>0.48</td>
<td>0.78</td>
</tr>
<tr>
<td>5</td>
<td>454</td>
<td>2.17</td>
<td>0.459</td>
<td>0.42</td>
<td>0.70</td>
</tr>
<tr>
<td>10</td>
<td>357</td>
<td>2.17</td>
<td>0.592</td>
<td>0.36</td>
<td>0.65</td>
</tr>
</tbody>
</table>

Table 7.8  Values of friction angle $\phi_c$ at critical state and dilatancy parameters $Q$ and $R$
with different silt contents (after Salgado et al. 1999, Bandini 1999).

<table>
<thead>
<tr>
<th>Silt content (%)</th>
<th>$\phi_c$</th>
<th>$Q$</th>
<th>$R$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>29.0</td>
<td>9.0</td>
<td>0.49</td>
</tr>
<tr>
<td>5</td>
<td>30.5</td>
<td>9.0</td>
<td>-0.50</td>
</tr>
<tr>
<td>10</td>
<td>32.0</td>
<td>8.3</td>
<td>-0.69</td>
</tr>
</tbody>
</table>
Table 7.9 Values of f and g used in finite element analyses for silty sands.

<table>
<thead>
<tr>
<th>D&lt;sub&gt;R&lt;/sub&gt; (%)</th>
<th>5% silt</th>
<th>10% silt</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>f</td>
<td>g</td>
</tr>
<tr>
<td>30</td>
<td>0.98</td>
<td>0.15</td>
</tr>
<tr>
<td>50</td>
<td>0.97</td>
<td>0.18</td>
</tr>
<tr>
<td>70</td>
<td>0.96</td>
<td>0.21</td>
</tr>
<tr>
<td>90</td>
<td>0.95</td>
<td>0.24</td>
</tr>
</tbody>
</table>

Finite element meshes and stress states used in the analyses were the same as those for clean sands previously presented. The cone resistance q<sub>c</sub> at the pile base level was obtained using the program CONPOINT (Salgado et al. 1997a, b, 1998a, b) with soil intrinsic parameters given in Tables 7.7 and 7.8.

Table 7.10 and Figure 7.12 show the normalized base resistance q<sub>b</sub>/q<sub>c</sub> for silty sands with different relative densities and pile lengths. For both 5% and 10% silt contents, most values of q<sub>b</sub>/q<sub>c</sub> were in the 0.12 – 0.17 range. As discussed earlier, the influence of the confining stress on the value of q<sub>b</sub>/q<sub>c</sub> for clean sands was small irrespective of the relative density. For silty sands, however, the influence of the confining stress was more significant. As can be seen in Table 7.10, q<sub>b</sub>/q<sub>c</sub> for lower relative density (D<sub>R</sub> = 30%) decreases with increase in pile length. On the other hand, the value of q<sub>b</sub>/q<sub>c</sub> for higher relative density (D<sub>R</sub> = 70%, 90%) increases as the pile length increases. This observation was more pronounced at the 10% silt content. The value of q<sub>b</sub>/q<sub>c</sub> of the 20-m pile for D<sub>R</sub> = 90% with the 10% silt content was even greater than that for D<sub>R</sub> = 30%. These results may be due to the increasing influence of the confining stress with increasing silt content. As shown in (4.50) and (4.76), the parameter n<sub>g</sub> in Table 7.7 represents the effect of the confining stress on the elastic modulus. The higher the value of n<sub>g</sub>, the greater the influence of the confining stress on the value of q<sub>b</sub>/q<sub>c</sub>. As a result, the differences of q<sub>b</sub>/q<sub>c</sub> for the silty sand between different pile lengths were more pronounced than those for the clean sand.
Table 7.10 Values of $q_b/q_c$ for silty sands with different relative densities and pile lengths.

<table>
<thead>
<tr>
<th>Silt content</th>
<th>Pile length</th>
<th>$D_R$ (%)</th>
<th>$q_b$ (kPa) ($s/B = 5%$)</th>
<th>$q_b$ (kPa) ($s/B = 10%$)</th>
<th>$q_c$ (kPa)</th>
<th>$q_b/q_c$ ($s/B = 5%$)</th>
<th>$q_b/q_c$ ($s/B = 10%$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>5 m</td>
<td>30</td>
<td>828</td>
<td>1336</td>
<td>8012</td>
<td>0.103</td>
<td>0.167</td>
</tr>
<tr>
<td></td>
<td></td>
<td>50</td>
<td>1103</td>
<td>1760</td>
<td>11911</td>
<td>0.093</td>
<td>0.148</td>
</tr>
<tr>
<td></td>
<td></td>
<td>70</td>
<td>1405</td>
<td>2239</td>
<td>16960</td>
<td>0.083</td>
<td>0.132</td>
</tr>
<tr>
<td></td>
<td></td>
<td>90</td>
<td>1749</td>
<td>2839</td>
<td>22488</td>
<td>0.078</td>
<td>0.126</td>
</tr>
<tr>
<td></td>
<td>10 m</td>
<td>30</td>
<td>1194</td>
<td>1909</td>
<td>12509</td>
<td>0.095</td>
<td>0.153</td>
</tr>
<tr>
<td></td>
<td></td>
<td>50</td>
<td>1551</td>
<td>2486</td>
<td>17391</td>
<td>0.089</td>
<td>0.143</td>
</tr>
<tr>
<td></td>
<td></td>
<td>70</td>
<td>1961</td>
<td>3144</td>
<td>23321</td>
<td>0.084</td>
<td>0.135</td>
</tr>
<tr>
<td></td>
<td></td>
<td>90</td>
<td>2425</td>
<td>3902</td>
<td>29800</td>
<td>0.081</td>
<td>0.131</td>
</tr>
<tr>
<td></td>
<td>20 m</td>
<td>30</td>
<td>1668</td>
<td>2736</td>
<td>19705</td>
<td>0.085</td>
<td>0.139</td>
</tr>
<tr>
<td></td>
<td></td>
<td>50</td>
<td>2191</td>
<td>3556</td>
<td>25596</td>
<td>0.086</td>
<td>0.139</td>
</tr>
<tr>
<td></td>
<td></td>
<td>70</td>
<td>2763</td>
<td>4460</td>
<td>32390</td>
<td>0.085</td>
<td>0.138</td>
</tr>
<tr>
<td></td>
<td></td>
<td>90</td>
<td>3411</td>
<td>5469</td>
<td>39808</td>
<td>0.086</td>
<td>0.137</td>
</tr>
<tr>
<td></td>
<td>5 m</td>
<td>30</td>
<td>755</td>
<td>1253</td>
<td>8789</td>
<td>0.086</td>
<td>0.143</td>
</tr>
<tr>
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<td></td>
<td>50</td>
<td>1001</td>
<td>1641</td>
<td>12186</td>
<td>0.082</td>
<td>0.135</td>
</tr>
<tr>
<td></td>
<td></td>
<td>70</td>
<td>1283</td>
<td>2100</td>
<td>16231</td>
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<td>20716</td>
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<td>0.128</td>
</tr>
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<td>10 m</td>
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<td>1129</td>
<td>1865</td>
<td>14186</td>
<td>0.080</td>
<td>0.131</td>
</tr>
<tr>
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<td></td>
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<td>1505</td>
<td>2440</td>
<td>18329</td>
<td>0.082</td>
<td>0.133</td>
</tr>
<tr>
<td></td>
<td></td>
<td>70</td>
<td>1903</td>
<td>3076</td>
<td>22976</td>
<td>0.083</td>
<td>0.134</td>
</tr>
<tr>
<td></td>
<td></td>
<td>90</td>
<td>2341</td>
<td>3815</td>
<td>27963</td>
<td>0.084</td>
<td>0.136</td>
</tr>
<tr>
<td></td>
<td>20 m</td>
<td>30</td>
<td>1682</td>
<td>2809</td>
<td>22953</td>
<td>0.073</td>
<td>0.122</td>
</tr>
<tr>
<td></td>
<td></td>
<td>50</td>
<td>2224</td>
<td>3674</td>
<td>27783</td>
<td>0.080</td>
<td>0.132</td>
</tr>
<tr>
<td></td>
<td></td>
<td>70</td>
<td>2813</td>
<td>4629</td>
<td>32863</td>
<td>0.086</td>
<td>0.141</td>
</tr>
<tr>
<td></td>
<td></td>
<td>90</td>
<td>3456</td>
<td>5596</td>
<td>38029</td>
<td>0.091</td>
<td>0.147</td>
</tr>
</tbody>
</table>
Figure 7.12 Values of $q_b/q_c$ for silty sand.
7.8 Summary

