Final Report

FHWA/IN/JTRP-2004/30

DAMAGE ANALYSIS OF JOINTED PLAIN CONCRETE PAVEMENTS IN INDIANA

PART I
FINITE ELEMENT MODELING AND DAMAGE ANALYSIS

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Project No. C-36-46U
File No: 5-11-21
SPR-2643

Prepared in Cooperation with the
Indiana Department of Transportation and the
U.S. Department of Transportation
Federal Highway Administration

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Purdue University
West Lafayette, Indiana
August 2005
Damage Analysis of Jointed Plain Concrete Pavements in Indiana

Introduction

Current design procedures for concrete pavements do not account for several factors that can influence their service life. Such factors include the consideration of nonlinear thermal gradients through the depth of the slab and nonlinear damage accumulation. Furthermore, current design is based on the assumption that the cause of failure of pavements is top-to-bottom cracking. However, recent research has shown that bottom-to-top crack can also play an important role on the service life of concrete pavements.

The objectives of this study are twofold. First, all factors that negatively affect the service life of concrete pavements are investigated and the findings are integrated into a procedure for better predicting long-term performance of concrete pavements. Second, the developed procedure is implemented into a software tool, termed INDISLAB, for use by concrete pavement designers.

The analysis procedure is developed with the aid of sophisticated finite element techniques and a series of parametric studies. The findings from the analysis module are integrated into a nonlinear procedure for damage accumulation. In the development of the comprehensive 3D Finite Element (FE) model several issues are studied including the geometry of the model, mesh refinement, element selection, interaction between pavement components, and loading simulation. The developed model is then used in a number of parametric studies to investigate the effect of soil conditions, subbase and slab thickness, and slab length and stiffness on the developed stresses.

The software tool, INDISLAB, implements the findings from the analysis phase of the research. The goal of this tool is to help engineers better predict the behavior of jointed plain concrete highway pavements. INDISLAB is created to include a user-friendly Graphical User Interface (GUI) designed for INDOT’s utilization in regards to design of concrete highway pavements. The GUI’s main purpose is to serve as an interface to the finite element method software package chosen for this project, ANSYS. INDISLAB takes user provided information about the concrete slabs to create a finite element model, analyze it with ANSYS, and display the results.

Findings

Among other findings, it is established that for a given slab length, increasing the slab thickness beyond a certain limit is not justifiable. The developed FE model is also used to investigate the behavior of skewed concrete pavement slabs under several loading conditions. In particular, the crack patterns obtained from the FE analyses are compared to those observed in an actual skewed concrete pavement. It is found that the developed FE model is able to successfully predict the cause and orientation of the failure of this pavement section. An investigation of various existing fatigue equations is also carried out and a software tool is developed to perform both linear and nonlinear damage accumulation calculations. A case study of a pavement section on Interstate 70, which has failed prematurely, is created using the previously developed finite element techniques. The resulting stresses from the finite element analyses under various loading conditions are then used in the damage analysis of the pavement section. It is predicted that, irrespective
of how the damage is accumulated, the pavement should have failed at an early age. Nonlinear damage accumulation predicted that the failure would occur at an earlier age than linear damage accumulation, which is consistent with the observed behavior of the pavement section.

INDISLAB is developed as an easy-to-use interface for practicing engineers that makes use of 3D complex finite element techniques. The GUI has been thoroughly tested and has been found to be problem free. The layout of the GUI is easy to understand and aesthetically pleasing. The INDISLAB program provides a simple process for engineers to input information about a slab layout, analyze the model and view results of the solution to determine the durability of the slab(s). Unlike previously developed software, INDISLAB includes the ability to analyze pavements under nonlinear thermal loads and it is capable of accumulating damage nonlinearly.

**Implementation**

The developed software tool, INDISLAB, is expected to aid designers better understand the behavior of concrete pavements and, thus, helping produce designs of longer lasting concrete pavements. INDOT Research Division will readily use the developed software tool to analyze current pavements and influence future concrete pavement designs.

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### Abstract

Current design procedures for concrete pavements do not account for several factors that can influence their service life. In this work, these factors are investigated and the findings are integrated into a procedure for better predicting long-term performance of concrete pavements. To achieve this, sophisticated finite element techniques are employed and parametric studies are performed. The findings are then integrated into a nonlinear procedure for damage accumulation. In the development of the comprehensive 3D Finite Element (FE) model several issues are studied including the geometry of the model, mesh refinement, element selection, interaction between pavement components, and loading simulation. The developed model is then used in a number of parametric studies to investigate the effect of soil conditions, subbase and slab thickness, and slab length and stiffness on the developed stresses. Among other findings, it is established that for a given slab length, increasing the slab thickness beyond a certain limit is not justifiable. The developed FE model is also used to investigate the behavior of skewed concrete pavement slabs under several loading conditions. In particular, the crack patterns obtained from the FE analyses are compared to those observed in an actual skewed concrete pavement. It is found that the developed FE model is able to successfully predict the cause and orientation of the failure of this pavement section. An investigation of various existing fatigue equations is also carried out and a software tool is developed to perform both linear and nonlinear damage accumulation calculations. A case study of an pavement section on Interstate 70, which has failed prematurely, is created using the previously developed finite element techniques. The resulting stresses from the finite element analyses under various loading conditions are then used in the damage analysis of the pavement section. It is predicted that, irrespective of how the damage is accumulated, the pavement should have failed at an early age. Nonlinear damage accumulation predicted that the failure would occur at an earlier age than linear damage accumulation, which is consistent with the observed behavior of the pavement section.
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CHAPTER 1 INTRODUCTION

1.1 Background

Portland cement concrete slabs are widely used in highway pavements in many states in the United States. In the state of Indiana several of these pavement sections have experienced premature cracking. This has had a negative financial impact in the state’s economy. Thus, it is imperative that the behavior of these pavements be better understood and that design procedures be developed that can produce pavements with longer life expectancy.

The stresses in a pavement can be evaluated either by closed-form equations, influence charts, or finite element analyses. In the last two decades, the latter has become more accepted in pavement research and design. A popular and readily available pavement analysis program, currently used by the Indiana Department of Transportation (INDOT), is KENSLABS (Huang 1985). Other similar programs are also available, such as ILLI-SLAB (Tabatabaie, Barenberg 1979). The engines of these programs were developed in 1970’s and, thus, use several simplified assumptions to reduce computational time and memory requirements. More modern commercial finite element software packages have also been used to more accurately analyze concrete pavements.

In service stresses in concrete pavements are lower than concrete’s tensile strength. Thus, their failure can be attributed to fatigue. Consequently, in the design of these pavements the stresses produced by the different loadings must be evaluated and the allowable number of load repetitions to failure must then be calculated by means of a fatigue equation. The design life of a pavement can then be predicted by adding up the damage in concrete.

Many researchers have tried to establish a relation between applied stress and the number of load repetitions to failure in concrete beams. These equations have been
consistently improved to consider more factors such as stress range, rate of loading, material properties, and probability of failure. However, these equations have mainly been used in research and a comprehensive equation that can be used in concrete pavement design is still not available.

To predict the service life of a pavement, the damage ratio, which is the ratio between the predicted and allowable number of repetitions, is computed for different load groups at different time periods and summed over the year linearly. The service life of concrete is then defined as the number of years for which the damage ratio adds up to one.

1.2 Problem Statement

Concrete pavements in Indiana have experienced premature random transverse mid-panel cracking. This phenomenon has been observed under a variety of environmental and traffic conditions. In some cases the cracking has occurred within months of construction and opening of the lanes to traffic. The repair of the cracked pavements is expensive, time consuming, and causes severe traffic disruption. A survey from the states, which have sponsored research projects related to cracking of Jointed Plain Concrete Pavements (JPCP), showed that there is no one clear factor that can be identified as the major cause of transverse cracking of these pavements. According to most researchers, the combined mechanisms of curling of the concrete slab due to temperature gradients, and fatigue due to repeated traffic loads, has lead to the occurrence of the transverse cracks. In addition, the improper control of shrinkage of concrete at early stages of construction is also cited as an important cause of cracking in JPCP. In an attempt to rectify the problem, INDOT has made adjustments to the geometry of the concrete slabs and to the materials used for the slab and subbase. However, it has been observed that these changes have not always prevented mid-panel cracking. Thus, it is imperative that the factors involved in this phenomenon be identified and that an improved design procedure be devised.
As mentioned previously, currently available FE programs, which are widely used to analyze pavements, use several simplifying assumptions in an attempt to save computing time and memory requirements. In particular, in these programs, the sub-base/sub-grade model is greatly simplified by nonlinear springs, and thus they cannot develop friction. In addition, the slab is modeled using thin shell elements, which can only incorporate linear thermal gradients through the depth of the slab. Finally, their dowel bar model does not include the effect of binding due to curling of the slab caused by thermal changes. The interactions between dowel bar and concrete slab and the between adjacent slabs is highly simplified in these programs. More sophisticated modeling techniques, must thus, be adopted to better capture the behavior of concrete pavements.

In terms of the current design methods, the available fatigue curves ignore the effect of stress range or rate of loading and as a result they produce unrealistic results. In addition, when adopting linear cumulative damage, the effect of sequence and amplitude of stresses are not considered. This may produce unconservative designs in certain situations. Therefore, there is a pressing need for improved design methods that can provide better estimates for the service life of pavements and, consequently, more reliable rehabilitation schedules. Such an improvement requires the ability to perform accurate finite element analyses of JPCP subjected to various wheel loads, subbase/subgrade conditions, environmental conditions, and dowel bar interactions. Finally, to accurately estimate pavement life expectancy, the developed stresses must be used in conjunction with an appropriate fatigue equation and damage must be accumulated nonlinearly.

1.3 Objective

The primary objectives of this study are to identify the main factors affecting mid-panel cracking in JPCP and to develop improved tools for better predicting the service life of these pavements. This is achieved by developing improved finite element models, which are analyzed under various conditions. The reliability of these models is examined through case studies of actual pavements sections.
1.4 **Approach**

The research was divided into main two phases. In the first phase a comprehensive FE model was developed and analyzed. The input data in this phase included pavement geometry, material properties, and environmental conditions, and the output data were the stresses developed in the pavement. In the second phase a thorough survey of the available fatigue equations for concrete and of the methodologies for accumulation of damage in concrete pavements was performed. The input data in this phase were the stresses obtained in the first phase and the output was the expected service life of the concrete pavement.

In the first phase, several aspects affecting the accuracy of the developed FE model were studied. The main concern in this phase was to develop the most accurate model possible while minimizing the number of degrees of freedom. As a first step, the main issues concerning the modeling of dowel bars, component interactions, subgrade and subbase modeling, and aggregate interlock were identified and appropriate decisions were made. The next step in this phase consisted of determining how to accurately apply traffic and environmental loads. This is a major issue since due to environmental loading the concrete slab may curl and lose full contact with the subbase. Consequently, in order to properly evaluate the stresses, a nonlinear combination of traffic and environmental loads is necessary. In addition, to create realistic climatic data, the Integrated Climatic Model (ICM) program was used and shrinkage was modeled by means of an equivalent temperature gradient through the depth of the concrete slab. Equivalent tire loads were also calculated and the effect of footprint shape, slab element types, and mesh size in the developed traffic stresses were studied. In particular, a skewed concrete pavement in northern Indiana was used as a case study to verify the developed FE model. More specifically, the transverse cracking patterns obtained from the FE analyses were compared to those observed in the field with good agreement. In the final step of phase one, several parametric studies were performed to investigate the effect of stiffness, depth, and length of both subbase and concrete slab on the developed stresses for both cases of positive and negative temperature gradient combined traffic loading. In
particular, optimal values of slab thickness under different conditions were determined. Finally, the effect of pumping in the developed the stresses was also investigated.

In the design of pavements, it is generally assumed that in cases where the traffic loading governs pavement behavior, the minimum stresses are close to zero. However, the results from the simulations performed in the first phase of this study revealed that this is not true when significant thermal loadings are present. Consequently, to accurately predict the service life of concrete pavements, there is a need to use a fatigue equation that considers the effect of range of loading. In the first step of phase two, a comprehensive literature review of available fatigue equations and damage accumulation techniques was performed. Different equations were compared and their ranges of applicability were identified. Furthermore, a procedure for the nonlinear accumulation of damage was selected from the experimental results of two different research works. This procedure was then implemented into a software tool using Visual Basic. In the final step of this phase, a case study of a pavement section on I-70 was developed. Various loading conditions were considered and both linear and nonlinear damage accumulation were calculated using the developed tool. The impact of using different fatigue equations and the difference between linear and nonlinear damage accumulation were thoroughly investigated.

1.5 Organization

This report contains eight chapters. The first chapter provides introductory remarks about the research. The second chapter discusses issues related to the development of the finite element model, while the third chapter provides an in-depth discussion of the modeling decisions used for the applied loads. In Chapter 4, the various performed parametric studies are described, with which the effect of stiffness, depth and length of both the subgrade and concrete slab on the developed stresses were investigated. Furthermore, it also presents the results on the effect of pumping on the stresses. The fifth chapter discusses the effect of each loading on the stresses developed in the concrete slab through a case study. The sixth chapter provides a comprehensive literature review and a critique of the available fatigue equations for concrete. In addition, in this chapter, linear
and nonlinear techniques for accumulating damage in concrete pavements are presented. In the seventh chapter, a case study is developed. The adequacy of each fatigue equation is discussed, and the results of the two types of damage accumulation are presented. Finally, Chapter eight summarizes the findings and conclusions from this research. It also provides recommendations related to the improvement of the service life of concrete pavements.
CHAPTER 2 FINITE ELEMENT MODELING

2.1 Introduction

To evaluate the service life of concrete pavements, an accurate procedure to calculate the stresses is necessary. The first step of the procedure adopted for this calculation consists of developing an accurate and reliable FE mesh. This implies that appropriate element types, material models, and dimensions are defined at this stage. The second step consists of preparing appropriate loading data and determining suitable boundary conditions. Nonlinear temperature gradient, concrete shrinkage, and traffic loadings are considered in this step. The final step of the procedure involves solving the model, calculating the stresses, and interpreting the results. In this chapter the first step of the adopted procedure is discussed.

Slabs, subbase, subgrade, dowel bars and tie bars are the main parts of a concrete pavement. Two issues were addressed during the selection of the elements to model these components. First, the element types were investigated. Finite element meshes using 20-node and 8-node solid brick elements were considered for modeling the slabs, subbase and subgrade. Initially, 20-node solid elements were selected, since a coarser mesh of these elements provided accurate responses. However, when models involving more than three slabs were tested, it was found that due to memory limitations, it was more appropriate to use a combination of 20-node and 8-node solid brick elements. That is, the higher order elements were used where the stresses were evaluated. The mesh refinement was carefully considered in all cases, in order to achieve the required accuracy. This was done by evaluating the results for different levels of mesh refinement, which led to the optimum number of elements in each case.

Second, the appropriate material models were investigated. In particular, the need for use of nonlinear elements was studied. Through numerical simulations, it was found that the maximum compressive and tensile stresses in the slab were well below concrete’s...
yield stress and tensile strength, respectively. Thus, the use of nonlinear elements was found to be unjustified and linear elements were adopted.

The next step in the development of the finite element model addressed the interactions between the different components of the pavement. First, the concrete slab-subbase interaction was addressed. Thermal effects can cause concrete slabs to curl up and lose contact with the sub-base. In order to model this, different nodes were used for slab and sub-base at the interface and contact elements were adopted to model the interaction between concrete slab and subbase. Second, the interaction between adjacent slabs was addressed. Two adjacent slabs act as separate entities, and their interaction occurs through aggregate interlock and dowel or tie bars. Aggregate interlock was modeled by placing several non-linear springs between adjacent slabs. Dowel bars and tie bars were modeled using beam elements, which were connected to the slab also by means of nonlinear springs. Finally, the interaction between subgrade and subbase was addressed. Since no separation is expected between sub-grade and sub-base, interface nodes were shared by both mediums. It should be noted, though, that the two layers might have different material properties. In the following subsections the various modeling decisions adopted in the developed finite element model are discussed in detail.

2.2 Dowel Bars

As the length of the slabs increase the thermal stresses also increase. In practice, two methods are used to prevent thermal cracking. The first method consists of using steel reinforcement and the second method consists of introducing joints. The latter is the most common in Indiana, and it is thus considered in this study. In this case when traffic load is applied close to a joint, the slab deforms and a vertical gap is formed between adjacent slabs. Dowel bars and aggregate interlock transfer the shear forces to adjacent slabs, thus decreasing differential settlement. This eliminates the impact when a vehicle goes from one slab to the other. In addition, the stresses are reduced as the load is distributed between two slabs instead of one.
2.2.1 Theoretical Model for Dowel and Tie Bars

Porter et al. (2001) give an overview of the state-of-knowledge on theoretical modeling of dowel bars. Based on their report, Timoshenko’s (1925) model of a beam on an elastic foundation can be used to simulate the behavior of a dowel bar in concrete. The deflection of a beam on an elastic foundation can be found solving the following differential equation:

\[
EI \frac{d^4 y}{dx^4} = -ky
\]

Equation 2-1

Where \( k \) is the stiffness modulus of the foundation, \( I \) is the Moment of Inertia, \( E \) is the modulus of elasticity, and \( y \) is the beam deflection. Frieberg (1940) suggested the following equation for the deflection on the face of a joint in a beam with semi-infinite length.

\[
y_0 = \frac{P_t}{4\beta^3EI} (2 + \beta z)
\]

Equation 2-2

where,

\[
\beta = \sqrt[4]{\frac{K_0 b}{4EI}} = \text{relative stiffness of dowel bar encased in concrete (in.}^{-1}) \text{) Equation 2-3}
\]

\( K_0 \) = modulus of dowel support (pci)
\( b \) = dowel bar width (in.)
\( E \) = modulus of elasticity of dowel bar (psi)
\( I \) = moment of inertia of the dowel bar (in.\(^4\))
\( P_t \) = load transferred through the dowel bar (lb.)
\( z \) = joint width (in.)

Porter et al. (1999) showed that Frieberg’s equation can be used with little to no error if the \( \beta L \) value is greater than 2, where \( L \) is the length of dowel bar in each section.
In a further development, Davis (2001) calculated the relative deflection of adjacent slabs, based on Figure 2-1, as

\[
\Delta = 2y_0 + z\left(\frac{dy_0}{dx}\right) + \delta + \frac{P_\tau z^3}{12EI}
\]

Equation 2-4

where \( y_0 \) = deflection at the face of the joint

\[
\delta = \frac{\lambda P_\tau z}{AG} = \text{Shear deflection}
\]

Equation 2-5

\( P_\tau \) = load transferred by dowel bar (lb)

\( \lambda \) = form factor, equal to 10/9 for solid circular section

\( A \) = cross sectional area of the dowel bar (in\(^2\))

\( G \) = shear modulus (psi)

In case of a small joint width, the deflection due to the slope of the dowel bar and the flexural deflection of the dowel bar between the joints are both negligible. This is because the joint width term in the equation is to the cubic power. In this case the deflection can be obtained as

\[
\Delta = 2y_0 + \delta
\]

Equation 2-6

When the stress caused by the contact between concrete slab and dowel bar is more than concrete’s bearing stress, the concrete around the dowel bar crushes and a void is created. As a result, the dowel bar no longer not act as per design and the load is not
completely transferred to the adjacent slab. This leads to more deflection of the loaded slab thus resulting in differential settlement between the two slabs.

The bearing stress at the face of the joint can be expressed as

\[ \sigma_b = K_0 y_0 \]  

Equation 2-7

Based on ACI code, the allowable bearing stress on concrete is equal to

\[ \sigma_a = \left( \frac{4 - b}{3} \right) f'_c \]  

Equation 2-8

where \( \sigma_a \) = allowable bearing stress

\( b \) = dowel bar width (in)

\( f'_c \) = compressive strength of concrete (psi)

Wheel loading is distributed among the dowel bars close to the applied load. Frieberg (1940) suggested that the dowel bars outside a radius of \( 1.8 l_r \) from the applied load do no affect the stresses and a linear stress distribution can be assumed inside this area. The parameter \( l_r \) is called the radius of relative stiffness, and is given by

\[ l_r = \frac{4 E_c h^3}{12(1 - \mu^2) K} \]  

Equation 2-9

where \( E_c \) = modulus of elasticity of the pavement concrete (psi)

\( h \) = pavement thickness (in.)

\( \mu \) = Poisson's ratio for concrete pavement

\( K \) = modulus of sub-grade reaction (pci)

Recent investigation by Tabatabaie et al. (1979) showed that an effective length of \( 1.0 l_r \) is more appropriate for recent pavements and dowel bars. They also showed that a linear distribution of shear stress could be observed under loading, as shown in Figure 2-2. The maximum shear load, which occurs beneath the applied load, can be calculated as

\[ P_c = \frac{P_w}{N_{\text{eff}}} \]  

Equation 2-10

where \( P_c \) = load transferred across the joint (lb)

\( P_w \) = applied wheel load (lb)

\( N_{\text{eff}} \) = number of effective dowel bars
2.2.2 Modeling of Dowel Bars

Since the stresses in the dowel bars are not the main concern in this study, which focuses on mid panel cracking, it was deemed unnecessary to use a detailed model for them. Instead, the dowel bars were modeled by beam elements. More specifically, the dowel bars were connected to the solid brick element nodes by two linear or nonlinear spring elements (the nonlinear springs were used to model the effect of concrete crushing) at directions parallel to the vertical axis (y) and to the transverse axis (z). At the slab connection joint, the dowel bars were connected to both sides of the joint as shown in Figure 2-3.
In Figure 2-3, the nodes with the same letter represent a single solid element node. The solid element nodes and dowel bar nodes have the same coordinates, however they are distinct nodes connected by springs. The solid nodes at the joint (for example nodes c and d in Figure 2-3) have the same coordinates, but they belong to different solid elements. It was assumed that there is no friction between the dowel bar and slab and horizontal connections were not used. Since concrete near the dowel bars may crush under bearing stresses, nonlinear springs were used to account for this effect (Figure 2-4).

![Figure 2-4 Force-Deflection of Nonlinear Spring](image)

2.2.3 Selection of Element Types

The shape functions for a 20-node solid element are quadratic, and they interpolate the translational degrees of freedom (Figure 2-5). Considering one edge of the element its deflection is given by

\[ V = -0.5 \xi (1 - \xi) V_1 + (1 - \xi^2) V_2 + 0.5 \xi (1 + \xi) V_3 \]  

Equation 2-11

where \( V \) = general displacement along the length of element
\( V_1, V_2, V_3 \) = displacement at nodes 1, 2, and 3, respectively
\( \xi \) = dimensionless parameter (natural coordinate)
The dowel bar is modeled by a 2-node beam element with both translational and rotational degrees of freedom in each node. This element’s shape functions are cubic, as seen in Figure 2-6 and the deflected shape is given by:

\[ V = \frac{1}{L^3} [(2x^3 - 3x^2L + L^3)V_1 + (x^3L - 2x^2L^2 + xL^3)R_1 + (-2x^3 + 3x^2L)V_2 + (x^3L - x^2L^2)R_2] \]

where  
- \( V \) = general displacement along the length of element  
- \( V_1, V_2 \) = displacement at point 1 and 2 respectively  
- \( R_1, R_2 \) = rotation at point 1 and 2 respectively  
- \( L \) = Length of the element  
- \( x \) = coordination of the required point

This discretization generates an incompatibility between the slab and dowel bar deflections. To decrease the effect of this incompatibility an increased number of elements in the region close to the dowel bar were adopted to more accurately model this interaction.
2.2.4 Modulus of Dowel Support

A comprehensive investigation for calculating the modulus of dowel support was performed at Iowa State University by Porter et al. (1999, 2001). Different types of steel rods with circular and elliptical cross sections were placed as dowel bars inside concrete specimens and the deflection of the section under loading was calculated (Figure 2-7).

![Figure 2-7 Element Loading, Porter et al., (2001)](image)

Using Equations 2-5 and 2-6 it is possible to find deflection at the face of the joint, which are repeated here for completeness. For a 2,000 lb loading the average deflections at the face of the joint were 0.001311 in and 0.000814 in and total relative deflections were 0.002642 in and 0.001642 in for 1-1/4”φ and 1-1/2”φ epoxy-coated steel respectively. Knowing \(y_0\), the modulus of dowel support \(K_0\) can be calculated using the Equation 2-2 numerically or graphically. \(K_0\), is equal to 1,800,000 and 2,100,000 for 1-1/4”φ and 1-1/2”φ epoxy-coated steel, respectively. Knowing \(K_0\) and \(y_0\), the bearing capacity can be calculated by the product \(K_0y_0\). If this value is more than the allowable stress of concrete, either the distance or size of the dowel bar should be changed or a void must be considered in the area.
2.2.5 Test Model

A simple model consisting of two adjacent half slabs was created to study the effect of mesh size and of the values of stiffness of the springs between dowel bars and slab on the accuracy of the results. The model is shown in Figure 2-8. A linear temperature gradient was applied through the depth of the slabs. The horizontal deflections of the slabs were restrained in the middle of the slab. A 5,000 lb/ft² pressure was applied at the end of one of the slabs to simulate the pressure of a wheel.

![Slab Model](image)

**Figure 2-8 Slab Model**

The stiffness values of the springs were varied from very large to medium to zero in three models. The deflections of the dowel bars and the associated elements in the slabs are shown in Figure 2-9 for the different spring stiffness values.

![Deflection of Dowel Bar and Slab](image)

**Figure 2-9 Deflection of Dowel Bar and Slab**
The length of the dowel bar and associated solid elements on both sides of the joint were divided to 2, 3, 4, 5 and 6 sections and the maximum stresses and deflection of the slab between 4 and 6 feet from the end of a dowel bar were compared. The results are shown in Table 2-1 and Table 2-2, respectively. Since, under bearing pressure, concrete surrounding a dowel bar may crush near the joints, the spring constants in this region were set to zero for comparison. The results are presented in the last two columns of Table 2-1 and Table 2-2. As it can be seen the stresses and deflections for the cases with very soft or very stiff springs are not sensitive to number of sections. For medium spring stiffness, the effect of the number of sections becomes more significant. Minor changes can be seen when the number of sections increases to five. Thus, the effect of element incompatibility on the stress and deflection of the slab can be lowered by increasing the number of elements near the dowel bars.

Table 2-1 The Effect of Number of Meshing of Both Sides on Stresses

<table>
<thead>
<tr>
<th></th>
<th>Stiff</th>
<th>Soft</th>
<th>Medium</th>
<th>Half-length</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Stress</td>
<td>Percent</td>
<td>Stress</td>
<td>Percent</td>
</tr>
<tr>
<td>2</td>
<td>40,500</td>
<td>99.51</td>
<td>79,800</td>
<td>100</td>
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<tr>
<td>3</td>
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<td>79,800</td>
<td>100</td>
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<tr>
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<td>100.00</td>
<td>79,800</td>
<td>100</td>
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<td>100</td>
</tr>
<tr>
<td>6</td>
<td>40,700</td>
<td>100.00</td>
<td>79,800</td>
<td>100</td>
</tr>
</tbody>
</table>

Table 2-2 The Effect of Number of Meshing of Both Sides on Deflections

<table>
<thead>
<tr>
<th></th>
<th>Stiff</th>
<th>Soft</th>
<th>Medium</th>
<th>Half-length</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Def</td>
<td>Percent</td>
<td>Def</td>
<td>Percent</td>
</tr>
<tr>
<td>2</td>
<td>28.90</td>
<td>100</td>
<td>46.90</td>
<td>100</td>
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<tr>
<td>5</td>
<td>28.90</td>
<td>100</td>
<td>46.90</td>
<td>100</td>
</tr>
<tr>
<td>6</td>
<td>28.90</td>
<td>100</td>
<td>46.90</td>
<td>100</td>
</tr>
</tbody>
</table>

Figure 2-10 and Figure 2-11 show the stresses and deflections for different meshes as a percent of exact result. If the element on both sides is divided into more than three sections, the error in both deflection and stress remains less than 1.5%.
2.2.6 Tie Bars

Tie bars connect adjacent slabs in longitudinal joints. They usually have a smaller diameter and are placed at a greater distance than dowel bars. In this study, they were modeled similarly to the dowel bars. Since tie bars can transfer horizontal forces between the slabs, nonlinear springs were added to the dowel bar model that allows for the development of the appropriate forces in that direction.
2.3 Slab-Subbase Interaction

One of the most important behaviors of concrete pavements that must be captured by the model is the separation between the concrete slab and the sub-base caused by thermal loads. At night, since the top of the pavement tends to be cooler than the bottom of the pavement, the slab tends to curl upwards and the edges lose contact with the subbase. During daytime, when the top of the pavement tends to be warmer than the bottom, the slab curls downwards and the middle of the slab loses contact with the subbase. This loss of contact by a portion of the slab with the subbase causes an increase in the stresses.

In this study, two methods for modeling the interaction between slab and sub-base were investigated. In the first method, this interaction was modeled by nonlinear springs. As it can be seen in Figure 2-12 the stiffness of the nonlinear springs is such that there would be no tension in the spring when the slab and sub-base separate and they would be very stiff when the two components press onto each other. The advantage of this model is that due to its simplicity, it requires a small amount of time to convergence. However, this method is not capable of modeling the friction between the slab and sub-base.

![Figure 2-12 Nonlinear Spring](image)

In the second method contact elements were adopted. Contact elements are highly nonlinear and demand significant computational time and memory. The regions that are in contact or separate are not known in the beginning of solution and vary based on the loads, material, boundary conditions, and other factors. Frictional behavior is also nonlinear and the frictional response can be discontinuous, making convergence difficult.
Rigid-to-flexible contact elements were used to simulate this interaction, since concrete has a much higher stiffness when compared to the subbase. More specifically, surface-to-surface, large sliding, semi-automatic contact elements were adopted. The friction between soil and concrete slab was incorporated to the model by assigning tangent of the friction angle of soil as the friction coefficient for the contact elements.

The bottom surface of the slab was defined as the "target surface" and the top of the subbase was defined "contact surface" thus forming a contact pair. Contact elements cannot penetrate the target surface; however, target elements can penetrate the contact surface. The use of surface-to-surface elements allowed the adoption of different element types for the slab and subbase. More specifically, lower order elements were used for the subbase and higher order elements were used for the slab, since the main goal is to determine the stresses in the slab. Furthermore, their use also enabled the consideration of large deformations with significant amount of sliding and friction, thus providing better contact results while using fewer contact elements and thus less memory. It also allowed the ability to control bonded contact and initial penetration. Since our target surface is flat, lower-order target elements (4-node quadrilateral elements) were found to be sufficiently accurate yet efficient. The latter is because these elements use less memory to calculate penetration and gap. It should be noted, however, that these elements may not produce a smooth meshed surface if the surface is curved. The commercial FE program ANSYS, which is used in this study, can generate the elements and recognize the proper target element shape based on the solid model. Contact elements have the same geometric characteristics and have the same order of the underlying elements. Since in the present studies involve the modeling of disconnected slabs, it was found that it is better to localize the contact zone by defining multiple target surfaces. Various contact factors are available in ANSYS. The most important factors are:

- Normal contact stiffness factor
- Allowable penetration factor
- Initial closure factor
- Allowable penetration range for initial penetration.
- Maximum contact friction factor
• Stiffness factor (applied when contact is lost)
• Tangent contact stiffness
• Cohesion sliding resistance factor

The contact stiffness is a critical factor in contact problems, since the amount of penetration between the two surfaces depends on this factor. Higher stiffness values decrease the amount of penetration but can lead to convergence difficulties. An optimum value must be chosen that create a high enough stiffness that contact penetration is acceptably small, but a low enough stiffness that the problem can converge. These values were obtained in this work by means of a series of numerical tests.

ANSYS uses the Coulomb friction model to develop the horizontal interaction between the surfaces. In this model, two contacting surfaces carry shear stresses up to a limit without sliding (sticking state). The shear stress at which sliding on the surface initiates is defined based on the material properties of the connecting surfaces. Once the shear stress is exceeded, the two surfaces can slide relative to each other (sliding state). Shearing stress can be defined by multiplying the coefficient of friction to the normal stress in the sticking state and is equal to the maximum value at the sliding state. The consideration of friction by contact elements results in a non-symmetric stiffness matrix for the system. The solution of a non-symmetric system of equations is time consuming and has large memory requirements. In this case, a symmetric approximation of the stiffness matrix might be used. However, it was found that in some cases this caused a low rate of convergence.

Initial contact is the other important factor in building a contact model. Since the slabs are constrained only by contact elements, the model must be built so that the contact pairs are in contact in the beginning of the analysis. Large initial penetration between target and contact surfaces may occur in the model and contact elements may overestimate the contact forces, resulting in divergence of the solution. To prevent this, a small initial contact closure can be defined or an initial allowable penetration range can be specified which brings the target surface into a state of initial contact at the beginning of the analysis.
The convergence behavior for contact problems is highly dependent on the problem. Choosing a right value for time step size is critical to successfully obtain the solution. The time step must be small enough to ensure proper contact zone. The smooth transfer of contact forces is disrupted if the time step size is too large. If the contact status changes during the iteration process, discontinuity can occur.

2.4 Modeling of the Subgrade

The quality of subgrade gravelly affects the performance of a concrete pavement. It is thus one of the most important aspects in their design. A very weak subgrade can result in large deformations and stresses in a pavement, thus, reducing its service life and increasing repair costs. Thicker subbases and slabs have been found to help reduce these deflections and stresses.

2.4.1 The Available Models in FE

Researchers have developed different methods to model the subgrade in pavements. Khazanovich (1993) studied the different available models. The first and simplest model is called Dense Liquid Foundation (DL). In this model, the soil is simulated by a series of vertical springs. While this technique uses the least amount of memory of all methods, it is incapable of developing the shear interaction within the soil mass. The second and more frequently used model is called Elastic Solid (ES). This model is capable of better capturing the behavior the soil than DL, since it can develop shear interaction within the soil mass. However, its weakness it cannot accurately simulate the behavior of soil near the slab edges and corners. The DL model cannot predict deflections beyond the slab and the ES model over-predicts these deflections. As the dimension of the loaded area increases the degree of the conservativeness of the DL model decreases. The ES model requires a large amount of memory and, thus, it is needs more computational time than the DL model.

Ioannides et al. (1988) studied the effect of different factors on the accuracy of the solution for the different modeling techniques. They found that, for varying slab sizes, the
deflections and stresses in the subgrade obtained by the ES model converge to the exact solution from above, while the deflections and stresses in the slab converge from below. However, for different levels of mesh refinement and varying all of other parameters cause the solution to converge from below. It was also found that the bending stresses are very sensitive to the area that the load is distributed. This is because in the FE method, distributed loads are converted to nodal loads. They conclude that mesh refinement and the area of applied loading are the critical factors affecting bending stresses. Studies on the dimensions showed the necessity of restricting the elements’ aspect ratios to less than 2 to ensure adequate level of accuracy. Finally, their comparison between the ES and DL models showed that the results from the techniques are close for the interior loading. However, the bending stresses in the slab and subgrade stresses and deflections are significantly higher in the DL model than in the ES model.

Another popular technique used for modeling the subgrade is the Two Parameters foundation (TP) model. This model is an extension of the DL model and it incorporates a degree of shearing interaction into the calculation. The basic equation is defined as

$$q = kw - GV^2w$$

Equation 2-13

where $q$ is the subgrade reaction pressure, $w$ is the surface deflection, $k$ is the vertical spring stiffness and $G$ is the coefficient of shear interaction. It can be seen that the TP models reduces to the DL model if $G$ is set to zero. Estimating an exact value for $G$ is essential to accurately develop the soil behavior. According to Khazanovich (1993), in this model the deflections of the soil over the edge of the slab vanish faster than in the ES model. Studies by Ioannides (1985) show that coefficient $G$ tends to transfer some of the response from the interior to the edges of the slab. So by increasing $k$, the deflection of the edges tends to decrease rapidly.

