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MULTICYLINDER RECIPROCATING REFRIGERATING COMPRESSOR MODELING

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ABSTRACT

A simulation model for multicylinder reciprocating refrigerating compressors has been developed. In this model, the following concepts have been considered: real gas properties; heat transfer between the gas and the cylinder wall during the working process; heat and mass transfer between the suction gas and the gas in the clearance volume; heat transfer between the gas and plenum wall; gas leakage through the clearance between piston ring and the cylinder wall; and pressure variations in the suction and discharge plenums.

Several efficiencies have been discussed which can be used to evaluate the compressor performance and to optimize the compressor parameters.

NOMENCLATURE

A - area \(M^2\)
C - specific heat \(J/Kg\)
c - valve stiffness \(Kg/M\)
d - cylinder bore \(M\)
e - internal energy \(J/Kg\)
E - excergy \(J/Kg\)
F - heat transfer area or force \(M^2\) or \(N\)
h - heat transfer coefficient \(J/(M^2\cdot{}^\circ{}C)\)
H - valve Lift \(M\)
k - ratio of specific heats
L - connecting rod length M
M - mass Kg
N - the number of valve plates
p - pressure Bar or N/M²
Q - heat flow J
R - gas constant J/Kg⁰C
r - crank radius M
S - stroke M
T - absolute temperature K
t - temperature ⁰C or time S
U - the prepressing length of spring M
V - volume M³
v - specific volume M³/Kg
W - work J
Z - compressibility of gas refrigerant
Nu- Nusselt number
Pr- Prandlt number
Re- Reynolds number
RPM- the speed of the compressor revolution per minute
α - coefficient of flow through the valve
ω - angle velocity of crank shaft Deg/S
ρ - density Kg/(M³)
ε - pressure ratio
φ - the crank angle
λ - the ratio between the crank radius and connecting rod length
Δ - difference
ζ - coefficient of impulsive force

SUBSCRIPTS

b - brake
c - cylinder
d - discharge
e - evaporator
INTRODUCTION

Computer simulation and modeling of the reciprocating refrigeration compressor has been developing rapidly in recent years and has become a useful tool for analyzing and improving compressor performance. Different models for predicting the performance of the compressor have been developed which differ in the completeness of the modeling of the physical processes.

The modeling of a multicylinder refrigerating reciprocating compressor is described in this paper. The interface between the cylinders must be considered in any multicylinder compressor modeling. Since the gas pressure and the temperature in the cylinder will be affected by the pressure and temperature in the suction and discharge plenums.

Computer simulation and modeling of reciprocating compressors is a combination of mathematical modeling and numerical analysis. It is important to use mathematical equations which will represent all physical phenomena occurring in the compressor. Unfortunately, there are some that are difficult to include in our mathematical model, such as: the heat transfer between the gas and the valve passage and the lubricating oil dissolved into and disassociated from the refrigerant. However, it is also possible to neglect some insignificant physical phenomena in order to simplify the program and save computation time.

The numerical simulation method depends upon the model. A fourth order Runge-Kutta method is commonly used to solve the differential equations. This method is used in this program.
There are two kinds of multicylinder reciprocating compressor schemes used in refrigeration systems: the single stage and the multiple stage. In the former case, usually in the one stage refrigeration system, all cylinders are the same bore. In the latter case, usually in the two stage or three stage system, the bores could be different from each other, and have intercoolers connected between the different stage cylinders.

A typical system that was chosen for simulated is the vertical type, two cylinder, single stage, ring valve, reciprocating compressor (Fig.1) having a suction plenum around the cylinder and a discharge plenum above the cylinder head. The crank angle between the two cylinders is 180°.

If the cylinder bore and the stroke of two cylinders are the same, and the angle between two crank is $\gamma$, then the pressure ($P_c$), temperature ($t_c$), and the volume ($V_c$) of the first cylinder at the crank angle $\psi$ equal the $P_c$, $t_c$, $V_c$, of second cylinder at the crank angle $\psi + \gamma$ deg. (in this scheme, $\gamma$ equals 180°). Therefore,

\[
\begin{align*}
P(2)|_\psi &= P(1)|_{\psi + \gamma} = P(1)|_{\psi + 180^\circ} \\
t(2)|_\psi &= T(1)|_{\psi + \gamma} = T(1)|_{\psi + 180^\circ} \\
V(2)|_\psi &= V(1)|_{\psi + \gamma} = V(1)|_{\psi + 180^\circ}
\end{align*}
\]

It can be seen from above equations that all gas properties in the two cylinders will be the same if we compare the gas parameters $P(1)$, $V(1)$, $T(1)$ at the crank angle $\phi$ with the gas parameters $P(2)$, $V(2)$, $T(2)$ at the crank angle $\phi + 180^\circ$ respectively. This method is called the characteristic cylinder method [1,2].