The cone penetration test (CPT) resembles the vertical loading process on a pile. In this chapter, in order to take advantage of the CPT for pile design, load-settlement curves in terms of normalized base resistance \((q_b/q_c)\) versus relative settlement \((s/B)\) were developed. Although the limit state design concept for pile design has been used mostly with respect to either \(s/B = 5\%\) or \(s/B = 10\%\), the normalized load-settlement curves obtained in this study allow determination of pile base resistance at any relative settlement level within the \(0 - 20\%\) range. This is important, as it permits consideration of specific project features, related to the superstructure or other components of the facility.

In order to obtain the pile base load-settlement relationship, finite element analyses were performed with a 3-D non-linear elastic-plastic constitutive model. Three 60-cm diameter piles with lengths of 5 m, 10 m and 20 m were used in the analyses. The piles were positioned within a granular soil deposit with \(D_R = 30, 50, 70\) and \(90\%\). Because the soil conditions around the piles were assumed to be the same as those existing before pile installation, the results obtained represent those corresponding to non-displacement piles. The cone resistance \(q_c\) was calculated from the penetration resistance analysis of Salgado et al. (1997a) and used to normalize the load-settlement curves in order to express them in terms of \(q_b/q_c\) and \(s/B\).

Most \(q_b/q_c\) values obtained from the finite element and penetration resistance analyses fall within the \(0.07 - 0.13\) range for \(s/B = 5\%\) and the \(0.10 - 0.20\) range for \(s/B = 10\%\). The effect of relative density on the normalized base resistance was significant, while that of the confining stress at the pile base level was small. At higher relative densities, the value of \(q_b/q_c\) was smaller \((q_b/q_c = 0.12 - 0.13\) for \(D_R = 90\%\)) than at lower relative densities \((q_b/q_c = 0.19 - 0.2\) for \(D_R = 30\%\)). The effect of the coefficient of lateral earth pressure at rest \(K_0\) was also investigated. The value of \(q_b/q_c\) tends to decrease as the value of \(K_0\) increases. This trend was more pronounced at lower relative densities, and negligible for very dense sand.
Based on the results by De Beer (1984, 1988), the values of $q_b/q_c$ for displacement piles were obtained as well. The values of $q_b/q_c$ were typically in the 0.15 - 0.25 range for $s/B = 5\%$ and in the 0.22 - 0.35 range for $s/B = 10\%$.

The normalized base resistance $q_b/q_c$ for silty sands was also investigated. For both 5\% and 10\% silt contents, most values of $q_b/q_c$ were in the 0.12 - 0.17 range. These values are typically smaller than those for clean sands, which ranged from 0.12 to 0.21. The confining stress was another important factor for the value of $q_b/q_c$ of silty sand. For lower relative density ($D_R = 30\%$), the value of $q_b/q_c$ decreases as the pile length increases while that for higher relative density increases.
CHAPTER 8 ASSESSMENT OF PROPOSED NORMALIZED BASE RESISTANCE VALUES BASED ON CASE HISTORIES

8.1 Introduction

In this chapter, we reexamine the values of normalized base resistance $q_b/q_c$ presented previously. We do so in the context of a few case histories involving actual pile load tests, calibration chamber data, and other numerical analyses. Both non-displacement and displacement piles are addressed.

The case histories for non-displacement piles include two instrumented pile load tests on drilled shafts, a series of calibration chamber tests (Ghionna et al. 1994), and the numerical results of Simonini (1996). A load test on a drilled shaft performed at a site on the Georgia Tech campus in Atlanta, Georgia (Mayne and Harris 1993, Harris and Mayne 1994) and another performed at the University of São Paulo (USP), São Carlos, experimental field (Albiero et al. 1995) are analyzed.

For displacement piles, the load test on a steel H-pile performed at the Purdue University campus (Goble et al. 1972) and the load tests on the precast concrete piles carried out by NGI (Gregersen et al. 1973) were used. All the load tests used in this section were instrumented, so base load-settlement responses were recorded separately. The observed values of $q_b/q_c$ for each case are summarized in Table 8.1.
Table 8.1  Values of $q_o/q_c$ from load tests on non-displacement and displacement piles.