Studies by Ioannides (1985) also showed that, increasing $k$ and/or $G$ decreases the deflection, increasing $G$ flattens the deflection, increasing $k$ decreases the bending stresses, increasing $G$ increases the bending stresses specially in the interior part of the slab. They also showed that changes are more susceptible to variations in $k$ than to variations in $G$. In addition, they also found that both $k$ and $G$ are highly related to the
thickness and length of the pavement slab. They also showed that mesh size affects the results produced by the ES and TP methods to a same degree.

Two other methods are available methods to simulate the subgrade. They are the ZSS method, which is a combination of the DL and ES methods, and the Three Parameter Foundation method, which is a combination of the DL and TP methods. A comparison of the stresses produced for a certain deflection, revealed that that the DL method is the most conservative of all methods.

The $k$ values can be evaluated directly from the field plate loading test. However, this test is expensive and the $k$ values are usually estimated by correlating it to the results from simpler tests such as CBR or R value tests. Vesic and Saxena (1974) presented the following relation between the relative flexibility of the slab, modulus of subgrade reaction, and soil stiffness that leads to comparable stress results

$$k = \left(\frac{E_f}{E_c}\right)^{1/3} \frac{E_f}{h(1-\nu_f)^2}$$  

Equation 2-14

where $E_f$ and $\nu_f$ are the elastic modulus and Poisson’s ratio of the solid foundation, $E$ is the elastic modulus of concrete, and $h$ is the thickness of the slab. It was shown that applying forty two percent of this value results in comparable deflections between liquid foundation and solid model. Haung and Sharpe (1989) suggested that a factor of 1.75 to be applied to the previous equation if the load is near the edge of the slab.

2.4.2 The Effect of Depth of Subgrade

The subgrade dimension is unlimited; however modeling the whole subgrade in a FE model is not possible. As a result either infinite elements must be used or enough depth of soil must be modeled beyond which the results would not change. Studies by Davids (2000) showed that if enough regular elements are used the boundary condition does not affect the final results. Ioannides et al (1988) also examined the effect of subgrade depth on the developed stresses and deflection of slabs. They found that the effect of boundary condition decreases as the thickness of the modeled subgrade increases. They also found that the bending stress variation for depth of modeled soil
over 5 elements is less than one percent. However the deflection results converge at a very slower rate and a depth of 12 elements of soil must be used to assure the 1 percent accuracy. They also found that the deflection results converge from the bottom, which means that if a smaller depth is modeled it results in smaller deflection.

Alternatively, infinite elements can be used to model the subgrade. There are two types of infinite elements. In the first type the shape functions are changed in a way that the field quantity approaches infinity while the element itself remains of finite size. In the type, conventional shape functions are used for the field quantity, while a different set of shape functions is used for the element geometry, which cause the element size to reach infinity on one of its side. The later type is referred to as mapped infinite elements.

![Mapping functions](image)

**Mapping functions**

\[ M_1 = -\frac{2\xi}{1-\xi} \]
\[ M_2 = -\frac{2\xi}{1+\xi} \]
\[ M_3 = \frac{1-\eta}{1-\xi} \]
\[ M_4 = \frac{1+\eta}{1-\xi} \]

**Shape functions**

\[ N_1 = 0.25((-\xi + \xi^2)(1-\eta)) \]
\[ N_2 = 0.25((-\xi + \xi^2)(1+\eta)) \]
\[ N_3 = 0.5(1-\xi^2)(1-\eta) \]
\[ N_4 = 0.5(1-\xi^2)(1+\eta) \]
\[ N_5 = 0.25(\xi + \xi^2)(1-\eta) \]
\[ N_6 = 0.25(\xi + \xi^2)(1+\eta) \]

Figure 2-13 Two Dimensional Infinite Element (Cook et al 2001)

Figure 2-13 shows the shape functions for a two dimensional mapped infinite element. Notice that when the mapping shape functions are used to define geometry \( x = \sum M_i x_i \) one gets: \( x = x_1 \) when \( \xi \) equal to -1 and \( \eta \) equal to -1, \( x = x_3 \) when \( \xi \) equal to 0 and \( \eta \) equal to -1, and \( x = \infty \) when \( \xi \) equal to 1. It should be noted that because of the use of regular shape functions to define the field quantity \( f = \sum N_i f_i \) when \( \xi \) equal to 1 gives: \( f = f_5 \). In the formulation of the element matrices the ordinary shape functions are used.
everywhere except in the construction of the Jacobian, when the mapping shape functions are used.

In order to have an idea of the needed depth of the soil a simple model was developed (Figure 2-14). A 4ft by 6ft by 1ft slab situated on the soil was modeled and analyzed using the commercial software package ABAQUS. A linear temperature was applied to the concrete slab. Contact elements were used between the concrete and the soil. Two models were considered that used different element types at the bottom subgrade layer. In the first ordinary elements were used for this layer with bottom most nodes restrained in the vertical direction. In the second model infinite elements were used. It should be noted that no boundary conditions are required for these elements, since in this case the field quantity is implicitly set to zero at infinity.

Several cases were tested using the two modeling techniques. Varying depths and varying number of elements were considered. The resulting maximum stresses in the slab for these cases were compared. This was done because the maximum stress is the
most important parameter that determines the service life of a pavement. Table 2-3 shows the results. As can be seen from this table, if enough ordinary elements through the depth are used both methods give similar results. When the depth of the modeled soil is reduced, the resulting the stresses diverge from the exact solution much faster for the model with ordinary elements than for the model with infinite elements. In this study, enough ordinary elements are used, since the adopted software package, ANSYS, does not provide infinite elements in its element library.

Table 2-3 Effect of Soil Depth on Maximum Stress in the Slab

<table>
<thead>
<tr>
<th>Depth</th>
<th>Stress</th>
<th>Percent</th>
<th>Depth</th>
<th>Stress</th>
<th>Difference</th>
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<tr>
<td>5</td>
<td>1,384.62</td>
<td>0</td>
<td>8</td>
<td>1,384.56</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>1,389.56</td>
<td>0.36%</td>
<td>6</td>
<td>1,385.62</td>
<td>0.076%</td>
</tr>
<tr>
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<td>1,419.97</td>
<td>2.55%</td>
<td>4</td>
<td>1,388.90</td>
<td>0.31%</td>
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</tbody>
</table>

2.4.3 Variation in the Stiffness of Subgrade

The relative stiffness values of the slab, subbase, and subgrade affect the distribution of thermal and traffic stresses/strains within the slab. Thus, predicting a realistic value for each layer is a critical issue. Various factors including moisture content, plasticity index, and vertical stresses can affect the stiffness of subgrade and subbase. Ksaibati et al. (2000) have studied the effect of moisture content in the stiffness of the subbase. They found that moisture content increases as the depth of the ground water decreases. In particular, the results from their Dynaflect and FWD tests showed a reduction in modulus of elasticity of the slab. However, the two methods did not produce exactly the same results.

Seasonal variations have direct effects on subgrade and subbase stiffness. In general, the stiffness of soil decreases in thaw cycles and increases with freezing. Results of AASHTO road tests indicate that the reduced subgrade support during thaw periods do not have a significant effect on the required slab thickness. This may be due to the fact that thaw period is significantly shorter when compared to the freezing period. The PCA
method does not consider the variation of stiffness over the year. It recommends selecting a normal summer or fall subgrade support as a reasonable mean value instead of using a more sophisticated method.

The US Army Corps of Engineers suggest that in areas where frost is a major concern and has a significant effect, pavement thickness be calculated according to the reduced subgrade strength method. For frost-design purposes, soil is divided into seven groups and special recommendations are given for each group. If the natural subgrade CBR is less than the frost-area soil-support index then the CBR value governs the design, and if the natural sub-grade CBR is greater than the soil-support index, then the required thickness is determined by frost area indexes.

The Federal Aviation Administration (FAA) Rigid Pavement Design, recommend a deduction in $k$-value (subgrade modulus) for soil types fg-1 to fg-3 and total protection for soil type fg-4. The designer must suggest the level of frost protection used. If the designer elects to use the RDF Site Summary (RSS) method the input for subgrade modulus ($k$-values) must be reduced according to the Table 2-4. These values represent a weakened subgrade during the frost thaw period.

<table>
<thead>
<tr>
<th>Frost Code</th>
<th>Reduced Sub-grade $k$-value (ksi)</th>
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</thead>
<tbody>
<tr>
<td>F-1</td>
<td>50</td>
</tr>
<tr>
<td>F-2</td>
<td>35</td>
</tr>
<tr>
<td>F-3</td>
<td>25</td>
</tr>
<tr>
<td>F-4</td>
<td>Not Applicable</td>
</tr>
</tbody>
</table>

2.4.4 Stress Dependent Model (Resilient Modulus)

Ioannides et al (1988) studied and developed new FE models, which could incorporate an iterative procedure to account for the deflection dependent Resilient Modulus (RM) of reaction. The resilient modulus is defined as the ratio of repeated axial load by the resilient or recoverable strain, i.e.:
where, $E_R = \text{resilient modulus}; \sigma = \text{repeated axial stress}; \text{and } \varepsilon_R = \text{resilient (recoverable)}$ strain.

This parameter was recognized to more appropriately model the nonlinear response of the subgrade under rapidly moving loads. The resilient modulus of soil is not a constant value and can vary based on the stresses and deflections. A coarse subgrade support exhibits a higher response at higher magnitude of stress or deflection. The resilient modulus for granular material can be expressed as (Huang 1993).

$$E_R = K_I \theta^{K_2}$$

where $\theta$ is the sum of three principal stresses and $K_I$ and $K_2$ are experimentally derived constants.

Fine material exhibit a different behavior and the resilient modulus decreases with the increase in deviatoric stress. Thompson and Elliott (1985) studied the relation between the RM and repeated deviatoric stress. They indicated that the relation is bilinear and the RM at the breakpoint in the bilinear curve is related to resilient behavior more than other factors. Figure 2-15 defines the material constant in the general relationship between RM and deviator stress of the fine grained soils.

Figure 2-15 Variation of the Stiffness of the Soil Based on the Applied Stress
Thompson and Elliott (1985) also categorized fine-grained soils into four types of very soft, soft, medium, and stiff. The material constants for each of these categories are given in Table 2-5.

<table>
<thead>
<tr>
<th>Material</th>
<th>Property</th>
<th>RM₀</th>
<th>RMₑ</th>
<th>K₁</th>
<th>K₂</th>
<th>K₃</th>
<th>K₄</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stiff</td>
<td></td>
<td>17,002</td>
<td>7,605</td>
<td>12,340</td>
<td>6.2</td>
<td>1,110</td>
<td>178</td>
</tr>
<tr>
<td>Medium</td>
<td></td>
<td>12,342</td>
<td>4,716</td>
<td>7,680</td>
<td>6.2</td>
<td>1,110</td>
<td>178</td>
</tr>
<tr>
<td>Soft</td>
<td></td>
<td>7,682</td>
<td>1,827</td>
<td>3,020</td>
<td>6.2</td>
<td>1,110</td>
<td>178</td>
</tr>
<tr>
<td>Very Soft</td>
<td></td>
<td>5,662</td>
<td>1,000</td>
<td>1,000</td>
<td>6.2</td>
<td>1,110</td>
<td>178</td>
</tr>
</tbody>
</table>

Thompson et al. (1985) also studied the resilient properties of the various soils used in the state of Illinois. Since the moisture content in soil has a significant effect on the RM, they studied the amount of moisture in the studied pavements. The RM tests were performed at optimum value, since the majority of the soils had more moisture content than this optimum value. The results are shown in Table 2-6. Their studies showed that the RM of a fine material is highly dependent on the moisture content and higher moisture decreases the RM. The stress level related to the break point of the bilinear variation remained relatively constant with a small standard deviation regardless of moisture content. Soil properties that mainly affect and decrease the RM are low plasticity, low group index, high silt content, low clay content, low specific gravity, and high organic carbon content. The RM can be related to the static modulus as:

\[ E_R = 3.46 + 1.9 E \]

where \( E_R \) is RM in ksi and \( E \) is static modulus in ksi.

Ioannides et al (1988) studied the RM and derived a relation between the ER and the deflection for fine materials. They used this relationship in FE analyses. For a 9 kip load on a 20 ft by 12 ft pavement changing from a 120–150 psi/in static Winkler subgrade to the low stress level RM with initial value of 425 psi/in resulted in 7%
reduction in stress and 47% reduction in deflection ratio. This effect increases for higher loads, such as airplanes, and for flexible pavements. The number of iterations needed in this case is just one to reach an accuracy of 2%, while for the other loading and material up to 5 iteration are needed.

Table 2-6 Variation of the Stiffness of the Soil Based on Percent of Water (Thompson & Elliott)

<table>
<thead>
<tr>
<th>Material</th>
<th>$E_{\text{static}}$</th>
<th>$K_1$</th>
<th>$K_2$</th>
<th>$K_3$</th>
<th>$K_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Optimum w</td>
<td>2677 psi</td>
<td>8940 psi</td>
<td>62 psi</td>
<td>1210 psi</td>
<td>186 psi</td>
</tr>
<tr>
<td>Deviation</td>
<td>1374 psi</td>
<td>3400 psi</td>
<td>600 psi</td>
<td>90 psi</td>
<td></td>
</tr>
<tr>
<td>Opt + 1% w</td>
<td>2677 psi</td>
<td>7360 psi</td>
<td>59 psi</td>
<td>1135 psi</td>
<td>171 psi</td>
</tr>
<tr>
<td>Deviation</td>
<td>1374 psi</td>
<td>3400 psi</td>
<td>600 psi</td>
<td>110 psi</td>
<td></td>
</tr>
<tr>
<td>Opt +2% w</td>
<td>2677 psi</td>
<td>6220 psi</td>
<td>61 psi</td>
<td>1011 psi</td>
<td>170 psi</td>
</tr>
<tr>
<td>Deviation</td>
<td>1374 psi</td>
<td>3200 psi</td>
<td>600 psi</td>
<td>100 psi</td>
<td></td>
</tr>
</tbody>
</table>

The resilient modulus is mainly used for flexible pavements where the tire stresses are developed over a limited area and the resulting stresses are relatively high. It has been documented that the resilient deformation of a flexible pavement structure is responsible for their fatigue failure. The resilient modulus is not used in concrete pavements and a constant value for modulus of elasticity is preferred. This is because, in this case, the stresses produced by a wheel load are mainly distributed over a large area by the concrete slab and the resulting stresses vanish rapidly through the depth of the subgrade. It should be noted, however, that the resilient modulus has been used for concrete slabs in airports.

In the study performed by Ioannides (1988), the effect of subgrade stress dependency was explored by calculating the number of elements exceeding the 2 psi limit in $\sigma_0$, below which a constant modulus behavior is assumed. A relatively high load (32 ksi) was applied over relatively thin slab (8 in) at different positions: interior, edge, and corner. The results are discussed next.

For the case in which the load was applied at the interior of the slab, only 9% of the subgrade elements at the top 2.5 ft were affected and the change in modulus of elasticity
was less than 2%. As a result, the effect of stress dependency on both deflection and stresses was deemed negligible. Applying the load on an edge of the slab changed modulus of elasticity by 8% at the top 6.5 feet of soil. This resulted in an increase of the maximum deflection and bending stress 10 and 8%, respectively. This increase was calculated as the ratio between the stresses at the iteration when the difference between two iteration is less than 2% and the first iteration where the applied modulus of elasticity is constant throughout the subgrade. The deflection of the subgrade, located in 6 in from the loaded slab edge reduced to 50% of the deflection at the edge of the slab. This demonstrates the less significant structural contribution of the soil and tendency of the soil to act as a liquid foundation when under high stresses. Subgrade stress dependency under corner loading was found to deserve more serious consideration. Their studies revealed a 20% increase in deflection and 10% increase in bending stresses in this case. About 20% of the subgrade elements of the upper 5.5 ft soil were affected in this case.

Highway pavements are thicker (10–14 in) and are subjected to lesser loads (maximum load less than 16 ksi) than the slab studied by Ioannides (1988). Under these conditions, the variation of stresses in the subgrade resulting from wheel loading has a minor effect on modulus of elasticity of the subgrade. As a result changing the stiffness of subgrade which results in negligible increase in deflection and stresses is deemed unnecessary in the present investigation.

2.5 Aggregate Interlock

The length of the saw cut in the studied pavement is one fourth to one third of the slab depth. Thus, a rational assumption is that aggregate interlock may exist at the bottom part of the slab. The effect of aggregate interlock can be modeled using vertical nonlinear springs connecting the joint nodes with the same coordinates, as shown in Figure 2-16.

Aggregate interlock depends on the joint opening, which is a result of shrinkage of concrete and thermal contractions. Shrinkage of concrete depends on concrete material proportions, added admixture, curing, and environmental factors including humidity and
wind speed. Thermal contraction depends on the time of placement, ambient temperature, and concrete thermal factors such as coefficient of thermal expansion, absorbability, emissivity, and conductivity.

Reinhardt and Walraven (1982) studied and developed a theoretical model for aggregate interlock. They simplified more complicated nonlinear models into a very accurate linear relation that relates four variables: shear stress, shear displacement, normal stress, and crack opening. Their equation is given by

\[
\tau = -\frac{f'_{cc}}{30} + [1.8w^{-0.80} + (0.234w^{-0.707} - 0.20)f'_{cc}]\Delta \quad (\tau \geq 0) \quad (N/mm^2, mm) \quad \text{Equation 2-18}
\]

where \( \tau \) is the shear stress in N/mm\(^2\), \( f'_{cc} \) is concrete cube compressive strength in N/mm\(^2\), \( w \) is the crack opening in mm, and \( \Delta \) is the slip or shear displacement. This model, due to its simplicity is thus used in the present study.

![Figure 2-16 Modeling of Aggregate Interlock Action](image-url)
CHAPTER 3 LOADING

3.1 Thermal and Equivalent Shrinkage Loading

Temperature affects concrete pavements in two main ways. First, seasonal temperature variation may cause the concrete slab to contract. However, the friction between the slab and the subbase resist this contraction and create tensile stresses inside the slab. The second effect is caused by temperature gradient through the depth of the concrete slab. This nonlinear temperature gradient greatly affects the stresses inside the slab, in particular the mid-panel stresses. Since the stresses in the concrete slab have a direct effect on the pavement’s service life, it is crucial that the thermal gradient through the thickness of the slab be accurately modeled.

It must be noticed that the concrete on the top surface of the slab is in direct contact with environment. It absorbs solar radiation, interacts with the air, and is in direct contact with rain and snow. On the other hand, the temperature at the bottom of the slab changes much slower than the temperature on the surface. This is because thermal changes at the bottom occur solely due to heat transfer. This phenomenon causes a nonlinear thermal gradient through the depth of concrete. This is especially true in warm weather followed by sudden rain shower, which causes an abrupt drop of temperature at the top of the slab, while the bottom temperature remains constant. This nonlinear temperature gradient causes large stresses in the middle of slab.

The temperature gradient that causes the top slab surface to cool down is termed a negative temperature gradient. Similarly, the temperature gradient that causes the top slab surface to warm up is called a positive temperature gradient. Negative temperature differentials may cause the slab corners to curl upward creating tensile stresses at the top slab surface and compressive stresses at the bottom surface (Figure 3-1-a). Positive temperature gradients may cause the slab edges to curl downward (Figure 3-1-b). Under a
positive temperature gradient, the top of the concrete slab is in compression while the bottom is in tension.

![Diagram of temperature gradient](image)

Figure 3-1 Curling of the Slab, a) Negative Temperature Gradient, and b) Positive Temperature Gradient

3.1.1 Theoretical Studies

The Integrated Climatic Model (ICM) program was developed by Larson G., Dempsey B. J. (1986, 1987) to calculate the temperature gradient throughout the depth of the slab. ICM models climatic effects on pavements, using input pavement properties along with appropriate climatic data. The integrated model combines three separate models of climate effects on pavement: the Climatic-Materials-Structures Model (CMS) developed at the University of Illinois by Dempsey B. J. (1986, 1987); the Infiltration and Drainage Model (ID) developed at the Texas A&M University, Texas Transportation Institute; and the CRREL Frost Heave and Thaw Settlement Model developed at the United States Army Cold Regions Research and Engineering Laboratory (CRREL). The original version of the Integrated Model was developed at Texas A&M University, Texas Transportation Institute in 1989. Appendix 1 defines the theoretical background of the Climatic-Materials-Structures Model. The general input data are: length of analysis period, time increment for output and calculation, latitude to calculate solar radiation, temperature, rainfall, wind speed, percent sunshine, and water table depth for the given time increment. ICM can read the data from files or can produce the data through an interface. The program has the capability of converting daily data to hourly data as well.
as hourly data to daily data. If the daily data is available, the program asks for the times of maximum and minimum temperatures and interpolates the needed data based on available functions. Emissivity and surface short-wave absorptivity factors, layer properties including thickness, number of elements per layer, thermal conductivity, heat capacity, and initial conditions must also be provided. The program manipulates the data and creates tables or graphs of: thermal gradient, water content, pore pressure, and ice content over any time period in a specific node or over the total depth at a given time.

Based on the CMS program and using climatic data of Illinois, temperature gradients were calculated through the depth of a 9in concrete pavement by Thompson et al (1987). Typical diurnal effects are shown in Figure 3-2.

![Figure 3-2 Temperature Gradient (Urbana, Illinois Thompson et al 1987), a) November, b) July, and c) April](image-url)
3.1.2 Florida Studies

Experimental studies were performed in Florida to determine the actual temperature gradient in a PCC pavement. The thermal gradient in a 9 in concrete pavement was measured within a period of 9 months. Thermocouples were placed at 1, 2.5, 4.5, 6.5 and 8 inches below the surface of the pavement.

They suggested that temperature gradient could be approximated by a parabolic curve as

\[ t = A + B y + C y^2 \]  \hspace{1cm} \text{Equation 3-1}

where \( t \) = temperature (°F), \( y \) = location above the bottom of the pavement (in.)

\( A, B, C \) = factors fitted by linear regression

Figure 3-3 illustrates the results based on representative values from August 14, 1984.

![Figure 3-3 Temperature Gradient (August, Florida)](image)

According to the report by Richardson (1987), Bergstrom (1950) developed a method to isolate the stress due to non-linearity of temperature gradient. In general, the energy received at the surface of the concrete slab is propagated through the depth of slab by radiation and convention resulting in a temperature gradient. If the slab was fully restrained, a uniform stress distribution across the depth would be generated as shown in Figure 3-4-b. The total stress can be calculated by integrating the stress throughout the depth and dividing it by the depth of the slab. If this stress was subtracted from total stress, the result would be a curling moment in the slab. This moment could be calculated
by taking the stresses about any point in the slab. This moment would produce the curling stress distribution shown in Figure 3-4-c. If both axial and curling stresses were subtracted from the overall temperature stresses, the remaining would be a nonlinear temperature gradient through the depth of the pavement as shown in Figure 3-4-d.

Assuming a parabolic temperature distribution, each of the components of the thermal stresses (Figure 3-4) can be calculated as

\[
\sigma_A = E \alpha (A + 4.5B + 27C) \quad \text{Equation 3-2}
\]

\[
\sigma_C = E \alpha [(y-4.5)B + 9(y-4.5)C] \quad \text{Equation 3-3}
\]

\[
\sigma_{NL} = E \alpha C (y^2 - 9y + 13.5) \quad \text{Equation 3-4}
\]

where \( \alpha \) is the coefficient of thermal expansion and \( E \) is the modulus of elasticity.

If the above equations are be used for the Florida data on August 14, 1984 at 1:00 PM (Figure 3-3), the maximum compressive stress produces is 113 psi. The other values of the maximum stresses throughout the year were calculated and are presented in Table 3-1. Unlike the maximum compressive stress, which occurs at about noon throughout the year, the maximum tensile stress occurs sometime between 8pm to midnight depending on the time of the year. Except for August 14, 1984, in which the maximum tensile stress was 113 psi, at the other days the maximum tensile stress was relatively lower than the compressive stress. The reason for the high tensile stress on that day is presumed to be the occurrence of an evening thunderstorm.
Table 3-1 Maximum Calculated Stresses in Florida’s Pavement

<table>
<thead>
<tr>
<th>Date</th>
<th>Maximum Compressive Stress (psi)</th>
<th>Date</th>
<th>Maximum Tensile Stress (psi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>December 23</td>
<td>60</td>
<td>December 23</td>
<td>37</td>
</tr>
<tr>
<td>January 1</td>
<td>100</td>
<td>January 1</td>
<td>30</td>
</tr>
<tr>
<td>February 1</td>
<td>107</td>
<td>February 1</td>
<td>43</td>
</tr>
<tr>
<td>March 1</td>
<td>104</td>
<td>March 1</td>
<td>58</td>
</tr>
<tr>
<td>April 2</td>
<td>91</td>
<td>April 2</td>
<td>40</td>
</tr>
<tr>
<td>July 25</td>
<td>109</td>
<td>July 25</td>
<td>53</td>
</tr>
</tbody>
</table>

3.1.3 Studies Based on Illinois Data

Based on the Illinois data gathered by Thompson et al. (1987), Mohamed and Hansen (1996) suggested that a polynomial of third degree as the best function fitting the temperature profile. The general form of thermal distribution is given by

\[ T(z) = A + Bz + Cz^2 + Dz^3 \]  

Equation 3-5

where the constants A, B, C and D are given in Table 3-2 for the given data and temperature distribution is given in Figure 3-5.

![Figure 3-5 Temperature Gradient (April, Urbana, Illinois Hansen et al.)](image-url)
Table 3-2 Temperature Distribution in a 9 in PCC Slab During April in Urbana, Illinois

<table>
<thead>
<tr>
<th>Time</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>2:00 a.m.</td>
<td>2.691</td>
<td>-0.495</td>
<td>-0.164</td>
<td>0.033</td>
</tr>
<tr>
<td>6:00 a.m.</td>
<td>1.881</td>
<td>0.091</td>
<td>-0.169</td>
<td>0.012</td>
</tr>
<tr>
<td>10:00 a.m.</td>
<td>1.214</td>
<td>0.172</td>
<td>0.079</td>
<td>-0.041</td>
</tr>
<tr>
<td>3:00 p.m.</td>
<td>1.476</td>
<td>-0.529</td>
<td>0.269</td>
<td>-0.051</td>
</tr>
<tr>
<td>7:00 p.m.</td>
<td>2.929</td>
<td>-1.372</td>
<td>0.079</td>
<td>0.021</td>
</tr>
<tr>
<td>11:00 p.m.</td>
<td>3.321</td>
<td>-0.872</td>
<td>-0.119</td>
<td>0.035</td>
</tr>
</tbody>
</table>

3.1.4 Effect of Element Selection on Temperature Gradient Stresses

A test example was developed to study the effect of element selection on the resulting thermal stresses as well as to verify the suitability of the two available commercial software packages (ANSYS and ABAQUS). The example consisted of the 9ft by 10 ft by 1 ft slab shown in Figure 3-6. The horizontal displacements were restrained at both ends of the slab and the parabolic temperature gradient shown in Figure 3-7 was applied throughout the slab.

![Figure 3-6 Experimental Model for Temperature Gradient](image)

Since both ends of the slab are fixed symmetry is present and, thus, only half of the slab was modeled. The stresses can be evaluated as
\[ \sigma = E \alpha \Delta t \]  \hspace{1cm} \text{Equation 3-6}

where \( \sigma \) is the stress, \( E \) is the modulus of elasticity, \( \alpha \) is the coefficient of thermal expansion and \( \Delta t \) is the change in temperature.

![Temperature Gradient Applied Through the Depth](image)

Figure 3-7 Temperature Gradient Applied Through the Depth

The example slab was modeled and analyzed using both ANSYS and ABAQUS, and with 4-node brick element (12 elements through the depth) and with 8-node brick elements (6 elements through the depth). For a 4-node brick element, the displacements are linear and the strains, which are derivatives of displacements, are constant. Thus, in this case, the stress distribution should be constant throughout the element. By comparing the calculated results in the middle of the element, at the nodes, and at the Gauss points it was found that ABAQUS calculate the stresses at the Gauss points and linearly interpolate them to the other locations, i.e., middle of the element and nodes. Thus, in this case, ABAQUS produces a single stress for all of the nodes of one element. However, ANSYS gives different stresses at the different nodes of an element, which indicates that the original FE formulation has been modified.

For 8-node brick elements, the displacements are quadratic, which leads to linear strain and stress distributions. Thus, a linear temperature gradient should be applied within an element, otherwise the stresses would be inconsistent and there locking may occur inside the element. It was found that the ANSYS results were exact, even if when the applied thermal gradient was not linear. This indicates, once again, that ANSYS
elements are not based on the standard FE formulation, but rather have been extended to avoid potential locking issues.

Table 3-3 shows the applied temperature and stresses through the depth of the pavement obtained using the different elements and the two software packages. Figure 3-8 shows the stresses through the depth produced by using different elements. Two different stresses are given for each node in ABAQUS 4-node elements. One is related to the top element and the other one is related to the bottom element.

Table 3-3 Temperature Stresses in Different Elements

<table>
<thead>
<tr>
<th>Depth (in)</th>
<th>Temp</th>
<th>Real σ</th>
<th>ANSYS 4 nodes</th>
<th>ANSYS 8 nodes</th>
<th>ABAQUS 4 nodes</th>
<th>ABAQUS 8 nodes</th>
<th>Linear Temp</th>
<th>ABAQUS 8 nodes</th>
</tr>
</thead>
<tbody>
<tr>
<td>-6</td>
<td>24</td>
<td>69120</td>
<td>69120</td>
<td>69120</td>
<td>53280</td>
<td>67200</td>
<td>24</td>
<td>69120</td>
</tr>
<tr>
<td>-5</td>
<td>13</td>
<td>37440</td>
<td>37440</td>
<td>37440</td>
<td>53280,24480</td>
<td>38400</td>
<td>14</td>
<td></td>
</tr>
<tr>
<td>-4</td>
<td>4</td>
<td>11520</td>
<td>11520</td>
<td>11520</td>
<td>24480,1440</td>
<td>9600</td>
<td>4</td>
<td>11520</td>
</tr>
<tr>
<td>-3</td>
<td>-3</td>
<td>-8640</td>
<td>-8640</td>
<td>-8640</td>
<td>1440,-15840</td>
<td>-7680</td>
<td>-2</td>
<td></td>
</tr>
<tr>
<td>-2</td>
<td>8</td>
<td>-23040</td>
<td>-23040</td>
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<td>-15840,-27360</td>
<td>-24960</td>
<td>-8</td>
<td>-23040</td>
</tr>
<tr>
<td>-1</td>
<td>-11</td>
<td>-31680</td>
<td>-31680</td>
<td>-31680</td>
<td>-27360,-33120</td>
<td>-30720</td>
<td>-10</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>-12</td>
<td>-34560</td>
<td>-34560</td>
<td>-34560</td>
<td>-33120,-33120</td>
<td>-36480</td>
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<td>-34560</td>
</tr>
<tr>
<td>1</td>
<td>-11</td>
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<td>-31680</td>
<td>-31680</td>
<td>-27360,-33120</td>
<td>-30720</td>
<td>-10</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>8</td>
<td>-23040</td>
<td>-23040</td>
<td>-23040</td>
<td>-15840,-27360</td>
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<td>1440,-15840</td>
<td>-7680</td>
<td>-2</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>11520</td>
<td>11520</td>
<td>11520</td>
<td>24480,1440</td>
<td>9600</td>
<td>4</td>
<td>11520</td>
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<td>69120</td>
<td>69120</td>
<td>53280</td>
<td>67200</td>
<td>24</td>
<td>69120</td>
</tr>
</tbody>
</table>

Figure 3-8 Temperature Gradient in Different Elements
3.1.5 Shrinkage

Shrinkage of concrete is a function of the concrete mixture materials and on environmental conditions. The effect of uniform shrinkage is similar to the effect of uniform temperature change. Consequently, its effects can be divided into uniform, linear, and nonlinear components. Since the top of the pavement is in direct contact with the environment and is subjected to sunshine and wind, the top layer of concrete looses moisture at a greater rate than the lower layers. This causes the pavement to curl upwards. The amount of curling depends on various factors including concrete mixture, curing, time of pouring, humidity, and wind speed. In some cases, curling due to shrinkage may be so high that it totally compensates the effect of temperature during daytime. In this case, the concrete slab would not experience a downward curvature. It should be noted that the amount of curling and developed stresses may change with time as a result of stress relaxation.

Since thermal gradients and linear shrinkage have similar effects on the concrete slab, linear shrinkage is usually modeled as an equivalent thermal gradient through the depth of the slab. Reddy et al. (1963) estimated an equivalent thermal gradient of 0.065°C/mm to 0.13°C/mm, while Armaghani et al. (1987) suggested the much lower value of 0.022 °C/mm. Fang et al. (2001) estimated the effect of moisture to be equivalent to a linear thermal gradient equal to 0.10 to 0.027 °C/cm. Depending on the weather and curing method used on the surface, the nonlinear shrinkage can change significantly and the stresses on the surface can increase so much that crack initiation at the top surface can occur. Janssen (1987) has shown that a typical moisture gradient can be enough to develop 19 mm of crack at the top of a pavement due to locally high shrinkage. Since strength of concrete and shrinkage increases with time, timely curing of concrete becomes a greater issue. For high strength concrete this increase is mainly uniform and the effect of strength on linear and nonlinear shrinkage is still unclear and a current topic of research. Similarly to thermal gradients, constant longitudinal stresses are restricted by weight and friction, while linear shrinkage causes curling.
3.2 Tire Pressure

The stresses developed under wheel loading combined with other stresses, such as thermal stress, are the main reason for concrete slab cracking. These stresses tend to increase as the slab becomes thinner and the subgrade and subbase materials become softer. In order to accurately predict the service life of a concrete pavement, an accurate model of wheel loading is necessary, since this loading can greatly affect the developed stresses. One method that can be used to improve the accuracy of the model consists of refining the finite element mesh by either increasing the number of elements or by using higher order elements. However, such a refinement should not be done indiscriminately. A main concern in finite element modeling consists of capturing an accurate enough mesh with minimum number of degrees of freedom. The following subsections provide detailed discussion on tire loading simulation. In particular, they explore the parameters that affect the of this simulation are

3.2.1 Tire Loading Area

Often time, when attempting to apply a wheel loading on a FE model, one may find that the tire print does not exactly coincide with an element in the FE mesh (Figure 3-9-a). In this case, one may opt to approximate the load so that it is completely applied on an element or elements. Alternatively, one may choose to more accurately simulate this loading by computing work equivalent nodal loads for the brick elements subjected to a partial pressure. The latter method is adopted in this study and it is thus described next.

Since in the finite element method, loads can only be applied at degrees of freedom, surface tractions \( b \) are replaced by work equivalent nodal loads, i.e.,

\[
P_{b} = \int_{A} f^{T} b dv
\]

Equation 3-7

where \( P_{b} = \) equivalent nodal loads

\( f \) = displacement shape function matrix

\( b = [ 0 \ 0 \ p ]^{T} = \) pressure loading matrix

\( p \) = tire pressure
For a 20-node brick element the shape functions are of the form:

\[
f = \begin{bmatrix} f_1 & 0 & 0 \\ 0 & f_1 & 0 \\ 0 & 0 & f_1 \end{bmatrix} \quad (i = 1,...,8)
\]

Equation 3-8

where

\[
f_i = \frac{1}{8}(1 + \xi_0)(1 + \eta_0)(1 + \zeta_0)(\xi_0 + \eta_0 + \zeta_0 - 2) \quad (i = 1,...,8)
\]

\[
f_i = \frac{1}{4}(1 - \xi^2)(1 + \eta_0)(1 + \zeta_0) \quad (i = 9,11,17,19)
\]

\[
f_i = \frac{1}{4}(1 - \eta^2)(1 + \xi_0)(1 + \zeta_0) \quad (i = 10,12,18,20)
\]

\[
f_i = \frac{1}{8}(1 - \xi^2)(1 + \xi_0)(1 + \eta_0) \quad (i = 13,14,15,16)
\]

Equation 3-9

where \(\xi, \eta,\) and \(\zeta\) are the three-dimensional natural coordinates for the element.