If the cylinder bore and the stroke of two cylinders are not the same, the relationship will be different from above.

In order to use the same mathematical equations in one program, it was necessary take the different cylinders as an array in the two cylinder, single stage case:

\[
\begin{align*}
\text{Crank angle} & \quad \psi(2) = \psi(1) + \gamma \\
\text{Cylinder bore} & \quad D(2) = D(1) \\
\text{Stroke} & \quad S(2) = S(1)
\end{align*}
\]
This method is more general than the characteristic cylinder method.

MATHMATICAL MODEL

1. Cylinder Model

1.1 The energy conservation equation

The first law of thermodynamics applied to the cylinder control volume (Fig. 2) is

\[ \dot{Q}(t) = M(i)C_v(i)\dot{T}(i) + \dot{W}(i) \]  
\[ \dot{v}(i) = \frac{\dot{Q}(i)}{M(i)C_v(i)} - \frac{Z(i)RT(i)}{C_v(i)v(i)} \dot{v}(i) \]

1.2 The cylinder volume equation is

\[ V(i) = \frac{\pi}{4} D^2(i) S(i) \left\{ V_0 + \frac{1}{2} \left[ 1 - \cos(\phi(i)) \right] 
+ \frac{\dot{v}(i)}{8} \left[ 1 - \cos(2\phi(i)) \right] \right\} \]

1.3 Real gas equation

The properties p, v, T, h, s of real gas refrigerants can be determined by either the Martin-Hou equations [3] or the Rumbusch equations [4,5]. They can also be found from other equations [6,7]. After selecting the P, V, T relationship for a given refrigerant, the compressibility coefficient \( Z \) can be found [8].

\[ Z(i) = \frac{p(i) v(i)}{R T(i)} \]

1.4 Heat transfer equation

The equation of heat transfer between the gas and the cylinder wall is

\[ \dot{Q}(i) = h(i)F(i) T_w(i) - \dot{T}(i) \]
The coefficient of heat transfer can be calculated by the correlation

\[ N_u = A \Re B \Pr C \]  

(11)

The constants, \( A, B, C \), of eq. (11) can be found in literature \([9,10]\).

The temperature distribution on the wall depends on the pressure ratio \( (\varepsilon) \), the relative distance \( (\bar{X}) \), and the suction plenum temperature \( (t_{s.pl}) \). There are correlations which can be used to calculated the temperature distribution on the cylinder wall and cylinder head \([10]\).

1.5 Suction gas and cylinder gas mixture

In the suction process, the temperature variation of the gas is caused by two actions; one is the heat transfer between the gas and the cylinder wall, (equation (10)) and the other is the mixing of the suction gas and the cylinder gas.

The mixed gas in the cylinder can be calculated by using the heat and mass balance equations.

\[
\tau(i)^{(2)} = [M(i)^{(1)} T(i)^{(1)} + (M(i)^{(2)} C_p(i)^{(2)} \nonumber \\
\tau(i)^{(2)} T_{s.pl} V (M(i)^{(2)} C_p(i)^{(2)}) ]
\]

(12)

where (1) - refers to the gas state in cylinder(i) before mixture.

(2) - refers to the gas state in cylinder(i) after mixture.

1.6 The equations of valve action

The equations for calculating the suction and discharge processes of the compressor depends the construction type and motion of the valves. In this ring type valve modeling, one degree of freedom is considered.