<table>
<thead>
<tr>
<th>Pile Type</th>
<th>Test Name</th>
<th>$q_o/q_c$ $(s/B = 5%)$</th>
<th>$q_o/q_c$ $(s/B = 10%)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Non-Displacement Pile (Drilled shaft)</td>
<td>Georgia Tech Test</td>
<td>0.18</td>
<td>0.26</td>
</tr>
<tr>
<td></td>
<td>USP Test</td>
<td>0.09</td>
<td>0.20</td>
</tr>
<tr>
<td></td>
<td>Simonini’s Analysis</td>
<td>N/A</td>
<td>0.17</td>
</tr>
<tr>
<td>Displacement Pile (Driven pile)</td>
<td>Purdue Test</td>
<td>0.27</td>
<td>0.37</td>
</tr>
<tr>
<td></td>
<td>NGI Test A</td>
<td>0.32</td>
<td>0.36</td>
</tr>
<tr>
<td></td>
<td>NGI Test B</td>
<td>0.32</td>
<td>0.36</td>
</tr>
<tr>
<td></td>
<td>NGI Test C</td>
<td>0.30</td>
<td>0.39</td>
</tr>
<tr>
<td></td>
<td>NGI Test D</td>
<td>0.43</td>
<td>0.47</td>
</tr>
</tbody>
</table>
8.2 Non-Displacement Piles

8.2.1 Georgia Tech load test

The Georgia Tech load test was described in the previous section. The representative value of \( q_c \) for base resistance calculation, an average value between the level of the pile base and a level about one-and-a half to two pile diameters below the pile base, is 6.5 MPa. The measured base loads for the shaft at 5 and 10% relative settlement are 580 and 810 kN, corresponding to \( q_b/q_c \) values of 0.18 and 0.26 respectively. Since the relative density of the soil around the pile base was not determined, the relationship between \( q_c \) and the relative density \( D_R \) suggested by Salgado et al. (1997b) was used. The estimated relative density of the soil around the pile base with a \( K_0 \) value of 0.41 was about 20 – 30%, representing a loose state. Using Table 7.4, the corresponding values of \( q_b/q_c \) at \( s/B = 5 \) and 10% can be found as 0.12 – 0.13 and 0.20 – 0.21, respectively. The difference between measured and predicted values of \( q_b \) in this example appears to be small (20 – 30% underprediction).

8.2.2 São Paulo load test

The USP, São Paulo, test site is located near downtown São Carlos, in the state of São Paulo, Brazil (Teixeria and Albiero 1994). The site geology consists of two clayey sand layers separated by a thin layer of pebbles. The upper layer is a reddish material of Cenozoic age, collapsible upon inundation (Vilar 1979), with void ratios close to 1. The lower layer is a brownish residual soil from a sandstone of the Bauru formation. The water table oscillates, but is typically around 10 meters deep. The base of the test shaft was placed at a depth of 10 m and the shaft diameter was 50 cm. The measured base loads corresponding to \( s/B = 5 \) and 10% are 50 and 114 kN, respectively. The load for 10% relative settlement was obtained from a quick maintained load test (QMLT).
performed after a slow maintained load test (SMLT) up to 5% relative settlement. The value of \( q_c \) was obtained from the CPT sounding provided in Albiero et al. (1995) as 2.8 MPa. This yields \( q_b/q_c \) values of 0.09 and 0.20 for \( s/B = 5 \) and 10%, respectively. The measured \( q_b/q_c \) values for this case were also favorably compared with the results in Table 7.4, namely \( q_b/q_c = 0.09 - 0.10 \) for \( s/B = 5\% \) and \( q_b/q_c = 0.14 - 0.17 \) for \( s/B = 10\% \) for medium dense sand.

8.2.3 Simonini’s results

Simonini (1996) carried out finite element analyses to obtain the base resistance of non-displacement piles in sands. The pile was modeled as having a diameter equal to 1 m and a length equal to 30 m. The soils were a dry sand deposit with \( \phi_c = 33^\circ \), \( D_R = 90\% \) and soil unit weight \( \gamma_m = 16.5 \text{ kN/m}^3 \). The value of the base resistance \( q_b \) corresponding to \( s/B = 10\% \) was found to be 8.6 MPa.

In order to obtain the cone resistance \( q_c \) at the pile base level, the penetration resistance analysis of Salgado et al. (1997a, b), available in the program CONPOINT (Salgado et al. 1997a, b), was used. The soil properties used to obtain the cone resistance were the same as those used in the finite element analyses. Combining the base resistance value with \( q_c \), the value of \( q_b/q_c \) is obtained as 0.17. This \( q_b/q_c \) value is slightly higher than that of Table 7.4, equal to approximately 0.13 for a relative density of \( D_R = 90\% \).

8.2.4 Calibration chamber plate load tests

A series of 30 calibration chamber load tests were carried out by Ghionna et al. (1994) in sand samples with two different relative densities, dense \( (D_R = 90\%) \) and
medium dense ($D_R = 50\%$). These tests simulate the loading of the base of drilled shafts. Calibration chamber size effects on the values of $q_b$ were shown earlier to be small. Figure 8.1 shows the values of $q_b/q_c$ for the calibration chamber tests corresponding to $s/B = 5$ and 10\%. The cone resistance $q_c$ used to prepare Figure 8.1 was obtained from the penetration resistance analysis of Salgado et al. (1997a, b).

As can be seen in Figure 8.1, the values of $q_b/q_c$ are in the 0.09 – 0.14 range for $D_R = 50\%$ and 0.07 – 0.10 range for $D_R = 90\%$ at $s/B = 5\%$, and in the 0.11 – 0.19 range for $D_R = 50\%$ and 0.10 – 0.14 range for $D_R = 90\%$ at $s/B = 10\%$. Although the results for $D_R = 50\%$ show more scatter than those for $D_R = 90\%$, the average values of $q_b/q_c$ for both relative densities are in reasonable agreement with the proposed results given in Table 7.4.

8.3 Displacement Piles

8.3.1 Purdue University load test

The Purdue University load test was performed on the western side of the Purdue University campus in West Lafayette, Indiana (Goble et al. 1972). This site is located on the edge of a large terrace along the Wabash river with a variable depth of weathered loess covering stratified sand and gravel layers. A 15-m long steel H-pile (10HBP57) was driven using a DELMAG D-12 diesel hammer and load tested. H-piles are sometimes referred to as small displacement piles because their relatively small cross-section does not cause as much disturbance and densification of the surrounding soil during the installation process as concrete or pipe piles would.

The base resistance $q_b$ and relative settlement $s/B$ were calculated based on the equivalent circular base area transformed from the half perimeter area of the H-pile. The obtained $q_b$ values for $s/B = 5\%$ and 10\% were 2.43 MPa and 3.37 MPa, respectively. Because the standard penetration test was used instead of cone penetration test, the cone
Figure 8.1 Values of $q_b/q_c$ in calibration chamber plate tests for (a) $s/B = 5\%$ and (b) $s/B = 10\%$. 
resistance \( q_c \) was estimated from the SPT blow count at the pile base level based on the SPT-CPT correlation of Robertson and Campanella (1983). The estimated \( q_c \) value at the pile base level was about 9.0 MPa. The values of \( q_b/q_c \) corresponding to \( s/B = 5 \) and 10% were 0.27 and 0.37 respectively. These results appear to be near the upper range of the values of \( q_b/q_c \) for displacement piles shown in Table 7.6.