Figure 3-9 Simulation of Tire Loading on FE Meshing, a) Wheel Loading on FE Meshing, b) Partial Mesh Loading, c) Joint Number, and d) Equivalent Load
The limits of the surface integral given by Equation 3-7 depend on the mesh and on the shape of loading. For a rectangular loading over a rectangular element (Figure 3-9) the equivalent loads can be evaluated by multiplying a factor of 24pxy to the following:

\[ F_1 = [(a^2-b^2+2b-2a)(d^3-c^3)+ (a^3-b^3+1.5 b^2-1.5a^2)(d^2-c^2)+ 2b^3 (d-c)- 2a^3 (d-c)-6b (d-c)+ 6a (d-c)] \]

\[ F_4 = [(b^2- a^2+2b-2a)(d^3-c^3)+ (a^3-b^3+1.5 b^2+1.5 a^2)(d^2-c^2)+2b^3 (d-c)- 2a^3 (d-c)- 6b (d-c)+ 6a (d-c)] \]

\[ F_5 = [(b^2+a^2+2b-2a)(d^3-c^3)+ (b^3- a^3+1.5 b^2-1.5 a^2)(d^2-c^2)+2b^3 (d-c)-2a^3 (d-c)- 6b (d-c)+ 6a (d-c)] \]

\[ F_8 = [(b^2- a^2+2b-2a)(d^3-c^3)+ (a^3-b^3+1.5 b^2-1.5a^2)(d^2-c^2)+ 2b^3 (d-c)- 2a^3 (d-c)- 6b (d-c)+ 6a (d-c)] \]

\[ F_2 = [(2b^3- 2a^3- 6b+ 6a)(d^2-c^2)- 4b^3 (d-c)+ 4a^3 (d-c)+12b (d-c)-12a (d-c)] \]

\[ F_3 = [(2b^3- 2a^3- 4b+ 4a)(d^3-c^3)-6b^2 (d-c)+6a^2 (d-c)+12b (d-c)-12a (d-c)] \]

\[ F_6 = [(2a^2- 2b^2- 4b+ 4a)(d^3-c^3)+ 6b^2 (d-c)- 6a^2 (d-c)+ 12b (d-c)-12a (d-c)] \]

\[ F_7 = [(2a^2- 2b^2+ 6b - 6a)(d^2-c^2)- 4b^3 (d-c)+ 4a^3 (d-c)+ 12b (d-c)-12a (d-c)] \]  

Equation 3-10

where, as illustrated in Figure 3-9, 2x and 2y are the dimension of the shell element and \( F_1 \) to \( F_8 \) are the equivalent nodal loads and \( a, b, c, \) and \( d \) are element dimensions. Thus, if a pressure \( p \) is applied over the entire surface area of an element, the resulting equivalent nodal forces are given by:

\[ F_1 = F_4 = F_5 = F_8 = -0.333 \text{ pxy}, \quad F_2 = F_3 = F_6 = F_7 = 1.333 \text{ pxy} \]  

Equation 3-11

### 3.2.2 Integration of the Pressure over the Footprint

The integration limits in Equation 3-7 depend on the mesh and on the shape of loading. The easiest way to compute this integral is to change the coordinates to a square unit coordinates (Figure 3-10). By transforming the coordinates Equation 3-7 can be rewritten as

\[ P_i = p \int_A f_i |J| d\xi d\eta \]  

Equation 3-12

Where \(|J|\) is the determinant of Jacobian matrix and \( \xi \) and \( \eta \) are dimensionless natural coordinates (Figure 3-10). The integration for a circular load applied on a skewed element was solved as an example. When the applied load is applied as shown in Figure 3-10, the following is true:
\[ x_2 = (Y + y) \tan \alpha, \quad y_1 = r \cos \alpha, \quad x_3 = r \sin \alpha \]

\[ x_1 + x_2 = \sqrt{r^2 - (Y + y)^2} \Rightarrow x_1 = \sqrt{r^2 - (Y + y)^2} - (Y + y) \tan \alpha \quad \text{Equation 3-13} \]

Next define the following quantities:

\[ m = r / X, \quad n = Y / r, \quad f = mn = Y / X \quad \text{Equation 3-14} \]

Substituting these into Equation 3-13, results in the following:

\[ x_1 / X = m \sqrt{1 - (n + ny / Y)^2} - f (1 + y / Y) \tan \alpha \quad \text{Equation 3-15} \]

\[ y_1 / Y = \cos \alpha / n \quad \text{Equation 3-16} \]

In this case the integral limit for \( \xi \) and \( \eta \) is

\[ \xi_{\min} = -X / X, \quad \xi_{\max} = (-X + x_1) / X \quad \text{Equation 3-17} \]

\[ \eta_{\min} = -Y / Y, \quad \eta_{\max} = (-Y + y_1) / Y \quad \text{Equation 3-18} \]

which results in the following:

\[ -1 \leq \xi \leq -1 + m \sqrt{1 - (n + n\eta)^2} - f (1 + \eta) \tan \alpha \quad \text{Equation 3-19} \]

\[ -1 \leq \eta \leq -1 + \cos \alpha / n \quad \text{Equation 3-20} \]
The integration for the obtuse angle is similar to the previous calculation. If we apply a total circular pressure on a mesh of four elements the previous results can be added up to produce the equivalent nodal forces, which are shown in Figure 3-11.

Figure 3-11 Equivalent Nodal Loads for a Circular Pressure applied on a Skewed Mesh

Equivalent loads for regular rectangular meshes can be evaluated by simply replacing $\alpha$ with zero. The coefficients corresponding to different ratios of $m$ and $f$ are summarized in Table 3-4. Using these coefficients, the equivalent nodal loads can be calculated as $P_i = 4 \times p \times X \times Y \times \text{co}_i$.

Table 3-4 Equivalent Nodal Loads Coefficients

<table>
<thead>
<tr>
<th>X/Y</th>
<th>r/2X</th>
<th>Node 1</th>
<th>Node 2</th>
<th>Node 3</th>
<th>Node 4</th>
</tr>
</thead>
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<tr>
<td>0.3</td>
<td>1</td>
<td>0.1168</td>
<td>0.0888</td>
<td>0.0113</td>
<td>0.0188</td>
</tr>
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<td></td>
<td>0.75</td>
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<td></td>
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<td>0.0433</td>
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<td>0.0127</td>
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<td>0.7</td>
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</tr>
<tr>
<td></td>
<td>0.75</td>
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<td>0.0790</td>
<td>0.0193</td>
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<tr>
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<tr>
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<td>0.0002</td>
<td>0.0023</td>
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<td>0.0605</td>
<td>0.0098</td>
<td>0.0253</td>
</tr>
<tr>
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<td>0.0688</td>
<td>0.0188</td>
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<td>0.0085</td>
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<td>0.0001</td>
<td>0.0012</td>
</tr>
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<td>1</td>
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<td>0.2083</td>
</tr>
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<td>0.75</td>
<td>0.2000</td>
<td>0.1010</td>
<td>0.0395</td>
<td>0.1010</td>
</tr>
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<td>0.5</td>
<td>0.1208</td>
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<td>0.0390</td>
<td>0.0047</td>
<td>0.0005</td>
<td>0.0047</td>
</tr>
</tbody>
</table>
3.2.3 Parametric Studies

Parametric studies on the effect of footprint, mesh size, element type, and slab thickness on the developed stresses were carried out in this investigation. A 3.66 m (12 ft) by 3.66 m (12 ft) concrete slab with 31 Mpa modulus of elasticity was modeled over a 0.3 m (12 inch) subbase and 3 meters of 6.895 kPa subgrade. Since it is known that the stresses are higher for softer subgrades, a medium soft soil was chosen.

3.2.3.1 Footprint

The shape of an actual tire footprint is close to an eclipse. However, representing an eclipse in a FE model is not straightforward. In general, an equivalent rectangular or circular footprint would be preferred. In this section, the effect of the shape of a footprint on the developed stresses is studied. This is done by considering the five different footprints shown in Figure 3-12. The first footprint consisted of a square and two semicircles, which is the closest shape to an actual footprint. The two tire footprints were simulated by two of these shapes. The second was similar to the first, but two rectangular shapes with the same area and width replaced the two semicircles. The third model replaced the two-tire footprints with two circles with each with the same area as the footprint used in the first model. In the fourth model a single circle representing the footprint of both tires was used. Finally, the fifth and simplest model, replaced the two tire footprint with a single square shape. In all cases a uniform pressure was applied throughout the footprint area.

3.2.3.2 Mesh Size

To study the effect of mesh size on the developed stresses under wheel pressure three different mesh sizes were considered. The first mesh consisted of 300mm by 300 mm elements. The other two meshes were constructed with element sizes one half and a quarter of the first mesh. The equivalent nodal loads for each mesh and for each footprint were calculated using the method described in Section 3.2.1.
In addition, two types of elements were considered, namely 8-node and 20-node solid elements. It should be noted that the number of degrees of freedom (dof) when a volume is meshed with 20-node solid elements is four times larger than the number of dof if the volume is meshed with the same number of 8-node solid elements. Furthermore, the number of dof on the surface of the 20-node solid element mesh is 3.67 times larger than the number of dof on the surface of the 9-node solid element mesh.

3.2.3.3 Slab Thickness

The tire pressure tends to be distributed over a larger area in thicker slabs. This results in decreased stresses in these cases. To verify this effect, four different thickness
values were studied in the model with the largest mesh size. The thickness values considered were: 375 mm, 300 mm, 225 mm, and 150 mm.

3.2.3.4 Results

Tables 3-5 and 3-6 present the results of the parametric studies. In particular, the maximum stresses in the x- and z- directions, which correspond to the transverse and longitudinal directions in the slab, are reported.

As can be seen from Table 3-5, the first two footprints (Figure 3-12) produce very similar results, i.e., the difference between these results is less than 0.2% for all of the cases except one. The third footprint generates slightly higher stresses (1 to 6 percent higher). The fourth footprint produces maximum stresses between 10 to 40 percent higher than the first two footprints, while the stresses generated by the fifth footprint are slightly lower than those produced by the fourth footprint (between 2 to 5 percent).

Table 3-5 The Effect of Footprint, Slab Thickness, and Element Type on Developed Stresses (Mesh Size 300 mm x 300 mm)

<table>
<thead>
<tr>
<th>Elemen Type</th>
<th>Depth(mm)→</th>
<th>150 mm</th>
<th>225 mm</th>
<th>300 mm</th>
<th>375 mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Load↓</td>
<td>σ_x(kPa)</td>
<td>σ_z(kPa)</td>
<td>σ_x(kPa)</td>
<td>σ_z(kPa)</td>
<td>σ_x(kPa)</td>
</tr>
<tr>
<td>A1</td>
<td>2386</td>
<td>2215</td>
<td>1242</td>
<td>1158</td>
<td>737</td>
</tr>
<tr>
<td>A2</td>
<td>2387</td>
<td>2211</td>
<td>1243</td>
<td>1156</td>
<td>737</td>
</tr>
<tr>
<td>A3</td>
<td>2432</td>
<td>2216</td>
<td>1265</td>
<td>1158</td>
<td>749</td>
</tr>
<tr>
<td>A4</td>
<td>2600</td>
<td>2600</td>
<td>1348</td>
<td>1348</td>
<td>797</td>
</tr>
<tr>
<td>A5</td>
<td>2539</td>
<td>2539</td>
<td>1319</td>
<td>1319</td>
<td>780</td>
</tr>
</tbody>
</table>

From Table 3-6, it can be seen that the results produced by the 20-node solid element meshes produces 70 to 80 percent higher stresses than the less accurate 8-node solid element meshes. It should be noticed that this difference is for the less refined mesh. The difference decreases slightly as the thickness of the slab increases. As it was mentioned previously, thicker slabs tend to better distribute the stresses, resulting in less
sensitivity to tire pressure modeling. For the second mesh size, the developed stresses produced by the 8-node solid element mesh are only about 10% lower than those generated by the 20-node solid element mesh. This indicates that doubling the number of elements in the 8-node solid element mesh has significantly improved its accuracy. For the third mesh refinement the developed stresses produced by the 8-node solid element mesh are only about 5% lower than those generated by the 20-node solid element mesh.

The improvement in the results with mesh refinement was of about 30% for the 8-node solid element mesh and 10% for the 20-node solid element mesh. This is because even the coarse 20-node mesh is already accurate enough. It should be noted that the 300 by 300 mm mesh of 20-node elements has about the same number of degrees of freedom as the 150 by 150 mm mesh of 8-node elements. Comparing the results of these two meshes, it can be seen that the 20-node element mesh produces stresses 10% higher than the more refined 8-node element mesh. This difference reduces to 5% when the stresses generated by the 150 by 150 mm mesh of 20-node elements are compared with those produced by the 75 by 75 mm mesh of 8-node elements. Thus, it can be concluded that for the same number of degrees of freedom using 20-node elements is more accurate, and thus should be preferred.

Table 3-6 The Effect of Footprint, Mesh Size, and Element Type on Developed Stresses (Depth of Slab 150 mm)

<table>
<thead>
<tr>
<th>Element Type</th>
<th>Mesh size→</th>
<th>300mm x 300mm</th>
<th>150mm x 150mm</th>
<th>75mm x 75mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Load ↓</td>
<td>σ_x(kPa)</td>
<td>σ_z(kPa)</td>
<td>σ_x(kPa)</td>
<td>σ_z(kPa)</td>
</tr>
<tr>
<td>8 Node</td>
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<td></td>
</tr>
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<td>3159</td>
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<td>2600</td>
<td>3430</td>
<td>3430</td>
</tr>
<tr>
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</tr>
<tr>
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</tr>
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<td>A4</td>
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<td>-----</td>
</tr>
<tr>
<td>A5</td>
<td>3737</td>
<td>3737</td>
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</tr>
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</table>
CHAPTER 4 PARAMETRIC STUDIES

Various factors can affect the service life of the concrete slab. These factors include the geometry of the slabs, environmental conditions, applied loading, and material properties. In general, the goal is to improve the service life of a pavement while minimizing the cost. The knowledge how each of these factors affects the stresses developed in the pavement allow for the optimization of concrete pavement design. However, as it is discussed later in this chapter, these factors also affect each other. For example, changing material properties and geometry can lead to different results for different loadings. For instance, a thicker slab can reduce the developed traffic stresses, but it cannot reduce the thermal stresses. Consequently, for a better understanding of the behavior, the effect of each parameter on different loading conditions must be studied separately. In this chapter, the effect of subbase and slab thickness, slab length and stiffness on the developed stresses is studied. In addition, the effect of soil properties and soil pumping on the stresses are discussed.

4.1 The Effect of Subbase and Slab Thickness, Length and Stiffness on Developed Stresses

During its service life, concrete pavements are subjected to various loads including self-weight, traffic and thermal loads, and shrinkage. The combination of positive thermal gradient (i.e., the bottom of the slab is cooler than the top) and traffic loading causes maximum tensile stresses at the bottom of the slab. Concrete pavements are mainly designed based on these stresses, since the majority of the observed cracks in the field are bottom to top cracking. It is accepted that increasing the thickness and stiffness of the concrete slab and of the subbase can decrease these stresses and, consequently this type
of cracking. This is because, in this case, traffic loading is responsible for most of these stresses and thermal stresses are of secondary importance.

Negative thermal gradients, on the other hand, can produce high stresses on the top of concrete slab. These stresses increase as the thickness and the stiffness of the concrete slab increase and, in general, are pretty insensitive to subbase thickness. Combining these stresses with the tensile stresses due to traffic loading on top of the slab and non-uniform shrinkage stresses can result in top to bottom cracking of the concrete slab.

In this section, the effects of concrete strength, and slab and subbase thickness, length, and stiffness on the stresses in concrete pavements are investigated. This is achieved through a series of simulations using a three-dimensional finite element model of a single slab pavement. By varying the thickness and strength of the concrete slab and of the subbase and by using different material properties for the subgrade, the effect of each of these factors was studied. This has enabled the determination of optimal values of slab thickness for which the development of bottom to top and top to bottom cracking is minimized. More specifically, for a 9-in thick slab the Equivalent Shrinkage Gradient (ESG) was changed and the sensitivity of the stresses on the top of the slab to the ESG was studied. Furthermore, given subbase thickness, optimal values of slab thickness were determined.

4.1.1 Developed Finite Element Model

The model developed in this study consists of a single concrete slab over subbase and subgrade layers, which can have distinct properties. The concrete slab is modeled using eight 8-node brick elements through its depth. The subbase and subgrade are modeled slightly wider than the slab to enable a better distribution of the stresses. In the following sub-sections, finite element modeling issues such as material models, the interaction between subbase and slab, boundary conditions, and loading combinations are discussed in detail.
4.1.1.1 Material Models

Simulations of a concrete pavement under service loads and thermal effects revealed that the stresses developed both in the soil and in the concrete slab are effectively smaller than the yield stresses of the two materials. Thus, it can be ascertained that the main reason for concrete pavement cracking is fatigue. As a result, linear elastic material models were used in the developed FE model. The modulus of rupture and the modulus of elasticity of concrete were estimated using the following equations:

\[ S_c = 21\sqrt{f'_c} \quad \text{(SI unit)} \quad \text{Equation 4-1} \]

\[ E_c = 150,000\sqrt{f'_c} \quad \text{(SI unit)} \quad \text{Equation 4-2} \]

where, \( f'_c \): concrete compressive strength, \( S_c \): modulus of rupture of concrete, \( E_c \): concrete Young’s modulus. The adopted material properties are given in Table 4-1 where \( \nu \): Poisson ratio, \( E \): Young’s modulus, \( \gamma \): unit weight, and \( \alpha \): coefficient of thermal expansion. It must be noticed that the minimum 28-day value for modulus of rupture of concrete pavement design in Indiana is 4,482 kPa (650 psi).

<table>
<thead>
<tr>
<th>Location</th>
<th>Material</th>
<th>( f'_c ) (MPa)</th>
<th>( S_c ) (kPa)</th>
<th>( E ) (MPa)</th>
<th>( \nu )</th>
<th>( \gamma ) (kg/m(^3))</th>
<th>( \alpha ) (1/°F)</th>
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<td>Crushed Stone</td>
<td>NA</td>
<td>NA</td>
<td>345</td>
<td>0.35</td>
<td>2240</td>
<td>NA</td>
</tr>
<tr>
<td>Subgrade</td>
<td>Med-stiff Clay</td>
<td>NA</td>
<td>NA</td>
<td>172</td>
<td>0.4</td>
<td>1760</td>
<td>NA</td>
</tr>
<tr>
<td>Subbase</td>
<td>Crushed Stone</td>
<td>NA</td>
<td>NA</td>
<td>241</td>
<td>0.35</td>
<td>2240</td>
<td>NA</td>
</tr>
<tr>
<td>Subgrade</td>
<td>Med Clay</td>
<td>NA</td>
<td>NA</td>
<td>69</td>
<td>0.4</td>
<td>1760</td>
<td>NA</td>
</tr>
<tr>
<td>Subbase</td>
<td>Crushed Stone</td>
<td>NA</td>
<td>NA</td>
<td>69</td>
<td>0.35</td>
<td>2240</td>
<td>NA</td>
</tr>
<tr>
<td>Subgrade</td>
<td>Med-soft Clay</td>
<td>NA</td>
<td>NA</td>
<td>28</td>
<td>0.4</td>
<td>1760</td>
<td>NA</td>
</tr>
</tbody>
</table>
4.1.2 Loading

Concrete slabs are mainly subjected to the following loads: (a) self-weight, (b) traffic load, (c) temperature gradients, and (d) shrinkage. The following subsections detail how each of these conditions was handled in this study.

4.1.2.1 Boundary Conditions

For both the subbase and subgrade, the degrees of freedom perpendicular to the model’s boundaries were restricted. If a sufficient depth of soil is modeled, restraining the displacements of the bottom most layer of soil has no effect on the stresses developed in the slab (Ioannides 1988). Thus, in this study, all degrees of freedom of the bottom layer subgrade were also restricted.

Under thermal loads, portions of the concrete slab may lose contact with the subbase. Thus, to capture this behavior, it is necessary to appropriately model the interface between concrete slab and subbase. This is achieved here by incorporating contact elements in this interface. The friction between soil and concrete slab was incorporated to the model by assigning tangent of the friction angle of soil as the friction coefficient for the contact elements.

4.1.2.2 Traffic Load

A 142 kN (32-kips) single axle load was applied to the concrete slab. The tire pressure was assumed to be 765 kPa (111 psi) and consequently the tire contact area is 46,450 mm² (72 in²). The area and shape of the tire footprint are important when the stresses of concern are those within a few elements from the line of load application. In these regions, tensile stresses due to traffic loading occur at the bottom of the slab. However, they have little effect on the stresses far from the wheel loading. In this case, tensile stresses due to traffic loading occur at the top of the slab. This effect increases as the area of the loading becomes smaller and the thickness of the slab decreases.
4.1.2.3 Thermal Loads

Previous research by Thompson et al. (1987) showed that the temperature change in the supporting layers of a concrete pavement is very small. Thus, in this work, the temperature differential is applied solely to the concrete slab.

In general, thermal gradients can be decomposed into uniform, linear, and nonlinear components, which are responsible for the development of axial, curling, and nonlinear stresses, respectively. The axial stresses depend on the difference in temperature between the time of pavement construction and the time when the stresses are calculated. The effect of these stresses is limited by the friction between the slab and subbase. The maximum friction force that can develop is given by

\[ F_{\text{max}} = \mu \gamma \frac{A L}{2} \]  

where \( \mu \) is the coefficient of friction between the slab and subbase, \( A \) is the area of the slab cross-section, and \( L \) is the length of the slab. Considering the coefficient of friction to be the tangent of the soil friction angle, the maximum stress can be determined by dividing this force to the cross section area. For a 5.49 m-long (18 ft) slab the maximum developed stress is 53.8 kPa (7.8 psi). Thus, it can be concluded that the stress produced by a uniform temperature change is negligible when compared to the modulus of rupture of concrete and further adjustment to the Coulomb friction factor was deemed unnecessary. It should be noted, however, that the axial stress would increase considerably if the dowel bars were to restrict the free displacement of the slab. In fact, such a restriction could cause extremely large stresses in the slab. For example, for concrete with modulus of elasticity of 31,030 MPa (4,500 ksi), a 27.8°C (50°F) change in temperature creates a tensile stress of 9,308 kPa (1,350 psi), which is significantly higher than the modulus of rupture of concrete. Such a stress would cause immediate cracking of the slab. In real pavements, however, the stress level depends on the amount of restriction induced by the dowel bars. In this study, the dowel bars were assumed to perform properly, and this restriction was neglected.

Linear thermal stresses cause the pavement to curl. The thermal gradient that causes the top slab surface to cool down is referred to as a negative temperature gradient (Figure 4-1-a). Similarly, the temperature gradient that causes the top slab surface to warm up is
called a positive temperature gradient (Figure 4-1-b). Negative temperature differentials may cause the slab corners to curl upwards creating tensile stresses on its top surface and compressive stresses on its bottom surface. Positive temperature gradients may cause the slab edges to curl downwards. Under a positive temperature gradient, the top of the concrete slab is in compression while the bottom is in tension. The temperature gradient through the depth of the slab depends on environmental conditions including the ambient temperature, percentage of sunshine, wind velocity, slab dimensions, and concrete properties including emissivity and conductivity.

![Figure 4-1 Positive and Negative Temperature Gradient, Developed Stresses and Corresponding Traffic Loading, a) Negative Temperature Gradient, b) Positive Temperature Gradient](image-url)

Figure 4-1 Positive and Negative Temperature Gradient, Developed Stresses and Corresponding Traffic Loading, a) Negative Temperature Gradient, b) Positive Temperature Gradient
The nonlinear part of the thermal gradient induces stresses in the slab without causing either axial or curling deformation. These stresses increase the tensile curling stresses in case of negative thermal gradient and decrease the tensile curling stresses in the case of positive thermal gradient.

The Integrated Climatic Model (ICM) program developed by Larson and Dempsey (1993) at the University of Illinois at Urbana-Champaign was developed to model climatic effects on pavements. The ICM program uses input pavement properties and appropriate climatic data to calculate thermal gradients through the depth of pavements. In this study, the ICM program with the available climatic data from Illinois was used to compute the thermal gradient through the various depths of the studied concrete slab. With this data, the peak temperature gradients were calculated through the depth of the pavement. The resulting thermal gradients are shown in Figure 4-2-a and 4-2-b for positive and negative temperature changes, respectively. As can be seen from this figures the thermal gradient at the top of the different slabs are relatively close to each other. This seems logical, since for thicker slabs the bottom layer of the concrete plays a similar role as the top layer of soil for a thinner slab. As the thickness of the slab increases the nonlinearity of the thermal gradient also increases.

4.1.2.4 Shrinkage

As mentioned previously, the effect of shrinkage may be simulated as a thermal gradient. In this study, an equivalent linear thermal gradient of 0.022°C/mm (1°F/in) was applied to the concrete slab (Figure 4-2-c). The sensitivity of the stresses to the change in equivalent shrinkage gradient for a 228 mm (9 in)-slab was studied and the results, which are summarized in Table 4-2 and Figure 4-2-d, are discussed in detail in Section 4.1.4.
Figure 4-2 Temperature Gradient Developed by ICM, a) Negative, b) Positive, c) Positive & Negative Temperature Gradient Plus Equivalent Shrinkage, and d) Total Negative Temperature Gradient for Various Equivalent Shrinkage Gradient.

Table 4-2 Effect of Equivalent Shrinkage Gradient on the Slab Stresses and Stress Ratios

<table>
<thead>
<tr>
<th>Equivalent temp→ (°C/mm)</th>
<th>Stress (kPa)</th>
<th>Ratio %</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.0 0.01 0.02 0.03 0.04</td>
<td>0.0 0.01 0.02 0.03 0.04</td>
</tr>
<tr>
<td>Soil type↓</td>
<td>Medium-soft</td>
<td>2620 2805 2980 3130 3275</td>
</tr>
<tr>
<td></td>
<td>Medium</td>
<td>2490 2765 3025 3275 3500</td>
</tr>
<tr>
<td></td>
<td>Medium-stiff</td>
<td>2510 2875 3205 3515 3780</td>
</tr>
</tbody>
</table>
4.1.3 Load Combinations

The developed finite element model was analyzed under several combinations of the following loads: thermal, shrinkage, and traffic. Each of these loads created stresses in the pavement that counteracted each other or added up to one another. Thus, to evaluate the worst case scenario, appropriate load combinations were applied.

Under a positive temperature gradient, the top of the concrete slab is in compression while the bottom is in tension. The amount of tension and compression increases from the edges towards the middle of the slab. Linear shrinkage causes the pavement to curl upwards and produces stresses that counteract the thermal stresses. Tire loading creates maximum tension in the bottom of the slab at the line of load application. To create maximum tensile stress due to the combination of positive and traffic loading (e.g., combined positive load) the tire loading must be applied in the middle of the slab (Figure 4-1-b).

Under a negative temperature gradient, the bottom of the concrete slab is in compression while the top of the slab is in tension. The middle of the slab has the highest tensile and compressive stresses. Linear shrinkage increases the stresses, thus the maximum shrinkage effect must be applied to maximize the stress. The compressive stresses developed under the line of load application decrease the tensile stresses due to negative thermal gradient. However, when the tire load is applied at the edge of the slab it creates tension at the top in the middle of the slab. Thus, to create the maximum tensile stress due to the combination of negative and traffic loading (e.g., combined negative load) the tire load must be applied at the edge of the slab (Figure 4-1-a).

4.1.4 General Results

The main concern in concrete pavement design and construction is to increase the pavement’s service life by preventing the occurrence of cracking. A solution that has been traditionally used for achieving this is to adopt thicker slabs. Alternatively, higher strength concrete has also been used. However, in this case, the higher shrinkage rate and decreased creep relaxation typical of these materials can adversely affect a concrete pavement performance especially in its early ages. Furthermore, the higher costs
associated with the use of higher strength concrete have also been a great concern. Research by Byrum et al. (1997) showed that the effect of Portland Cement Concrete strength on the performance of pavements is highly dependent on environmental conditions. More specifically, they found that in cooler-wet no-freeze environments the higher strength concrete improved performance, while in hot regions, a lower cement content material with lower modulus of elasticity performed better. This is mainly because of shrinkage cracking for high temperature conditions.

In this study, the developed 3D FE model of a single concrete slab pavement was analyzed for different load combinations, material properties, subbase thickness, and three different types of soil using the commercial software ANSYS (1995). Figure 4-1-a and Figure 4-1-b show obtained sample deflections under combined negative and positive load obtained, respectively. The results from these analyses are discussed in the following subsections.

4.1.4.1  **Effect of Shrinkage**

The total temperature gradient with different equivalent shrinkage gradients were calculated (Figure 4-2-d) and applied to a 228 mm (9-inch) deep concrete slab. The concrete strength $f'_c$ was taken as 30,400 MPa (4,415 ksi). The maximum stresses and maximum stress ratios (the ratio of the maximum stress to the modulus of rupture) were calculated and are summarized in Table 4-2. As it can be seen, both the stresses and the stress ratios are greatly affected by the shrinkage gradient. The stress ratio increases by about 4, 6, and 7.5 percent of the MR for medium-soft medium and medium-stiff soil, respectively, while the equivalent shrinkage increases by about 0.011ºC/mm (0.5 ºF/in).

4.1.4.2  **Effect of Soil Type**

The stresses obtained for three types of soil are summarized in Table 4-3. It can be seen that for thinner slabs the stresses are very sensitive to the stiffness of the soil, i.e., the stresses and stress ratios greatly increase for softer soils. As the thickness of the slab increases this sensitivity decreases until it reaches a point that the stresses become
insensitive to the soil type following which, the reverse effect is observed. For thin slabs, traffic loading governs the developed stresses. Consequently, even for the case of combined negative temperature gradient and traffic, a stiffer subgrade results in a reduced maximum stress in the slab. As the thickness of the slab increases the negative thermal loading stresses becomes more significant. As a result, higher stresses are developed in thicker slabs over stiffer subgrade.

4.1.4.3 Effect of Concrete Stiffness and Thickness

With the exception of a few cases, for thicker slabs over soft subgrade, if all other conditions remain unchanged, the bending stresses for both the negative and positive combined situations increase with concrete strength and consequently with its modulus of elasticity (Table 4-3). This tendency is observed for all slab thicknesses. The effect of slab thickness on stresses depends on the loading combination. For positive combined loading the developed stresses are significantly less sensitive to the stiffness of concrete as the slab thickness increases. Under negative combined loading, on the other hand, the stresses remain sensitive to the stiffness of concrete when the slab thickness is increased. This is because the stresses caused by negative combined stresses are mainly due to the temperature gradient, while traffic loading is responsible for the stresses produced by positive combined loads. When all other conditions are unchanged, as the thickness of the concrete slab increases the bending stresses in the positive combined load decrease. The rate of reduction is higher for stiffer concrete. The bending stresses produced by the negative combined load increase slightly for stiffer subgrade or remain constant for slabs over softer subgrades.

For medium or medium stiff soil, the minimum stress for positive load combination occurs in the softest thickest concrete slab. However, for negative load combination, the minimum stress takes place in the softest thinnest concrete slab. In a medium soft soil, the minimum stress for both positive and negative combined condition occurs in the softest thickest concrete slab.

The modulus of rupture of concrete increases with the square root of concrete’s compressive strength. However, the change in stress ratio (the ratio of maximum stress
to modulus of rupture) shows a different trend. As it can be seen from Table 4-3 and Figure 4-3, the stress ratio decreases with the increase in concrete stiffness. This means that the rate of increase in stiffness is higher than that in stresses. It should be pointed out that, in this study, the rate of shrinkage has been considered the same for the concrete materials of different strength. In reality, however, it is accepted that increasing concrete’s stiffness often increases shrinkage. Thus, if such an increase were considered, the observed trend would be less severe. Finally, it should be mentioned that using higher strength concrete may not be economical and the observed benefits may not be justified.

![Graphs showing stress ratio vs depth for different concrete strengths](image)

Figure 4-3 The Optimum Thickness of the Slab Based on the Slab Depth and Concrete Stiffness, a) Medium Soft Subgrade, b) Medium Subgrade, and c) Medium Stiff Subgrade
<table>
<thead>
<tr>
<th>Load case</th>
<th>Positive combined</th>
<th>Negative combined</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Stress (kPa)</td>
<td>Stress (kPa)</td>
</tr>
<tr>
<td></td>
<td>Ratio %</td>
<td>Ratio %</td>
</tr>
<tr>
<td>Depth (mm)</td>
<td>150 225 300 375</td>
<td>150 225 300 375</td>
</tr>
<tr>
<td>Soil Type</td>
<td>F'c (MPa)</td>
<td>S_c (kPa)</td>
</tr>
<tr>
<td>1</td>
<td>27.6 3490</td>
<td>2468 1944 1524 1027</td>
</tr>
<tr>
<td>2*</td>
<td>34.5 3900</td>
<td>2703 2151 1648 1083</td>
</tr>
<tr>
<td>3**</td>
<td>41.4 4270</td>
<td>2923 2331 1744 1124</td>
</tr>
<tr>
<td>4***</td>
<td>48.3 4610</td>
<td>3151 2482 1834 1158</td>
</tr>
<tr>
<td>5**</td>
<td>55.2 4940</td>
<td>3365 2620 1910 1179</td>
</tr>
<tr>
<td>6**</td>
<td>62.1 5230</td>
<td>3565 2724 1972 1200</td>
</tr>
<tr>
<td>7***</td>
<td>68.9 5520</td>
<td>3744 2779 2027 1207</td>
</tr>
<tr>
<td></td>
<td>27.6 3490</td>
<td>2951 2213 1634 1034</td>
</tr>
<tr>
<td>2*</td>
<td>34.5 3900</td>
<td>3206 2372 1724 1055</td>
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<tr>
<td>3**</td>
<td>41.4 4270</td>
<td>3434 2510 1800 1062</td>
</tr>
<tr>
<td>4***</td>
<td>48.3 4610</td>
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<td>5**</td>
<td>55.2 4940</td>
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</tr>
<tr>
<td>7***</td>
<td>68.9 5520</td>
<td>4144 2930 1993 1062</td>
</tr>
</tbody>
</table>

* medium stiff subgrade  
** medium subgrade  
*** medium soft subgrade

4.1.4.4 Effect of Subbase Thickness and Stiffness

The effect of the thickness and stiffness of the subbase over a soft subgrade was studied for different slab thicknesses and under various loading conditions. The stresses developed in the slab decreased for thick stiff layers of subbase over a soft subgrade. A stiffer subbase tends to reduce the deflection of the slab and, thus, distributes the traffic pressure over a larger area causing lower levels of stresses. On the other hand, as the
thermal gradient causes the slab to curl, the slab and subbase may lose full contact and a stiffer subbase may reduce the contact area resulting in higher stresses in the slab.

The effect of the subbase on the stress distribution when the pavement is subjected to various load combinations depends on the intensity of each load. The stresses at the bottom of the slab are mainly caused by traffic loading and a better subbase tends to reduce these stresses. Conversely, the stresses at the top of the slab are mainly caused by temperature and a better subbase may not reduce the stresses or even have an adverse effect.

As discussed previously, the slab thickness has a major impact on the developed stresses. Traffic stresses increase in a thinner slab while thermal stresses remain constant or may even decrease. Therefore, a better subbase can significantly decrease the stresses in a thin slab. The next subsections describe the effects of different loading combinations on the stresses developed in the slab for varying subbase thickness and stiffness over a soft subgrade ($E = 7$MPa). Two values of subbase stiffness were considered: ordinary subbase ($E = 240$ MPa) and stiff subbase ($E = 3450$ MPa).

4.1.4.4.1 Traffic Loading

As can be seen from Table 4-4 when the slab is subjected only to traffic loading, increasing the thickness of the slab and of the subbase, and the stiffness of the subbase reduces the maximum stress in the slab. These results are summarized in Figure 4-4-a and Figure 4-5-a.