1.6.1 Mass flow through the valves

Assuming the gas flow as an adiabatic reversible process and using the fluid dynamic equations, gives:

\[
\frac{dM_{s.v}(i)}{d\varphi} = \frac{1}{\omega_{c.v} A_{s.v}(i) \frac{b_s^{(i)}}{b_{s.pl}}} \sqrt{\frac{2 K}{R T_{s.pl}}} \nonumber \\
x \left( \frac{\frac{b(i)}{b_{s.pl}}}{\frac{b_{s.pl}}{b_{s.pl}}} \right)^{2/k} - \left( \frac{\frac{b(i)}{b_{s.pl}}}{\frac{b_{s.pl}}{b_{s.pl}}} \right)^{(\kappa+1)/k}
\]

(13)
Discharge valve
\[
\frac{dM_{dv}(i)}{d\varphi} = \frac{1}{\omega(i)} A_{dv}(i) \dot{p}(i) \sqrt{\frac{2K}{(k-1) R T(i)}} \times \sqrt{\left(\frac{\hat{p}_{el}(i)}{\hat{p}(i)}\right)^{2K} - \left(\frac{\hat{p}_{el}(i)}{\hat{p}(i)}\right)^{(k+1)/K}}
\tag{14}
\]

1.6.2 Valve motion equations

By neglecting the heat transfer during the Runge-Kutta integration, the first law of thermodynamics can be applied.

Considering the gas flow through the valves as an adiabatic process and using continuity of gas flow, gives for the valves:

Suction valve
\[
\frac{d\psi_{sv}(i)}{d\varphi} = \frac{-K}{\omega(i) V(i)} \left[ \psi_{sv}(i) \frac{w(i) dV(i)}{d\varphi} \right] - \sqrt{\frac{2RK}{k-1} \frac{\psi_{sv}(i) \frac{w(i) dV(i)}{d\varphi} \sqrt{1 - \frac{\psi_{sv}(i)^{(k-1)/K}}{(k-1)/K}}}{\psi_{sv}(i) \frac{w(i) dV(i)}{d\varphi} \sqrt{1 - \frac{\psi_{sv}(i)^{(k-1)/K}}{(k-1)/K}}}}
\tag{15}
\]

Discharge valve
\[
\frac{d\psi_{dv}(i)}{d\varphi} = \frac{-K}{\omega(i) V(i)} \left[ \psi_{dv}(i) \frac{w(i) dV(i)}{d\varphi} \right] + \sqrt{\frac{2RK}{k-1} \frac{\psi_{dv}(i) \frac{w(i) dV(i)}{d\varphi} \sqrt{1 - \frac{\psi_{dv}(i)^{(k-1)/K}}{(k-1)/K}}}{\psi_{dv}(i) \frac{w(i) dV(i)}{d\varphi} \sqrt{1 - \frac{\psi_{dv}(i)^{(k-1)/K}}{(k-1)/K}}}}
\tag{16}
\]

1.6.3 Valve force equations

From a force balance on the valve, the inertia force equals the applied force of the gas minus the force of spring
\[
\ddot{H} = \frac{1}{M_v(i)} \varepsilon(i) A_v(i) \dot{A}_v(i) - \frac{1}{M_v(i)} C_v(i) [\dot{H}(i) + \dot{U}(i)]
\tag{17}
\]

Assuming the ratio of the gas pressure in the cylinder and the gas pressure in the plenum can be expressed as \(\Phi\), gives for the:

Suction valve
\[
\frac{d^2H_{sv}(i)}{d\varphi^2} = \frac{(1 - \psi_{sv}(i)) \hat{p}_{el}(i) \varepsilon_{sv}(i) A_{sv}(i) C_{sv}(i) (H_{sv}(i) + U_{sv}(i))}{M_v(i) \omega^2(i)}
\tag{18}
\]

and for the discharge valve
\[
\frac{d^2H_{dv}(i)}{d\varphi^2} = \frac{(\psi_{dv}(i) - 1) \hat{p}_{el} \varepsilon_{dv}(i) A_{dv}(i) - C_{dv}(i) (H_{dv}(i) + U_{dv}(i))}{M_v(i) \omega^2(i)}
\tag{19}
\]
1.7 Gas leakage equation

Several references can be used for estimating the gas leakage as in [11,12,13].

The result of the calculations shows that the leakage gas is much smaller than the gas in the cylinder. In this case, it can be neglected.

