2004

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MODIFIED WILSON- PLOT TECHNIQUE FOR HEAT EXCHANGER PERFORMANCE: STRATEGIES FOR MINIMIZING UNCERTAINTY IN DATA REDUCTION

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ABSTRACT

The so-called ‘modified Wilson-Plot’ technique is commonly accepted as the preferred method for interpreting air-side heat transfer performance data for liquid- and refrigerant-to-air heat exchangers. Indeed, there are good reasons to believe Wilson-Plot techniques are superior to alternative data interpretation schemes. Unfortunately, Wilson-Plot methods are conceptually and computationally more complex than the other methods and it is not possible to simply extend the single-sample uncertainty analysis of Kline and McClintock to the modified Wilson-Plot technique. A rational approach to uncertainty analysis for the modified Wilson-Plot is not available in the literature, but if this method—accepted as superior to others—is to be widely adopted, a well-defined approach to uncertainty analysis is needed. The purpose of this paper is to provide an uncertainty analysis appropriate for the modified Wilson-Plot technique.

1. INTRODUCTION

In modeling and analysis of heat transfer equipment used in refrigeration and air conditioning systems, often the most important goal is to obtain heat transfer coefficient for processes such as convection, condensation and boiling. Accurate heat transfer coefficients are also required to obtain correlations suitable for design purposes. For making design decisions during design optimization the uncertainties associated with transport coefficients and correlations become important. However, many researchers using modified Wilson-Plot techniques do not report uncertainties in the extracted heat transfer coefficients, instead they simply report measurement uncertainties and energy-balance discrepancies. This situation is partly due to the fact that simple, rational approaches to uncertainty analysis for Wilson-Plot data reduction procedures do not exist in literature. The Wilson-Plot method and its variants are widely used for extracting heat transfer coefficients and exponents for heat transfer correlations, and this paper addresses uncertainty issues pertaining to the modified Wilson Plot as compared to more direct approaches for extracting the heat transfer coefficient.

1.1 Brief Literature Review

Following the work by Wilson (1915) establishing his method for data reduction (now called the Wilson Plot), most early papers on the topic were dedicated to applications and extension of the method. Katz and Geist (1948) used this method to obtain condensing coefficients for finned tubes in a vertical row. Young and Wall (1957) modified the Wilson procedure to develop tube- and shell-side heat transfer coefficient correlations for concentric pipe heat exchangers. The modification mainly consisted of introducing the Sieder and Tate (1936) equation—as apposed to Wilson’s dimensional use of Dittus-Boelter—and extracting the two constants from the slope and ordinate intercept of the Wilson Plot for the outside and inside heat transfer coefficients, respectively. Thus, this approach allowed estimation of two unknown parameters compared to one in the original method. Further modifications allowed for determination of up to three parameters. Briggs and Young (1969) proposed an iterative method (with double linear regression) to determine both heat transfer coefficients without knowing one of the exponents on the Reynolds number. The original method required knowledge of the exponents on the Reynolds number. This modification proves useful if the tube side correlation is not known, as is often the case with compact heat exchangers with proprietary tube internal design. Khartabil et al, (1988) also presented a similar iterative scheme to extend the
method to four or five unknowns. The fifth unknown is the tube wall resistance which may not be known in some tube geometries such as fluted tube. The relative merits and assessment of above mentioned methods has been presented by Shah (1990). It is pertinent to mention that in the modified techniques, equal weight is given to all data points and the issue of experimental uncertainty is not addressed at all. In a somewhat related context, Wojs and Tietze (1997) studied the effects of temperature uncertainty on the heat transfer coefficient calculated using modified Wilson plot method. Their simulations show that a root-mean-square deviation of 0.1K in temperature measurements can introduce an error of ~ 20% on the determined convection coefficient. Deviations of about 1K could render the method useless altogether. Ravi Kumar et al. (2001) found that the modified Wilson-Plot method under predicts the condensing-side heat transfer coefficients by 7.5 – 25% for different fluids compared to respective experimental measurements; however the uncertainty analysis was not presented. Recently, Rose (2004) further amplified the effect of thermal measurement accuracy on Wilson-Plot results and presented general guidelines for assessing the accuracy of data followed by some examples of estimating uncertainty of the condensing coefficient on an internally cooled horizontal tube. Recently, Styrylska and Lechowska (2003) presented the unified Wilson plot method with embedded uncertainty analysis to estimate four unknown parameters of the Nusselt correlations for both fluid streams of a heat exchanger along with their associated uncertainties. The calculation of uncertainties as part of data reduction was an improvement; however, the method is complex to implement and is limited to single-phase flow conditions.