From Table 4-4, it can also be seen that the thinner the slab, the more the stresses are sensitive to the increase in subbase thickness and stiffness. This means that a better subbase tends to reduce the maximum stress at a higher rate for thin slabs under traffic loading. Accordingly, the effect of slab thickness on the stresses developed by traffic loading decreases as the subbase becomes stiffer and/or the depth of the subbase increases.
4.1.4.4.2 Positive Temperature Gradient and Traffic Loading

The behavior of a concrete pavement under the combination of a positive temperature gradient and traffic loading is different than that of when it is subjected to traffic loading alone. The findings for this load combination are shown in Figure 4-4-c and Figure 4-5-c, for ordinary and stiff subbases, respectively. Since the traffic loading governs the pavement response, the thickness of the slab has a significant effect on the stresses. As the subbase becomes stiffer the stresses decrease at a lower rate than for the case when traffic loading is acting alone. This is especially true for thicker slabs, since in this case the traffic stresses are less important. For thinner slabs, increasing the subbase thickness decreases the stresses, while for thicker slabs the opposite is observed. This is because traffic stresses are significantly higher than thermal stresses in thin slabs and while thermal stresses are more important in thicker slabs. Finally, the effects of the subbase thickness and stiffness decrease as the thickness of the slab increases.

4.1.4.4.3 Negative Temperature Gradient

As it can be seen from Figure 4-4-b and Figure 4-5-b, the stresses developed when the slab is subjected only to a negative temperature gradient are relatively independent of the slab thickness. In this case, a stiffer and thicker subbase increases the developed stresses. As it was mentioned before, this is mainly because of the reduction in the contact area between slab and subbase for a stiffer subbase.

4.1.4.4.4 Negative Temperature Gradient and Traffic Loading

As seen in Figure 4-4-d and Figure 4-5-d, when the negative temperature gradient is combined with traffic loading, for thicker slabs the stresses, in general, tend to decrease slightly, especially when a thin ordinary subbase is used. However, this is not true for thick stiff subbase. In thin slabs the effect of traffic loading in the stresses is higher than the thermal loading. As a result for a thinner slab, the thicker and stiffer the subbase the more the stresses are reduced. As the slab becomes thicker the traffic stresses decrease, a
thicker and stiffer subbase tends to increase the stresses due to the combination of these two types of loads.

Table 4-4 The Effect of Subbase Stiffness and Thickness on Developed Stresses

<table>
<thead>
<tr>
<th>Subbase Depth (mm)</th>
<th>Loading</th>
<th>Traffic</th>
<th>Maximum stress (kPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Subbase Stiffness (MPa)</td>
<td>Slab Depth (mm)</td>
<td>240</td>
</tr>
<tr>
<td>600</td>
<td></td>
<td></td>
<td>1014 400</td>
</tr>
<tr>
<td></td>
<td>37.5</td>
<td>30</td>
<td>1427 579</td>
</tr>
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<td></td>
<td>30</td>
<td>22.5</td>
<td>1889 558</td>
</tr>
<tr>
<td></td>
<td>22.5</td>
<td>15</td>
<td>2751 517</td>
</tr>
<tr>
<td></td>
<td>15</td>
<td></td>
<td>1110 614</td>
</tr>
<tr>
<td>450</td>
<td></td>
<td></td>
<td>1593 827</td>
</tr>
<tr>
<td></td>
<td>37.5</td>
<td>30</td>
<td>2172 1000</td>
</tr>
<tr>
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<td>3372 1020</td>
</tr>
<tr>
<td></td>
<td>22.5</td>
<td>15</td>
<td>1234 807</td>
</tr>
<tr>
<td></td>
<td>15</td>
<td></td>
<td>1793 1158</td>
</tr>
<tr>
<td>300</td>
<td></td>
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<td>30</td>
<td>4082 2213</td>
</tr>
<tr>
<td></td>
<td>30</td>
<td>22.5</td>
<td>1358 965</td>
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<tr>
<td></td>
<td>37.5</td>
<td>30</td>
<td>4833 4061</td>
</tr>
</tbody>
</table>
Figure 4-4 The Effect of Ordinary Subbase Thickness and Slab Depth on Developed Stresses, a) Traffic Load, b) Negative Temperature Gradient, c) Positive Temperature + Traffic Load, and d) Negative Temperature + Traffic Load
Figure 4-5 The Effect of Stiff Subbase Thickness and Slab Depth on Developed Stresses, a) Traffic Load, b) Negative Temperature Gradient, c) Positive Temperature + Traffic Load, and d) Negative Temperature + Traffic Load

Figure 4-6 The Optimum Thickness of the Slab Based on the Slab Depth and Depth of Subbase, a) Ordinary Subbase, b) Stiff Subbase
4.1.5 Optimal Slab Thickness

During its service life, a pavement undergoes various cycles of varying loading combinations. Thus, an accurate estimation of a pavement’s service life requires the application various load combinations with different thermal load gradients and various traffic loading conditions. This is an impractical proposition, since it would require an awesome number of simulations. However, investigating the cases that produce the maximum stresses can provide valuable insight on the service life of pavements.

As mentioned previously, it was found that although increasing the thickness of the slabs reduces the stresses for positive combined loading, it increases or does not affect the stresses for negative combined loading. Thus, indiscriminately increasing the thickness of the slab can result in top to bottom cracking, which is highly undesirable. Thus, ideally, the best design can be achieved by selecting a slab thickness for which the same level of stresses are developed for both the positive and negative combined load cases. Increasing the slab depth beyond this optimal value would not increase the service life of the pavement.

For a specified optimal thickness, either the stiffness of concrete must be increased or the stiffness and depth of the subgrade must be changed or the length of the slab must be decreased until an appropriate stress ratio is obtained. Each of these cases is discussed in the following subsections.

4.1.5.1 Concrete Stiffness

As can be seen from Figure 4-3, higher optimum thickness results for softer subgrade/subbase, higher traffic loading, and milder weather conditions. On the other hand, concrete with higher shrinkage potential subjected to severe weather conditions (high temperature gradients) demands a thinner slab. For the cases analyzed in the present study relatively high thermal gradients were applied and the subgrade and subbase were modeled by the ES method. It was found that the optimum slab thickness for medium subgrade is around 190 mm (Figure 4-3-b).

It should be noted that Rao et al. (2001) observed that upward curling and warping relaxes as a result of creep in concrete at the end of the two years. This tends to reduce
the stresses in the concrete slab due to negative load combination. Thus, in this case, to improve the service life of the concrete, a slightly higher thickness is recommended.

4.1.5.2 Subbase Thickness

Figure 4-6-a and Figure 4-6-b show the results for the combined positive and negative loading cases on a slab for varying subbase thicknesses. As can be seen, for a certain thickness of subbase, there is an optimum value of slab thickness. This optimum value corresponds to the point where the positive and negative thermal stresses curves encounter. For each case, increasing the slab thickness beyond this optimal value does not reduce the negative combined stresses.

Table 4-4 summarizes the results for various subbase thickness and stiffness values. Interestingly, it can be observed that the stresses corresponding to the optimal slab thickness are virtually the same for the different conditions. Also, as the thickness and stiffness of the subbase increases the optimum slab thickness decreases. For ordinary subbase, the optimum slab thickness does not change significantly for the different subbase conditions. Therefore, employing a thick layer of ordinary subbase over a soft subgrade is not justified. For thin layers of subbase, the optimum slab thickness and its corresponding stresses are the same for both ordinary and stiff subbases. Thus, using a stiff subbase in this case is not warranted. In conclusion, using a thin ordinary subbase in combination with a slab of optimum thickness is expected to provide the best alternative for improved service life of a pavement.

4.1.5.3 Closing remarks

A 3D finite element model of a concrete pavement was developed and analyzed for different load combinations, various material properties, and different soil types. The effect of the subbase and concrete slab and thickness on the stress ratio and consequently on the service life of the pavement was investigated. It was found that the indiscriminate increase in the slab thickness is not always justified and can adversely affect the performance of a concrete pavement by increasing the possibility of top to bottom cracking. This becomes more important when the subgrade is stiff or the pavement is
subjected to severe environmental conditions. Optimal values for the concrete slab thickness were obtained by considering the conditions associated with both top to bottom and bottom to top cracking.

4.1.6 The Effect of Slab Length on Thermal and Traffic Stresses

To study the effect of the length of the slab on the developed stresses various FE models were developed for the same example used to investigate the effect of slab thickness. Two different slab thicknesses, i.e. 9 and 14 inches, were used. In the first series of models the modulus of elasticity of the subgrade and subbase were set to 10,000 pci and 35,000 pci, respectively and the results for this case are referred to as “Stiff”. In the second series of models the modulus of elasticity of the subgrade and subbase were set to be 1,000 pci and 10,000 pci, respectively and the results for this case are called “Soft”. The maximum temperature gradient obtained using the ICM program was applied to the 14 in slabs. The effect of shrinkage was added only in the negative case. For the 9 in slab the temperature gradient of the top of the 15 in slab was used. This assumption is justified as the previous studies with different thicknesses showed similar temperature gradient in the top of the slabs; however the stresses may be slightly overestimated. The axle load of the HS20 truck was applied in the middle of the slab for the case of positive temperature loading and at the edge of the slab for negative temperature loading case.

Since thermal stresses are the main concern in the negative temperature gradient, the effects of temperature alone and combined temperature and traffic loading were considered. On the other hand, traffic stresses are more important in the positive temperature gradient case. Thus, the effects of traffic loading alone and combined temperature and traffic loading were considered in this case. The obtained stresses for all cases are summarized in Figure 4-7, Figure 4-8, Table 4-5, and Table 4-6. Since the effect of slab length in the positive and negative cases is different the results are discussed in the separated sub-sections.
4.1.6.1 **Negative Temperature Case**

The results for the negative thermal gradient are shown in Figure 4-7 and Table 4-5. As it can be seen, as the length of the slab decreases the stresses also decrease. However, the amount of stress reduction for stiff and soft subgrade, for thick and thin slabs, and for temperature only and in combination is very different.

In all cases a stiffer subgrade/subbase increases the stresses. The rate of increase of stress in soft and stiff subgrade is close when the other conditions remain unchanged. As it could be predicted, the slabs subjected to both temperature gradient and traffic loading experience the largest stresses. When only temperature loading is applied, the thicker slab experiences higher stresses than the thinner slab. However with the application of traffic loading, the stresses developed on the thick and thinner slabs are closer. In this situation, as the length of the slab is increased, the thermal stresses tend to overcome the stresses due to traffic loading. This results in higher stresses in the thicker slab. Conversely, when the length of the slab is decreased, the traffic loading stresses overcome the thermal stresses, and as a result it produces larger stresses in the thinner slab.

As the length of the slab decreases, the rate of change in stress in the 14 in slab is less than the rate of change in stress in the 9 in slab in all studied situations. Figure 4-7 and Table 4-5 shows that the smallest and largest change in stress due to a decrease in length occur for the 14 in slab under temperature loading and in 9 in slab under combined loading, respectively. The effect of subgrade stiffness in the rate of reduction is found to be negligible.

4.1.6.2 **Positive Temperature Case**

The results for the positive thermal gradient case are shown in Figure 4-8 and Table 4-6. As can be seen, the stresses in the 14 in slab are smaller than those in the 9 in slab. This is because the stresses produced by traffic loading dominate in the positive combined case. When traffic loading is the only applied load, the stresses are independent of the length of the slab. This result does not come as a surprise since traffic loading tend to produce highly localized stresses. When temperature loading is added to traffic loading, the stresses tend to increase and the length of the slab has an effect on the
produced stresses. As can be seen the stresses increase with the length of the slab. The rate of increase is higher for the 9 in slab than for the 14 in slab. This is mainly because of the relatively higher temperature gradient applied in the 9 in slab. If traffic loading is the only loading present, the stresses in the slab over a softer subgrade are higher than those in the slab over stiffer subgrade. On the other hand, when the temperature gradient is applied the opposite trend is observed. This is because the thermal stresses tend to increase over a stiffer subgrade.

Table 4-5 The Effect of the Length of the Slab on the Developed Stresses in Case of Negative Temperature Gradient

<table>
<thead>
<tr>
<th>Thickness (in)</th>
<th>Length (ft)</th>
<th>Stress (psi)</th>
<th>Percent</th>
<th>Stress (psi)</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>W+T</td>
<td>T</td>
<td></td>
<td>W+T</td>
<td>T</td>
</tr>
<tr>
<td>14</td>
<td>20</td>
<td>761</td>
<td>100</td>
<td>593</td>
<td>100</td>
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<td>18</td>
<td>698</td>
<td>92</td>
<td>554</td>
<td>93</td>
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<tr>
<td></td>
<td>16</td>
<td>637</td>
<td>84</td>
<td>518</td>
<td>87</td>
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<tr>
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<td>14</td>
<td>576</td>
<td>76</td>
<td>487</td>
<td>82</td>
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<tr>
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<td>12</td>
<td>524</td>
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<td>458</td>
<td>77</td>
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<tr>
<td>9</td>
<td>20</td>
<td>829</td>
<td>100</td>
<td>559</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td>18</td>
<td>741</td>
<td>89</td>
<td>502</td>
<td>90</td>
</tr>
<tr>
<td></td>
<td>16</td>
<td>656</td>
<td>79</td>
<td>449</td>
<td>80</td>
</tr>
<tr>
<td></td>
<td>14</td>
<td>565</td>
<td>68</td>
<td>398</td>
<td>71</td>
</tr>
<tr>
<td></td>
<td>12</td>
<td>475</td>
<td>57</td>
<td>353</td>
<td>63</td>
</tr>
</tbody>
</table>

4.1.6.3 Minimum Stress

The results discussed in the previous sections indicate that although increasing the thickness of the slab can mainly decrease the stresses produced traffic loading, it does not reduce thermal stresses. They also show that decreasing the length of the slab can greatly decrease the thermal stresses, but it does not reduce the stresses caused by traffic loading. To reduce the combined stresses a combination of the two methods must be adopted.

The stresses in the negative case are mainly caused by temperature loading, while traffic loading is the primary cause of stresses in the positive case. As mentioned above, the length of the slab has a direct effect on thermal stresses, but it does not affect traffic
stresses. Consequently, decreasing the length of the slab reduces the stresses in negative case at a higher rate than the stresses in the positive case. With this knowledge and considering the optimum values obtained previously, it can be concluded that the optimum thickness of the slab increases as the length of the slab decreases.

Figure 4-7 The Effect of the Length of the Slab on the Developed Stresses in Case of Negative Temperature Gradient

Figure 4-8 The Effect of the Length of the Slab on the Developed Stresses in Case of Positive Temperature Gradient
Table 4-6 The Effect of the Length of the Slab on the Developed Stresses in Case of Positive Temperature Gradient

<table>
<thead>
<tr>
<th>Thickness (in)</th>
<th>Length (ft)</th>
<th>Stiff W+T</th>
<th>Stress (psi)</th>
<th>Percent</th>
<th>Stiff W</th>
<th>Stress (psi)</th>
<th>Percent</th>
<th>Soft W+T</th>
<th>Stress (psi)</th>
<th>Percent</th>
<th>Soft W</th>
<th>Stress (psi)</th>
<th>Percent</th>
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<td>143</td>
<td>100</td>
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<td>105</td>
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<td>18</td>
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<td>105</td>
</tr>
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<td></td>
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<td></td>
</tr>
<tr>
<td></td>
<td>18</td>
<td>226</td>
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<td>902</td>
<td>96</td>
<td>464</td>
<td>103</td>
<td>757</td>
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<td>831</td>
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<td>464</td>
<td>58</td>
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</tbody>
</table>

4.2 Effects of Soil on the Stresses

The quality of subgrade material has a direct effect on the stresses and deflection of concrete pavement slabs. The magnitude of this effect depends on the type of loading applied to the pavement. In this Section the effect of soil type on the stresses in pavements is investigated.

Davids (2000) studied the effects of foundation models (layered elastic and dense liquid) and properties on the response of jointed plain concrete pavements subjected to axle and thermal loads. They found out that with the layered elastic foundation model, a stiffer foundation significantly decreased the shear stress in the dowel bars. However, the model with Winker foundation could not capture this effect, and the shear stresses in the dowel bars were slightly higher than those in produced in the elastic foundation model. This discrepancy tends to decrease when the finite element mesh is refined near the dowel bars. Furthermore, they found that changing the bottom layer of the foundation boundary conditions had no effect on the stresses if enough foundation depth is adopted. This result was also obtained by the Ioannides et al. (1988).

In these studies it was found that since the contact area of slabs over stiffer subbase under thermal loading is reduced, the resulting stresses tend to be higher. On the other
hand, a stiffer subbase provides better support for axle loading, thus, decreasing the associated stresses, since in this case the slab and the subbase are in full contact. These results were obtained with both liquid foundation and elastic foundation, and the change in stresses was found to be higher in the model using liquid foundation. Since under thermal loading the effect of subbase stiffness is the opposite than under traffic loading, for a combination of axle and thermal loading depends on the relative intensity of each of these loads.

Pumping usually occurs near the joints of a pavement slab and causes the deterioration of the concrete in this region. This results in the lack of support causing the concrete slab to act as a cantilever beam under an axle load. This phenomenon can increase the stresses near the edges beyond concrete’s tensile limit, thus, resulting in the development of cracks. Pumping can also increase the permanent faulting of slabs. The higher stresses produced by pumping on the dowel bars may lead to crushing of the concrete surrounding the bar, which can adversely affect load transfer. It should be noted however that this effect is localized near the joints and have a secondary effect on the stresses in the middle of the pavement slab.

4.2.1 Parametric Studies on the Effect of Relative Stiffness

In this research, a detailed investigation of the effect of the relative stiffness between the slab, subbase, and subgrade on the distribution of thermal and traffic stresses/strains within the slab is carried out through a series of parametric studies. To achieve this, three different models were developed. Each model consists of 6 slabs in two lanes and three shoulders. It also includes tie bars and dowel bars. The subbase and subgrade are modeled using solid brick elements. The contact between slab and subbase is simulated using contact elements.

The first model was that of a typical pavement with an ordinary subbase (E = 35 ksi) and medium-stiff subgrade (E = 10 ksi). The second model was similar to the first one, but the medium-stiff subgrade was replaced by medium-soft soil (E = 5 ksi). The third model was similar to the second model, but the subbase was removed and the slab was supported directly by medium-soft soil. Six types of loading were applied to each
model. They were: positive and negative temperature gradients, two different wheel loads, and the combinations of temperature gradient and wheel loads. Wheel locations were chosen in such a way that traffic stresses and thermal stresses add up to each other.

Figure 4-9 illustrates the applied positive temperature loading. It also shows wheel placement and lines in which stresses are measured. In this case, the wheels were placed in the middle of the slab.

![Figure 4-9 Definition of Applied Loads and Stress Lines, a) Line 1 and Line 2, and b) Positive Temperature Gradient](image)

Figure 4-10 shows the stress distribution and relative vertical deflection for the subgrade and the slabs under positive temperature gradient loading. As can be seen, the positive thermal gradient forces the slabs to curl down and as a consequence the pavement loses contact with the subbase towards the middle of each slab. Since the subbase and subgrade in the first model are stiffer, the contact area is smaller and resulting in a larger free span. As expected, this results in higher stresses. Similar results can be observed when comparing the second and third models. It should be noted that the third model does not have a subbase and, thus, the contact area is even larger than the second model and the free span is smaller, thus, resulting in even lower stresses.
Figure 4-10 Stress and Deflection for Positive Temperature Gradient with no Wheel Loading, a) Stress Along Line 1, b) Deflection Along Line 1, c) Stress Along Line 2, and d) Deflection Along Line 2

Figure 4-11 shows the stress distribution and relative vertical deflection of the subgrade and slabs under traffic loading. Since no temperature gradient was applied, the slabs did not curl and were in full contact with the subbase. In this case, the first model provided the best support and, consequently, developed the smallest deflection. The stresses in the third model, which did not have a subbase, were much higher and their variation was much sharper than that of the stresses in the first and second models. A comparison between first and second model shows that a poor subgrade tends to increase the stresses and a good subbase tends to reduce the effect of subgrade stiffness on the stresses.
Figure 4-11 Stress and Deflection with Wheel Loading and no Temperature Gradient, a) Stress Along Line 1, b) Deflection Along Line 1, c) Stress Along Line 2, and d) Deflection Along Line 2

Figure 4-12 shows the stress distributions and relative vertical deflections for the subgrade and slabs for the combination of traffic and thermal loading. Since in this case slab and subbase are not in full contact this is a problem nonlinear problem, i.e., the effects of thermal and wheel loading cannot be simply superimposed. This can be observed by comparing the stress results of the combined loading with the addition of stresses in each case. The subbase/subgrade stiffness has an opposite effect on the stresses due to positive thermal loading and to traffic loading. The contribution of each of these loads on the stresses when they are applied in combination is not clear, i.e., it depends on the magnitudes of each of the loads. If the magnitude of the thermal gradient stresses is higher than that of the traffic stresses, the overall stresses are lower in the
second and third model. If the opposite is true, the stresses are higher in less supported pavements.

Figure 4-12 Stress and Deflection under Positive Temperature Gradient and Wheel Loading, a) Stress Along Line 1, b) Deflection Along Line 1, c) Stress Along Line 2, and d) Deflection Along Line 2

Figure 4-13 illustrates the applied negative temperature loading. It also shows wheel load placement and the lines in which stresses were measured. In this case, the wheels were placed near the edge of the slab and the temperature of the top of the slab is lower than the bottom of the slab.
Figure 4-13 Definition of Applied Loads and Stress Lines, a) Line 1 and Line 2, and b) Negative Temperature Gradient

Figure 4-14 presents the stress distributions and the relative vertical deflections of subgrade and slabs due to the negative temperature loading. As can be seen, the negative temperature gradient causes the slabs to curl upwards. As a result the pavement slab loses contact with the subbase at its edges and corners. As it was the case for the positive temperature gradient case, in the present case, the stresses are higher for the first model. This is because the subbase and subgrade were stiffer causing the contact area to be smaller and the free span to be larger. Similar results are observed when comparing the second and third models. Since the third model does not include a subbase, the contact area is even larger than the second model and the free span is smaller, thus resulting in lower stresses.
Figure 4-14 Stress and Deflection under Negative Temperature Gradient with no Wheel Loading, a) Stress Along Line 1, b) Deflection Along Line 1, c) Stress Along Line 2, and d) Deflection Along Line 2

Figure 4-15 shows the stress distribution and relative vertical deflection of subgrade and slabs due to traffic loading. Since temperature gradient is not present, the slabs do not curl and are in full contact with the subbase. In this case, the first model has the best support and the minimum deflection. This tends to increase the serviceability and ridability of the pavement structure and tends to reduce bumping between the adjacent slab, thus, potentially decreasing joint deterioration. It should be noted that the stresses developed in the third model (model without sub-base) are higher than those developed in the first and second models.
Figure 4-15 Stress and Deflection with no Temperature Gradient, a) Stress Along Line 1, b) Deflection Along Line 1, c) Stress Along Line 2, and d) Deflection Along Line 2

Figure 4-16 shows the stress distribution and relative vertical deflection of subgrade and slabs for a combination of traffic and negative thermal gradient loading. Similarly to positive combined load, the two loadings have opposite effects in the different models and the combined stresses depend on the relative intensity of the two types of loading. If the thermal gradient stresses overcome the traffic stresses, the second and third models experience less stresses. On the other hand, if the traffic stresses overcome the thermal stresses, the less supported pavements experience less stresses.
Figure 4-16 Stress and Deflection under Negative Temperature Gradient with Wheel Loading, a) Stress Along Line 1, b) Deflection Along Line 1, c) Stress Along Line 2, and d) Deflection Along Line 2

Table 4-7 and Table 4-8 compare the maximum stress for three types of soil and under various loading conditions for positive temperature gradient and negative temperature gradient, respectively. The difference between the linear and nonlinear combination of stresses is also compared in these tables. As can be seen, combining the stresses linearly is unconservative, since the resulting stresses are significantly lower than the more realistic stresses produced by the actual combined load (nonlinear combination).
Table 4-7 Developed Stresses in the Slab Under Positive Temperature Gradient

<table>
<thead>
<tr>
<th>Pos. gradient Stress( psi)</th>
<th>Line 1</th>
<th>Line 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thermal</td>
<td>75</td>
<td>92</td>
</tr>
<tr>
<td>Percent</td>
<td>82%</td>
<td>100%</td>
</tr>
<tr>
<td>Wheel (mid.)</td>
<td>69</td>
<td>50</td>
</tr>
<tr>
<td>Percent</td>
<td>138%</td>
<td>100%</td>
</tr>
<tr>
<td>Linear comb.</td>
<td>144</td>
<td>142</td>
</tr>
<tr>
<td>Percent</td>
<td>102%</td>
<td>100%</td>
</tr>
<tr>
<td>Non linear comb.</td>
<td>179</td>
<td>207</td>
</tr>
<tr>
<td>Percent</td>
<td>87%</td>
<td>100%</td>
</tr>
</tbody>
</table>

Table 4-8 Developed Stresses in the Slab Under Negative Temperature Gradient

<table>
<thead>
<tr>
<th>Neg. gradient Stress( psi)</th>
<th>Line 1</th>
<th>Line 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thermal</td>
<td>185</td>
<td>203</td>
</tr>
<tr>
<td>Percent</td>
<td>91%</td>
<td>100%</td>
</tr>
<tr>
<td>Wheel (mid.)</td>
<td>64</td>
<td>45</td>
</tr>
<tr>
<td>Percent</td>
<td>142%</td>
<td>100%</td>
</tr>
<tr>
<td>Linear comb.</td>
<td>249</td>
<td>248</td>
</tr>
<tr>
<td>Percent</td>
<td>101%</td>
<td>100%</td>
</tr>
<tr>
<td>Non linear comb.</td>
<td>260</td>
<td>264</td>
</tr>
<tr>
<td>Percent</td>
<td>98%</td>
<td>100%</td>
</tr>
</tbody>
</table>

4.3 Pumping

During the past decades truckloads have increased steadily resulting in ever increasing stresses in the subgrade. Thus, the quality of subgrade has become an important factor in pavement performance. The increase in loading has resulted in pumping of the subgrade under the pavement, which can further decrease a pavement’s service life. To prevent this, granular base courses of varying thickness were applied under the concrete slabs to prevent the loss of subgrade support. Various studies were performed to obtain the optimum subbase thickness.
Based on Huang (1993) pumping is defined as the ejection of water and subgrade soil through joints, cracks, and along the edges of pavements caused by downward slab movements due to heavy axle loads. Pumping can be recognized by surface staining or accumulation of material on the surface close to cracks. However, in some cases, mainly when stabilized bases are used, pumping can occur without a sign. Another sign of pumping is ejection of water from the joints by heavy traffic loads after a rainstorm.

When a heavy traffic load passes, the slab deforms. This deformation increases as the result of temperature curling in the pavement. Thus, subgrade goes under a plastic deformation and a void space may develop which leads to water entrance. As a truckload passes by and goes to the leading slab the trailing slab rebounds and a vacuum is created under the slab sucking the fine material from underneath the leading slab (Figure 4-17). As a result the direction of the water flow is in the direction opposite to traffic. The voids enlarge as a result of reduction of fine material. Voids cause non-uniform slab support, which increase stresses and strains in the pavement and may lead to premature cracking and faulting.

![Direction of water](image_url)

**Figure 4-17 The Mechanism of Pumping**

It must be noticed that pumping is related more to pavement deflections than to stresses. The most critical pavement deflection occurs at a slab corner when an axle load is placed at the joint near to a corner, thus resulting in the most amount of pumping near that corner. The thermal gradient affects the amount of deflection. Under positive
temperature gradient the pavement tends to deflect downwards and the slab edges experience an initial downward deflection. Applying load at the edges at this time tends to increase the deflection of the edges. During the nighttime the negative temperature gradient causes the middle of the slab to experience the largest deflection. Applying the axle load in the middle of the slab may increase the deflection in this situation, but the resulting deflection in the middle is much smaller than the edge deflection. According to Gulden et al. (1983) the maximum pumping occurs at the edges of the pavement and at the transverse joints. The void tends to initiate near the edges of the slab and to propagate towards the slab centerline.

4.3.1 Prevention

In order to prevent pumping it is important to understand the factors that cause the phenomenon to occur. The first and most important factor that contributes to pumping is free water. The amount of free water can be controlled by limiting the size and condition of cracks and joints using some type of sealant. In fact, if the pavement materials are well drained and the joints are well sealed no pumping is expected to occur.

Passing vehicles are the second factor that contributes to this phenomenon. The higher their number and weight the larger is the pavement deflection, which directly impacts the initiation and/or progression of pumping. In general, as a preventive measure, for a high number of heavy vehicles the depth of the concrete slab and the number of dowel bars are increased to ensure the best load transfer through the slabs.

The third factor that contributes to pumping is erodibility of the foundation material. If the subgrade and subbase materials are erodible, applying a treated base can help prevent pumping.

The risk of the occurrence of pumping can be decreased if appropriate slab properties are adopted. More specifically, a thicker slab of shorter length and with higher modulus of elasticity, in which excellent load transfer mechanisms are installed, can significantly reduce deflection and consequently pumping.

Finally, environmental conditions contribute to pumping. This is because the amount and distribution of precipitation affect the free water present in the pavement,
ambient temperature and sun exposure determine the thermal gradient through the depth of the slab, which affects pavement deflection, and freeze-thaw cycles contribute to the erodibility of the subbase. The effect of these parameters in the development of pumping of the concrete pavements was studied by VanWijk et al. (1989).

4.3.2 Pumping in Codes

It is widely accepted that the main reason for concrete pavement failure is fatigue. Consequently most of the design criteria of the concrete pavement adopted in design codes are based on fatigue of concrete. In the PCA (1984) code an additional criterion was added which is based on the erosion of the corner of the slabs resulting from pumping. However, the method is based on the erodible subbases of AASHO Road Test and is not comprehensive. The subbase materials in the AASHO road tests were erodible and most of the failures were the result of pumping. A comprehensive modeling should take into account all factors that contribute to pumping such as water, rate of ejection, erodability of subgrade material, and the number and magnitude of loads. However, the PCA equation was obtained by relating the performance of the pavement to the rate of work, which was defined as a function of the radius of relative stiffness. This is in agreement with the fact that a thin slab with a shorter deflection basin receives a faster load punch than a thicker slab. The following equations were developed to compute the allowable load repetitions:

\[
\log N = 14.524 - 6.777 (C_1 P - 9.0)^{0.103} \quad \text{Equation 4-4}
\]

where \( N \) is the allowable number of load repetitions based on a PSI of 3.0, \( C_1 \) is an adjustment factor with a value of 1 for untreated subbases and 0.9 for stabilized subbases, and \( P \) is the rate of work or power defined by:

\[
P = 268.7 \frac{p^2}{hk^{0.73}} \quad \text{Equation 4-5}
\]

where \( p \) is the pressure on the foundation under the slab corner in psi, \( h \) is the thickness of slab in inches, and \( k \) is the modulus of subgrade reaction in pci.
The equation for erosion damage is given by:

\[
\text{Percent erosion damage} = 100 \sum_{i=1}^{m} \frac{C_2 n_i}{N_i} \quad \text{Equation 4-6}
\]

where \(C_2 = 0.06\) for pavements without concrete shoulders and 0.94 for pavements with tied concrete shoulders. The percent erosion damage should be less than 100%.

Darter et al., (1985) developed the COPES pumping models. Unfortunately, the effect of erodibility of the base course was not included in these models. The equation for Jointed Plain Concrete Pavements is as follows:

\[
\text{PI} = (N_{18})^{0.443} [-1.479 + 0.255(1 - S) + 0.0605(P)^{0.5} + 52.65(H)^{-1.747} + 0.0002269(FI)^{1.205}] \quad \text{Equation 4-7}
\]

where

- \(\text{PI} = \) pumping index rated on a scale of 0 to 3, with 0 for no pumping, 1 for low-severity pumping, 2 for medium-severity pumping, and 3 for high-severity pumping
- \(N_{18} = \) number of equivalent 18-kip single-axle loads in millions
- \(S = \) soil type based on AASHTO classification, with 0 for coarse-grained soils (A-I to A-3) and 1 for fine-grained soils (A-4 to A-7)
- \(P = \) annual precipitation in cm
- \(H = \) slab thickness in inches
- \(FI = \) freezing index in degree days

4.3.3 Developed Model

To study the effect of pumping on the stresses developed in the middle of the slab a FE model was developed. The model consisted of three concrete slabs over subbase and subgrade layers, with distinct properties. The concrete slab was modeled using eight and five 20-node brick elements through its depth. The subbase and subgrade were modeled using 8-node brick elements slightly wider than the slab. Linear elastic material models were used in the developed FE model. The modulus of elasticity of concrete, subbase and
subgrade were assumed to be 31,922 MPa, 241 MPa, and 69 MPa, respectively. For both the subbase and subgrade, the degrees of freedom perpendicular to the model’s boundaries and all degrees of freedom of the bottom layer subgrade were restricted. Contact elements were used to model the interaction between the subbase and slab.

To model the pumping effect the contact element in the corners and edges were removed, resulting in no connection between the slab and soil in this area (Figure 4-18). The area of unsupported slab can be correlated to the amount of pumped material and the size (thickness) of the void. Unfortunately up to now, no accurate experiments are available that have measured the area and thickness of voids generated by pumping. Thus, these are assumed quantities in this work. A 142 kN (32-kips) single axle load was applied to the concrete slab as the tire loading. The maximum positive and negative temperature gradients developed by ICM program were applied as the temperature loads. In case of negative temperature gradient shrinkage was also applied through a linear temperature gradient, as discussed previously. Table 4-9 summarizes the results of the pumping effects for the two studied cases, i.e. pumping in the corners and edges and pumping middle of the slab at the edge. As can be seen from this table, if no temperature is present and the wheel load is applied at the edge, the stresses increases as the pumping increase. This is due to the increase in the free span. The rate of stress increase is higher
for the model without dowel bars. This is because dowel bars transfer the load to the adjacent slab and also change the behavior of the end of the slab from free end to only moment free end. This leads to a reduction in the stresses where the subbase and slab are in contact.

Table 4-9 The Effect of Pumping on Developed Stresses

<table>
<thead>
<tr>
<th>Thickness</th>
<th>Temperature</th>
<th>Load</th>
<th>Dowel</th>
<th>Ordinary</th>
<th>Pump Edge</th>
<th>Pump Middle</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thin</td>
<td>Negative</td>
<td>Edge</td>
<td>No</td>
<td>510</td>
<td>510</td>
<td>510</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Yes</td>
<td>503</td>
<td>503</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Positive</td>
<td>Middle</td>
<td>No</td>
<td>385</td>
<td>392</td>
<td>450</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Yes</td>
<td>390</td>
<td>399</td>
<td></td>
</tr>
<tr>
<td></td>
<td>No Temp.</td>
<td>Edge</td>
<td>No</td>
<td>144</td>
<td>135</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Yes</td>
<td>146</td>
<td>138</td>
<td></td>
</tr>
<tr>
<td>Thin</td>
<td>Positive</td>
<td>Middle</td>
<td>Yes</td>
<td>694</td>
<td>71</td>
<td></td>
</tr>
<tr>
<td></td>
<td>No Temp.</td>
<td>Middle</td>
<td>Yes</td>
<td>285(582*)</td>
<td>383(648*)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Edge</td>
<td>Yes</td>
<td>131(216*)</td>
<td>184(243*)</td>
<td></td>
</tr>
</tbody>
</table>

As it can be seen, for the negative temperature gradient, the difference between the model that considered pumping and the one that did not is negligible. This does not come as a surprise, since under negative temperature gradient the slab tends to curl upward resulting in loss support at the edges. Consequently, there is little difference between the regular model and the one in which the soil under the edges are pumped. In the case of positive temperature gradient, if pumping occurs in the corners and edges, a small change in the stresses resulted. It should be noted that pumping in the corner has no effect on the stresses developed in the middle of the slab. Also, the effect of the edge pumping on the stresses in this location is low. The maximum change in stresses in this location resulted when pumping at the edge in the middle of the slab was considered. Since there is in no support under the traffic load, the slab bends under the load and acts like a beam and the flexural stresses are increased significantly.
CHAPTER 5 CASE STUDY - CRACK PATTERN IN SKEWED PAVEMENTS

5.1 Introduction

A section of State Route 49 in Valparaiso, Indiana is under reconstruction due to premature failure. The pavement is a skewed Jointed Plain Concrete Pavement (JPCP) from the Reference Post 29+73 north of US-30 to the Interstate 80/90 interchange. This section of JPCP was constructed in 1987 with 4 lanes and functions as a rural major collector (RMC) road. The weighted average AADT (Annual Average Daily Traffic) in 1999 was 23,000 with 14% truck traffic. The 1999 PCR (Pavement Condition Rating) was 86 (fair condition) but the PQI (Pavement Quality Index) was 69 (poor condition). The low PQI was mostly from the mid-slab cracks and joint faulting with lane drop-off almost in the whole length of the pavement section. These skewed JPCPs failed prematurely -- within 3-4 years of construction. Interestingly, the pavements generally cracked perpendicularly to the direction of traffic, not parallel to the joints, which are skewed. These JPCPs are now being replaced. Studies were performed in order to ascertain the main reason of the failure and the ability of the 3D Finite Element (FE) analysis techniques in predicting the cause and orientation of the cracking. In particular, the crack patterns due to the effects of nonlinear thermal gradients and traffic loading were studied. With the developed finite element model, the effects associated with each crack pattern observed in the field were determined.