8.3.2 NGI load tests

The NGI load tests were performed on precast-concrete piles driven into a very loose deposit of quite homogeneous sandy soil. The site is located in a small island, Holmen, in the middle of the Drammen river near the city of Drammen, Norway. The soil layer consists of uniform, loose upper sand layer down to 30 m, underlain by a clay layer and finally bedrock. From the geological history of this area, the subsoil condition is believed to be normally consolidated (Gregersen et al. 1973). Four instrumented piles with two lengths, 8 and 16 m, were tested. The four piles are referred to by the letters A, B, C and D in Table 8.1. Piles A and C had the same length (8 m); pile A had a diameter of 28 cm, while pile C was a tapered pile with diameter varying from 28 cm at the pile top to 20 cm at the pile base. Piles B and D had the same length (16 m); pile B had a diameter of 28 cm, while pile D was made by connecting piles A and C, resulting in a pile with a diameter of 28 cm from the top to half the pile length, and then tapered with a diameter of 20 cm at the pile base. The cone resistances at depths 8 m and 16 m were 3.1 MPa and 5.0 MPa. The values of \( q_b/q_c \) for each pile are given in Table 8.1. Although pile D shows an exceptionally high value of \( q_b/q_c \), most of the \( q_b/q_c \) values fall within the 0.29 – 0.32 range for \( s/B = 5\% \) and the 0.36 – 0.39 range for \( s/B = 10\% \). Considering that the soil is in a loose state, these measured values agree well with the values proposed for displacement piles in Table 7.6.
8.4 Summary

In order to verify the normalized base resistance value $q_b/q_c$, several case histories for both non-displacement and displacement piles were examined. For non-displacement piles, the observed results of field pile load tests, finite element analysis, and calibration chamber plate load tests were compared with the $q_b/q_c$ values presented in Chapter 7. Overall, the observed values of $q_b/q_c$ were in good agreement with the proposed values given in Chapter 7.

For displacement piles, 5 pile load tests were investigated. Although the values of $q_b/q_c$ for displacement piles were established based on the results of non-displacement piles, the comparison between the measured and the proposed $q_b/q_c$ values showed reasonably good matches.
CHAPTER 9 PILE DESIGN USING CPT RESULTS

9.1 Introduction

In this chapter, the design of piles using CPT results is discussed. Both base and shaft resistances are addressed. Since the standard penetration test is still widely used in practice, the correlation between the SPT blow count N and the cone resistance $q_c$ is also discussed. A proper SPT-CPT correlation makes it possible for CPT-based methods to be used when only SPT results are available.

A computer program developed for the estimation of pile load capacity based on CPT results is introduced. In this program, base and shaft resistances can be estimated using different methods.

9.2 Determination of Base and Shaft Resistances

9.2.1 Base resistance

As discussed in Chapter 2, there are several methods available for pile design using CPT results (DeRuiter and Beringen 1979, Schmertmann 1978, Aoki and Velloso 1975, Bustamante and Gianeselli 1982). All of these methods define the base resistance in terms of the cone penetration resistance $q_c$ and correlation parameters. Since these methods were developed under different conditions, the selection of the method should be
made with consideration of the differences and recommendations of the methods. The important differences between the methods include (Bandini and Salgado 1998):

(1) the criterion adopted to define pile load capacity;
(2) the type of equipment used to obtain $q_c$;
(3) the selection and relative importance of $q_c$ values above and below the pile base;
(4) soil types and conditions under which the methods were developed.

Regarding the criterion adopted to define the pile load capacity, CPT-based pile design methods do not clearly define the pile load capacity. Although for small-diameter piles the differences in pile capacity are not large, as the diameter increases the way in which load capacity is defined becomes critical. A relative settlement-based criterion for pile load capacity definition is favored, as discussed previously.

Following Franke (1991), piles must typically undergo relative settlements $s/B$ greater than 10% to reach a limit state, including either a loss of functionality of structures or damage to the superstructure and/or foundations. Settlements less than 10% of the pile diameter, in some design situation, may cause the foundations or the supported structures to reach a limit state. Hence, in the evaluation of the value of the relative settlement associated with the limit states design of piles, the type, functionality, location, and importance of structures should also be taken into account.

Figure 9.1 shows a simple example for estimating pile base resistance using CPT results. The pile is a drilled shaft having a length of 10 m and a diameter of 50 cm. The soil is a normally consolidated, medium dense sand. The cone resistance $q_c$ was assumed varying linearly with depth from 1 MPa at the surface to 10 MPa at the pile base level. For these given pile and soil conditions, the base resistances $q_b$ were calculated using different methods.
- Dutch method: \( q_b = \frac{8.2 + 10.9}{2} = 9.55 \) MPa

- Aoki and Velloso’s method: \( q_b = \frac{q_c}{3.25} = \frac{10}{3.25} = 3.07 \) MPa

- LCPC method: \( q_b = k_c \times q_{ca} = 0.4 \times 10 = 4.0 \) MPa

- Proposed method: \( q_b = 0.16 \times 10.9 = 1.74 \) MPa

Figure 9.1 Estimation of pile base resistance using different methods.
For the method proposed in this study, the value of $q_b/q_c$ was equal to 0.16 corresponding to the medium dense condition. Although the profile of cone resistance in this example is extremely simplified, the values of $q_b$ obtained using different methods show a quite wide range. The highest value of $q_b$ equal to 9.55 MPa was obtained from the Dutch method while the lowest one equal to 1.74 MPa was obtained from the proposed method in this study. It should be noticed that the lowest base resistance obtained from the proposed method was based on the relative settlement $s/B$ equal to 10%.

9.2.2 Shaft resistance

The shaft resistance of piles in most cases is fully mobilized well before the maximum base resistance is reached (Franke 1993). It follows that shaft resistance can be estimated with greater simplicity, at least from a conceptual point of view, than base resistance.

Based on the review of the methods in Chapter 2, Aoki and Velloso’s method (Aoki and Velloso 1975) and the LCPC method (Bustamante and Gianeselli 1982) appear to be effective approaches to estimate the shaft resistance using CPT results. Figure 9.2 shows a simple example of estimation of shaft resistance using these methods. Soils consist of two clayey layers and three sandy layers. Pile condition is the same as that for the example of base resistance estimation discussed in the previous section. The representative cone resistance $q_{ci}$ for each sub-layer was obtained from the center of the layer. The calculated values of shaft resistance are given in Figure 9.2.