The theory of errors in measurements and their propagation to the computed results is not new and dates back as far as to the work of Airy (1879). The approach to single-sample uncertainty analysis in the engineering literature is found in works of Kline and McClintock (1953) and Moffat (1988). James et al. (1995) present a useful discussion pertaining to issues related to uncertainty analysis in thermal systems design with an application to heat exchanger performance predictions.

1.2 Objectives

Uncertainty analysis is not merely a procedure for providing some measure of the scatter in the results, but it is also a useful tool for “judging” whether the observed scatter was “reasonable”. It may be added that uncertainty analysis should also guide the experimenter in developing strategies for minimizing error in planning experiments— one of the goals of this work. In this paper, we focus in the uncertainties that result from a modified Wilson-Plot approach. The role and merits of Chi-square fitting is explained for calculation of overall uncertainty and strategies for minimizing this overall uncertainty are quantitatively discussed with experimental data and examples. It is observed that distribution of individual uncertainties of data-points and their spatial proximity (on the Wilson Plot) has significant implications on overall uncertainty. The validity and accuracy of the modified Wilson Plot is accessed in general terms and guidelines are presented for planning experiments so as to minimize uncertainty. For simplicity the discussions are limited to single phase flow conditions for both fluids in a gas-liquid heat exchanger; however the conclusions are valid in general and extend naturally to two-phase flows.

2. DATA REDUCTION: SINGLE-SAMPLE UNCERTAINTY

In simple schemes to interpret heat exchanger performance data, the air and refrigerant energy balances are used to calculate a heat transfer rate for the heat exchanger. With a single-phase flow on both the air- and refrigerant-side surfaces the equations take the following forms:

\[ q_{air} = m_{air} c_{p,air} (T_{air,in} - T_{air,out}) , \]  

\[ q_{ref} = m_{ref} c_{p,ref} (T_{ref,out} - T_{ref,in}) , \]  

and

\[ q = \frac{q_{air} + q_{ref}}{2} . \]  

In these equations, the measured mass flow rates for the air and refrigerant streams are \( m_{air} \) and \( m_{ref} \), respectively. The measured air and refrigerant inlet and outlet temperatures are \( T_{air,in} \), \( T_{air,out} \), \( T_{ref,in} \), \( T_{ref,out} \), and \( T_{ref,out} \), the specific heats \( c_{p,air} \) and \( c_{p,ref} \) are taken as known. The overall heat exchanger conductance, \( UA \), is then determined from an \( \varepsilon \)-NTU relation or an LMTD formula, such as...
\[ q = U A F \Delta T_{LM} \]  

with \( F \) known for the particular flow arrangement, and the log-mean temperature difference dependent only on the measured air and refrigerant temperatures. Finally, the air-side heat transfer coefficient, \( h_o \), is determined using the general resistance relation, and the following equations for the total thermal resistance.