This chapter details the three-dimensional finite element model developed this case study. The specific issues that are discussed include the modeling of: materials, infinite subgrade, slab-subbase interaction, and joint, dowel and tie bars, and aggregate interlock. Issues related to the application of nonlinear thermal gradient and wheel loading are also presented. Furthermore, the effect of soil stiffness on the stresses in the concrete slab under various loading conditions using the developed finite element model was investigated and the results are also reported in this chapter.
5.2 Existing Pavement

The studied skewed jointed pavement is located on SR-49 in the state of Indiana, from 1.29 km north of US-30 to I-80/90. The 1.65 km portion of the southbound lanes from E400N to E500N was investigated in this work. Soil borings taken in the vicinity indicate the native subsoils were medium-stiff clay; however the main reason for the poor performance of the pavement was believed to be due to poor subbase and subgrade conditions. In this work, the subgrade material for the representative model was assumed to be medium-stiff clay. The thickness of the concrete slab and the granular subbase were 25.4 cm (10 in) and 15.2 cm (6 in), respectively.

For the SR-49 project, a skew angle of approximately 15° was used at the pavement joints. A repeating cycle of four joint spacing of 3.66 m (12 ft), 3.96 m (13 ft), 5.79 m (19 ft), and 5.49 m (18 ft) were used. A visual inspection of the pavement was performed. It revealed that the cracks in the existing slabs were more likely in the longer concrete slabs (5.49 m and 5.79 m) than in the shorter slabs (3.66 m and 3.96 m). In the current study, eight slabs, with emphasis on the longer slabs, were considered. Four sections with four different lengths were chosen for the main FE model. The basic dimensions of the studied pavement section are shown in Figure 5-1.

5.3 Finite Element Model

A main goal of this study was to develop a comprehensive finite element model that could capture all significant factors affecting the stress distributions in concrete pavements. To achieve this, a three-dimensional finite element model was employed. It is well known, however, that these types of models consume a great amount of static and dynamic computer memory, which is limited by hardware constraints. Thus, there was a limitation in the total number of degrees of freedom that could be adopted. On the other hand, the accuracy of the results of a finite element analysis increases with mesh refinement, i.e, with the number of degrees of freedom in the mesh. Therefore, a main concern in the development of the present 3D FE model was to minimize the amount of memory usage without jeopardizing accuracy. To do so, various simpler 3D FE models
were developed and studied using both ANSYS Version 5.2 (1995) and ABAQUS Version 5.7 (1996).

![Figure 5-1 Dimensions of the Pavement Section](image)

The mesh sizes of the slabs, subbase, subgrade, and dowel and tie bars were refined to the point where the change in the calculated maximum stress, which is the main concern of this study, was less than 3%. This level of accuracy was deemed adequate, and no further refinement was considered necessary. The developed finite element mesh is shown Figure 5-2.

The boundary conditions of the model were set as follows. The edges of the subbase and subgrade boundaries were constrained in their normal direction, and no restraints were applied at the concrete slab boundaries. All translational degrees of freedom were restrained at the bottom most layer of the subgrade. The dowel bars were connected to the slab in the x-direction only at one end.
The following subsections provide the details of the developed finite element model. More specifically, issues related to the modeling of the slab and soil materials properties, subgrade, subbase-slab interaction, joint between concrete slabs, dowel and tie bars, and aggregate interlock are thoroughly discussed.

Figure 5-2 Developed Finite Element Mesh

5.3.1 Material Properties

Under service loads and thermal gradients and shrinkage effects, the stresses developed in the soil and in the concrete slab were found to be effectively smaller than the yield stresses of the two materials. Thus, it can be ascertained that the main reason for the cracking of the pavement was fatigue. As a result, linear elastic material models were used in the developed FE model. The adopted material properties are given in Table 5-1, where E: Young’s modulus, v: Poisson ratio, γ: unit weight, and α: coefficient of thermal expansion. The modulus of rupture of concrete was taken as 4,482 kPa (650 psi). This is the 28-day value used for concrete pavement design in Indiana.
Table 5-1 Material Properties

<table>
<thead>
<tr>
<th>Location</th>
<th>Material</th>
<th>E(Mpa)</th>
<th>ν</th>
<th>γ(kg/m³)</th>
<th>α(1/°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slab</td>
<td>Concrete</td>
<td>27580</td>
<td>0.15</td>
<td>2270</td>
<td>1.08E-5</td>
</tr>
<tr>
<td></td>
<td>Crushed Stone</td>
<td>138</td>
<td>0.35</td>
<td>2240</td>
<td>-------</td>
</tr>
<tr>
<td>Subgrade</td>
<td>Medium Clay</td>
<td>35</td>
<td>0.4</td>
<td>1770</td>
<td>-------</td>
</tr>
<tr>
<td>Dowel bar</td>
<td>Steel</td>
<td>200000</td>
<td>0.3</td>
<td>7470</td>
<td>0.90E-5</td>
</tr>
</tbody>
</table>

5.3.2 Subgrade Modeling

Solid elements were used to model the subgrade. It must be noticed that using solid elements instead of customary spring elements (liquid foundation) can greatly affect the results. The main difference arises from the fact that the deflection of a certain point in a liquid foundation depends only on the forces applied at that point, and not on the forces applied at the other points. The solid model, on the other hand, can develop shear, which transmit loads to the other nodes in the mesh, thus more accurately representing the actual field conditions.

The modulus of elasticity of soil cannot be directly evaluated in the field. The main parameter that can be measured in the field is the modulus of subgrade reaction or California bearing ratio. In this study, the modulus of elasticity of soil was back calculated from the modulus of subgrade reaction using the relation developed by Vesic and Saxena (1974).

\[
k = \left(\frac{E_f}{E_c}\right)^{1/3} \frac{E_f}{h(1-\nu_f)^2}
\]

A Poisson ratio of 0.4 was adopted for the subgrade. The modeled depth of the subgrade was determined by developing models with different thicknesses and comparing the maximum stress in the slab, since this is the quantity of interest in this study. A 1.5 m subgrade depth gave stress results within one percent of accuracy. Further reduction of the depth of the modeled soil caused a significant variation in the axial stresses in the
slab. It should also be noted that to predict deflection of the slab with the same level of accuracy a higher depth of soil would be necessary.

5.3.3 Slab-Subbase Interface Modeling

The interaction between the subbase and slab was studied using two approaches. In the first approach, nonlinear spring elements were adopted. These elements are such that they are able to transfer the load where the slabs and subbase are in contact and become inactive where the slabs and subbase are separated. In the second approach, the interface between the concrete slab and the subbase was modeled using contact and target elements. These nonlinear elements can carry only compression, thus allowing unrestrained separation of the concrete slab and subbase when tensile strains arise between contact and target surfaces. The main difference between the two models is that contact and target elements are also capable of simulating the effect of friction, sliding, and bonding. In order to capture the effect of friction, eight-node surface-to-surface contact and target elements were utilized in the developed FE model. The target elements were overlaid on the bottom surface of the concrete slab whereas the contact elements were placed on the top surface of the subbase. The initial value of Coulomb friction coefficient was taken as the tangent of the soil friction angle.

5.3.4 Joint Modeling

Adjacent concrete slabs were modeled separately, and did not share nodes. It was assumed that the shear forces were transferred from one slab to the other at the skewed joints through the dowel bars and through aggregate interlock, and at the longitudinal joints through the tie bars.

Since in the actual pavement most of the observed cracks occurred towards the middle of the slabs, the use of an intricate solid model for the dowel and tie bars was not deemed necessary. The dowel and tie bars were modeled by simple three dimensional beam elements.
5.3.4.1 Dowel and Tie Bars

The dowel bars are connected to the solid brick element nodes by two nonlinear spring elements. The definition of the model is discussed in section 2.2.2.

For tie bar, nonlinear springs with very high stiffness values were also employed at every node in the direction parallel to the tie bar. This allowed for the development of horizontal friction forces. The stiffness coefficients of the springs were determined based on the results from a study performed at Iowa State University by Porter et al (2001). Concrete crushing near the dowel bars was captured in the developed finite element model by setting the stiffness of the spring to zero when its relative stress was more than maximum bearing stress of concrete. To combat the incompatibility between the solid elements and the beam elements and to more accurately model the interaction between slab and dowel bar an increased number of elements in the region close to the dowel bar were adopted.

5.3.4.2 Aggregate Interlock

The stiffness of the springs used to simulate aggregate interlock was evaluated using Equation 2-32 and by multiplying the obtained shear stress by the corresponding tributary area. A 1 mm joint opening was assumed based on an estimated average of 15°C change in slab temperature from the time of placement to the time of calculating the stress in addition to an average shrinkage strain of 0.0001 mm/mm.

The Load Transfer Efficiency (LTE) was calculated from the results obtained by the developed finite element model. According to Ioannides and Korovesis (1990), LTE is defined as a parameter that measures the load transfer from the loaded side to the unloaded side of a crack or joint, and it is given by

\[ LTE = \frac{\delta_U}{\delta_L} \]  

Equation 5-1

Where \( \delta_U \) and \( \delta_L \) are the unloaded and loaded deflection, respectively. A 100% LTE indicates that both sides of a crack or joint equally share the loads. A 0% LTE indicates that no loads are transferred from the loaded side to the unloaded side. The LTE
value depends on the gap between the dowel bar and the surrounding concrete. It also depends on the joint opening when aggregate interlock is considered -- the closer the gap and/or the joint opening, the higher the LTE value. Since the studied pavement was not old, a relatively high value for LTE was expected. In the present study, the values of $\delta_u$ and $\delta_L$ obtained from the finite element analyses resulted in an average value of LTE around 95%, which attests to the validity of our assumptions.

5.4 Mesh Refinement

The adequacy of the selected slab mesh size was investigated by comparing the stresses and deflections that it produced for a 254 mm slab under circular loading with a well-known analytical solution. The equivalent nodal loads due to a 689-kPa pressure of a 304.8 mm diameter circle were applied at the center of the slab. The slab was supported by vertical springs, whose stiffness values were obtained by multiplying 27.1 MN/m$^3$ (modulus of subgrade reaction) by the appropriate tributary area. The slab lengths were chosen to be large enough to prevent the effect of the edges on the stresses.

Various models were developed and the stresses and deflections obtained using different element types were compared. The Westergaard (1926) analytical calculation of the stress and deflection in the interior of the slab under a circular load area was used as the exact solution. It was found that for the same number of elements, 8-node solid elements predicted stresses approximately 32% smaller than those obtained from the Westergaard solution, 20-node solid elements were off by 0.3 and 2.2% for rectangular and skewed elements, respectively, and shell elements were off by less than 1%. Thus, compared to shell elements, solid elements are stiffer and need finer meshing to produce results with the same level of accuracy. This problem becomes even greater when the area under the loading is smaller than the area of the element and when 8-node elements are used.

Based on the above results, two distinct mesh refinements were adopted in different parts of the mesh. The main slab, i.e., the one in which the traffic load was applied and the stresses were measured, was meshed using 20-node elements. Each of the
other seven slabs was meshed using the same number of 8-node elements as the main slab, since further refinement of these meshes did not have much impact on the accuracy of the produced stresses. This was done to produce a finite element mesh of manageable size, while including the interaction between adjacent slabs in the simulations.

5.5 Loading Cases

Four types of loading that have major effect on the stress distribution in pavements were considered in this study. They are self-weight, thermal loading, traffic loading, and shrinkage. Since self-weight is handled automatically by the finite element software and shrinkage can be modeled as a thermal gradient, the discussion in this section focuses on finite element issues related to the application of thermal and traffic loads. Temperature differentials were applied only on the concrete slab as the temperature change in the supporting layers of a pavement is very small (Thompson et al 1987).

A given temperature change can be decomposed into constant, linear, and nonlinear components. The constant temperature change is the temperature difference between pavement installation time and the time under consideration. The deformation produced by this temperature change is limited by the friction developed between the slab and subbase. In this study the maximum stress developed by friction was calculated to be 33 kPa, which is around one percent of the maximum allowable tensile stress of concrete. Since these stresses were relatively small, further adjustment in Coulomb friction factor was deemed unnecessary. It should be noted that it is assumed that the dowel bars function as per design and do not restrict the displacement of the slabs. Such a restriction would cause a significant increase in the stresses produced by a uniform temperature change.

The Integrated Climatic Model (ICM) program developed by G. Larson and B. J. Dempsey (2003) at the University of Illinois at Urbana-Champaign was used to calculate the thermal gradient through the depth of the concrete slab for selected positive and
negative temperatures at the uppermost surface of the slab. Figure 5-3 shows the thermal gradients used in this investigation and Table 5-2 provides the numerical data.

<table>
<thead>
<tr>
<th>Depth (mm)</th>
<th>Negative Temperature (°C)</th>
<th>Equivalent Shrinkage (°C)</th>
<th>Negative Temp. + Shrinkage (°C)</th>
<th>Positive Temperature (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-11.1</td>
<td>-5.6</td>
<td>-16.6</td>
<td>14.2</td>
</tr>
<tr>
<td>3.175</td>
<td>-9.0</td>
<td>-4.9</td>
<td>-13.8</td>
<td>8.4</td>
</tr>
<tr>
<td>6.35</td>
<td>-7.1</td>
<td>-4.2</td>
<td>-11.3</td>
<td>4.3</td>
</tr>
<tr>
<td>9.525</td>
<td>-5.4</td>
<td>-3.5</td>
<td>-8.9</td>
<td>1.9</td>
</tr>
<tr>
<td>12.7</td>
<td>-3.9</td>
<td>-2.8</td>
<td>-6.7</td>
<td>0.6</td>
</tr>
<tr>
<td>15.875</td>
<td>-2.7</td>
<td>-2.1</td>
<td>-4.8</td>
<td>0.0</td>
</tr>
<tr>
<td>19.05</td>
<td>-1.6</td>
<td>-1.4</td>
<td>-3.0</td>
<td>-0.2</td>
</tr>
<tr>
<td>22.225</td>
<td>-0.7</td>
<td>-0.7</td>
<td>-1.4</td>
<td>-0.2</td>
</tr>
<tr>
<td>25.4</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
</tbody>
</table>

The uniform temperature difference between the time of the placement of the slab and the time under consideration was not available. However, as discussed previously, constant temperature and shrinkage can only change the resulting maximum stresses around one percent of the maximum allowable tensile stress of concrete. Consequently the temperature in the bottom of the slab was set to be zero and the temperature of the rest of the slab was modified accordingly using the temperature gradients provided by the ICM program. In order to accurately apply a nonlinear thermal gradient through the depth of the finite element mesh representing the concrete slab, an adequate number of nodes must be adopted. In this work this was achieved using eight elements through the depth of the slab. Shrinkage was modeled as a thermal gradient. Different levels of shrinkage were considered for the different load combinations.
Figure 5-3 Temperature Gradients Through the Depth of Concrete Slab

Since the studied pavement was skewed, the skewed element shape was transformed to a system of dimensionless coordinates (i.e., natural coordinates) and the limits of the integration were accurately defined. In this study, the AASHTO HS-20 truck wheel load was used as the traffic load, for which a 140 kN (32 kips) load is carried by the four wheels in a single axle. Nodal equivalent loads of 4 rectangular pressure footprints were applied on the pavement where the stresses near the tire loading were the main concern. The tire pressure was assumed to be 765 kPa (111 psi) and consequently the tire contact area was taken as 0.0465 m² (72 in²). Equivalent nodal loads, for 8- and 20-node skewed elements subjected to a partial circular and rectangular pressures were calculated. The loading footprints are summarized in Figure 5-4.

5.6 Load Combinations

The 3D FE model of the skewed JPCP was analyzed for a combination of thermal, shrinkage, and truck loads. Each of these load conditions can create stresses on the pavement that can counteract or combine with each other. Linear shrinkage causes the pavement to curl upwards and the produced stresses counteract the positive thermal stresses. In the case of positive temperature gradient no shrinkage effect was considered.
Tire loading creates maximum tension in the bottom of the slab in the line of load application. To create maximum tensile stress due to the combination of positive thermal gradient and traffic loading the tire loading was applied in the middle of the slab.

In the case of negative temperature gradient shrinkage effect was applied to produce the maximum tensile stress. In this study, a thermal gradient of 0.022 °C/mm was adopted to simulate this maximum shrinkage effect.

To increase the thermal stresses a single axle load was applied near the skewed joint. If another single axle load were to be applied on the other side of the slab a larger tensile stress (approximately 12% higher) would be produced. However the possibility of this situation is lower. Figure 5-4 summarizes the traffic load locations for the applied load combinations.

Figure 5-4 Traffic Loading Locations for the Development of Maximum Stresses, a) Two Edge Loading, b) One Edge Loading, and c) Middle Loading

5.7 Stress Results Under Various Loading and Soil Conditions

The effect of soil stiffness on the stresses developed in a skewed pavement due different loading conditions was investigated. The loading conditions considered were the following:

- Thermal gradients: positive and negative.
- Traffic loading applied in different positions of the slab that cause maximum tensile stress. These loads are shown schematically in Figure 9, together with their denominations.
- Combined thermal and traffic loads. Three cases were considered:
  - Negative thermal gradient with one edge load
Negative thermal gradient with two edge loads

Positive thermal gradient with middle load

The results are summarized in Table 5-3. The modulus of elasticity of the subbase was taken as 68.95 kPa (10 psi) and three values for the modulus of elasticity of the soil were adopted: 6.89 kPa (1 psi), 34.47 kPa (5 psi), and 68.95 kPa (10 psi), for soft, medium, and stiff soil, respectively.

Table 5-3 Maximum Stresses under Various Loading for Soils with Different Stiffness

<table>
<thead>
<tr>
<th>Soil</th>
<th>Maximum tensile stress (kPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Neg. temp.</td>
</tr>
<tr>
<td>Soft</td>
<td>1710</td>
</tr>
<tr>
<td>Med.</td>
<td>2296</td>
</tr>
<tr>
<td>Stiff</td>
<td>2537</td>
</tr>
</tbody>
</table>

As it can be seen from Table 5-3, similar to rectangular pavement, thermal loading produces higher stresses when a stiffer soil is adopted. However, the rate of change is small. More specifically, when the modulus of elasticity of the subgrade is increased by ten-fold, the stresses due to a positive temperature gradient increase only by 7% and those due to negative temperature gradient increase by 48%. This behavior seems reasonable, since under thermal gradients the subbase and the slabs are not in full contact and the stiffer the subgrade the smaller the contact area, which results in higher stresses.

When traffic loading is applied alone, the slab and the subbase are in full contact. In this case, a stiffer subgrade provides a better support for the slab, thus resulting in a better load distribution and less deflection. As can be seen from Table 5-3, in this case the stresses decrease at a significantly higher rate with the increase in stiffness of the subgrade material, than that observed for thermal loading acting alone.

The stresses produced by combining thermal and traffic loading are different from the summations of the stresses resulting from each of these loadings acting separately. This happens for two reasons. First the location of the maximum stresses is different for the different loading conditions: the maximum positive thermal stresses occur between
the middle and bottom of the slab while the maximum traffic loading stresses occur at the bottom of the slab. Second, the model behaves nonlinearly. As it can be seen for the combined traffic and negative thermal gradient loading the stresses increase as the subgrade becomes stiffer, while for the combined traffic and positive thermal gradient loading the opposite trend is observed. This is because the stresses in the combined positive loading the traffic loading stresses dominate over the thermal stresses. Thus the behavior in this case follows the same trend and the one observed when traffic loading acts alone, i.e. the stresses increase when the soil stiffness decreases. On the other hand, the stresses in the combined negative loading are mainly caused by the thermal gradient, which governs the behavior, i.e., the stresses increase when the soil stiffness increases.

5.8  Analyses and Results

In this section additional results obtained from the developed FE model, which was analyzed under traffic loads and nonlinear thermal gradients are reported. In particular, the results related to the crack patterns due to the effects of nonlinear thermal gradients and traffic loading are discussed in detail. Finally, the developed FE model is used to determine the effects associated with each crack pattern observed in the field. Throughout this study, the 5.79 m slab is referred to as the “main slab”, since this is the slab where traffic loads were applied and stresses were measured.

5.8.1  Transverse Cracks

From the visual survey, the observed cracks were classified into five major types, according to their orientation, as the following (Figure 5-5):

A: Completely perpendicular to the lane
B: Perpendicular to the lane at the edges, opposite direction of the skewed joint around the middle of the slab
C: Perpendicular to the lane at the edges, parallel to the direction of the skewed joint around the middle of the slab
D: Irregular
E: Perpendicular to the lane at one edge, opposite the skew on the other side

Figure 5-5 The Classification of the Existing Transverse Cracks, a) Type A Crack, b) Type B Crack, c) Type C Crack, d) Type D Crack, e) Type E Crack, f) Schematic Illustration of the Crack Classification

Figure 5-5-f shows a schematic illustration of these crack patterns. Although the majority of the cracks were located near the midspan of the concrete slabs, some of the
cracks were located close to the skewed joints (Figure 5-5-e). In Table 5-4, the statistics of the crack distribution is presented.

Table 5-4 Distribution of the Crack Types

<table>
<thead>
<tr>
<th>Crack Type</th>
<th>Number</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>No cracks</td>
<td>106</td>
<td>35</td>
</tr>
<tr>
<td>A</td>
<td>139</td>
<td>46</td>
</tr>
<tr>
<td>B</td>
<td>19</td>
<td>6</td>
</tr>
<tr>
<td>C</td>
<td>23</td>
<td>8</td>
</tr>
<tr>
<td>D</td>
<td>6</td>
<td>2</td>
</tr>
<tr>
<td>E</td>
<td>7</td>
<td>2</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>300</strong></td>
<td><strong>100</strong></td>
</tr>
</tbody>
</table>

5.8.2 General Results

The developed model is able to capture a number of characteristics of JPCP that affect their behavior under a variety of loading conditions. In general, the meshes developed for the slabs, subbase, subgrade, and dowel and tie bars were refined to the point where the change in the calculated maximum stress, which is the main concern of this study, was less than 3%.

It was found that under service loads, and thermal and shrinkage effects, the stresses in the soil and in the concrete slab were effectively smaller than the yield stresses of the two materials. Thus, linear elastic material models were used in the developed FE model.

Eight-node surface-to-surface contact and target elements were utilized to capture the interaction between the concrete slab and the subbase. The initial value of Coulomb friction coefficient was considered to be the tangent of soil friction angle.

The dowel and tie bars were connected to the solid brick element nodes by two sets of nonlinear spring elements. The stiffness coefficients of the springs were determined based on experimental data available in the literature. Concrete crushing near the dowel bars was also captured in the developed finite element model. This was done by setting the stiffness of the spring to zero when its relative stress was more than maximum
bearing stress of concrete. The effect of aggregate interlock was modeled using vertical nonlinear springs connecting the joint nodes with the same coordinates.

Through numerical experiments, it was found that modeling a depth of 1.5 m of subgrade with ordinary FE resulted in stresses within one percent of accuracy. The mesh near the footprint of the truck loading must be sufficiently refined if 8-node brick elements are used and the stresses near the footprint are the major consideration. Insufficient refinement resulted in lower stresses in the traffic direction. The use of fewer 20-node brick elements is recommended as an alternative. If the stresses far from the truck-loading footprint are the major concern, this refinement of the mesh has minor effect on the results.

The effect of soil stiffness on the stresses developed due different loading conditions was investigated. It was found that the stresses developed when positive thermal gradients are combined with the traffic loading increase when the soil stiffness decreases. This is because in this case the stresses due to traffic loading dominate over the thermal stresses. On the other hand, when negative thermal gradients are combined with traffic loading, the produced stresses are mainly caused by the thermal gradient, which governs the behavior. Thus, in this case, the stresses increase when the soil stiffness increases. It must be noticed that this results is for this particular depth and as the thickness increases the thermal stress becomes more important.

Figure 5-6 Deformed Configuration of the Pavement Slabs under Positive Temperature Gradient
Under negative thermal gradient, the concrete slabs curled upward and the corners of the slabs lost contact with the subbase. In this case, the maximum vertical gap between the slab and subbase was about 2.5 mm. The application of the traffic load decreased the gap near the load but did not entirely close it.

Under positive thermal gradient, the concrete slabs curled downward and the interior part of the slabs lost contact with the subbase (Figure 5-6). The maximum gap between the slab and subbase was about 0.4 mm. When the axle load was applied in the middle of the slab, the vertical gap closed causing full contact between the slab and the subbase. Figure 5-7-a through Figure 5-7-c show the stress distributions through the slab thickness in the longitudinal pavement centerline at different positions (see schematic of these positions in Figure 5-7-d). Figure 5-7-a and Figure 5-7-c show the results for positive and negative thermal gradient, respectively, while Figure 8-b depicts the stresses produced by wheel loading.

As it can be seen from Figure 5-7-a, for positive thermal gradient, the maximum tensile stress occurs near the middle of the depth of the slab. This is mainly because of the highly nonlinear positive thermal gradient. For equilibrium to be satisfied, no horizontal forces should develop at Point 1, since it is located at the edge of the slab. This is verified, since the net force, which is the sum of the hatched area in Figure 5-7-a, is zero. For negative thermal gradient, it can be observed from Figure 5-7-c that the stresses increase towards the center of the slab (Figure 5-7-d, Point 2) when compared to the stresses at the edges of the slab (Figure 5-7-d, Point 1). Figure 5-7-b shows the stresses produced by wheel loading. In particular, this figure shows that the tensile stresses developed at the bottom in the middle of slab due to a wheel loading applied in the middle of the slab is significantly higher than the tensile stresses developed at the top in the middle of the slab due to a wheel loading applied at the edge of the slab.

Through a number of numerical simulations, it was found that the maximum tensile stress resulting from the application of a wheel load occurred when the load was applied close to the longitudinal free edge of the slab (Figure 5-7-d, Point 4). It was also observed that the stresses produced when the wheel loading was applied near the middle longitudinal joint (Figure 5-7-d, Point 5) were higher than those produced when the
wheel loading was applied in the middle of the slab (Figure 5-7-d, Point 2). However, these stresses were smaller than those produced when the load was applied near the free edge (Figure 5-7-d, Point 4). This does not come as a surprise, since in the latter case one would expect that some of the force would be transmitted to the adjacent slab via the tie bars.

Figure 5-7 Axial Stress Through the Depth of the Pavement, a) Positive Thermal Gradient, b) Traffic Loading, c) Negative Thermal Gradient, and d) Location of Points

5.8.3 Cracking Patterns

As it can be seen from the statistical data given in Table 5-4, the majority of the observed cracks in the studied pavement were located near the midspan of the concrete slabs. As mentioned previously, this study focuses on the stresses in the 5.79 m slab shown in (Figure 5-1), which is referred as the “main slab”. The effect of the shoulder
was not considered in the analyses, since the actual pavement had an asphalt shoulder, which has little contribution to the pavement structure.

The resulting stresses in the main slab obtained from the simulations are depicted in Figure 5-8 and Figure 5-9. In these figures two types of plots are provided. One shows the variation of the principal stresses in the longitudinal direction in three positions: at the middle joint, at the middle of the slab, and at the free joint (Figure 5-1), and the other presents the orientations of the principal stresses along with schematics of crack patterns. The crack patterns were obtained by plotting lines in the direction perpendicular to the largest principal stress on the slab in every position. Depending on the loading case, tensile stresses occurred either at the top or at the bottom of the slab. Figure 5-8 presents the results for the loading cases in which tensile stresses occur at the top of the slab. These loading cases include negative thermal gradient, wheel loading applied at the edge of the slab, and the combination of these two loads. Figure 5-9, on the other hand, presents the results for the loading cases in which tensile stresses occur at the bottom of the slab. These loading cases include positive thermal gradient, wheel loading applied at the middle of the slab, and the combination of these two loads. The comparison of these figures with the observed cracking pattern provides insight on the causes of crack pattern formation. This is further discussed later on this section.

As it can be seen from the stress distribution plots in Figure 5-8 and Figure 5-9, in all cases the stresses near the middle longitudinal joint are not smooth due to the presence of the tie bars. In this case, stress concentrations occur in the neighborhood of the tie bars, i.e., the stresses reach a peak value near the tie bars and decrease in value as the distance to tie bar increases.

Comparing Figure 5-8-a and Figure 5-9-a, it can be seen that the stresses caused by negative thermal gradient are significantly higher than those caused by positive thermal gradient. Furthermore, it can be observed that the stresses due to negative thermal gradients reach a maximum in the middle of the slab and decrease smoothly towards the slab’s longitudinal edges. The stress distribution due to a positive thermal gradient, on the other hand, shows a different trend. The stresses are larger near the longitudinal edges and decrease towards the middle of the slab.
Figure 5-8 Principal Stress Distribution (PSD) and Crack Pattern (CP) on the Top of the Slab

a) PSD under Negative Thermal Gradient, b) CP under Negative Thermal Gradient, c) PSD under Edge Wheel Load, d) CP under Edge Wheel Load, e) PSD under Combined Loading, and f) CP under Combined Loading
In both cases, the magnitudes of the stresses in the middle joint fall between the magnitudes of the stresses in the middle of the slab and on the free edge. This is mainly due to the load transferred by the tie bars to the adjacent slab.

Wheel loads, in general, create high tensile stresses at the bottom of the slab and compressive stresses at the top of the slab close to the location where the load is applied.
These stresses decrease as the distance from the load increases until they become zero and finally change direction (Figure 5-8-c and Figure 5-9-c). As it can be seen, the maximum tensile stress at the bottom of the slab (Figure 5-9-b) is much higher than the maximum tensile stress at the top of the slab (Figure 5-8-b). This seems quite reasonable, since in the former situation the tensile stresses occur in the line of the application of the wheel load in the middle of the slab, while in the latter case the wheel load is applied at the edge of the slab and the tensile stresses occur in the middle of the slab. In both cases, the stresses near the free longitudinal joint are higher than in the other two positions. In addition, in this case, the stresses increase as the load is applied closer to the free edge.

In order to produce maximum tensile stresses within the slab, the negative thermal gradient was combined with the wheel loading applied at the edge of the slab (Figure 5-8-e). Likewise, the positive thermal gradient was applied in conjunction with wheel loading applied in the middle of the slab (Figure 5-9-e). In the first case, the thermal stresses were significantly higher than the stresses produced by traffic loading, while in the second case the opposite trend was observed. As a result negative thermal effects govern the stresses in the first case while traffic loading governs the stresses in the second case. In the combined case of negative thermal variations and traffic loading, the stresses in the middle of the slab and at the free edge were similar in magnitude. However, for the case in which positive thermal gradients and wheel loading were combined, the stresses near the longitudinal edge were significantly higher than those in the middle of the slab.

Figure 5-8 and Figure 5-9 show plots of crack patterns obtained from stress values for the three loading cases considered. For both negative and positive thermal gradients the crack pattern is similar to those classified as crack types B and C in Figure 5-5-a, Figure 5-5-b, and Figure 5-5-c. However, for the negative thermal gradient case the inclination of the crack is much higher (Figure 5-8-b, Figure 5-9-b). The stress pattern resulting from the application of the traffic load in the middle of the slab (Figure 5-9-d) shows a tendency for crack development perpendicular to the lane of traffic (crack type A). However, the stress pattern resulting from the application of traffic load at the edge of the slab shows (Figure 5-8-d) a rather different tendency. As it can be seen from Figure 5-8-d the crack pattern tends to be perpendicular to the lane of traffic on one edge
and highly inclined on the other edge (crack type E). The cracking pattern produced due to the combination of positive thermal gradient and traffic loading tends to be perpendicular to the traffic direction (Figure 5-9-f). For the negative thermal gradients and traffic load combination a propensity for inclined cracking is observed (Figure 5-8-f).

As discussed in the first of the two-paper series, the performed numerical tests indicate that stresses in the slab due to traffic loading tend to increase for pavements with soft subgrade material, while stresses due to thermal gradients increase with the length of the slab. Since, in the field, a much higher number of cracks were observed in longer slabs than in shorter slabs in the field, it can be concluded that the unavoidable thermal stresses were a main culprit for the premature cracking observed in the longer slabs. Furthermore, the high number of cracks perpendicular to the traffic direction indicates that positive thermal gradients combined with traffic loading were the main reason for the widespread cracking. In the simulations performed in this study, the stresses generated by the combination of positive thermal gradient and traffic loading were of the same order of magnitude as those produced by the combination of negative thermal gradient and traffic loading. This indicates that the soil in the actual pavement was likely softer than the one used in the simulations, since a lower modulus of elasticity of subgrade would significantly increase the resulting stresses.

From the simulation results it can be concluded that Type B and Type C crack patterns result from the combination of negative thermal gradient and traffic loading. In these cases top to bottom cracking occurs, which is a secondary concern in the cracking of concrete pavements.

Finally, cracks of Type E likely occurred in the studied pavement due to traffic loading applied on the edges of the slab. It should be noted that the number of cracked slabs with this type of cracking was much smaller than the other types.
5.8.4 Conclusions

The developed three-dimensional finite element model was successfully used for predicting the cause and orientation of the failure of the studied pavement section. The main conclusions that can be drawn from this investigation are the following.

- The maximum stress due to the application of a positive thermal gradient occurred in the middle of the longitudinal edges of the slab, while the maximum stress due to the application of a negative thermal gradient occurred in the middle of the slab. Furthermore, the stresses produced by negative thermal gradients were much higher (5 to 6 times) than those developed by positive thermal gradients. This difference increased even further when the effect of shrinkage was considered.

- Traffic loading caused significantly large tensile stresses at the bottom of the slab in the line of application of the load and significantly smaller (2.5 times) tensile stresses at top of the slab faraway from the location of load application.

- When negative thermal gradient was combined with traffic load, the maximum tensile stress was obtained when the traffic load was applied on the transverse edge of the slab. The combination of positive thermal gradient with traffic load, on the other hand, produced maximum tensile stress when the wheel load was applied in the middle of the slab near the longitudinal edge of the slab.

- A larger area of the slab lost contact with the subbase due to positive thermal gradients than due to negative thermal gradient. However, the magnitude of the gap in the latter case was higher. The application of traffic loading removed the gap in the positive thermal gradient case, but it only slightly changed the gap in negative thermal gradient case. Consequently no separation between slab and subbase was observed in the combined positive and traffic loading case. However the effects of the separation between slab and subbase governed the behavior in the combined negative temperature and traffic loading case.

- The performed field investigation showed that a fewer number of shorter slabs were cracked when compared to the number longer slabs that showed cracking. Through finite element analyses, it was found that thermal gradient stresses increase with the
length of the slab. Thus, this shows the importance of the effect of thermal stresses on the cracking of the longer slabs.

- Numerical tests indicate that the stresses produced by traffic loading are sensitive to the stiffness of the subgrade and subbase materials. The stresses increase as the stiffness of the sublayers decreased. When traffic loading was applied in the middle of the slab in combination with positive thermal gradients, bottom to top cracks occurred. In this case, the stresses due to the traffic loading were much higher than those due to the thermal gradient. Therefore, in this case, the quality of the subbase and subgrade are of utmost importance. Furthermore, in this case it was observed that the principal stress patterns were such that the resulting crack pattern would likely be perpendicular to the traffic direction. In the actual pavement, most of the observed cracking was perpendicular to the traffic direction. This leads to the conclusion that the premature cracking in the field was due to the weak performance of the subgrade and subbase materials.