In Figure 9.2, the shaft resistances of layer 2, 4, and 5 for the LCPC method were taken as the limit values given in Table 2.5 because the calculated shaft resistances were greater than the limit values. The estimated value of the shaft resistance from Aoki and Velloso’s method was significantly lower than that from the LCPC method.
Figure 9.2 Estimation of pile shaft resistance using different methods.

<table>
<thead>
<tr>
<th>Layer</th>
<th>$q_c$ (kPa)</th>
<th>$\alpha$ (%)</th>
<th>$F_2$</th>
<th>$q_{si}$ (kPa)</th>
<th>$q_c$ (kPa)</th>
<th>$k_s$</th>
<th>$q_{si}$ (kPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1900</td>
<td>1.4</td>
<td>6.5</td>
<td>4.1</td>
<td>1900</td>
<td>60</td>
<td>31.6</td>
</tr>
<tr>
<td>2</td>
<td>2500</td>
<td>6.0</td>
<td>6.5</td>
<td>23.1</td>
<td>2500</td>
<td>40</td>
<td>35.0</td>
</tr>
<tr>
<td>3</td>
<td>5500</td>
<td>1.4</td>
<td>6.5</td>
<td>11.8</td>
<td>5500</td>
<td>100</td>
<td>55.0</td>
</tr>
<tr>
<td>4</td>
<td>6000</td>
<td>6.0</td>
<td>6.5</td>
<td>55.4</td>
<td>6000</td>
<td>60</td>
<td>35.0</td>
</tr>
<tr>
<td>5</td>
<td>9100</td>
<td>1.4</td>
<td>6.5</td>
<td>19.6</td>
<td>9100</td>
<td>100</td>
<td>80.0</td>
</tr>
<tr>
<td>$\Sigma$ &amp; 114.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>236.6</td>
<td></td>
</tr>
</tbody>
</table>

B = 50 cm

Drilled Shafts

Aoki-Velloso method

LCPC method
This indicates that Aoki and Veloso’s method produces very conservative results in the estimation of the shaft resistance compared to the LCPC method. The LCPC method was studied by several authors (Briaud et al. 1989, Milovic and Milovic 1993), and was found to be satisfactory.

9.2.3 Factor of safety

The selection of factors of safety in pile design is an important. In general, large factors of safety may be required in the following cases (Canadian Geotechnical society 1992):

(1) friction piles in clay;
(2) sites where only a limited number of tests are performed and where soil conditions are variable;
(3) piles in loose sands and silts for which the capacity may decrease with time.

When the pile load capacity is determined from field tests including in-situ tests and pile load tests, factors of safety in the 2 to 3 range have been proposed (Broms et al. 1988, Canadian Geotechnical society 1992). Table 9.1 shows the values of factor of safety proposed by Canadian Geotechnical society (1992) as a function of the type of field test, based on the ultimate pile load capacity corresponding to an ultimate limit state. In the table, \( f_p \) is referred to as the resistance modification factor representing the ratio of allowable pile load capacity to ultimate pile load capacity.

Some authors have proposed the use of the partial factors of safety. As mentioned earlier, the shaft resistance, in most cases, is fully mobilized before the base resistance reaches a limit state. Based on this observation, the use of separate factors of safety for the base and the shaft resistance is sometimes suggested to artificially account for the different rates of mobilization of shaft and base resistance.
Table 9.1  Resistance modification factor $f_p$ and factor of safety for different field tests (after Canadian Geotechnical Society 1992).

<table>
<thead>
<tr>
<th>Type of filed test</th>
<th>$f_p$</th>
<th>Factor of safety</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cone penetration test</td>
<td>0.5</td>
<td>2</td>
</tr>
<tr>
<td>Standard penetration test</td>
<td>0.3</td>
<td>3.3</td>
</tr>
<tr>
<td>Static pile load test (routine test)</td>
<td>0.5</td>
<td>2</td>
</tr>
<tr>
<td>Static pile load test (high technical level test)</td>
<td>0.6</td>
<td>1.7</td>
</tr>
<tr>
<td>Dynamic analysis using measured data of strain and acceleration</td>
<td>0.5</td>
<td>2</td>
</tr>
</tbody>
</table>
According to Franke (1993), the allowable design pile load capacity is given by:

\[ Q_d = \frac{Q_u}{FS_g} = \frac{Q_b}{FS_b} + \frac{Q_s}{FS_s} \]  \hspace{1cm} (9.1)

where

- \( Q_d \) = allowable design pile load capacity
- \( Q_u \) = unfactored pile load capacity
- \( FS_g \) = global factor of safety
- \( FS_b \) = factor of safety for the base resistance
- \( FS_s \) = factor of safety for the shaft resistance
- \( Q_b \) = base resistance from a limit state
- \( Q_s \) = shaft resistance.

Since \( Q_u = Q_b + Q_s \) \hspace{1cm} (9.2)

the following relationship can be obtained:

\[ FS_g = \frac{FS_b \cdot FS_s \cdot (1 + \frac{Q_b}{Q_b})}{FS_s + FS_b \cdot \frac{Q_s}{Q_b}} \]  \hspace{1cm} (9.3)

If \( FS_g = 2 \) and \( FS_s = 1 \) are assumed, \( FS_b \) is obtained as in Table 9.2. From (9.3) and Table 9.2 it can be seen that for a constant global safety factor no unique value of factor of safety for the base resistance is obtained when partial safety factors are used and \( Q_b/Q_s \) is allowed to vary.
Table 9.2  Partial factor of safety for the base resistance.

<table>
<thead>
<tr>
<th>$Q_s/Q_b$</th>
<th>0</th>
<th>0.2</th>
<th>0.4</th>
<th>0.8</th>
<th>0.6</th>
<th>1.0</th>
<th>&gt; 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_{S_b}$</td>
<td>2</td>
<td>2.5</td>
<td>3.33</td>
<td>5</td>
<td>10</td>
<td>$\infty$</td>
<td>&lt; 0</td>
</tr>
</tbody>
</table>

Since the goal in design is to achieve a target level of safety (say $F_{S_g} = 2$), the use of partial safety factors is yet variable, as fixed values of $F_{S_b}$ and $F_S$ would produce different levels of global safety depending on the relative contribution of base and shaft resistance to pile capacity.

9.3 Use of SPT Blow Counts in CPT-based Method

The method for estimating pile load capacity proposed in this study is based on the cone penetration resistance $q_c$. Although the CPT is a superior test for pile design, SPT is still widely used in many geotechnical engineering projects. If the proper correlation between the SPT blow count $N$ and the cone penetration resistance $q_c$ can be established, the CPT pile design methods can also make use of the SPT blow count $N$ data. This approach may be subjected to more uncertainties, but is believed to be useful in design.

