Homogenized estimates for soft fiber-composites and tissues with two families of fibers

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ABSTRACT

The macroscopic response of hyperelastic fiber composites is characterized in terms of the behaviors of their constituting phases. To this end, we make use of a unique representation of the deformation gradient in terms of a set of transversely isotropic invariants. Respectively, these invariants correspond to extension along the fibers, transverse dilatation, out-of-plane shear along the fibers, in-plane shear in the transverse plane, and the coupling between the shear modes. With the aid of this representation, it is demonstrated that under a combination of out-of-plane shear and extension along the fibers there is a class of nonlinear materials for which the exact expression for the macroscopic behavior of a composite cylinder assemblage can be determined. The macroscopic response of the composite to shear in the transverse plane is approximated with the aid of an exact result for sequentially laminated composites. Assuming no coupling between the shear modes, these results allow to construct a closed-form homogenized model for the macroscopic response of a fiber composite with neo-Hookean phases. A new variational estimate allows to extend these results to more general classes of materials. The resulting explicit estimates for the macroscopic stresses developing in composites and connective tissues with one and two families of fibers are compared with corresponding finite element simulations of periodic composites and with experimental results. Estimates for the critical stretch ratios at which the composites loose stability at the macroscopic level are compared with the corresponding numerical results too. It is demonstrated that both the primary stress–strain curves and the critical stretch ratios are in fine agreement with the corresponding numerical results.