- When the combination of traffic loading on the transverse edge and negative thermal gradient was applied to the model, top to bottom cracking occurred. In addition, the stresses due to the thermal loading, which is sensitive to the length of the slab, were much larger than those due to traffic loading. The obtained principal stress pattern indicates a high likelihood of inclined cracking. In the actual pavement a smaller number of slabs cracked in this manner. This indicates that cracking due to negative thermal gradients are of secondary concern.

Finally, the consistency between the observed field cracking patterns and the results from the numerical tests further validates the developed finite element model. Furthermore, with this model it was possible to get better insight on the effects of the different factors involved in the cracking of the concrete pavements.
CHAPTER 6 DAMAGE OF CONCRETE PAVEMENTS

6.1 Literature Review and Preliminary Analysis

The design life of a pavement is not uniquely defined. Some engineers use this term when there is a need for an overlay while others believe that the useful life of a concrete pavement is not over until the first or second overlay is complete (Fordyce, P and Yrjanson W.A. 1969). In this work, the design life of a concrete pavement is assumed to be the period that a pavement can be used without severe damage.

Different factors, including the type of subbase, type of shoulder, pavement thickness, and weather conditions, affect the design life of a pavement. Stresses in concrete pavements are usually much lower than the yield stress of concrete. Thus fatigue is the main reason for the initiation and development of cracks. During the last five decades many researchers have tried to establish a relation between maximum tensile stress and number of load repetitions, which leads to concrete failure. It was found that flexural stresses in concrete could be repeated indefinitely without any failure if they did not exceed 45-50 % of the modulus of rupture of concrete. However, if the tensile stresses increase beyond this limit, the number of load repetitions to failure decreases rapidly.

Different investigations have been conducted to characterize the behavior of plain concrete under variable loading. Next a brief review of these investigations is provided.

6.1.1 Review of Previous Synthesis Studies

Several state DOT’s and FHWA gathered and summarized the results of available investigations and recommended methods for designing concrete pavement. These results are provided in the next sub-sections. The following terms are commonly used and thus are defined here for brevity:

- Stress ratio = ratio of repetitive stress to ultimate steady load strength.
- Fatigue strength = number of repetitions to failure for a given amount of stress.
- Endurance limit = maximum stress ratio at which concrete shows elastic behavior, i.e., unlimited number of loads can occur without any loss of fatigue resistance (Fordyce, P and Yrjanson W.A. 1969).

6.1.1.1 Darter (1977)

In Darter (1979) the available research at the time was reviewed and a complete procedure for the design concrete pavements under heavily trafficked roadways was presented. Different aspects of design such as structural design based on serviceability and on fatigue, and design of joints, shoulders, and subsurface drainage were considered. As part of this work, a computer program was developed to aid the design procedure.

He suggested that the results of fatigue failure laboratory beams subjected to high repetitive flexural stresses cannot be used directly in the design of pavements. Although application of heavy truck traffic in concrete pavements caused fatigue failure in several road tests and many in-service PCC slabs, no correlation results between laboratory and field fatigue results were attempted. A summary of the results of laboratory studies of on fatigue properties are provided in Darter (1977) and are as follows:

1- Stress ratio is the main parameter in calculating the number of load repetitions to failure.

2- Laboratory experiments showed that even with up to 20 million load applications no fatigue strength limits could be found. After 10 million repetitions of load application, the ratio of strength to the static ultimate strength was approximately 55%.

3- The range of load or the ratio of flexural stress at minimum load to flexural stress at maximum load had a significant effect on the number of load repetitions. If the lower stress is near zero or is reversed, the number of repetitions to failure was much lower than if the lower stress was near the maximum stress.

4- The results of laboratory experiments showed that when applying loads with different amplitude in different sequences of load application caused different levels of fatigue damage. A non-linear model must, thus, be developed to consider these effects when accumulating damage. However, it was also shown that the inaccuracy in the linear
damage accumulation equations was much less than the variability in the calculation of
the static strength and the fatigue equation of PCC. Hence, it seems reasonable to use
Miner’s linear accumulative method when a comprehensive nonlinear technique is not
available.

5- The experimental results obtained by different researchers vary. These results
showed a high variability of fatigue life of concrete specimens with about a 100 percent
coefficient of variation. This means that a large numbers of concrete beams reached
failure under the same conditions that other identical concrete beams were able to resist.

6- The rest period after the loading period increased the cycles to failure in the tested
specimens. It seems that during this period the specimens experienced some level of
recovery.

7- Some limited tests showed that specimens with higher moisture content had lower
fatigue strength, while other results showed that while moisture content had a negative
effect on fatigue life it did not affect fatigue strength significantly. Overall, the effect of
moisture on fatigue behavior was not completely determined and more experiments were
deemed necessary.

8- During the experiments it was found that concrete strength and modulus of rupture
increased as time passed, thus increasing the fatigue life. However, when the modulus of
rupture at the specific time was used to compute the stress ratio, the fatigue strength was
unaffected.

Fatigue data for plain concrete beams from four studies (Kesler 1953, 1970,
Raithby and Galloway 1974, Ballinger 1971) were gathered by Darter (1977). A least
square regression curve was fit through the data and the following equation was obtained:

\[
\log_{10} N = 17.61 - 17.61 (R)
\]

Equation 6-1

where  \( N = \) number of stress applications to failure of beam

\( R = \) ratio of repeated flexural stress to modulus of rupture

Since this equation is a mean regression curve, for a certain ratio of stresses the
calculated number of load repetitions (N) results in the failure of half of the specimens, or
in other words, the failure probability is 50%.
Many differences between laboratory and field conditions exist, which most likely lead to different fatigue results. Darter (1977) summarized the probable differences in fatigue life of laboratory tested beams and of field slabs and the relative significance is given in the Table 6-1, where $N_s$ is the number of load repetitions to failure in field slabs and $N_b$ is the number of load repetition to failure in beams.

Table 6-1 Probable Differences in the Fatigue Life of Laboratory Beams and Field Slabs

<table>
<thead>
<tr>
<th>Field slab condition</th>
<th>Fatigue Life</th>
<th>Significance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Variability of PCC Strength</td>
<td>$N_s &lt; N_b$</td>
<td>Very important</td>
</tr>
<tr>
<td>Thermal Gradient Curve</td>
<td>$N_s &lt; N_b$</td>
<td>Very important</td>
</tr>
<tr>
<td>Thickness Variation</td>
<td>$N_s &lt; N_b$</td>
<td>Important</td>
</tr>
<tr>
<td>Loss of Support</td>
<td>$N_s &lt; N_b$</td>
<td>Important</td>
</tr>
<tr>
<td>Durability of PCC</td>
<td>$N_s &lt; N_b$</td>
<td>Important</td>
</tr>
<tr>
<td>Thermal Shrinkage</td>
<td>$N_s &lt; N_b$</td>
<td>Less important</td>
</tr>
<tr>
<td>Moisture Gradient Warp</td>
<td>$N_s &gt; N_b$</td>
<td>Important</td>
</tr>
<tr>
<td>Rate of Loading</td>
<td>$N_s &gt; N_b$</td>
<td>Important</td>
</tr>
<tr>
<td>Rest Periods</td>
<td>$N_s &gt; N_b$</td>
<td>Important</td>
</tr>
<tr>
<td>Age of PCC (strength gain)</td>
<td>$N_s &gt; N_b$</td>
<td>Very important</td>
</tr>
<tr>
<td>Stress Ratio</td>
<td>$N_s &gt; N_b$</td>
<td>Less important</td>
</tr>
<tr>
<td>Moisture of Slab</td>
<td>Unknown</td>
<td>Unknown</td>
</tr>
<tr>
<td>Scale Effects</td>
<td>Unknown</td>
<td>Unknown</td>
</tr>
<tr>
<td>Drying Shrinkage</td>
<td>Unknown</td>
<td>Unknown</td>
</tr>
<tr>
<td>Load Effects</td>
<td>Unknown</td>
<td>Unknown</td>
</tr>
</tbody>
</table>

Based on the variability and limitation of the results from laboratory experiments, Darter (1977) suggested that experimental curves be calibrated based on field data in order to be used in fatigue damage analysis. He suggested that the following equation, which has a failure probability of 24%, be used for design purposes

$$\log N_d = 16.61 - 17.61 (R)$$

Equation 6-2

where $N_d$ is number of load repetition to failure and $R$ is the stress ratio. It should be noted that this failure probability is much higher than that of PCA (1984). The fatigue curve used in design by PCA (1984) has a failure probability lower than 5%. Since the uncertainty in ultimate static strength of specimens causes the variation of fatigue strength results, using a lower failure probability curve does not seem logical. As it was
mentioned before, the modulus of rupture is not constant and increases with time. To include this effect in fatigue damage analysis, Darter (1977) suggested the following equation:

\[ F = F_A (F_{28}) \]  

Equation 6-3

where

\[ F_A = 1.22 + .17 \log_{10} T - .05 (\log_{10} T)^2 \]  

Equation 6-4

\( T = \) time since slab construction in years

\( F = \) modulus of rupture at time \( T \)

\( F_{28} = \) modulus of rupture at 28 days

\( F_A = \) ratio of the modulus of rupture at time \( T \) to the modulus of rupture at 28 days

The modulus of rupture can be estimated as:

\[ \text{Mod. Of Rupture (psi)} = 10 [\text{Compressive Strength (psi)}]^{0.5} \]  

Equation 6-5

Since during the life of a pavement, curling stresses and loading stresses occur with different frequency, they cannot be directly added to obtain the combined overall resulting stress. To account for this, Darter calculated an adjustment factor based on the available results to correct total combined stress.

As it was mentioned previously, Equation 5-2 had a failure probability of 24%. This means that a computed damage of 1.0 in design corresponds to 24% of the slabs failing due to fatigue. Hence, a safety factor must be applied to the computed damage to provide a lower probability of failure.

Darter (1977) defined an appropriate accumulated fatigue limiting damage for use in zero maintenance design by gathering field data, calculating total accumulated fatigue damage and plotting of cracking index versus total accumulated fatigue damage. The results of these plots showed that a correlation exists between transverse slab cracking and computed fatigue damage in the slab. Hence, depending on the amount of cracking that is acceptable, the appropriate limiting fatigue damage can be selected for design. A limiting damage value for a zero maintenance design of \( 10^{-4} \) was chosen to provide a high reliability in which the pavement slabs do not exhibit cracking during their life period.
6.1.1.2 Packard and Tayabji (1985)

Packard and Tayabji (1985), two members of the Portland Concrete Association, also developed a new thickness design procedure for highway and street concrete pavements. In their report, they discussed the basis of design and developed details including the effect of shoulders and dowel bars among other issues. Their fatigue results were based on the studies by Kesler (1953, 1970), Fordyce (1969), and Ballinger (1971). They suggested that flexural fatigue be applied to edge-load stresses. Their equation for fatigue damage design is as follows:

\[
\begin{align*}
0.55 < S & : \quad \log_{10} N = \frac{(0.9718 - S)}{0.0828} \\
0.45 < S < 0.55 & : \quad N = \left(\frac{4.2577}{(SR - 0.4325)}\right)^{1.268} \\
S < 0.45 & : \quad N = \text{Unlimited}
\end{align*}
\]

Equation 6-6

where  
- \( SR \) = flexural stress divided by the 28-day modulus of rupture  
- \( N \) = allowable number of load repetitions  
- \( S \) = Stress ratio

The failure probability in this method is less than 5%. In other words, if the number of applied load repetitions for a specific stress ratio is calculated by this equation, less than 5% of specimens would experience failure. So, it is reasonable that for a design problem, cumulative damage based on Miner’s rule (1945) be considered as 100%. Packard and Tayabji (1985) suggested that this design method is conservative.

6.1.1.3 Fordyce and Yrjanson (1969)

Fordyce and Yrjanson (1969) also reviewed and summarized the available research on concrete pavement design. In their review they cover the effect of concrete properties, sub-grade support, fatigue, and traffic on pavement design. The experimental results for specimens under repeated loading showed an increase in deformation, which is due to the decrease in the elasticity modulus. This increase becomes very significant at the end of the specimen’s life.

Fordyce and Yrjanson (1969) gathered the results of the following three important experiments:
1- Extensive tests performed at Karsrule, Germany by Probst et al (1931).
2- Investigation by Illinois Division of Highways.
3- Investigation at Purdue University by Hatt (1924, 1925) and Crepps (1923).

The equipment used at Purdue University was developed to produce complete reversal of stress during each load cycle. However, in a real pavement complete stress reversal does not occur, i.e., stress reversal is usually a small fraction of the main stress. The Illinois study did not consider stress reversal. Thus, it can be concluded that the actual fatigue behavior of a pavement must be between the Illinois and the Purdue results. Table 6-2 provides a summary of the results of these investigations.

Table 6-2 Previous Fatigue Investigation (Fordyce and Yrjanson 1969)

<table>
<thead>
<tr>
<th></th>
<th>Karsrule result</th>
<th>Illinois Research</th>
<th>Purdue Research</th>
</tr>
</thead>
<tbody>
<tr>
<td>Endurance limit</td>
<td>Endurance limit between .5 and .6</td>
<td>Endurance limit between .5 and .6</td>
<td>Endurance limit between .5 and .65 for four month old beams</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Endurance limit between .54 and .55 for six month old beams</td>
</tr>
<tr>
<td>Non elastic behavior</td>
<td>Non elastic behavior above endurance limit</td>
<td>Non elastic behavior above endurance limit</td>
<td>Non elastic behavior above endurance limit</td>
</tr>
<tr>
<td>above endurance limit</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Elastic behavior below</td>
<td>Elastic behavior below endurance limit</td>
<td>Elastic behavior below endurance limit</td>
<td>Elastic behavior below endurance limit</td>
</tr>
<tr>
<td>endurance limit</td>
<td></td>
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</tr>
<tr>
<td>Better fatigue strength for specimens tested first at ratios below the endurance limits</td>
<td>Modulus of rupture increased in cases that the specimens where subjected to 1,000,000 load repetition below the endurance limits</td>
<td>Applying initial repetitive load less than endurance limits increase endurance limit stress ratios</td>
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<tr>
<td>Increased results in fatigue strength due to elastic recovery during rest periods</td>
<td>Modulus of rupture increased in cases that the specimens where subjected to 1,000,000 load repetition below the endurance limits</td>
<td>Rest period increases fatigue strength</td>
<td></td>
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<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Increasing stress ratios above endurance limits decreases fatigue strength</td>
<td>Increasing stress ratios above endurance limits decreases fatigue strength</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
During a pavement’s service life, there are rest times between the maximum stresses in pavement. Thus, the fatigue strength of an actual pavement should be higher than those obtained for experimental specimens. This leads to the conclusion that the experimental values are conservative. Research by Hilsdorf and Kesler (1966) showed that using various stress rates affect the fatigue response greatly. This is especially important for design of pavements, which are subjected to different load amplitudes. Based on these facts, constant probability curves were developed by Fordyce and Yrjanson (1969). The curve that is the nearest to the design curve has a constant probability of 5%. The PCA (1984) design method uses a conservative curve (5% probability of cracking) and unlike other design methods (Darter 1977) it does not apply safety factor for design of a pavement.

6.1.2 Effect of Various Parameters on Fatigue of Concrete Beams

6.1.2.1 Uniaxial Tension, Flexural Test and Splitting Tests

Since developing a state of uniaxial constant tension in concrete is very difficult, not many uniaxial tension tests have been conducted. In Molinas (1992) the results from previous investigation on the fatigue behavior of concrete were summarized. Highlights of this report are discussed in the next paragraphs.

Based on the report by Molinas (1992), Morris and Garret (1981) conducted tests with mortar, in which the minimum and maximum applied stresses were 5% and 70% to 85% of the static tensile strength, respectively. Unfortunately, their results were too scattered and they could not reach any reliable conclusions. This was mainly attributed to brittleness of the mortar.

The uniaxial fatigue test done by Kolas and Williams (1978) on typical highway pavement samples were also discussed in Molinas (1992). Fine and coarse aggregate were used in their experiments. For the same amount of applied stress, coarse-graded concrete showed a longer fatigue life.

Saito and Imai (1983) tested saturated specimens, cured under the water, with a minimum and maximum tensile stresses of 8% and 75% to 87.5% of the static tensile
strength, respectively. An equation for an arithmetic average value was obtained, which is given by:

\[ S = -4.12 \log (N) + 98 \]  \hspace{1cm} \text{Equation 6-7}

Based on the report by Molinas (1992), a comprehensive uniaxial test was done by Cornelissen (1984). In this work, different curing conditions and different maximum and minimum stresses including stress reversal were applied on specimens. As it can be seen in Figure 6-1 decreasing the stress range increase the fatigue life for a given maximum stress, and stress reversal leads to a drastic reduction in fatigue life even if the minimum compressive stress is small.

The flexural test is the most convenient fatigue tensile test and the majority of published results of fatigue tensile behavior of concrete are in this category. Furthermore, it is more accurately represents the behavior of concrete in an actual pavement. The results of these tests are covered in Chapter 6.1.3.1 of this report.

Molinas (1992) studied various works on fatigue tests. Based on this report, under the same conditions, the number of cycles to failure in a flexural test is significantly higher than that in a uniaxial tension test (Cornelissen 1984). The difference was
attributed partly to the stress gradient across the cross section and partly because of the
effect of applied lowest tensile strength. Ople and Hulsbos (1966) found that as the load
eccentricity and thus stress gradient increased, the number of cycles to failure increased
as well. Furthermore, studies by Dillmann (1981) showed that during fatigue tests some
relaxation occurred and the fibers with the greatest strain underwent the largest stress
relaxation. This relaxation caused a stress redistribution, which allowed the section to
carry more of the applied load.

Linger and Gillespie (1966) and Tepfers (1979) performed splitting tests on
concrete beams. They concluded that the mechanism of fatigue is the same in both
compression and tension.

6.1.2.2 The Effect of Air Content and Water Cement Ratio

Klaiber and Lee (1982) considered the effect of air content on the fatigue strength
of concrete. Entrained air is added to concrete to increase the resistance of the concrete
against freezing and thawing. It also improves the workability and decreases the
segregation of the freshly mixed concrete. It was found that the fatigue strength of
concrete decreases as the air content increases regardless of water cement ratio and
aggregate type. It was also determined that the difference is higher as the stress ratio
decreases. This is the case for pavement design.

Zhang et al. (1996, 1997) observed inconsistencies in the results of previous
investigations on the effect of water-cement ratio (w/c) on concrete fatigue strength. For
instance, Graf and Brenner (1934) suggest smaller fatigue strength with increased w/c,
while Clemmer (1922) suggests higher fatigue strength with richness of cement, and
Klaiber and Lee (1982) concluded a reduction in flexural strength with w/c, when w/c is
less than 0.4, and no change in flexural strength for w/c more than 0.4. Based on previous
work and the scatter in fatigue results caused by variability of the concrete static strength
f_c, Zhang et al (1996, 1997) suggested that parameters controlling the static strength (w/c, cement content, curing condition, and concrete age) do not have an explicit effect if the
fatigue strength is expressed in terms of static strength. Comprehensive fatigue tests with
different amounts of w/c were performed to support this proposition. The average value
of the material constant $\beta$ in Equation 6.16 was obtained from 209 sets of data as .0807 with a coefficient of variation of 7.2%. This confirmed the proposition that concrete with different w/c ratios have the same fatigue relations.

6.1.2.3 Aggregate Type

Different investigations have been conducted to consider the effect of aggregate type on the fatigue strength of concrete, which have produced different findings. Raithby and Galloway (1974) studied this effect and observed a similar mechanism of damage in concrete beams failed under fatigue tests, static flexural tests, and indirect tensile tests in specimens with flint gravel aggregate. Very few aggregates were fractured. Cracks progressed around the aggregate and considerable loss of adhesion was observed between aggregate and mortar. On the contrary, most of the concrete beams with limestone aggregate failed in tension and no bond failure was observed in all three kinds of tests mentioned for gravel aggregate. The flexural and fatigue strength of limestone was more than flint gravel, but the indirect tensile strength was the same.

Klaiber and Lee (1982) also studied the effect of coarse aggregate type. At high stress levels, concrete with gravel showed a better fatigue strength than that of concrete with limestone. At lower stress levels, the difference was not significant. Fine aggregate type did not seem to have an effect on fatigue behavior. In this case, since the air content increases, the failure of concrete occurs at the aggregate-cement boundary instead of through the aggregate particles. For similar air content, the failure surface in gravel tends to be in the aggregate-cement interface and the failure surface in limestone tends to be in through the aggregate. Zhang et al (1996, 1997) gathered and summarized the previous works and their findings are shown in Table 6-3.

Zhang et al (1996, 1997) performed comprehensive experiments with gravel, slag and clay ceramsite to study the effect of aggregate type on concrete fatigue strength. They found that a slightly higher amount of material constant $\beta$ in Equation 6.16 can be used for higher strength aggregate ($\beta = 0.081$), however, the results of the experiments showed a mean value of $\beta=0.070$ for the ceramsite aggregate, which is significantly less than that for the both ordinary and heavy weight concrete. This results in higher fatigue
strength of the lightweight aggregate concrete, although the modulus of rupture and compression strength is significantly less than normal concrete. A comparison between the results of the previous investigations (Table 6-3) with this recent investigation showed that results of tensile loading of lightweight aggregate could be predicted by Equation 6.16 and the material constant $\beta = 0.07$. The value of $\beta$ in lightweight concrete is more dependent on the composition of the aggregates than in normal weight concrete.

<table>
<thead>
<tr>
<th>Researcher</th>
<th>Type of test</th>
<th>Comparison of Lightweight and normal weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>Williams (1943)</td>
<td>Flexural fatigue</td>
<td>Fatigue strength of lightweight is inferior to normal weight</td>
</tr>
<tr>
<td>Tepfers and Kutti (1979)</td>
<td>Compressive</td>
<td>No obvious difference</td>
</tr>
<tr>
<td>Sparks (1973, 1982)</td>
<td>Compressive</td>
<td>No obvious difference</td>
</tr>
<tr>
<td>Cornellissen (1984)</td>
<td>Tension</td>
<td>No obvious difference</td>
</tr>
<tr>
<td>Cornellissen (1984)</td>
<td>Tension-comp</td>
<td>Fatigue strength of lightweight is inferior to normal weight</td>
</tr>
<tr>
<td>Saito (1984)</td>
<td>Tensile - Saturated</td>
<td>Fatigue strength of lightweight is higher to normal weight</td>
</tr>
<tr>
<td>Waagaard (1981, 1986)</td>
<td>Saturated-reinforced</td>
<td>Fatigue strength of lightweight is higher to normal weight</td>
</tr>
</tbody>
</table>

6.1.2.4 Moisture Condition and Age

Raithby and Galloway (1974) studied the effect of moisture conditions, age, and rate of loading on fatigue strength of concrete. All the tests were carried out on 102 x 102 x 510 mm beams. The test specimens were cured under water before being tested between 3 and 28 days. The minimum load was slightly more than zero and the maximum load varied from 55% to 95% of the static failing load. To consider the effect of moisture conditions, specimens were tested under four different conditions:

1) Saturated during the test
2) Dried one week in the laboratory environment before test
3) Dried in an oven for one week at 105 °C
4) Oven dried and resoaked in water for three weeks before test

The strength and endurance values of the specimens in the first group were intermediate between the high strength of third group and low strength of second group. The static strength of the specimens in the fourth group was between that of the specimens in the first and the third groups, but their fatigue strength was similar to that of specimens in the first group. It was observed that the scatter in the fatigue results for the first group was less than for the other groups.

Raithby and Galloway (1974) studied the effects of age of concrete in the fatigue strength. Specimens with different curing times up to three years were subjected to static and fatigue load. The results of the fatigue tests showed a considerable increase in strength with age. However, when the results were plotted in terms of modulus of rupture at appropriate times, the curves for the different ages were very close to one another. It was concluded that it is possible to derive a particular curve for fatigue behavior of concrete at any age. Their study was limited to up to three years.

6.1.3 The Effects of Loading on Fatigue Strength

6.1.3.1 Range of Loading

Murdock and Kesler (1958) studied the effect of range of stresses on the fatigue strength of plain concrete. All of the specimens were 6 x 6 x 64 inches plain concrete beams. The ratio of the minimum to maximum stress was kept constant except for one case, in which the minimum stress was kept constant. The tests were divided into four groups. In the first group the minimum stress was kept constant at 70 psi and the ratio of minimum to maximum stress (R) ranged between 0.13 and 0.18. In the other three groups the ratio, R, was held constant and equal to 0.25, 0.50 and 0.75 for the second, third, and forth group, respectively. It was found that fatigue strength at ten million load repetitions was 61%, 63%, 73%, and 85% of the static ultimate strength for the first, second, third and forth group, respectively.
Figure 6-2 summarizes the results of the effect of the range of stress on the behavior of concrete under fatigue loading. In this figure S is the Range of stress and n is the number of cycles to fatigue failure. From this figure, for each group an equation can be derived, i.e.:

First group  Stress ratio (percent) = \(-4.7 \log (n) + 94.5\)  
Equation 6-8

Second group  Stress ratio = \(-4.5 \log (n) + 94.5\)  
Equation 6-9

Third group  Stress ratio = \(-3.1 \log (n) + 95\)  
Equation 6-10

Fourth group  Stress ratio = \(-1.6 \log(n) + 96\)  
Equation 6-11

The scatter in the results for the fourth group of tests was much higher than in the other groups. This is because in this case all of the specimens were tested at a rather high percentage of their static ultimate strength.

To understand the effect of concrete moisture condition on the fatigue strength separate computations were carried out for both dry tests and sealed tests. The results of these computations showed that the differences were negligible.

![Figure 6-2 Effect of the Range of Stress on the Behavior of Plain Concrete under Fatigue Loading (Murdock and Kesler 1958)](image-url)
The ratio of stress to static ultimate strength in which the strain remains elastic was found to be approximately 56% when the minimum stress was equal to zero. The following equation was used to approximate the fatigue strength:

\[ F_{10} = 0.56 + 0.44M \text{ or } F_{10} = \frac{1.3}{3.3-R} \quad 0 < M, R < 1 \]

\[ F_{10} = 0.56 \quad -0.56 < M < 0 \text{ & } -1.0 < M < 0 \]  \hspace{1cm} \text{Equation 6-12}

where \( M = \) the ratio of flexural stress at minimum load to modulus of rupture

\( R = \) the ratio of flexural stress at minimum load to the flexural stress at maximum load

\( F_{10} = \) the ratio of flexural stress at minimum load to modulus of rupture at ten million repetitions of stress

Cornelissen and Reinhardt (1984) performed constant and variable loading tests on plain concrete to study the fatigue behavior of concrete under direct tension and tension-compression loading. They suggested that the tensile properties of concrete are related to initiation and propagation of micro cracks affecting the stiffness, damping, and durability of concrete and to the bond between aggregates and cement. One of the main points of these experiments was to determine the effect of minimum stress of concrete on its fatigue behavior. For a constant amount of maximum tension stress, the minimum stress was changed. The minimum stresses were taken from a certain amount of tension to zero and then to a certain amount of compression. The minimum amount in compression was small and this forced all of the specimens to break in tension.

In order to calculate the static strength, 500 direct tensile strength, 300 compressive strength, and 300 splitting tests were performed. An ultimate strain of 0.01% and an average strength of 2.5 N/mm² were obtained for pure tension.

For dynamic loading, the maximum stress was between 40% and 90% of the static tensile strength \((f_{ctm})\). The minimum stress for both dry and sealed experiments was applied at four fixed levels: 40, 30, 20, and 0 percent of \(f_{ctm}\). To consider the effect of compressive stress in tensile fatigue, minimum stress levels of 10%, 15%, 20%, and 30% of the static compressive strength \((F'_{cm})\) were applied. The following equations were derived from the tensile tests \((\sigma_{\text{min}} > 0)\):
\[ \log N = 14.81 - 14.52 \frac{\sigma_{\text{max}}}{f_{\text{ctm}}} + 2.79 \frac{\sigma_{\text{min}}}{f_{\text{ctm}}}, \]  

Dry specimens

\[ \log N = 13.92 - 14.52 \frac{\sigma_{\text{max}}}{f_{\text{ctm}}} + 2.79 \frac{\sigma_{\text{min}}}{f_{\text{ctm}}}, \]  

Sealed specimens

\[ \sigma_{\text{max}} = \text{maximum stress level} \]

\[ \sigma_{\text{min}} = \text{minimum stress level} \]

\[ N = \text{number of cycles to failure} \]

In both cases the 90\% confidence regions correspond to \( \log N \pm 1.74 \). Another, similar expression was derived for tensile-compression tests (\( \sigma_{\text{min}} < 0 \)):

\[ \log N = 9.36 - 7.93 \frac{\sigma_{\text{max}}}{f_{\text{ctm}}} - 2.59 \frac{|\sigma_{\text{min}}|}{f_{\text{ctm}}}, \]  

Dry and Sealed

Equation 6-14

The 90\% confidence regions correspond to \( \log N \pm 1.38 \).

Figure 6-3 shows the Modified Goodman Diagram. The vertical axis shows the ratio of the maximum applied stresses to maximum tensile strength. It should be noted that the maximum applied stress is tensile. The horizontal axis shows the ratio of the minimum stresses to the maximum tensile or compression strength. Unlike maximum applied stress, the minimum applied stress can be tensile (right side of the diagram) or compressive (left side of the diagram). The diagram is developed based on the different number of loads to failure. As the minimum stresses decrease for the same number of load to failure a smaller maximum load is obtained. The reduction in the rate of maximum load greatly increases, since the minimum stress is compressive (Figure 6-3). This effect is more significant for long duration tests (\( \log N > 5 \)) because the maximum tensile stress is relatively low and the effect of compression is more important. They suggested that this phenomenon maybe related to the interaction between the micro-cracks under tensile and compressive loading. It is important to point out that these micro-cracks have different orientation.
In dynamic tests the minimum and maximum stresses were calculated as a percentage of static strength. This percentage was estimated from the static test results performed on other specimens from the same batch of concrete. This process introduced some error in the stress-strength level and caused a scatter in the results. Thus, when comparing these results with those obtained by other investigators, this error must be considered. It was shown that a considerable part of scatter is because of the adjustment in the stress levels.

Cornelissen and Reinhardt (1984) calculated the longitudinal strain during dynamic tests (Figure 6-4). They found that in case of repeated tension, the strain remained linear during most of fatigue life. However, in the beginning and ending stage nonlinear behavior (rapid change in strain) was observed. For compression-tension experiments a non-recoverable compressive strain was observed during the first cycles, which remained constant. Meanwhile, an increasing tensile strain was observed at maximum tensile stress. Near failure, the tensile strain grew rapidly and a strong deviation from linearity was observed. Based on these results, the maximum and minimum strains were plotted against the ratio of number of applied cycles to number of failure cycles. These diagrams were
divided into primary, secondary and tertiary parts. In the primary part, which is about 10 percent of the total life, concrete cracks caused a nonlinear increase in strain. In the secondary part, the pre-existing cracks extended into a stable configuration causing a linear increase in strain. Finally in the tertiary part, the pre-existing cracks propagate unstably leading to failure.

![Figure 6-4 Development of Strains at Maximum and Minimum of the Cyclic Stress in Constant Amplitude Tests (Cornelissen and Reinhardt 1984)](image)

When constant sustained loading is applied on concrete the strain increases with time. This behavior is called creep at constant loading. A similar phenomenon occurs when concrete is subjected to dynamic loading, but in this case it is called cyclic creep. Cyclic creep is caused partly by micro cracking, especially when the stresses are high, and partly by other factors.

There is a close relation between the creep rate (in the part of Figure 6-4 where the strain increase is linear) and the total life. Based on experiments, the following equation was derived by Cornelissen and Reinhardt (1984), which relates the number of loading cycles (N) to the rate of maximum strain $\varepsilon_{\text{sec}}$: 
\[
\log N = -3.25 - 0.89 \log \varepsilon \quad \text{Equation 6-15}
\]

It should be noted that no significant difference was observed between dry and sealed concrete or tension and tension-compression tests. This equation is independent of stress and, therefore, it is not affected by the errors of stress-strength level.

Zhang et al (1996, 1997) performed the most recent comprehensive research on this subject that has been reported in the literature. They studied the effect of water-cement ratio, aggregate type, loading sequence, the validity of Miner’s linear cumulative damage equation, and the effect of the tension-compression strength ratio on concrete fatigue strength. In their work, fatigue strength is defined in two ways: \( S_{\max}/f_c \) for a given number of cycles, \( N \), or a number of cycles, \( N \), for a given \( S_{\max}/f_c \).

Considering the effect of loading frequency and stress reversal, the following equation was proposed:

\[
\frac{S_{\max}}{f_c} = C_f [1 - (1 - R') \beta \log N] \quad \text{Equation 6-16}
\]

where

- \( f_c \) = static strength of concrete
- \( N \) = number of load cycles to failure
- \( S_{\max} \) = the maximum stress
- \( \beta \) = a material constant determined as .087 for ordinary concrete
- \( C_f \) = a loading frequency coefficient given by:
  \[
  C_f = ab \log f + c \quad \text{Equation 6-17}
  \]
  where
  - \( f \) = the loading frequency in Hz
  - \( a, b, c \) = material constants determined as \( a = 0.249, b = 0.920, \) and \( c = 0.796 \)
  - \( R' \) = specialized stress ratio, which takes into account stress reversal, given by:
\[ R' = R = \frac{S_{\text{min}}}{S_{\text{max}}} \quad \text{for } R \geq 0 \]
\[ R' = \frac{|f_{c_{\text{max}}}|}{f_{c_{\text{min}}}} R \quad \text{for } R < 0 \]

Equation 6-18

where \( S_{\text{min}} \) = the minimum stress

\( f_{c_{\text{min}}}, f_{c_{\text{max}}} \) = static strength corresponding to \( S_{\text{min}} \) and \( S_{\text{max}} \), respectively.

It should be noted that tensile stresses are to be taken as positive.

They found that for flexural stress reversal, tension dominates. Hence \( f_{c_{\text{max}}} = f_r \), the static modulus of rupture, and \( f_{c_{\text{min}}} = f'_c \), the static compressive strength. The ratio \( |f_{c_{\text{max}}}/f_{c_{\text{min}}}| \) then becomes the tension-compression strength ratio \( f_{tc} = |f_r/f'_c| \).

Zhang et al (1996, 1997) reviewed previous research on the effect of loading frequency on the fatigue strength of concrete. Based on their review, the first studies were done by Hatt (1924, 1925) and Crepps (1923) which concluded that stress reversal had no effect on the fatigue life of concrete. However, Clemmer (1922) and Clifford (1924) found that it may actually have a small influence. Based on two sets of loading combinations, Tepfers (1979) observed that the effect of fatigue reversal on fatigue strength of concrete was too small to be considered in a fatigue equation. Hsu (1981) performed the first systematic studies on plain concrete. However, not enough data was available at the time to reach a general conclusion. Cornelissen (1984) considered the detrimental effect of stress reversal on the fatigue strength of plain concrete by conducting two series of experiments. Some of their results are shown in Figure 6-3 and were discussed.

Based on comprehensive tests, Zhang et al (1996, 1997) proposed Equation 6-16. They compared their equation with the results from several previous investigations to verify if their equation was in agreement with them. They found that they correctly predicted the behavior of concrete in compression fatigue tests, flexural fatigue tests, loading frequency, and stress reversal. The details of these findings are discussed further in the following paragraphs.

To verify their equation for the flexural fatigue effect, they compared their result with the outcome of Murdock and Kesler (1958). In this work tests on 1626 x 152 x 152
mm specimens were conducted for four-stress ratios ($R' = 0.15, 0.25, 0.50, 0.75$) and for a loading frequency varying slightly from 6.7 to 7.3 Hz. A good agreement was observed between results.

To verify their findings related to stress reversal, they compared their results with those of Mc Call (1958), in which 368 x 76 x 76 mm air-entrained concrete beams were subjected to complete stress reversal at a frequency of 30 Hz. They also compared their results with those obtained by Cornelissen’s (1984) from direct tension compression tests. A good agreement was once again observed in all cases.