A useful CPT-SPT correlation was proposed by Robertson and Campanella (1983). According to Robertson and Campanella (1983), the ratio of the cone resistance $q_c$ to the SPT blow count $N$ is not a constant, varying with the mean grain size ($D_{50}$). Since considerable scatter was found in the values of the $q_c/N$ ratio for silty clay, they suggested that the $q_c/N$ ratio be used for sandy soils. Figure 9.3 shows the $q_c/N$ ratio with respect to mean grain size.

As can be seen in the figure, the value of $q_c/N$ increases as the mean grain size increases. For sands, the values of the $q_c/N$ ratio were in the 4–6 range while those for silty sand were in the 3–4 range.
Figure 9.3  CPT-SPT correlation with the mean grain size (after Robertson and Campanella 1983).
For further investigation of the value of the $q_c/N$ ratio, both CPT and SPT were performed at the same site. The site was located at US State route 66 over Garvin Street, Evansville, Indiana. The tests were performed under the North East end of the bridge, on the grassy area next to the bridge wall. Both CPT and SPT boring logs indicate that the test site mostly consists of sandy soil down to about 20 m. The ground was very hard to penetrate in the first few feet, making it difficult to anchor down the CPT rig. The starting test depth appeared to be at the same elevation as all the surrounding terrain. No appreciable presence of fill material was observed.

Two sand samples were obtained using the CPT soil sampler. Both samples were taken at a depth of from 14.0 m down to 14.7 m. Some of the standard laboratory tests were performed with these soil samples, including grain size distribution analysis, the maximum and minimum void ratio tests, and the specific gravity test. The mean grain size ($D_{50}$) was about 0.3 mm, and the maximum and minimum void ratios were 0.826 and 0.454, respectively.

Figure 9.4 shows the SPT blow counts $N$ and the cone resistance $q_c$ with depth. The corresponding values of $q_c/N$ for this site are given in Table 9.3. Since the SPT $N$ values in Figure 9.3 were based on the energy ratio of 55%, the values of $N_{60}$, which was corrected for the energy ratio of 60%, were used for the calculation of the $q_c/N$ ratio in Table 9.3. As can be seen, the lowest value of $q_c/N$ was about 1.15 at a depth of 4.33 m and the highest one was about 14.96 at a depth of 13.47 m. Most the $q_c/N$ ratios, however, were observed in the 3 – 8 range. The averaged $q_c/N$ ratio without including the lowest and highest values of $q_c/N$ was approximately 5.63. This value appears to be in reasonable agreement with the value for sand given in Figure 9.3 by Robertson and Campanella (1983). This result is one more indication that the correlation of Figure 9.3 can be used for CPT pile design using the SPT blow count $N$ for Indiana soils.
Figure 9.4  Cone resistance $q_c$ and SPT blow count $N$ with depth.
Table 9.3 Correlation between CPT and SPT.

<table>
<thead>
<tr>
<th>Depth (m)</th>
<th>N</th>
<th>$N_{60}^a$</th>
<th>q$_c$ (bar)$^b$</th>
<th>q$<em>c$/N$</em>{60}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.21</td>
<td>36</td>
<td>33.12</td>
<td>236.9</td>
<td>7.12</td>
</tr>
<tr>
<td>0.98</td>
<td>17</td>
<td>15.64</td>
<td>112.0</td>
<td>7.16</td>
</tr>
<tr>
<td>1.12</td>
<td>40</td>
<td>36.80</td>
<td>120.0</td>
<td>3.26</td>
</tr>
<tr>
<td>2.80</td>
<td>11</td>
<td>10.12</td>
<td>26.7</td>
<td>2.63</td>
</tr>
<tr>
<td>4.33</td>
<td>15</td>
<td>13.80</td>
<td>16.0</td>
<td>1.15</td>
</tr>
<tr>
<td>5.85</td>
<td>10</td>
<td>9.20</td>
<td>49.8</td>
<td>5.40</td>
</tr>
<tr>
<td>7.38</td>
<td>18</td>
<td>16.56</td>
<td>85.0</td>
<td>5.10</td>
</tr>
<tr>
<td>8.90</td>
<td>37</td>
<td>34.04</td>
<td>138.5</td>
<td>4.10</td>
</tr>
<tr>
<td>10.42</td>
<td>40</td>
<td>36.80</td>
<td>55.6</td>
<td>1.51</td>
</tr>
<tr>
<td>11.95</td>
<td>13</td>
<td>11.96</td>
<td>88.7</td>
<td>7.4</td>
</tr>
<tr>
<td>13.47</td>
<td>8</td>
<td>7.36</td>
<td>110.0</td>
<td>14.96</td>
</tr>
<tr>
<td>14.99</td>
<td>33</td>
<td>30.36</td>
<td>146.6</td>
<td>4.8</td>
</tr>
<tr>
<td>16.52</td>
<td>15</td>
<td>13.80</td>
<td>120.0</td>
<td>8.7</td>
</tr>
<tr>
<td>18.04</td>
<td>27</td>
<td>24.84</td>
<td>156.7</td>
<td>6.3</td>
</tr>
</tbody>
</table>

$N_{60}^a$ = SPT N value corrected for the energy ratio of 60%

bar$^b$ = 100 kPa
9.4 Program CONEPILE

For the effective use of the CPT-based pile design method proposed in this study in practice, it was programmed together with other methods discussed herein in program "CONEPILE". Calculation of both base and shaft resistances is addressed in the program. The methods available in the program CONEPILE are:

for the calculation of the base resistance,

   (1) Aoki and Velloso's method;
   (2) LCPC method;
   (3) proposed method;

for calculation of the shaft resistance,

   (1) Aoki and Velloso's method;
   (2) LCPC method.

Program CONEPILE can be used to calculate base and shaft resistances for both non-displacement and displacement piles. Users can select a preferred method to use for a given soil condition and pile type.

The program CONEPILE consists of two different programs: a user-friendly interface program for pre- and post-processing, and the FORTRAN code used in actual calculation. The user-friendly interface program was developed using Visual-C for easy input and output data processing.

For Aoki and Velloso's method and the LCPC method, the correlation parameters given in Tables 2.1, 2.2, 2.5, and 2.6 were included in the program CONEPILE as a form of table. For the calculation of base resistance using the proposed method, regression equations matching with the normalized load-settlement curves shown in Figure 7.9 were used. The regressions were made with fourth order polynomials, and the square of the coefficient of correlation ($r^2$) were about 0.999. Several intermediate steps were also
used in the program. These include the estimation of the relative density, the correction for the coefficient $K_0$ of lateral earth pressure at rest, and the calculation of the base resistance ratio for displacement piles. Since the effect of the pile length or the confining stress at the pile base level appeared to be small (as can be seen in Figure 7.10), it was not considered for calculating the base resistance.