\[ \frac{1}{U A_T} = R_{\text{ref}} + R_{\text{cond}} + R_{\text{air}} \]  

where, \( R_{\text{ref}} = \frac{D_i}{k_{\text{ref}} N_{\text{uref}} A_{\text{ref}}} \), with \( D_i \) the inside tube diameter, \( k_{\text{ref}} \) the coolant conductivity, \( A_{\text{ref}} \) the tube-side area, and \( N_{\text{uref}} \) the refrigerant-side Nusselt number is determined from a correlation for the tube-side flow. The conduction resistance \( R_{\text{cond}} \) is usually negligible but can be included with a simple analytical expression, and the air-side resistance is \( R_{\text{air}} = 1/(h_o \eta_o A_o) \) determined by iteration, because the surface efficiency \( \eta_o \) depends on \( h_o \). The air-side resistance can be obtained from the calculated values of the total and refrigerant resistances,

\[ R_{\text{air}} = R_T - R_{\text{ref}} \]  

where the \( R_T = 1/U A_T \). Standard error-propagation analysis leads to

\[ \sigma_{R,air}^2 = \left( \frac{\partial R_{air}}{\partial R_T} \sigma_{R,T} \right)^2 + \left( \frac{\partial R_{air}}{\partial R_{\text{ref}}} \sigma_{R,\text{ref}} \right)^2, \]

Using Equation (6)

\[ \sigma_{R,air}^2 = \sigma_{R,T}^2 + \sigma_{R,\text{ref}}^2. \]

Rearranging gives

\[ \left( \frac{\sigma_{R,air}}{\sigma_{R,T}} \right)^2 = 1 + \left( \frac{\sigma_{R,\text{ref}}}{\sigma_{R,T}} \right)^2. \]

Equation (9) indicates that \( \sigma_{R,air}/\sigma_{R,T} > 1 \), for a non-zero uncertainty in \( R_{\text{ref}} \). Thus, the error in the air-side resistance is greater than that of the total thermal resistance. The significance of the Wilson plot is in reducing the error in \( R_{\text{air}} \) to less than that determined from Equation (9). The error in refrigerant resistance, \( R_{\text{ref}} \), is influenced by the tube geometry and the accuracy of the Nusselt number correlation. Higher values of \( \sigma_{R,\text{ref}} \) result in larger uncertainties in the air-side resistance. If the errors due to the refrigerant side are eliminated, the error in the air-side resistance becomes equal to that of the total resistance.

In the data-interpretation and error-analysis approach outlined above, calculating \( h_o \) and the uncertainty in \( h_o \) is relatively straightforward, because the calculation procedure is simple and depends on direct measurements of flow rate and temperature and uncertainties in those data.

### 3. DATA REDUCTION: MODIFIED WILSON- PLOT METHOD

The modified Wilson-Plot method essentially consists of determining individual heat transfer resistances from an overall resistance by extrapolating the refrigerant-side resistance to zero. In this method, the air-side resistance, \( R_{\text{air}} \), is extracted in a different way. Data are collected by holding the air-side conditions (Reynolds number) fixed, while the tube-side conditions are varied. The resulting data are then plotted as shown in Figure 1, and by extrapolating to \( 1/N_{\text{uref}} = 0 \), \( R_{\text{air}} \) is determined. This method ostensibly removes uncertainties associated with \( N_{\text{uref}} \) by extrapolating the tube-side resistance to zero. From Equation (5) it is obvious that for a negligible conduction resistance, the measured \( 1/U A_T = R_{\text{air}} \) upon extrapolating to \( R_{\text{ref}} = 0 \).

Unfortunately, uncertainties in the Wilson-Plot approach are not as straightforward as in the single sample method. Measurement uncertainties in refrigerant and air temperatures and flow rate have a direct impact on the slope and the ordinate intercept of the Wilson Plot. Moreover, the magnitude and relative distribution of each point on the Wilson plot also affects the uncertainty in the ordinate intercept. Often, experimentalists impose a single slope to
Wilson-Plot data sets to extract $h_o$. It is also unclear what impact such an approach has on uncertainty. The overall impact of all these factors on the uncertainty in the computed $h_o$ has not been quantitatively evaluated or reported in open literature.