Shi et al (1993) considered that three forms of fatigue equations are generally used. Most of these equations provide a relationship between $S$, the maximum loading stress to modulus of rupture, and $N$, the number of load repetitions necessary to cause failure. The non-dimensional parameter, $S$, is used to eliminate influences such as water cement ratio, type and gradation of aggregates, and type and amount of cement. The equation is known as the Wholer equation and is given by:

$$S = \frac{\sigma_{\text{max}}}{MR} = a - b \log(N)$$  \hspace{1cm} \text{Equation 6-19}

where $a$ and $b$ are experimental coefficients, which vary with loading condition, i.e., compression, tension, or flexure.

This equation was used by PCA (1984) and Darter (1977) to predict the fatigue life of concrete pavements. The second form of the equation, which is a modification of the previous one, considers the ratio $R$ of minimum to maximum stress $\sigma_{\text{min}}/\sigma_{\text{max}}$. This term is included to consider the effect of actual loading in which the minimum stress is not equal to zero. This is because in concrete pavements the minimum stress is not equal to zero, because of the presence of thermal loads. In its third form, the fatigue equation is a power formula. It is used by pavement researchers and was developed based on findings obtained on concrete pavement slab sections in the various AASHTO road test experiments. This equation is as follows:
\[ S = \frac{\sigma_{\text{max}}}{MR} = AN^{-B} \]  

Equation 6-20

where \( A \) and \( B \) are constant numbers. Previous laboratory tests and field data showed that no fatigue limit (a stress below which the fatigue damage does not occur) existed for concrete up to 10 million load cycles. Shi et al (1993) showed that the only fatigue equation that could satisfy this boundary condition was Equation 6-20. Thus, they developed a new method for calculating the fatigue strength of plain concrete, which incorporates the effect of stress ratio and stress level. In their work, experiments were conducted to incorporate the term \( R \) in this equation. Defining the new term equivalent fatigue life \( E \text{N} \) as \( N^{1-R} \) and using Weibull distribution the following equation was proposed:

\[ S = AN^{-0.0422(1-R)} \]  

Equation 6-21

where \( A \) is a coefficient which varies with survival probability \( SF (n) = P (N>n) \) and should be chosen for desired survival probability level from Table 6-4.

<table>
<thead>
<tr>
<th>SF</th>
<th>0.95</th>
<th>0.90</th>
<th>0.80</th>
<th>0.70</th>
<th>0.60</th>
<th>0.50</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.9611</td>
<td>0.9843</td>
<td>1.0067</td>
<td>1.0190</td>
<td>1.0289</td>
<td>1.0380</td>
</tr>
</tbody>
</table>

A comparison of this equation with the previous laboratory tests showed a good agreement between the results. However it should be noted that the results from the laboratory tests were different from the field performance–based curve. This is due to the difference in loading conditions. The main advantage of the following equation is that it can be adjusted with the field data by simply calibrating the \( A \) and \( B \) terms.

6.1.3.2 Rate of Loading

Since fatigue experiments take a considerable time, Kesler (1953) carried out studies to determine the effect of speed of testing on fatigue behavior of plain concrete. One hundred 6 x 6 x 64 inches specimens were loaded with frequencies from 70 to 440
cycles per minute (c.p.m.) for two different concrete strengths of 3,500 and 4,500 psi. The remaining specimens were brought to failure under static third point loading. The average of the two modulus of rupture was chosen as the static flexural strength of the specimen. The greatest change in strength of concrete due to the rate of loading occurred for the specimens who were tested under 230 cpm. The results showed 7 percent difference from the average for concrete beams that failed in a low cycle of loading repetition and less than 4 percent for concrete beams that failed after 1,000 cycles of load repetition. It was concluded that the relation between stress ratio and number of cycles is independent of concrete strength. It was also observed that the maximum variation in fatigue strength for different frequencies was less than 5 percent, which is negligible. The final result was that the speed of testing had particularly no effect on fatigue behavior.

Figure 6-5 Average Curves for the Three Different Type of Loading

The average curves for all the experiments were plotted as shown in Figure 6-5. From this, the following equation was obtained:

\[ \text{Stress ratio \ (percent)} = -3.4 \ \log(N) + 73.2 \]  
Equation 6-22

Hsu (1981) studied the effect of rate of loading on fatigue strength. He concluded that the duration of loading has an effect on low cycle fatigue (N<1000 cycles), but it is
negligible for high cycle fatigue \((1000 < N < 10,000,000)\). He proposed the use of two different equations, one for low-cycle fatigue and the other one for the high-cycle fatigue range, as can be seen in Figure 6-6.

![S-N Curves for Concrete Subjected to Flexural Fatigue Including Rate of Loading (Hsu 1981)](image)

Raithby and Galloway (1974) found that the rate of loading has a great effect on the static strength of specimens. However, the effect of rate of loading on fatigue strength was not significant. They found that if the rate of loading (in static tests) is comparable to rate of loading of fatigue tests, the flexural stress is 50 percent more than that measured at a standard rate of loading. Fatigue tests were conducted at 4 and 20 Hz on both saturated and dry surface specimens. Although the fatigue strength of the 20 Hz experiment was slightly higher, the differences were considered statistically insignificant.

Zhang et al (1996, 1997) reviewed previous investigations on the effect of loading frequency. Based on their review, Graf and Brenner (1934) were the first to study the effect of rate of loading on fatigue strength. For frequencies ranging from 4.5 to 7.5 Hz little effect on the fatigue life was observed. For frequencies less than .16 Hz, the fatigue life decreased. Hanson et al. (1974) and Murdock (1965) suggested that if the maximum stress ratio \(S\), is less than 0.75, changing the frequency from 1 Hz to 15 Hz has little influence on fatigue strength. Sparks et al (1973, 1982) also found that for stress ratios
less than 0.75, frequencies between 0.1 and 100 had no effect on N (number of cycles to failure). This investigation did not consider the effect of frequency on fatigue strength. The first equation that considers the effect of frequency was proposed by Furtak (1984). A frequency influence coefficient, \( C_f \), was added to the equation resulting in the following:

\[
\frac{S_{\text{max}}}{f_c} = CN^{-A}(1 + B \log N)C_f
\]

Equation 6-23

where \( C_f = 1 + a (1 - bR) \log f \)

Equation 6-24

\( A, B, C, a, \) and \( b = \) material parameters which can be determined through experiments.

6.1.3.3 Rest Period

Raithby and Galloway (1974) applied one-second rest periods to consider the effect of rest on fatigue strength of concrete. Although a slight reduction was observed in fatigue strength, the difference was not deemed statistically significant. The reason for the discrepancy between these results and the results from Kesler’s (1953) experiment was assumed to be the rest duration. The rest duration is 1 second in the Raithby and Galloway (1974) experiment and between 1 and 27 seconds in the Kesler (1953) experiment.

Hilsdorf and Kesler (1966) performed a comprehensive investigation on the effect of rest period on fatigue strength. They observed that in previous research, constant maximum and minimum values were used, which does not accurately represent the actual structure undergoing random varying load. Furthermore, they realized that there was no reasonable method to correlate simplified laboratory tests and real situations. Based on these facts, two series of tests were conducted to evaluate the effect of rest periods and changing the amplitude of minimum and maximum loads on fatigue strength. 185 moist cured specimens were subjected to flexural loads with a constant ratio of the minimum to maximum load equal to 0.17.

To study the effect of rest periods on fatigue strength of concrete, during the Hilsdorf and Kesler (1966) experiment, specimens were subjected to 4,500 load cycles
and then a sustained constant load equal to the minimum limit of the repeated load for different times of 1, 5, 10, 20, or 27 minutes. This dynamic load and rest period was applied on the specimens continuously until the specimens failed.

It was found that periodic rests tend to increase the fatigue strength for a given stress level. The effect was found to be considerably higher for larger number of cycles to failure, N. However, for a small value of N the difference was found to be not significant. The fatigue strength also increased with the length of rest time up to five minutes. For longer rest periods it remained constant. A statistical analysis showed the difference more clearly. For instance the probability of failure for a specimen subjected to one million cycles of loading at a stress level of 0.65 was 0.75 for continuous loading, and 0.22 for periodic rest and loading. It was observed that the extreme fiber strains increased when the number of cycles increased. Failure happened when tensile strain reached a value of .00025 regardless of the applied stress level. During the rest period, the deformation decreased when the rest period increased. Therefore, the number of cycles necessary to reach the failure strain also increased.

6.1.4 Comparison between Fatigue Failure Equations

Different equations were proposed to describe the fatigue behavior of plain concrete under different circumstances. To compare the results equations derived for similar situations are considered separately. The equations for flexural test with a probability of 50% are compared in Table 6-5 where,

- \( S \) = the ratio of maximum stress to modulus of rupture
- \( N \) = number of load cycle to failure

These equations are drawn in Figure 6-7. As it can be seen in the diagram the empirical AASHTO equation is different from the laboratory equations. One of the reasons might be that the AASHTO equation does not exclude the effect of thermal stresses. It should be noted that the results of AASHTO for low number of cycles appear to be non-conservative when compared to the others.
Table 6-5 Flexural Fatigue Equations with a Probability of 50%

<table>
<thead>
<tr>
<th>Investigator</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Darter</td>
<td>Flexural test, $\sigma_{\text{min}} = 0$ $S = 100 - 5.678 \log (N)$</td>
</tr>
<tr>
<td>Murdock and Kesler</td>
<td>Flexural test, $\sigma_{\text{min}} = 0.13 \sim 0.18$ $S = 94.5 - 4.7 \log(N)$</td>
</tr>
<tr>
<td>AASHTO</td>
<td>Flexural test, $\sigma_{\text{min}} = 0$ $S = 233(\log(N))^{-0.8184}$</td>
</tr>
<tr>
<td>Zhang et al.</td>
<td>Flexural test, $\sigma_{\text{min}} = 0$ $S = 107.353 - 8.66 \log(N)$</td>
</tr>
<tr>
<td>Shi et al.</td>
<td>Flexural test, $\sigma_{\text{min}} = 0$ $S = 103.8 N^{-0.0422}$</td>
</tr>
</tbody>
</table>

Figure 6-7 Comparison of Flexural Test Equations, $P_{\text{failure}} = 50\%$, $\sigma_{\text{min}} = 0$

As it can be seen from Figure 6-7, Shi’s results are near to Darter’s for the low number of cycles and near to Zhang’s for high number of cycles. The equations for uniaxial tests with the minimum stress of zero and probability of 50% are compared in Table 6-6. These results have also been plotted in Figure 6-8. It should be noted that the flexural test results by Shi et al. (1993) have been added in the figure for comparison purposes.
Table 6-6 Uniaxial Fatigue Equations with Minimum Stress of Zero

<table>
<thead>
<tr>
<th>Investigator &amp; Conditions</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Imai et al.</td>
<td>$S = 98.73 - 4.12 \log (N)$</td>
</tr>
<tr>
<td>Cornelissen &amp; Reinhardt</td>
<td>$S = 101.997 - 6.887 \log(N) + 19.215 S_{\text{min}}$</td>
</tr>
<tr>
<td>Cornelissen &amp; Reinhardt</td>
<td>$S = 95.868 - 6.887 \log(N) + 19.215 S_{\text{min}}$</td>
</tr>
<tr>
<td>Cornelissen &amp; Reinhardt</td>
<td>$S = 118.032 - 12.610 \log N - 32.661</td>
</tr>
</tbody>
</table>

As it can be seen from this diagram, the results obtained by Shi et al. (1993) fall between those by Cornelissen and Reinhardt (1984) for dry and sealed specimens. The results from the experiments performed by Saito and Imai (1983) are the least conservative, especially when the number of cycles increases. From the results by Cornelissen and Reinhardt (1984), it can be seen that tension-compression has a negative effect on fatigue strength when the number of cycles increases. The equations developed for a probability of 5 or 24% cracking are shown in Table 6-7.

Figure 6-8 Comparison of Uniaxial Test Equations, $P_{\text{failure}} = 50\%$, $\sigma_{\text{min}} = 0$
Table 6-7 Fatigue Equations with Probability of 5~24% of Failure

<table>
<thead>
<tr>
<th>Investigator</th>
<th>Conditions</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Darter</td>
<td>Flexural test, $\sigma_{\text{Min}} = 0$</td>
<td>$S = 94.321 - 5.678 \log (N)$</td>
</tr>
<tr>
<td></td>
<td>$\sigma_{\text{Min}} = 0$</td>
<td>$S = 143(\log(N))^{-0.8184}$</td>
</tr>
<tr>
<td>AASHTO</td>
<td>Flexural test, $\sigma_{\text{Min}} = 0$</td>
<td>$S = 96.11 N^{-0.0422}$</td>
</tr>
<tr>
<td>Shi</td>
<td>$\sigma_{\text{Min}} = 0$</td>
<td>$S = 97.18 - 8.28\log(N)$ $124500 &gt; N$</td>
</tr>
<tr>
<td></td>
<td>$S = 43.25 + \frac{425.77}{N^{0.306}}$ $62,790,000 &gt; N &gt; 124500$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$S &lt; 0.45$ $N &gt; 62,790,000$</td>
<td></td>
</tr>
</tbody>
</table>

These equations are plotted in Figure 6-9. As in the previous case (Figure 6-7), the empirical AASHTO equation differs from the laboratory equations. The results obtained by Shi et al (1993) are very close to those produced by the PCA (1984) equation. It can be seen in the diagram that Darter’s (1977) equation is above the other curves. This is because the probability of cracking in Darter’s equation is higher than the others.

Figure 6-9 Comparison of Fatigue Equations, $P_{\text{failure}} = 5$ or 24%, $\sigma_{\text{Min}} = 0$

6.1.4.1 Fatigue Equations for Different Ranges of Load

To consider the effect of stress level, results of four investigations were compared for three different stress levels. The ratios of minimum to maximum stress considered were 25, 50, and 75 percent and the equations are summarized in Table 6-8.
Table 6-8 Fatigue Equations with Different Range of Loading

<table>
<thead>
<tr>
<th></th>
<th>R = 25%</th>
<th>R = 50%</th>
<th>R = 75%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Zhang</td>
<td>107.35 – 6.49 x Log N</td>
<td>107.35 – 4.33 x Log N</td>
<td>107.35 – 2.17 x Log N</td>
</tr>
<tr>
<td>Shi</td>
<td>103.8 N^{-0.03165}</td>
<td>103.8 N^{-0.0211}</td>
<td>103.8 N^{-0.0155}</td>
</tr>
<tr>
<td>Cornelissen</td>
<td>107.14 - 7.23 x Log N</td>
<td>112.84-7.61 x Log N</td>
<td>119.97 - 7.62 x Log N</td>
</tr>
<tr>
<td>Murdock</td>
<td>94.5 - 4.5 x Log N</td>
<td>95 - 3.1 x Log N</td>
<td>96 - 1.6 x Log N</td>
</tr>
</tbody>
</table>

Figure 6-10 Comparison of Available Equations, \( P_{\text{failure}} = 50\% \), \( \sigma_{\text{min}} = 25\% \) \( \sigma_{\text{max}} \)

Figure 6-11 Comparison of Fatigue Equations, \( P_{\text{failure}} = 50\% \), \( \sigma_{\text{min}} = 50\% \) \( \sigma_{\text{max}} \)
These results for stress ratio of 25%, 50%, and 75% have been plotted in Figure 6-10, Figure 6-11, and Figure 6-12, respectively. As can be seen the results obtained by Murdock and Kesler (1958) for low cycle and Cornelissen (1984) for high cycle are the most conservative, while the results the equation derived by Shi et al (1993) fall between the other two curves. The results from Zhang et al. (1996) equation are the least conservative of all results.

![The effect of Stress Level](image)

**Figure 6-12 Comparison of Fatigue Equations, P_{\text{failure}} = 50\%, \sigma_{\text{min}} = 75\% \sigma_{\text{max}}**

6.1.4.2 Fatigue Equations with Different Probability and Frequency

Some of the fatigue equations consider probability of failure implicitly. Two set of equation are compared in Table 6-9 for different failure probability. The results of these equations are plotted in Figure 6-13. It can be observed that AASHTO equation is slightly more affected by the probability of failure than Shi et al. (1993) equations.

<table>
<thead>
<tr>
<th></th>
<th>P= 50%</th>
<th>P= 20%</th>
<th>R= 5%</th>
</tr>
</thead>
<tbody>
<tr>
<td>AASHTO</td>
<td>233(\log(N))^{0.8184}</td>
<td>189(\log(N))^{0.8184}</td>
<td>143(\log(N))^{0.8184}</td>
</tr>
<tr>
<td>Shi et al</td>
<td>103.8 N^{-0.0422}</td>
<td>100.67 N^{-0.0422}</td>
<td>96.11 N^{-0.0422}</td>
</tr>
</tbody>
</table>
The results of Zhang et al. (1996) for two frequencies have been plotted in Figure 6-14. A change in frequency from 0.01 to 20 Hz changes the stress ratio by about 7 percent for the same number of cycles to failure.

6.1.4.3 Proposed Fatigue Equation

Comparing the previous diagrams, the equation proposed by Shi et al. (1993) seems to provide the most reliable and practical results. This equation considers the effect of loading level and provides the ability to define the desired probability of failure. In
order to consider the effect of loading frequency, the coefficient $C_f$ in the equation by Zhang et al. (1996) can be added to the equation proposed by Shi et al. (1993) as follows:

$$S = C_f A N^{-0.0422(1-R)}$$  

Equation 6-25

where

$$C_f = 0.9315 \ (a \ b^{-\log f} + c)$$  

Equation 6-26

The coefficient 0.9315 is used to produce $C_f = 1$ when the loading frequency is 20 Hz, which is the test frequency used by Shi et al. (1996). The material constants $a$, $b$, and $c$ are determined as $a = 0.249$, $b = 0.920$, and $c = 0.796$ and the coefficient $A$ can be chosen for a desired survival probability level from Table 6-4. This modified version of Shi’s equation is, thus, selected in this work for the cumulative damage calculations performed in the case study presented in Chapter 7 of this report.

### 6.2 Cumulative Damage of Concrete under Fatigue Loading of Variable Amplitude

In the design of concrete pavements based on fatigue behavior, a year is divided into 12 or 24 periods, and different loads (e.g., different truck loads) are applied to the pavement. A foundation seasonal adjustment factor is defined to each period and the maximum tensile stress in the concrete pavement is calculated. Knowing the tensile stress, the allowable number of load repetitions can be calculated based on Darter (1977), Packard and Tayabji (1985) or any other available equation. These equations have been developed using a third point repeated flexural loading on a beam, where the stress ratio (the ratio of stress to modulus of rupture) is plotted against number of repetitions to failure in log scale. The damage ratio, which is the ratio between the predicted and allowable number of repetitions, is computed for each load group in each period and summed over the year using the Palmgren-Miner (1945) equation, which is given by:

$$D_r = \sum_{i=1}^{n} \sum_{j=1}^{m} \frac{n_{i,j}}{N_{i,j}}$$  

Equation 6-27

where

$n_{i,j} = \text{the predicted number of load repetitions for load } j \text{ in period } i$
$N_{ij} =$ the allowable number of load repetitions for load j in period i

The design life of the pavement can then be predicted as $1/D_r$.

The above method is widely used in pavement design. However, this method has some limitations. Miner (1945) theory is based on the assumption that fatigue damage occurs in proportion to the work, done on specimens by the applied load and can be calculated by linear summation of the ratio of the number of applied cycles at a particular stress to the number of cycles at the same stress, which results failure. Miner’s equation does not consider the effect of sequence and amplitude of stresses, which is a critical factor in concrete pavements. Stresses caused by temperature variation along the depth of pavement are considerably high and concrete undergoes different stress amplitudes and sequence. As a result linear cumulative damage is not conservative in all conditions.

Several research studies have been performed to evaluate the effect of repetitive loads in concrete. These studies were conducted for a constant repetitive amount of stress. The effect of different parameters such as water ratio, minimum stress ratio and frequency of loading were evaluated through these studies. These experiments showed that fatigue strength of concrete is independent of nominal strength, air-entrainment, and type of aggregate and frequency of load repetition when expressed in terms of the static ultimate strength; however, as discussed previously, concrete’s fatigue strength depends on rest time and on minimum and maximum stresses (Ballinger 1971).

Since 1970, experiments with different stress level have been carried out. Investigators studied the effect of sequence and amplitude of stresses on the number of cycles to failure. In this work, these results are incorporated in a nonlinear damage accumulation procedure.

### 6.2.1 Previous Studies

#### 6.2.1.1 Ballinger (1971)

Ballinger (1971) tested specimens subjected to both constant and variable dynamic loading. Variable dynamic loading tests were performed on 6 x 6 x 64 in. and 6 x 6 x 32
in. beams. Static experiments led to a great disparity in ultimate static strength, so a multiple-term regression equation was chosen to relate static tests to the average flexural strength of the halves of the broken 64 in. beams. For the dynamic constant loading test, change in the stress (increase and decrease by different amounts) and change in the number of cycles were considered. The dynamic constant loading test showed that the S-N relationship consisted of two parts, a straight descending line that is the main portion of the diagram and an initial curved part that represents a low cycle fatigue region. The presence of the curved part was justified by the fact that the ultimate strength of concrete is related to both fracture as well as fatigue. The straight line portion can be expressed mathematically by the following equation:

\[
\text{Stress, percent ultimate} = 92.6 - 2.7(\log \text{cycles}) \pm 1.914 \quad \text{Equation 6-28}
\]

The plus or minus sign indicates the 99 percent confidence limits. The result of this study was compared with those from previous studies. It was found that the variation in the results for a specific ratio of stress was much smaller than that from previous studies. This is because an appropriate regression is adopted in the current study. The Miner rule was applied to the final results and it was noted that damage at failure ranged from 0.01 to 384. The scatter is due to errors incurred when predicting an accurate ultimate strength.

For variable loading dynamic tests, two phases of loading were considered. Different magnitude of loads and different number of cycles were applied at each level. In twenty cases a lower load was applied first and greater loads were applied afterwards. In eight cases the loads were decreased. To compare the results of variable loading fatigue equation with the constant loading fatigue equation, the number of first loading cycles was transformed to the equivalent number of second loading cycles. This was done using the following equations:

\[
d_1 = \frac{n_1}{N_1}
\]

\[
d_1 (N_2) = n'_1
\]

\[
n'_1 + n_2 = n_2
\]

\[
\text{Equation 6-29}
\]

where

\[
n_1 = \text{number of applied loading cycles at the first stress level}
\]

\[
N_1 = \text{number of cycles to failure at the first stress level}
\]
\( n_2 = \) number of applied loading cycles at the second stress level
\( N_2 = \) number of cycles to failure at the second stress level
\( n^*_2 = \) equivalent total number of cycles at the second stress level

The equivalent total number of cycles at the second stress level versus that stress level was plotted and the equation for the regression line was calculated as:

\[
\text{Stress, percent ultimate} = 92.7 - 1.88 (\log \text{cycles}) \pm 1.132 \quad \text{Equation 6-30}
\]

This line was compared with the previous S/N line and its 99 percent confidence band. It was observed that the recent regression lines fall in the 99 percent band throughout most of its length. Comparing two equations with confidence limits showed that the slopes of two lines are not statistically different and their intercepts at the vertical axis are almost identical. Adding a comparison of standard error to this, it was concluded that the variable load test data were not significantly different from the S/N test data. It was concluded that the Miner hypothesis represents the cumulative damage of plain concrete in a reasonable manner. Finally, it was recommended that because the strength of the concrete is variable, the stress in plain concrete should be kept at a level less than 70 percent of concrete static ultimate strength. By doing so, fatigue would not become a critical issue.

### 6.2.1.2 Hilsdorf and Kesler (1966)

Hilsdorf and Kesler (1966) determined the effect of variable loading on fatigue behavior of concrete by applying three different types of loading conditions. In the first and second types, dynamic loading was applied in two sequences. In the first type of loading, the stress ratio was increased from \( S_1 \) to \( S_2 \) after applying \( n_1 \) cycles of stress level. In the second type of loading the stress ratio was decreased. In the third type of loading, the specimens were subjected to repeated changes of maximum and minimum stress levels. This was done by two different methods. In the first method, the duration of each load application was fixed and the amount of increase in stress was variable. In the second method, the amount of increase in stress was fixed and the duration of each load application was different.
It was found that for $S_1 < S_2$ the fatigue life of concrete decreases, while for $S_1 > S_2$ the fatigue life of the concrete increases. This result is in contradiction with the results given by Oh (1991) and Zhang et al (1997). A likely reason for this discrepancy is that the minimum applied stress was not constant in Hilsdorf and Kesler (1966) experiments (the ratio of minimum to maximum load was constant). Also, changing the minimum stress affects the fatigue strength of concrete leading to different results. In the case where the loading change was frequently repeated, it was noticed that for lower cycles the Miner (1945) rule was not conservative and for higher cycles it was too conservative.

6.2.1.3 Cornelissen and Reinhardt (1984)

Cornelissen and Reinhardt (1984) also studied the effect of variable loading on the fatigue behavior of concrete. Their tests were divided into two groups. In the first group, the maximum stress was more than 0.7 $f_{ctm}$, for which cracking of concrete is predominant. In the second group, the maximum stress was less than 0.7 $f_{ctm}$, which stress redistribution is more important. In all cases, the minimum stress remained constant at 0.4 $f_{ctm}$, 0.2 $f_{ctm}$, 0, 0.05$f'_{cm}$ and 015$f'_{cm}$, where $f_{ctm}$ and $f'_{cm}$ are the static tensile and compressive strength of concrete, respectively.

It was determined that in general using Miner’s equation to accumulate damage lead to a value that exceed one. However, their results were highly scattered. This complicated their comparison with the results from other investigations and the estimation of the effect of minimum stress on the fatigue of concrete. This scatter was attributed to the uncertainties in the stress-strength level. In order to avoid the scatter, the relation of cycles to failure with secondary creep rate was evaluated. The scatter in these results was much smaller and, as mentioned before, this is because the actual stress levels are taken into account in this case. It was concluded that Miner’s rule is, in general, conservative and minimum levels of stress had little effect on the results. It was also found that the evaluation of the secondary creep rate gives a more accurate estimation of fatigue life of concrete.
6.2.1.4 Oh (1991)

Oh (1991) performed the most comprehensive investigation on the effect of variable loading on the fatigue of concrete. Through loading application in two and three stages with different amplitudes (Figure 6-15), it was found that the linear damage theory by Miner (1945) was not directly applicable to concrete. The results from Miner’s rule were not conservative when the amplitude of load was gradually decreased. In other words, the sum of the cumulative damage was less than one in this case. On the other hand, Miner’s sum was more than one when the amplitude of load was gradually increased. Motivated by these results, they derived a nonlinear damage theory that considered the effect of the load magnitude and sequence of load application.

Eighty concrete specimens with dimension of 100 x 100 x 500 mm were selected for their experiments. Four different types of loading were applied and are summarized in Figure 6-15.

In the first type of loading, the maximum stresses in the beam were increased from 0.75 of the modulus of rupture ($f'_{r}$) to 0.85 $f'_{r}$ after the number of load application in the first loading level was about 20 percent of number of loads to failure. In the second type of loading, the maximum stresses in the beam were decreased from 0.85 $f'_{r}$ to 0.75 $f'_{r}$ after the number of load applications was about 20 percent of the number of loads to failure in the first loading level. In the other two cases, the load amplitude was increased or decreased in two steps. First, the stresses were increased from 0.65 $f'_{r}$ to 0.75 $f'_{r}$ (after the number of load applications was 10 percent of the number of loads applications that caused failure in the 0.65 $f'_{r}$ load level). The stresses were then increased from 0.75 $f'_{r}$ to 0.85 $f'_{r}$ (after the number of load applications was 10 percent of the number of load applications that caused failure in the 0.75 $f'_{r}$ load level). In the fourth case, the loading order was the opposite of the third case. It was observed that in cases one and three the sum of cumulative damage is greater than one and in cases two and four it was less than one.
Nonlinear damage behavior can be explained through strain-cycle diagrams (Figure 6-16). These diagrams depict the increase of strain against the increase in damage for concrete specimens. Based on Holeman’s (1982) results, there are three phases with increasing strain. In the beginning and at the end of load cycle a rapid increase in strain is observed. In the middle stage, the rate of increase is fairly constant. The diagram shown in Figure 6-16 is based on Holeman’s (1982) equations. It can be seen that the path of damage is different for different stress ratios.

Figure 6-16 Fatigue Damage Curves for Various Stress Levels (Oh 1991)

Figure 6-16 indicates that if one goes from point A to B on the $S = 0.75$ line, $N/N_f$ is twenty percent. Increasing the amplitude of loading and the stress from $S = 0.75$ to $S =$
0.85 and considering that the amount of damage is constant takes one from point B to point C. By applying load on the beam one would advance from point C to point D (failure point). As can be seen from the diagram, cumulative linear damage would give 0.20 + 0.87 = 1.07, which is more than one.

On the other hand, if one goes from point A to C on the S = 0.85 line, N/N_f is thirteen percent. Decreasing the amplitude of loading and the stress from S = 0.85 to S = 0.75 and considering that the amount of damage is constant one goes from point C to point B. By applying a load on the beam one would go from point B to point D (failure point). As can be seen from the diagram, cumulative linear damage would give 0.13 + 0.80 = 0.93, which is less than one. This is the main reason why nonlinear damage accumulation produces different results than linear damage accumulation.

Although the previous description was a comprehensive way to define fatigue behavior of concrete structures, it is not useful for practical purposes. Thus, a simpler nonlinear method for calculating the remaining life of concrete under fatigue loading was developed by Oh (1991). This simple method considers the effect of magnitude and sequence of loading. The basic concept of this theory is that the damage caused by n_i cycles of stress level S_i is equal to the damage of n_1 cycles of stress level S_1. Using this concept one can write an equation to find an equivalent number of cycles n_{ie} at a reference stress level S_1 that would produce the same damage as n_i cycles of operation at the actual stress level S_i, i.e.:

\[ n_{ie} = n_i \left( \frac{S_i}{S_1} \right)^p \]  

Equation 6-31

where

- \( n_{ie} \) = number of cycles at a reference stress level \( S_1 \)
- \( n_i \) = number of actual cycles at a stress level \( S_i \)
- \( S_1 \) = reference stress level
- \( S_i \) = any stress level
- \( P \) = parameter obtained experimentally

The damage ratio is given by:
\[ D_i = \left( \frac{n_{ie}}{N_1} \right) \] 

Equation 6-32

Failure will occur when the summation of the damage ratios is equal to one, i.e.:

\[ \Sigma D_i = D1 + D2 + D3 + \ldots + D_i = 1 \] 

Equation 6-33

or

\[ \left( \frac{n_{ie}}{N_1} \right) + \left( \frac{n_{2e}}{N_1} \right) + \ldots + \left( \frac{n_{ie}}{N_1} \right) = 1 \] \[ \rightarrow n_{ie} + n_{2e} + \ldots + n_{ie} = N_1 \] 

Equation 6-34

Plugging Equation 6-31 into Equation 6-34, one obtains:

\[ n_i \left( \frac{S_i}{S_1} \right)^p + n_2 \left( \frac{S_2}{S_1} \right)^p + \ldots + n_i \left( \frac{S_i}{S_1} \right)^p = N_1 \] 

Equation 6-35

This equation can be rewritten as:

\[ n_{ir} = (N_1 - n_i) \left( \frac{S_i}{S_1} \right)^p - n_2 \left( \frac{S_2}{S_1} \right)^p - n_3 \left( \frac{S_3}{S_1} \right)^p - \ldots - n_{i-1} \left( \frac{S_{i-1}}{S_1} \right)^p \] 

Equation 6-36

Defining \( n_{ir} \) as:

\[ n_{ir} = k N_1 \] 

Equation 6-37

one can obtain the following:

\[ k = \left( 1 - \frac{n_i}{N_1} \right) \left( \frac{S_i}{S_1} \right)^p - \frac{n_2}{N_1} \left( \frac{S_2}{S_1} \right)^p - \frac{n_3}{N_1} \left( \frac{S_3}{S_1} \right)^p - \ldots - \frac{n_{i-1}}{N_1} \left( \frac{S_{i-1}}{S_1} \right)^p \] 

Equation 6-38

So the remaining life of the pavement can be calculated by multiplying coefficient \( k \) to the reference fatigue life \( N_1 \). The results from the experiments were used to calculate the quantity \( p \) in the previous equations and it was observed that four different cases lead to nearly the same result (the average value was 18.21). The remaining life of the pavement was compared between theory and the experiment. Good results were observed and it was concluded that the proposed nonlinear method produces realistic results for concrete structures.
6.2.1.5 Zhang, Philips and Wu (1997)

Zhang et al. (1997) gathered and summarized the previous works on the effect of loading sequence on cumulative damage as shown in Table 6-10. In their experiments, two different loading sequences were applied to the concrete specimens to investigate the effect of load sequence on damage of concrete. The load amplitude ranged from 0.75 \( f_r \) to 0.85 \( f_r \) and from 0.85 \( f_r \) to 0.7 \( f_r \) and 0.725 \( f_r \). The number of loadings in the first amplitude was near to half of the strength (\( n_1 / N_1 = 0.5 \)) and stress ratio, \( R' \), was kept equal to 0.2. For the low to high stress sequence, the obtained damage values were from 1.23 to 1.76 with an average of 1.48. For the high to low stress, damage values changed from 0.43 to 0.88, averaging 0.71.

They argued that high stresses cause rapid propagation of micro-cracks at the cement paste aggregate interfaces, or through weak aggregate particles, which tend to accelerate the rate of damage in the subsequent stress levels. However, lower stresses were found to relieve internal residual stresses and to stabilize micro-cracks, thus reducing the damage ratio. Unfortunately, the scatter of their results prevented any conclusive findings. However, they observed that in addition to maximum stress, minimum stress is also important factor in the fatigue behavior of concrete. The authors recommend further investigation in this matter.

<table>
<thead>
<tr>
<th></th>
<th>Two &amp; three stage loading</th>
<th>S_1 &lt; S_2 → D &gt; 1, S_1 &gt; S_2 → D &lt; 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tepfers et al. (1973)</td>
<td>Different stage</td>
<td>Scatter in result – No contradiction to PM rule</td>
</tr>
<tr>
<td>Holemen (1982)</td>
<td>Compressive</td>
<td>S_1 &gt; S_2 → D &lt; 1</td>
</tr>
<tr>
<td>Cornelissen &amp; Reinhardt (1984)</td>
<td>Tension &amp; Tension-Comp.</td>
<td>Sequence has little effect- Miner is safe</td>
</tr>
</tbody>
</table>

A fairly recent branch of Mechanics, called Continuum Damage Mechanics, has provided a new tool for describing damage accumulation in solids. It is centered on the development and accumulation of micro-voids and micro-cracks. Investigators are
making efforts to describe the behavior of concrete based on this theory. As of now, the results of these investigations are too sophisticated to be used for practical purposes.

6.2.2 Stresses in Concrete Pavements

Concrete pavements undergo a variety of loadings during their life. The main loadings affecting their service life are: self-weight, thermal loads, wheel loads, and shrinkage. These loads impact the pavement in different ways. For instance, shrinkage stresses do not change significantly after the pavement’s initial life. Thermal stresses occur with a frequency of one day, while stresses due to traffic increase and decrease the maximum stress at much higher frequencies (Figure 6-17).

![Typical Stress-Time Diagram](image)

Figure 6-17 Typical Stress-Time Diagram

Unfortunately, the change of stresses with time in pavements is much different than the uniform change in stresses in laboratory tests. To consider these effects one has to use empirical equations or try to normalize laboratory test results. As it was seen in the previous sections, the effect of range and rate of applied loads, of rest periods, and of sequence of applied loads must be considered. This makes the prediction of the service life of a pavement very difficult. To have the best approximation of a pavement’s life, a reliable fatigue equation that considers the effect of these various parameters must be
used. The results from this equation must then be implied in a nonlinear damage accumulation method in order to produce reliable results.