2. Plenums Model

2.1 The energy conservation equation

2.1.1 Suction plenum

\[ \frac{dT_{s.pl(i)}}{d\phi} = - \frac{1}{M_{s.pl(i)} C_v(i)} \left[ C_p(i) \left( \frac{dM_{in}}{d\phi} \right) - \frac{\sum dM_s(i)}{T_{s.pl}} \right] + \frac{dQ(i)}{d\phi} \]  \hspace{1cm} (20)

2.1.2 Discharge plenum

\[ \frac{dT_{d.pl(i)}}{d\phi} = - \frac{1}{M_{d.pl} C_v(i)} \left[ C_p(i) \left( - \frac{dM_{out}}{d\phi} \right) - \frac{\sum dM_d(i)}{T_{i(i)}} \right] - \frac{dQ(i)}{d\phi} \]  \hspace{1cm} (21)

2.2 Mass flow through plenums

2.2.1 Suction plenum

\[ \frac{dM_{in}}{d\phi} = \frac{1}{\omega(i)} \cdot d_{s.pl} A_{s.pl} \int \frac{P_s - P_{s.pl}}{A_{s.pl}} \]  \hspace{1cm} (22)

2.2.2 Discharge plenum

\[ \frac{dM_{out}}{d\phi} = \frac{1}{\omega(i)} \cdot d_{d.pl} A_{d.pl} \int \frac{P_{d.pl} - P_d}{A_{d.pl}} \]  \hspace{1cm} (23)

THE COEFFICIENT OF PERFORMANCE

Several efficiencies 14 can evaluate the compressor performance from different points of view:

1. Isentropic Efficiency

\[ \eta_s = \frac{W_{isent}}{W_{actual}} \]  \hspace{1cm} (24)

where \( W_{isent} \) is the isentropic indicated work, and \( W_{actual} \) is the actual indicated work calculated from
the \( P - \phi \) diagram.

2. Coefficient of Performance

\[ \text{COP} = \frac{Q_E}{W_b} \]  

(25)

where \( Q_E \) is the refrigerating capacity of the compressor at the given rating and \( W_b \) is the work into in the compressor at the same rating.

3. Exergy Efficiency

\[ \eta_e = \frac{E_e - E_s}{W_{\text{actual}}} \]  

(26)

where \( E \) is the exergy which was calculated by the real gas equations.

4. Volumetric Efficiency

\[ \eta_v = \frac{\dot{M} \cdot V}{V_s} \]  

(27)

THE NUMERICAL SOLUTION

A fourth order Runge-Kutta integrating method was used for solving the above differential equations. Several subroutines were used in this program, such as the real gas properties subroutine, the suction and discharge processes subroutine, the re-expansion and compression subroutine and the gas leakage subroutine. The heat transfer equation where included in the re-expansion and compression subroutine, the process of suction gas and cylinder gas mixture and the process of heat transfer during the suction and the discharge process were included in the program.

The example compressor chosen to simulate is a two cylinder, single stage, ring valve, R12 refrigerating compressor, \( D=0.1 \) M, \( S=0.07 \) M, RPM=1440 R.P.M., the ratio between the clearance and the cylinder volume equals 0.04, the evaporating pressure \( p_e=1.825 \) Bar, \( (t_e=-15^\circ C) \), the condensing pressure \( p_k=8.453 \) Bar, \( (t_k=35^\circ C) \).

Fig.3 - Fig.7 show the \( p - \phi \) diagram, \( T - \phi \) diagram, \( M - \phi \) diagram and \( H - \phi \) diagram, which were plotted by computer.

It can be seen by comparing the Fig.3 and Fig.7 that the pressure variation will effect to the \( p - \phi \)
diagram, and also effect to the efficiency of compressor.

CONCLUSIONS

1. A computer modeling for the multicylinder refrigerating reciprocating compressor has been developed. The use of real gas properties produced results closer to the real processes.

2. The general cylinder method is necessary for modeling a multicylinder compressor which has different cylinder diameters.

3. The gas parameters in the cylinder and the efficiencies are effected by the gas parameters in the suction and discharge plenums.

4. Further investigations should be done which refer to developing the mathematical equations of the heat transfer between gas and valve passage, developing the mathematical equations described the mechanism of refrigerant solubility in oil during the working processes, and developing the equation described the compressor mechanical efficiency at different rating.

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Fig. 1 Two Cylinders, One Stage Compressor Scheme (The Crankangle Between Two Cylinders is 180°)

Fig. 2 The Cylinder Control Volume, Suction Plenum Control Volume and Discharge Plenum Control Volume.
Fig. 3 The $P-\psi$ Diagram

Fig. 4 The $T-\psi$ Diagram
Fig. 5 The M - ϕ Diagram

Fig. 6 The Suction and Discharge Valve Displacements
Fig. 7. The $P - \psi$ Diagram ($P_{\mu} = \text{const}$)