![Figure 1. Modified Wilson Plots for a sample heat exchanger at different air-side Reynolds numbers. For a given air flow rate, the air-side resistance, $R_{as}$, is the ordinate-intercept of the straight line](image)

### 4. STRATEGIES FOR MINIMIZING UNCERTAINTY

In this section we consider the various factors which affect the uncertainty of Wilson plot results and also propose methods to estimate the influence of those parameters on overall uncertainty. Once the influence of controlling factors have been quantitatively characterized, the experimenter can make judgments as to data-reduction procedure, i.e., whether to adopt a single-sample or Wilson-Plot approach. In general terms, in all Wilson plot methods the accuracy of results depends on:

1. number of data points for each Wilson line
2. range and spatial separation between each data point (along the abscissa)
3. the single-sample uncertainty for each data point and the distribution of uncertainty
4. validity and accuracy of Nusselt correlation used for one or both fluids
5. number of constants to be determined. Rose (2004) points out that attempting to determine more constants than the accuracy, number and range of the data justify, may give widely erroneous results.

Given a set of observations on $(1/UA_i, 1/Nu_i)$ and the standard deviation ($\sigma_i$) associated with $1/UA_i$, Equation (5) describes the straight line Wilson Plot model to which the data need be fit with two adjustable parameters. Commonly, a simple least-squared-error method is employed to minimize the sum of squares of residuals of $1/UA$.

Although simplistic, the major disadvantage of this approach is assigning the same weight to all points and neglecting individual uncertainties of data points. A more robust and useful fitting procedure is the weighted-least-squared-error or Chi-square fitting which provides a statistical measure of goodness-of-fit. The goodness-of-fit parameter provides a quantitative measure of how well the data fit the model and whether the fit is reasonable or outright wrong. Assuming that the measurement errors are normally distributed, the Chi-square merit function is defined as (Press et al., 1992)

$$\chi^2(a,b) = \sum_{i=1}^{N} \left( \frac{y_i - a - bx_i}{\sigma_i} \right)^2,$$

where $a$ and $b$ are maximum likelihood parameter estimations of ordinate intercept and slope respectively. If the errors are not normally distributed, then the estimations are not maximum likelihood, but may still be useful in a practical sense. Equation (6) is minimized to determine $a$ and $b$ as given below:
\[
a = \frac{S_{xx}S_y - S_xS_{xy}}{\Delta},
\]

and

\[
b = \frac{SS_{xy} - S_xS_y}{\Delta},
\]

where,

\[
\Delta = SS_{xx} - (S_x)^2,
\]

\[
S = \sum_{i=1}^{N} \frac{1}{\sigma_i^2} \quad S_x = \sum_{i=1}^{N} \frac{x_i}{\sigma_i^2} \quad S_y = \sum_{i=1}^{N} \frac{y_i}{\sigma_i^2},
\]

and

\[
S_{xx} = \sum_{i=1}^{N} \frac{x_i^2}{\sigma_i^2} \quad S_{xy} = \sum_{i=1}^{N} \frac{x_iy_i}{\sigma_i^2}.
\]

Considering the data to be independent, the probable uncertainties in the estimates of \(a\) and \(b\) are given by:

\[
\sigma_a^2 = \frac{S_{xx}}{\Delta},
\]

and

\[
\sigma_b^2 = \frac{S}{\Delta}.
\]

The goodness-of-fit (GOF) of the model can be estimated using

\[
GOF = Q\left(\frac{N - 2}{2}, \chi^2\right),
\]

where \(N\) is number of data points and \(Q\) is incomplete gamma function. A detailed discussion of the interpretation and role of GOF in validating this model is presented in Press et al. (1988). It may be pertinent to note that GOF can also be used to ascertain how many data points are necessary for a reasonably good representation of a Wilson Plot using a trial and error procedure. For example if 8 data points are available, GOF can be calculated successively using say 4 points then 5 and so on. If 5 data points yields a reasonably good GOF and no significant improvement is observed with more data points in the Wilson Plot, then the remaining experiments can be performed at this benchmark marked number of data points. It is expected that experiments conducted with higher accuracy will need fewer data points and this procedure can assist in making quantitative decisions about the adequacy of number of data points necessary for a particular experimental set up and conditions.