The estimation of relative density was made using the following relationship by Salgado et al. (unpublished paper):

$$q_c = C_1(P_a)^{1-C_2} \sigma_h^2 \exp(C_3 D_R)$$  \hspace{1cm} (9.4)

where $q_c =$ cone resistance; $C_1$, $C_2$, and $C_3 =$ correlation parameters; $P_a =$ reference stress $= 100 \text{ kPa} = 0.1 \text{ MPa} = 1 \text{ tsf}$; $\sigma_h =$ effective horizontal stress; and $D_R =$ relative density. The correlation parameters $C_1$, $C_2$, and $C_3$ are given in Table 9.4. Using (9.4), the relative density can be obtained for a given stress state and cone resistance $q_c$ (which need to be assigned as input data for the program CONEPILE). The estimated relative density is then used for calculating the normalized base resistance $q_b/q_c$.

The correction for $K_0$ was based on the relationship shown in Figure 7.11. Because the effect of $K_0$ was small for relative densities greater than 70%, it was only considered for relative densities less than 70%. Figure 9.5 shows the procedure for the estimation of the base resistance using the proposed method. More detailed information for the program CONEPILE are given in Appendix.
Table 9.4 Correlation parameters for estimation of relative density.

<table>
<thead>
<tr>
<th></th>
<th>Friction angle at the critical state ($\phi_c$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$30^\circ$</td>
</tr>
<tr>
<td>$C_1$</td>
<td>40.0</td>
</tr>
<tr>
<td>$C_2$</td>
<td>0.524</td>
</tr>
<tr>
<td>$C_3$</td>
<td>0.0195</td>
</tr>
</tbody>
</table>
Figure 9.5 Estimation of the base resistance for a given soil condition.
9.5 Summary

In this chapter, the design of piles using CPT results was discussed. Since the CPT methods available for pile design were developed under different conditions, the selection of the method should take into account the differences and recommendations of the methods.

From the example for calculating the base resistance, it was observed that the highest value of $q_b$ was obtained from the Dutch method while the lowest one was obtained from the method proposed in this study. The lowest base resistance obtained from the proposed method was based on a relative settlement $s/B$ equal to 10%. For the shaft resistance, Aoki and Velloso’s method produced lower values compared with the LCPC method. A global factor of safety (and not partial base and shaft factors of safety) was found to be more correct for use in pile design.

For more general use of CPT-based pile design methods, a correlation between the SPT blow count $N$ and the cone resistance $q_c$ can be used. Field SPT and CPT test data for an Evansville sandy soil site suggests the correlation between $q_c$ and $N$ proposed by Robertson and Campanella (1983) is likely applicable to Indiana sandy deposits.

The program CONEPILE developed for the estimation of the pile load capacity was introduced. This program has a user-friendly interface, which controls an underlying FORTRAN-based DLL.
10.1 Summary

Pile foundations are often used for supporting large, heavy superstructures at sites where competitively priced shallow foundations would lead to excessive settlements. Based on the method of installation, piles are classified as either displacement or non-displacement. The bearing capacity of both displacement and non-displacement piles consists of both base resistance and side resistance. The side resistance of piles is in most cases fully mobilized well before the maximum base resistance is reached. As the side resistance is mobilized early in the loading process, the determination of pile base resistance is a key element of pile design.

Pile design methods using in-situ test results have been mainly based on the standard penetration test (SPT) and the cone penetration test (CPT). SPT blow counts result from dynamically driving a standard sampler one foot into the ground. Such a process is not well related to the quasi-static pile loading process. Static cone penetration, on the other hand, is better related to the pile loading process. The test is performed quasi-statically and resembles a scaled-down pile load test. In the present study, in order to take advantage of the CPT for pile design, load-settlement curves of axially loaded piles bearing in sand were developed in terms of normalized base resistance \((q_b/q_c)\) versus relative settlement \((s/B)\). Cone resistance \(q_c\) used to normalize the load-settlement curves was determined from the penetration resistance analysis of Salgado et al. (1997a) using the program CONPOINT. The limit state concept has been proposed as a modern design approach, in which the adequate technical quality of
foundations and superstructures is considered. Although the limit state design concept for pile design has been used mostly with respect to either \( s/B = 5\% \) or \( s/B = 10\% \), the normalized load-settlement curves obtained in this study allow determination of pile base resistance at any relative settlement level within the \( 0 - 20\% \) range. The normalized base resistance for both non-displacement and displacement piles were addressed.

In order to obtain the pile base load-settlement relationship, a 3-D non-linear elastic-plastic constitutive model was used in finite element analyses. The 3-D non-linear elastic-plastic constitutive model takes advantage of the intrinsic and state soil variables that can be uniquely determined for a given soil type and condition. This non-linear stress-strain model represents changes of elastic parameters (the shear modulus \( G \) and the bulk modulus \( K \)) according to the stress level before a failure condition is reached. For the description of failure and post-failure soil response for the three-dimensional stress state, the Drucker-Prager plastic model with non-associate flow rule was used.

Calibration chamber tests have been used to investigate both the load-settlement response of the base of non-displacement piles and cone penetration resistance under a variety of conditions. Because calibration chambers have finite sizes, the possibility of size and boundary effects arises. If size effects are well understood, calibration chambers can be used to experimentally assess pile base resistance under controlled conditions. A series of calibration chamber tests were modeled and analyzed using the finite element approach with the 3-D non-linear elastic-plastic stress-strain model. The analytical results were compared with the measured values of plate resistance in calibration chamber plate load tests. The predicted load-settlement curves showed good agreement with measured load-settlement curves. Calibration chamber size effects were also investigated for different relative densities and boundary conditions using the finite element analysis.

For effective use of the CPT pile design methods in practice, the method proposed in this study and some of existing methods reviewed in study were programmed with the user-friendly interface procedure. This program can be used in practice to estimate pile
load capacity for a variety of pile and soil conditions without significant difficulties for input and output procedure.

10.2 Conclusions

Based on findings of the present study, the following conclusions are drawn.