Now, consider an approach to improve the uncertainty in the air-side resistance using the Wilson-Plot method. The error in \(R_{air}\) as given by Equation (16) and using Equations (13) and (15) can be expressed as

\[
\sigma_{R\_air}^2 = \frac{\sum x_i^2}{\sigma_i^2} \left(\sum \frac{1}{\sigma_i^2} \sum \frac{x_i^2}{\sigma_i^2} - \left(\sum \frac{x_i}{\sigma_i^2}\right)^2\right).
\]

For the special case of constant uncertainty, \(\sigma_i = \sigma_{R\_T}\) for all points,

\[
\sigma_{R\_air}^2 = \sigma_{R\_T}^2 \frac{\sum x_i^2}{n\sum x_i^2 - (\sum x_i)^2} = \sigma_{R\_T}^2 \frac{\sum x_i^2}{n\sum (x_i - \bar{x})^2}.
\]
Thus,

\[
\frac{\sigma_{R,air}}{\sigma_{R,T}} = \left( \frac{1}{n} \sum \left( x_i - \bar{x} \right)^2 \right)^{1/2}.
\] (21)

Equation (21) can also be written as

\[
\frac{\sigma_{R,air}}{\sigma_{R,T}} = \frac{1}{n} + \frac{\bar{x}^2}{n} \sum \left( x_i - \bar{x} \right)^2.
\] (22)

Some measures to reduce the ratio \( \frac{\sigma_{R,air}}{\sigma_{R,T}} \) in Equation (21) are:

1. Increase the range of tube-side flow rates so that \( \sum \left( x_i - \bar{x} \right)^2 \).
2. Use high tube-side flow rates to increase \( \bar{x} \).
3. Increase the number of points obtained.

Note that the Wilson plot is most effective when \( \frac{\sigma_{R,air}}{\sigma_{R,T}} < 1 \). In this case, the errors in air side resistance due to \( \sigma_{R,ref} \) are eliminated, and the errors due to \( \sigma_{R,T} \) are reduced. The ratio \( \frac{\sigma_{R,air}}{\sigma_{R,T}} \) can still be greater than unity, and less than that determined from individual points. Figure 2 (a) shows the effect of increasing the range of tube-side flow rates on \( \frac{\sigma_{R,air}}{\sigma_{R,T}} \) for a fixed number of points; the effect of increasing the average tube-side flow rate is also shown. The effect of collecting more data is shown in Figure 2(b).

![Figure 2](image-url)  
Figure 2. (a) Effect of the Nu\(^{-1}\) range for the same n. The effect of the average Nu is also shown. (b)Figure shows the effect of the number of points.

It is useful to consider whether a preferred distribution of uncertainty among the data points exists, one yielding a lower overall uncertainty in \( a \), i.e. lower \( \sigma_{R,air} \). As it turns out preferred uncertainty distributions do exist. Qualitatively, this can be best understood with reference to Figure 4, a cartoon showing two data points. It can be seen that if the two error bars on the extreme ends of the data set are such that one nearest to the ordinate is smaller, then the uncertainty in \( a \) is smaller than if the error bars are switched. This observation suggests that relative distribution of error bars of data points does effect the overall uncertainty in the ordinate intercept, and hence that of heat transfer coefficient. In order to further assess this idea statistically, we consider two cases. In first case, let \( \sigma_j \) and \( \sigma_i \) be the standard deviations associated with first and second data points with their x-axis coordinates denoted by \( x_1 \) and \( x_2 \) respectively. The overall uncertainty in ordinate intercept is given by Equation (19) and can be written as,

\[
\sigma_{R,air}^2 = \frac{S_{xx}^+}{\Delta^+} = \frac{\sigma_j^2 x_1^2 + \sigma_i^2 x_2^2}{(x_1 - x_2)^2}.
\] (22)

In the second case, let \( \sigma_2 \) be associated with \( x_i \) and \( \sigma_j \) with \( x_2 \). In this case applying Equation (19) gives,
\[ \sigma_{R,\text{air}}^{-2} = \frac{S_{xx}^-}{\Delta} = \frac{\sigma_1^2 x_1^2 + \sigma_2^2 x_2^2}{(x_1 - x_2)^2}. \]  

(23)

Taking a ratio of equation (22) and (23) yields,

\[
\frac{\sigma_{R,\text{air}}^{2+}}{\sigma_{R,\text{air}}^{-2}} = \frac{\left(\frac{\sigma_2}{\sigma_1}\right)^2 x_1^2 + x_2^2}{x_1^2 + \left(\frac{\sigma_2}{\sigma_1}\right)^2 x_2^2}
\]

(24)

Now, by equating equation (24) with unity it is found that for \( \frac{\sigma_{R,\text{air}}^{2+}}{\sigma_{R,\text{air}}^{-2}} < 1 \),

\[ \sigma_2^2 > \sigma_1^2 \text{ (for } \forall x_1 \neq x_2). \]  

(25)

Hence, the overall uncertainty in ordinate intercept can be minimized if the more accurate data points fall closer to the y-axis. In order to check the validity of this assertion for more data points than two, several numerical examples were considered in which the individual errors were distributed with an increasing or decreasing order with respect to the x-axis. In all cases, it was found that the distribution with relatively higher accuracy points near the y-axis was more favorable. While performing wind-tunnel experiments on heat exchangers, typically as the refrigerant flow rate is increased, the temperature difference between refrigerant inlet and outlet decreases, causing the errors due to thermometry to become increasingly important. Hence a compromise is needed: high refrigerant mass flow rates cause large uncertainties as the y-axis is approached, but approaching the y-axis reduces uncertainty (see above).

Another issue often confronting the experimentalist trying to measure heat exchanger performance in a wind tunnel is the choice of tube-side Nusselt-number correlation. The information on uncertainty associated with the correlation is important to calculate the individual uncertainties in the data point. For a round-tube geometry, the well-known Gneilinski correlation is preferred if the flow is maintained in the turbulent regime. In many cases, such as with flat-tube or brazed-plate heat exchangers, the internal tube geometry is complex and there exists a dearth of tube-side performance correlations. In such situations a modified Wilson Plot may not be the best procedure for data reduction and constrained optimization methods as suggested by Styrylaska and Lechowska (2003) may be preferred.

Figure 4. Qualitative representation of effect of uncertainty distribution on the overall uncertainty of the ordinate intercept.

International Refrigeration and Air Conditioning Conference at Purdue, July 10-15, 2004
5. CONCLUSIONS

• The issue of quantitative assessment of uncertainty in data reduction procedures pertaining to Wilson-Plot methods is studied and important experimental factors affecting uncertainty is identified.
• A simple but complete uncertainty analysis for the modified Wilson-Plot method is presented within the framework of a Chi-square method.
• On the basis of the Chi-square analysis, strategies for minimizing the uncertainty in ordinate intercept are presented. In particular it was found that closer proximity of x-axis points to the ordinate and greater relative spread helps improve the error in ordinate intercept.
• The error in ordinate intercept is also minimized if points with smaller errors lie closer to the ordinate than those with larger error. Using this knowledge to design experiments requires a compromise in tube-side mass flow rate.
• Depending on the scatter in the data, in certain situations it is possible for the uncertainty in heat transfer coefficient obtained from a Wilson Plot to be worse than that obtained from single-sample measurement.

REFERENCES