(1) The values of the parameters $f$ and $g$ for the 3-D non-linear elastic-plastic stress-strain model vary according to the relative density level. As the relative density increases, the value of $f$ decreases while the value of $g$ increases. This indicates that the ratio of the elastic modulus at failure to its initial value is higher for denser than for looser sand, and the rate of degradation of elastic modulus is higher for looser than for denser sand;

(2) Plate unit loads in calibration chamber tests tend to be lower (for BC1, BC4) and higher (for BC2, BC3) than pile base unit loads. The confining stress level at the pile base level also influences calibration chamber size effect. The calibration chamber size effects under BC1 were more pronounced at low confinement, corresponding to shorter piles, while size effects under BC2 were more pronounced at high confinement, corresponding to longer piles;

(3) The value of the normalized base resistance $q_b/q_c$ is not a constant, varying as a function of the relative density, the confining stress, and the coefficient of lateral earth pressure at rest;

(4) The values of the normalized base resistance $q_b/q_c$ for non-displacement piles fall within the 0.07 - 0.13 range for $s/B = 5\%$ and the 0.10 - 0.20 range for $s/B = 10\%$;
(5) The effect of relative density on the normalized base resistance \( q_b/q_c \) was most significant, while that of the confining stress at the pile base level was small. At higher relative densities, the value of \( q_b/q_c \) was smaller (\( q_b/q_c = 0.12 - 0.13 \) for \( D_R = 90\% \)) than at lower relative densities (\( q_b/q_c = 0.19 - 0.2 \) for \( D_R = 30\% \));

(6) The values of the normalized base resistance \( q_b/q_c \) for displacement piles are higher than those for non-displacement piles, showing typically the \( 0.15 - 0.25 \) range for \( s/B = 5\% \) and the \( 0.22 - 0.35 \) range for \( s/B = 10\% \);

(7) The value of the normalized base resistance \( q_b/q_c \) tends to decrease as the coefficient of lateral earth pressure at rest \( (K_0) \) increases. This trend is more pronounced at lower relative densities, and negligible for very dense sand;

(8) The values of the normalized base resistance \( q_b/q_c \) for silty sands are in the \( 0.12 - 0.17 \) range, depending on the relative density and the confining stress at the pile base level. The confining stress is another important factor that influences the value of \( q_b/q_c \) for silty sands. For lower relative density, the value of \( q_b/q_c \) decreases as the pile length increases while that for higher relative density increases;

(9) Field test results of both SPT and CPT performed at the same site show reasonable agreement with the correlation proposed by Robertson and Campanella (1983). This indicates that CPT pile design method can be used with the SPT blow count \( N \) for practical purpose, if the proper value of \( q_c/N \) can be obtained for a given soil condition.
10.3 Recommendations

(1) Since pile design methods using in-situ test results were developed under different conditions, the selection of the method should be made with consideration of the differences between the methods, such as:
- the criterion adopted to define pile load capacity;
- the type of equipment used to obtain $q_c$;
- the selection and relative importance of $q_c$ values above and below the pile base;
- soil types and conditions under which the methods were developed;

(2) Use of global factor of safety is more suited to pile design if a target level of safety is aimed in design irrespective of the relative contribution of base and shaft resistances to total pile capacity;

(3) The relative density is the most important factor influencing the base resistance of piles in sands. It is, therefore, recommended that the estimation of the relative density be made through a reasonable correlation with in-situ tests such as CPT, since it is very difficult to obtain undisturbed granular soil samples;

(4) Piles typically undergo relative settlements $s/B$ greater than 10% to reach a limit state corresponding to either a loss of functionality of structures or damage to superstructures and/or foundations. However, the evaluation of the relative settlement associated with the limit states design of piles should be done with consideration of the type, functionality, location, and importance of structures.
LIST OF REFERENCES


APPENDIX: THE PROGRAM CONEPILE

A.1 Introduction

The program CONEPILE is for calculating the pile load capacity including both the base and shaft resistances. The methods adopted in the program CONEPILE are:

- base resistance: Aoki and Velloso’s method; LCPC method; proposed method (Lee-Salgado method);
- shaft resistance: Aoki and Velloso’s method; LCPC method.

The details of each method can be found in Chapter 2. The program CONEPILE can calculate the base and shaft resistances for both non-displacement and displacement piles. Users can select the method desired to use for a given soil condition and pile type.

The program CONEPILE consists of two different programs: a user-friendly interface program for pre- and post-processing, and the FORTRAN code used in actual calculation. The user-friendly interface program was developed using Visual-C for easy input and output data processing.

A.2 Guidelines for Running CONEPILE

A.2.1 Starting the program

(1) Click the icon named “CONEPILE” in Windows.
(2) Click the “start” button (Figure A-1).
(3) Click the “exit” button if the program needs to be terminated (Figure A-1).
(4) Click the “continue” button after reading the introduction (Figure A-2).
Figure A-1. Start of the program CONEPILE.

Figure A-2. Introduction of the program.
A.2.2 Selection of the method for calculating the base resistance

(1) Click a button corresponding to the method to be used (Figure A-3).

(2) Aoki-Velloso method (Figure A-4):
   - Required input parameters: pile index number; pile diameter (B); cone resistance at the pile base level ($q_c$);
   - Click “pile index” button to find a pile index number for a given pile type;
   - Click “continue” after entering all required parameters.

(3) Lee-Salgado method (Figure A-5):
   - Required input parameters: pile index number; pile diameter (B); representative cone resistance between 0 and 2B below pile base level ($q_c$); effective vertical stress at the pile base level ($\sigma'_v$); coefficient of lateral earth pressure at rest ($K_0$); relative settlement ($s/B$); critical state friction angle for the material where pile base is located ($\phi_c$);
   - Click “continue” after entering all required parameters.

(4) LCPC method (Figure A-6):
   - Required input parameters: index numbers for pile and soil types; pile diameter (B); cone resistance at the pile base level ($q_c$);
   - Click “index box” button to find index numbers for given pile and soil types;
   - Click “continue” after entering all required parameters.
Figure A-3. Selection of the method for calculating the base resistance

Figure A-4. Aoki-Velloso method for calculating the base resistance.
Figure A-5. Lee-Salgado method for calculating the base resistance.

Figure A-6. LCPC method for calculating the base resistance.
A.2.3 Selection of the method for calculating the shaft resistance

(1) Click a button corresponding to the method to be used (Figure A-7).

(2) Aoki-Velloso method (Figure A-8):
   - Required input parameters: pile index number; pile diameter (B); number of layers for shaft resistance calculation; thickness, soil index number, and cone resistance $q_c$ of each sub-layer;
   - Click “pile index” and “soil index” buttons to find pile and soil index numbers for given pile and soil types;
   - Click “continue” after entering all required parameters;

(3) LCPC method (Figure A-9):
   - Required input parameters: pile diameter (B); number of layers for shaft resistance calculation; thickness, LCPC index number, and cone resistance $q_c$ of each sub-layer;
   - Click “LCPC index” button to find index numbers for given pile and soil types;
   - Click “continue” after entering all required parameters;

A.2.4 Output (Figure A-10)

(1) Click “show results” button to get the calculated base and shaft resistance.

(2) Click “end” button to terminate the program.
Figure A-7. Selection of the method for calculating the shaft resistance.

Figure A-8. Aoki-Velloso method for calculating the shaft resistance.
Figure A-9. LCPC method for calculating the shaft resistance.

Figure A-10. Output results.