6.2.3 Nonlinear Accumulation of Damage

A simple model for nonlinear accumulation of fatigue was discussed in section 6.2.1.4. Unfortunately, this simple model does not consider the effect of range of loading and is based on zero minimum stress, which is not a good assumption for pavements. The other method, which uses damage cycle diagram also discussed in section 6.2.1.4, is more complicated, but more realistic. Application of this method requires enough data on damage fraction-cycle ratio (the ratio of number of applied loading cycle to number of loading cycle to failure) curves of the concrete beams.

To understand the effectiveness of the comprehensive nonlinear damage accumulation model, the experimental results of Oh (1991) and Zhang et al. (1996, 1997) were compared with the developed theoretical results. Oh (1991) developed three cubic equation based on strain accumulation under various constant-amplitude cyclic loading which are given by:

For $S = 0.85$  \[ D = 1.727 \left( \frac{n}{N_f} \right)^3 - 3.625 \left( \frac{n}{N_f} \right)^2 + 2.898 \left( \frac{n}{N_f} \right) \]

For $S = 0.75$  \[ D = 0.757 \left( \frac{n}{N_f} \right)^3 - 1.589 \left( \frac{n}{N_f} \right)^2 + 1.832 \left( \frac{n}{N_f} \right) \]

For $S = 0.65$  \[ D = -0.712 \left( \frac{n}{N_f} \right)^3 + 1.495 \left( \frac{n}{N_f} \right)^2 + 0.217 \left( \frac{n}{N_f} \right) \]  \hspace{1cm} \text{Equation 6-39}

These equations were used to develop the theoretical nonlinear damage accumulation procedure. Using these equations in combination with the method based on the damage cycle diagram, the theoretical damage accumulation was calculated. Table 6-11 compares the result of the calculated stresses with the experimental data.
Table 6-11 Calculated and Experimental Result

<table>
<thead>
<tr>
<th></th>
<th>Oh (1991) experiments</th>
<th>Zhang et al. (1996, 1997) experiments</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Calculated</td>
<td>Experiment</td>
</tr>
<tr>
<td>1.10</td>
<td>1.33</td>
<td></td>
</tr>
<tr>
<td>0.88</td>
<td>0.76</td>
<td></td>
</tr>
<tr>
<td>1.13</td>
<td>2.01</td>
<td></td>
</tr>
<tr>
<td>0.71</td>
<td>0.96</td>
<td></td>
</tr>
</tbody>
</table>

From these results it can be conclude that using these equations and the damage cycle diagram provides good insight in the nonlinear damage accumulation in concrete. However, for a better correlation between theory and field application, better equations are still needed.

Molinas (1991) has also developed damage fraction-cycle ratio curves. Based on his report a general expression describing the damage as a function of minimum and maximum stresses was defined as

\[
D = \alpha_0 \sigma_{r \text{ max}}^2 + \alpha_1 \sigma_{r \text{ max}} \sigma_{r \text{ min}} + \alpha_2 (\sigma_{r \text{ max}} - \sigma_{r \text{ min}})
\]

Equation 6-40

where

\[
D = \text{damage at a given value of } N / N_f
\]

\[
\sigma_{r \text{ max}} = \text{ratio of applied maximum tensile stress to tensile strength of concrete}
\]

\[
\sigma_{r \text{ min}} = \text{ratio of applied minimum tensile stress to tensile strength of concrete}
\]

The coefficients \( \alpha_0, \alpha_1 \) and \( \alpha_2 \) can be calculated from Table 6-12.
Table 6-12 Values of Coefficient Defining Damage

<table>
<thead>
<tr>
<th>N_r = N / N_f</th>
<th>α_0</th>
<th>α_1</th>
<th>α_2</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.025</td>
<td>0.182</td>
<td>0.11</td>
<td>0.085</td>
</tr>
<tr>
<td>0.05</td>
<td>0.371</td>
<td>-0.084</td>
<td>0.02</td>
</tr>
<tr>
<td>0.1</td>
<td>0.191</td>
<td>0.28</td>
<td>0.225</td>
</tr>
<tr>
<td>0.15</td>
<td>-0.028</td>
<td>0.631</td>
<td>0.437</td>
</tr>
<tr>
<td>0.2</td>
<td>-0.053</td>
<td>0.698</td>
<td>0.479</td>
</tr>
<tr>
<td>0.3</td>
<td>-0.161</td>
<td>0.891</td>
<td>0.627</td>
</tr>
<tr>
<td>0.4</td>
<td>-0.525</td>
<td>1.26</td>
<td>0.978</td>
</tr>
<tr>
<td>0.5</td>
<td>-0.694</td>
<td>1.5</td>
<td>1.17</td>
</tr>
<tr>
<td>0.6</td>
<td>-0.666</td>
<td>1.49</td>
<td>1.2</td>
</tr>
<tr>
<td>0.7</td>
<td>-0.68</td>
<td>1.55</td>
<td>1.26</td>
</tr>
<tr>
<td>0.8</td>
<td>-0.762</td>
<td>1.69</td>
<td>1.38</td>
</tr>
<tr>
<td>0.85</td>
<td>-0.756</td>
<td>1.69</td>
<td>1.41</td>
</tr>
<tr>
<td>0.95</td>
<td>-0.81</td>
<td>1.77</td>
<td>1.59</td>
</tr>
<tr>
<td>0.975</td>
<td>-1.08</td>
<td>2.12</td>
<td>1.9</td>
</tr>
</tbody>
</table>

Based on this model and using the method explained in Section 6.2.1.4 and Figure 6-16 the results shown in Table 6-13 are obtained.

Table 6-13 Calculated and Experimental Result

<table>
<thead>
<tr>
<th>Byung Oh experiment</th>
<th>Zhang et al. experiment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Calculated</td>
<td>Experiment</td>
</tr>
<tr>
<td>1.07</td>
<td>1.33</td>
</tr>
<tr>
<td>0.92</td>
<td>0.76</td>
</tr>
<tr>
<td>1.12</td>
<td>2.01</td>
</tr>
<tr>
<td>0.84</td>
<td>0.96</td>
</tr>
</tbody>
</table>

Similar to the previous results, this method can better predict the nonlinear damage behavior of concrete. However, as mentioned previously, a more comprehensive equation to accumulate the damage more accurately is necessary.
CHAPTER 7  CASE STUDY – SERVICE LIFE OF PCC PAVEMENTS

Mid-panel random cracking has been observed in concrete pavements in several states in the United States, and in particular in Indiana, under a variety of environmental and traffic conditions. The repair of a cracked pavement is expensive, time consuming, and causes severe traffic disruption. There is no obvious trend that would indicate any one specific reason why this phenomenon is occurring in Indiana.

A comprehensive FE model can help to more accurately find the possible causes leading to pavement distresses and the sensitivity of the stresses to each factor that affects its behavior. By designing pavements in order to decrease these adverse effects, one is able to extend its service life.

Existing pavement design software packages rely on a number of simplifying assumptions resulting in inaccurate results. Thus, in order to accurately estimate a pavement’s life expectancy, more complex and reliable three-dimensional finite element models must be adopted. Such a model must be able to simulate subbase and subgrade separation, sub-base/slab separation and friction, and dowel bar and slab interaction. It must also be able to accurately model nonlinear thermal gradients and wheel loads. Furthermore, the developed stresses must be analyzed using a valid fatigue equation and damage must be accumulated nonlinearly. It is the hypothesis of this research that such a methodology provides more reliable findings.

A case study has been developed to evaluate the available fatigue equations and to study the difference between nonlinear and linear damage accumulations. The selected site was inspected, and a finite element model was developed. Studies were performed using the developed 3D model in conjunction with a nonlinear damage model. Results of the research showed the developed model was able to predict the likely causes of pavement cracking. In particular, the effects of nonlinear thermal gradients and traffic loading were studied. Using the stresses produced by the finite element analysis, various
fatigue equations were compared and the difference between nonlinear and linear accumulation of damage was studied.

7.1 Existing Pavement

The selected site was a section of the Interstate 70 East of Indianapolis, from mile marker 92.5 to 103.1. Both East and Westbound lanes were considered. This section has been experiencing extreme distress. The pavement section was installed in 1992 and it consisted of plain jointed 14 inch PCCP. The slabs were 20 feet long and 12 feet width and the shoulder was 10 feet width. The specification for pavement subbase was four inches of INDOT # 8 aggregate as a drainage layer and three inches of INDOT #53 aggregate as a separation layer. The specification for the pavement subgrade was six inches of special subgrade treatment. The six inches of soil underneath the subbase was compacted to 100% of maximum dry density.

Figure 7-1 A Sample of Mid Panel Cracking and Dowel Bar Retrofitting on Interstate I70

7.2 Site Inspection

Visual inspection determined that the eastbound section was much more severely damaged than the westbound section. At the time of inspection, the eastbound section had over 420 cracks, while the westbound section had approximately 60 cracks. A significant
number of panels had been replaced in both the east and westbound sections. Distressed sections that were not replaced were repaired with retrofit dowel bars (Figure 7-1). At the time of inspection, many of those replaced panels were already showing signs of distress. Most of the panels repaired with retrofit dowel bars have reopened and are faulting. There is a loss of subgrade support in many locations throughout the entire section. These panels are rocking and pumping under heavy truckloads.

7.3 FE Modeling Issues

Two different FE models were developed to simulate the inspected site. In the first model, the soil was modeled by nonlinear springs, which had the capability to separate from the slab. In the second model, soil was modeled by solid brick elements and contact elements were used for modeling the contact between the subbase and the slabs. The contact elements are capable of modeling the friction between the two surfaces as well as their separation. The thickness of the concrete slab, subbase and subgrade were 14 inch, 7 inch, and 100 inch, respectively, and the length of each slab is 20ft (Figure 7-2). The basic dimensions of the studied section are given in Figure 7-2.

Twenty-node brick elements were selected for the slab where the wheel load was applied, since, as discussed in Chapter 2, higher order elements are necessary to accurately obtain the stresses under tire loading. Eight-node elements were used to model the subbase, subgrade, and the other slabs. A 1-ft strip of soil, which included subbase and subgrade, was used on both sides in the longitudinal direction to allow for better stress distribution around the slab edges and to better model the continuity of soil. The general mesh can be seen in Figure 7.3. To allow separate behavior, the adjacent slabs coincident locations were modeled with different nodes. It should be noted that a more refined mesh was used near the edges of the slabs, since that is where the dowel bars were situated. This refinement enabled the accurate modeling of the interaction between dowel bars and slabs.

The following section describes the developed three-dimensional finite element model, which captures several different aspects that influence the behavior of the
pavement under various conditions, such as traffic loading, thermal gradients, and shrinkage. It also discusses the application of nonlinear thermal gradient, wheel loading and shrinkage. Finally, it presents the effect of soil stiffness on the stresses in the concrete slab under various loading conditions was studied and the results.

Figure 7-2 Finite Element Mesh

7.4 The Developed 3D Finite Element Model

The basic concepts of the developed FE model are similar to those previously discussed in Chapters 2 and 5. The model was developed in a way to minimize the amount of memory usage without jeopardizing accuracy. In terms of the boundary conditions, the edges of the subbase and subgrade boundaries were restrained in their normal direction, and no restraints were applied at the concrete slab boundaries. All translational degrees of freedom were restrained at the bottom most layer of the subgrade. Linear elastic material models were used. The adopted material properties are given in Table 7-1, where E: Young’s modulus, \( \nu \): Poisson ratio, \( \gamma \): unit weight, and \( \alpha \): coefficient of thermal expansion. The concrete material properties match those of concrete used by
INDOT. Since the evaluation of the subgrade and subbase properties through the available data has approximations, two sets of material were used to study the sensitivity of the results to these properties.

Table 7-1 Material Properties

<table>
<thead>
<tr>
<th>Location</th>
<th>Material</th>
<th>$f'_c$ (Mpa)</th>
<th>$S_c$ (kPa)</th>
<th>E (MPa)</th>
<th>v</th>
<th>$\gamma$ (kg/m$^3$)</th>
<th>$\alpha$ (1/°F)</th>
<th>K (N/mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slab</td>
<td>Concrete</td>
<td>45.507</td>
<td>4,482</td>
<td>31,900</td>
<td>0.15</td>
<td>2,400</td>
<td>11E-6</td>
<td>--------</td>
</tr>
<tr>
<td>Subbase</td>
<td>Crushed Stone</td>
<td>------</td>
<td>------</td>
<td>241</td>
<td>0.35</td>
<td>2,240</td>
<td>--------</td>
<td>1,750</td>
</tr>
<tr>
<td>Subgrade</td>
<td>Med Clay</td>
<td>------</td>
<td>------</td>
<td>69</td>
<td>0.4</td>
<td>1,760</td>
<td>--------</td>
<td>--------</td>
</tr>
<tr>
<td>Subbase</td>
<td>Crushed Stone</td>
<td>------</td>
<td>------</td>
<td>69</td>
<td>0.35</td>
<td>2,240</td>
<td>--------</td>
<td>875</td>
</tr>
<tr>
<td>Subgrade</td>
<td>Med-soft Clay</td>
<td>------</td>
<td>------</td>
<td>7</td>
<td>0.4</td>
<td>1,760</td>
<td>--------</td>
<td>--------</td>
</tr>
</tbody>
</table>

In the first model both linear and nonlinear springs were used. The main difference between the two is that the nonlinear springs are capable of modeling the separation of slab and subbase slab in case of tension while the linear springs are not. In the second model solid elements were used to model the subgrade. Each of the models used a different method to model the interaction between the subbase and slab. In the first model, nonlinear spring elements were adopted, and in the second model, contact and target elements were used. In all models adjacent concrete slabs were modeled separately, and did not share nodes. In the first model, the stiffness coefficients of the springs were determined based on the results reported by Porter et al. (2001). The effect of aggregate interlock was modeled using vertical nonlinear springs connecting the joint nodes with the same coordinates. A 1 mm joint opening was assumed based on an estimated average of 15°C change in slab temperature from the time of placement to the time when the stresses were developed in addition to an average shrinkage strain of 0.0001 mm/mm.

All four types of loading that have major effect on the stress distribution in pavements were considered in this study. The ICM program with the available climatic data from Illinois was used to compute the thermal gradient through the various depths of
the concrete slabs. With this data, the temperature gradient through the slab for a year was calculated. Based on the developed data the average temperature gradients and the standard deviation were evaluated through the depth of the pavement. Table 7-2 shows the results. Figure 7-3-a depicts the average value of the temperature within the slab for the different times of a day. Figure 7-4-b shows the average temperature and the average temperature ± one or two standard deviations at 2 am.

Table 7-2 The Average Temperature Gradient and the Standard Deviation

<table>
<thead>
<tr>
<th>Depth (mm)</th>
<th>Time 2:00</th>
<th>6:00</th>
<th>10:00</th>
<th>15:00</th>
<th>19:00</th>
<th>23:00</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Ave.</td>
<td>SD</td>
<td>Ave.</td>
<td>SD</td>
<td>Ave.</td>
<td>SD</td>
</tr>
<tr>
<td>0</td>
<td>-6.22</td>
<td>3.71</td>
<td>-5.07</td>
<td>3.49</td>
<td>5.81</td>
<td>6.51</td>
</tr>
<tr>
<td>44.45</td>
<td>-4.05</td>
<td>2.72</td>
<td>-4.46</td>
<td>2.66</td>
<td>1.89</td>
<td>4.13</td>
</tr>
<tr>
<td>88.9</td>
<td>-2.38</td>
<td>2.01</td>
<td>-3.32</td>
<td>2.06</td>
<td>-0.07</td>
<td>2.61</td>
</tr>
<tr>
<td>133.35</td>
<td>-1.20</td>
<td>1.51</td>
<td>-2.19</td>
<td>1.51</td>
<td>-0.78</td>
<td>1.67</td>
</tr>
<tr>
<td>177.8</td>
<td>0.46</td>
<td>1.14</td>
<td>-1.28</td>
<td>1.05</td>
<td>-0.81</td>
<td>1.09</td>
</tr>
<tr>
<td>222.25</td>
<td>-0.06</td>
<td>0.82</td>
<td>-0.64</td>
<td>0.71</td>
<td>-0.57</td>
<td>0.70</td>
</tr>
<tr>
<td>266.7</td>
<td>0.09</td>
<td>0.52</td>
<td>-0.23</td>
<td>0.43</td>
<td>-0.29</td>
<td>0.41</td>
</tr>
<tr>
<td>311.15</td>
<td>0.09</td>
<td>0.25</td>
<td>-0.03</td>
<td>0.21</td>
<td>-0.08</td>
<td>0.19</td>
</tr>
<tr>
<td>355.6</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>

Figure 7-3 Average Temperature Gradient, a) Different Times of a Day and b) At 2am
The AASHTO HS-20 truck wheel load was used as the traffic load. The single axle load 140kN (32kips) is carried by four wheels. The tire contact area was assumed to be 147 in\(^2\). The tire pressure of 109 psi was applied over one brick element. The traffic loads were applied at the edges of the slab for negative temperature gradients and in the middle of the slab for positive temperature gradients. As seen previously, these load combinations produce the highest stresses. Since the number of degrees of freedom has major effect in the developed stress under tire pressure, 20-node brick elements were used. Based on the results of the studies by Armaghani (1987), an equivalent thermal gradient of 0.022°C/mm (1°F/in) was applied to the concrete slab.

7.5 Model Verification

The results obtained using the two developed models were compared with the analytical results obtained by Westergaard (1926). It should be noted that in the analytical method a circular loading was applied and the boundaries were unlimited. In the FE models, however, the applied loading was rectangular and the boundaries were finite.

In the FE model, all springs in the liquid foundation model had the same spring constant. Therefore, a higher modulus of support reaction was assigned to the locations where the mesh was refined. This results in a reduction of stresses. On the other hand, tie bars transfer the loading to the other slabs, thus reducing the stresses in the FE model. The analytical equations by Westergaard (1926) for each type of loading are given by:

\[
\sigma = \frac{0.316P}{h^2} (4 \log \left( \frac{L}{b} \right) + 1.069) \quad \text{Equation 7-1}
\]

\[
\sigma = \frac{0.803P}{h^2} (4 \log \left( \frac{L}{a} \right) + 0.666 \left( \frac{a}{L} \right) - 0.034) \quad \text{Equation 7-2}
\]

where

\[
b = \sqrt{1.6a^2 + h^2} - 0.675h, \quad L = \left[ \frac{Eh^3}{12(1-\nu^2)k} \right]^{0.25} \quad \text{Equation 7-3}
\]

where \(L\) is the radius of relative stiffness, \(a\) is the radius of the circular loaded area, \(E\) is the modulus of elasticity, \(\nu\) is the Poisson ratio, and \(h\) is the thickness of the slab. The
load was applied in the two FE models in the 2 positions shown in Figure 7-5. The results are shown in Table 7-3, Table 7-4 for the liquid foundation model and the elastic solid model, respectively. Notice that Table 7-3 also shows the stresses obtained by Westergaard (1926). The load was always applied in the position 2, since the possibility of a passing vehicle over the edge is small. As the load gets closer to the edge of the slab the stresses increase.

Table 7-3 Comparison between Results from the Liquid Foundation Model and Analytical Model

<table>
<thead>
<tr>
<th></th>
<th>Load position</th>
<th>Stress (kPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Soil (1)*</td>
</tr>
<tr>
<td>Westergaard</td>
<td>Middle Tire</td>
<td>947</td>
</tr>
<tr>
<td></td>
<td>Edge Tire</td>
<td>1,993</td>
</tr>
<tr>
<td>Spring Model</td>
<td>Pos. 1 Edge</td>
<td>1,944</td>
</tr>
<tr>
<td></td>
<td>Pos. 2 Edge</td>
<td>1,648</td>
</tr>
<tr>
<td></td>
<td>Pos. 1 Middle</td>
<td>1,413</td>
</tr>
<tr>
<td></td>
<td>Pos. 2 Middle</td>
<td>1,351</td>
</tr>
</tbody>
</table>

* Modulus of support reaction is equal to 5,340 kN/m³, 10,680 kN/m³, 53,420 kN/m³ for soil (1), soil (2), soil (3) respectively
Table 7-4 Stresses Developed in the Elastic Solid Model

<table>
<thead>
<tr>
<th>Load position</th>
<th>Stress (kPa)</th>
<th>Soil (1)</th>
<th>Soil (2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pos. 1 Edge</td>
<td>1,178</td>
<td>1,634</td>
<td></td>
</tr>
<tr>
<td>Pos. 2 Edge</td>
<td>1,010</td>
<td>1,482</td>
<td></td>
</tr>
<tr>
<td>Pos. 1 Middle</td>
<td>778</td>
<td>1,130</td>
<td></td>
</tr>
<tr>
<td>Pos. 2 Middle</td>
<td>772</td>
<td>1,110</td>
<td></td>
</tr>
</tbody>
</table>

Soil(1): Subbase stiffness = 240 Mpa, Subgrade stiffness = 69 MPa
Soil(2): Subbase stiffness = 69 Mpa, Subgrade stiffness = 7 MPa

From Table 7-3 it can be seen that the stresses in the middle of the slab in the liquid foundation model are higher than the analytical model while the stresses in the edge are smaller. The latter is due to the higher modulus of support reaction on the refined edges and the effect of the tie bars. The stresses in the ES model are smaller than the stresses obtained by analytical model and the liquid foundation model. This is because the elastic soil can develop shear and consequently the load is distributed over a larger area, thus reducing stresses.

7.6 Cumulative Damage Evaluation

Concrete pavements undergo a variety of loadings during their service life. These include self-weight, thermal loads, wheel loads, and shrinkage. Each of these loadings varies differently with time. The change in shrinkage stresses is insignificant after a initial time. Thermal stresses vary with a frequency of one day and stresses due to passing wheels increase and decrease with a much higher frequency. The complicated geometry of slab components and their interaction, different type of loadings and various frequencies make an accurate evaluation of cumulative damage very difficult.

In the most common and simplest approach used in the design of a concrete pavement, a year is divided to 12 or 24 periods, and different loads (ex. different truck loads) are applied to the pavement. A foundation seasonal adjustment factor is defined for each period and the maximum tensile stress in the concrete pavement is calculated. With
the knowledge of the maximum tensile stress, the allowable number of load repetitions can be calculated based on Darter (1977), Packard and Tayabji (1985) or any other available damage equation. As mentioned previously, these equations have been developed using a third point repeated flexural loading on a beam, where the stress ratio (the ratio of stress to modulus of rupture) is plotted against number of repetitions to failure in log scale. The damage ratio, which is the ratio between the predicted and allowable number of repetitions, is computed for each load group in each period and summed over the year using Miner’s equation (Miner 1945).

The accuracy of this method can be improved. In this study, this was achieved in several different ways. First, the actual variation of the stresses for all load cases was used. In addition, the proposed fatigue equation (Section 6.1.4.3) was used. This equation considers the effect of rate of loading and that uses the actual stresses in the input, and, it is, thus, more accurate than the usually adopted fatigue equations. Finally, in this study, damage is accumulated nonlinearly. This improved method was applied to the case study to predict its service life.

7.6.1 Slab Deterioration

Various factors can cause the deterioration of a concrete slab. If unsuitable materials are used causing poured concrete to experience high shrinkage, then random cracking occurs. This cracking weakens the slab. In addition, if the soil is extremely weak or experiences pumping after construction, the loadings are not transferred properly and the significant local stresses occur which can lead to cracking and deterioration of the concrete slab. In the present study, only ordinary slabs (ordinary shrinkage) over ordinary soil (no pumping) were considered.
7.6.2 Evaluation of the Traffic

Traffic loading is one of the major factors affecting pavement performance. Thus, this load must be accurately considered in the design and when predicting the service life of a pavement.

For design purposes it is important to accurately predict the number of repetition of each axle loads during the pavement’s service life. Information about the traffic can be obtained from field measurements. The Average Annual Daily Traffic of Interstate I70 for the year 2000 in Indiana (2000) is summarized in the Table 7-5. This is the traffic that has been adjusted for daily (weekday versus weekend) and seasonal (summer versus winter) variations. An AADT equal to 53,490 can be observed for the specific given day and section considered in the case study. The number of repetitions per day for each type of axle can be determined as

\[(no) = (P)(AADT)(T)(A)\]  
Equation 7-4

where AADT is the adjusted annual daily traffic, T is the percentage of trucks in the AADT, A is the average number of axles per truck, and P is the percentage of the total number of repetitions of a certain load group.

<table>
<thead>
<tr>
<th>County</th>
<th>From:</th>
<th>To:</th>
<th>AADT</th>
</tr>
</thead>
<tbody>
<tr>
<td>Marion</td>
<td>Exit 90/Interstate 465</td>
<td>Exit 91/Post Road</td>
<td>75,920</td>
</tr>
<tr>
<td>Marion/Hancock</td>
<td>Exit 91/Post Road</td>
<td>Exit 96/Mount Comfort</td>
<td>53,490</td>
</tr>
<tr>
<td>Hancock</td>
<td>Exit 96/Mount Comfort</td>
<td>Exit 104/Indiana 9</td>
<td>37,460</td>
</tr>
</tbody>
</table>

The simplified axle load distributions for interstate highways are summarized in Table 7-6. Depending on the location along I-70, trucks account for about 21-32 percent of the traffic stream. The calculated number is in both directions over all traffic lanes. Thus, this number must be multiplied by the directional and lane distribution factors to
determine the actual number for the design lane. For each lane the number of load repetitions is given by:

\[ N = (\text{no})(D)(L) \]  

Equation 7-5

where \( D \) is the directional distribution factor, which is usually assumed to be 0.5 unless the traffic in two directions is different, \( L \) is the lane distribution factor, which varies with the volume of traffic and the number of lanes.

Table 7-6 Axle Load Distribution for Interstate Highways

<table>
<thead>
<tr>
<th>Axle Load (kips)</th>
<th>Axles per 1000 trucks</th>
<th>Axles per day in a lane</th>
<th>Axle Load (kips)</th>
<th>Axles per 1000 trucks</th>
<th>Axles per day in a lane</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Single axle</td>
<td>Tandem axle</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>16</td>
<td>57.07</td>
<td>281.4</td>
<td>24</td>
<td>71.16</td>
<td>350.8</td>
</tr>
<tr>
<td>18</td>
<td>68.27</td>
<td>336.6</td>
<td>28</td>
<td>95.79</td>
<td>472.2</td>
</tr>
<tr>
<td>20</td>
<td>41.82</td>
<td>206.2</td>
<td>32</td>
<td>109.54</td>
<td>540.0</td>
</tr>
<tr>
<td>22</td>
<td>9.69</td>
<td>47.8</td>
<td>36</td>
<td>78.19</td>
<td>385.5</td>
</tr>
<tr>
<td>24</td>
<td>4.16</td>
<td>20.5</td>
<td>40</td>
<td>20.31</td>
<td>100.1</td>
</tr>
<tr>
<td>26</td>
<td>3.52</td>
<td>17.4</td>
<td>44</td>
<td>3.52</td>
<td>17.4</td>
</tr>
<tr>
<td>28</td>
<td>1.78</td>
<td>8.8</td>
<td>48</td>
<td>3.03</td>
<td>14.9</td>
</tr>
<tr>
<td>30</td>
<td>0.63</td>
<td>3.1</td>
<td>52</td>
<td>1.79</td>
<td>8.8</td>
</tr>
<tr>
<td>32</td>
<td>0.54</td>
<td>2.7</td>
<td>56</td>
<td>1.07</td>
<td>5.3</td>
</tr>
<tr>
<td>34</td>
<td>0.19</td>
<td>0.9</td>
<td>60</td>
<td>0.57</td>
<td>2.8</td>
</tr>
</tbody>
</table>

To calculate the service life of a concrete pavement, the traffic growth over the pavement must be considered. This can be done by assuming a yearly rate of traffic growth. According to Darter et al. (1985) for four-lane highways with two lanes in each direction, the lane distribution factors range from 66% to 94% depending on the ADT. Table 7-7 summarizes the lane distribution factors range for different ADT values. In the present study, the AADT in one direction is \( 53,490/2 = 26,745 \). Thus, the appropriate value of \( L \) is 73%.
Table 7-7 The Directional Distribution Factor

<table>
<thead>
<tr>
<th>ADT</th>
<th>One-way</th>
<th>Two lanes in each direction</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Inner</td>
<td>Outer</td>
<td></td>
</tr>
<tr>
<td>2,000</td>
<td>6</td>
<td>94</td>
<td></td>
</tr>
<tr>
<td>4,000</td>
<td>12</td>
<td>88</td>
<td></td>
</tr>
<tr>
<td>6,000</td>
<td>15</td>
<td>85</td>
<td></td>
</tr>
<tr>
<td>8,000</td>
<td>18</td>
<td>82</td>
<td></td>
</tr>
<tr>
<td>10,000</td>
<td>19</td>
<td>81</td>
<td></td>
</tr>
<tr>
<td>15,000</td>
<td>23</td>
<td>77</td>
<td></td>
</tr>
<tr>
<td>20,000</td>
<td>25</td>
<td>75</td>
<td></td>
</tr>
<tr>
<td>25,000</td>
<td>27</td>
<td>73</td>
<td></td>
</tr>
<tr>
<td>30,000</td>
<td>28</td>
<td>72</td>
<td></td>
</tr>
<tr>
<td>35,000</td>
<td>30</td>
<td>70</td>
<td></td>
</tr>
<tr>
<td>40,000</td>
<td>31</td>
<td>69</td>
<td></td>
</tr>
<tr>
<td>50,000</td>
<td>33</td>
<td>67</td>
<td></td>
</tr>
<tr>
<td>60,000</td>
<td>34</td>
<td>66</td>
<td></td>
</tr>
</tbody>
</table>

7.6.3 Evaluation of the Stresses

To calculate the service life of a concrete pavement, damage is accumulated over a period of time. The increase in damage can be determined if the stress history of the pavement is known. To obtain the stress history of the pavement in the case study, the developed FE model was analyzed under appropriate loading conditions. The maximum and minimum stresses were obtained and used as input data for the damage analysis.

Concrete pavements undergo different loading conditions during their service life. Except for the self-weight, which is constant with time, other loadings such as thermal, traffic, and shrinkage loading vary with time. Since temperature gradients vary continuously in a concrete pavement, considering different combinations of thermal and traffic loadings requires an unlimited number of situations, which is clearly not feasible. As a result, a certain number of loading combinations were considered. The results from the analyses of the pavement under these loading conditions were used to extrapolate the amount of damage in other situations. This problem is a highly nonlinear and the stresses depend on the specific loading combinations. Thus, these results must be used with
caution. The larger the number of situations that are analyzed, the more accurate the result is. For the present case study, the following load conditions were considered:

a) For the negative thermal loading the temperature gradient at 2:00 am was chosen (Figure 7-5-b) since the maximum negative temperature gradient occurred at this time (Table 7-2). The stresses were calculated for three different types of soil and for a combination of edge loading and different temperature gradient in 2:00 am (average temperature, average temperature minus one standard deviation, and average temperature minus two standard deviations). The effect of shrinkage was calculated separately by adding an equivalent linear temperature gradient to each of the loadings. In a separate case, a gradient was applied alone in order to obtain the minimum stress.

b) For the positive thermal loading the temperature gradient at 3:00 pm was selected (Figure 7-5-a), since the maximum positive temperature gradient occurred at this time (Table 7-2). The effect of shrinkage was not included, since it counteracts the effects due to positive temperature gradient. In separate cases, the effect of temperature gradient alone and traffic loading were evaluated.

c) For a given thermal gradient, as the traffic loading moves, the slab undergoes different stresses. To study the effect of the loading location over the slab, the tire loading was placed over the edge, middle and quarter position on the slab and stresses under
various loading combination were evaluated. Figure 7-7 and Table 7-8 show the stress developed along the middle of the slab in the direction of the traffic under combination of positive temperature gradient and wheel loading.

Figure 7-6 Stresses Developed under Positive Temperature Gradient and Traffic Loading Applied at Different Loading Positions for Soil Stiffness of a) $K=1750$ N/mm, b) $K=8750$ N/mm, and c) $K=875$ N/mm

Table 7-8 The Maximum and Minimum Developed Stresses in Various Cases

<table>
<thead>
<tr>
<th></th>
<th>Negative + shrinkage</th>
<th>Positive</th>
<th>Pos. Temp. Gradient + Tridem axle</th>
<th>Tridem axle</th>
</tr>
</thead>
<tbody>
<tr>
<td>Soil 1</td>
<td>max: 2499 min: 1418</td>
<td>max: 2110 min: 232</td>
<td>min: 323 Max: 217 min: 1290 max: 2570</td>
<td>min: 0 max: 1 min: 1 max: 2036</td>
</tr>
<tr>
<td>Soil 2</td>
<td>max: 2749 min: 1744</td>
<td>max: 2451 min: 413</td>
<td>min: 413 Max: 2425 min: 1600 max: 2894</td>
<td>min: 0 max: 1 min: 1 max: 1564</td>
</tr>
<tr>
<td>Soil 3</td>
<td>max: 3197 min: 2388</td>
<td>max: 2840 min: 547</td>
<td>min: 547 Max: 2858 min: 2062 max: 3295</td>
<td>min: 0 max: 1 min: 1 max: 653</td>
</tr>
</tbody>
</table>
As can be seen as the traffic load passes by, the stresses change from the maximum of about 2.5 MPa when the loading is applied in the middle of the slab to zero when the loading was applied at the edge of the slab. This indicates that applying fatigue equations that are developed based on the zero minimum stresses is appropriate in this case. However, Figure 7-7 and Table 7-8 indicate that the stress developed along the middle of the slab in the perpendicular direction of the traffic under combination of negative temperature gradient and wheel loading. As the traffic load passes by, the stresses change from a maximum value of about 2.8 MPa in case of edge loading to 1.2 MPa in case of middle loading. In other words, although the traffic load tends to counteract the thermal stresses, it can not completely neutralize it. This indicates that in this case, only fatigue equations that consider the effect of range of loading should be used. In addition, as can
be seen from Table 7-8, the stresses tended to increase for stiffer soils. This is because the thermal stresses were dominant.

d) The stresses due to traffic loading depend on the intensity of the load. The changes in stress are more critical for the combination of positive temperature and traffic loading than for the combination of negative and traffic loading. This change is not linear since the slab and the subbase are partially in contact and the loading may change the contact area. The model was analyzed for the 32kips, 16 kips, and zero loading and the stresses were calculated. The results are shown in Figure 7-8. As it can be seen for the case of positive temperature gradient the stress drops from 2.2 MPa to zero as the traffic load changes from 71 kN to zero. On the other hand the maximum stress decreases from 2.8 MPa to 2.4 MPa as the loading changes. This indicates that the stresses in the positive combined case are mainly due to traffic loading while the stresses in negative combined case are mainly due to temperature loading.

![Figure 7-8 Stresses Developed under Tire Loads of Varying Intensity, a) Positive Temperature Gradient and b) Negative Temperature Gradient](image)

e) The ways in which the stresses change under tandem and tridem loading are different than for a single axle loading. To consider this in the stress history a combination of positive temperature gradient and tandem and tridem load was applied on the slab and the stresses were evaluated. The results for the tridem loadings are shown in Figure 7-9 and Table 7-8. When positive temperature gradient was present, the behavior
of all analyzed cases is similar behavior irrespective of soil stiffness. The stresses tend to increase until the first load reaches the middle of the slab. The stress then decreases by about 20 percent and once again increases to its maximum value when the second load reaches the middle of the slab. After that, the stress drops about 30 percent and reaches the maximum as the third load is in the middle of the slab. In this case, considering the tridem loading as three separate loads is not appropriate. In cases where the temperature is not present, the minimum stresses are lower and considering three different loads is more appropriate. However, the stresses in this case are lower than the combined situation.

Figure 7-9 Stresses Developed under Triple Axle Loads, a) Positive Temperature Gradient b) Traffic Load

7.6.4 Stress Development

The stresses developed under the loading combinations discussed in the previous sections were extrapolated, so that the stresses due to other loads could be predicted. As discussed in the previous section, the number of repetitions for each axle per day is known. The whole day was divided into six time periods with the medium at 2am, 6am, 10am, 3pm, 7pm, and 11pm. It was assumed that the traffic loading was constant during a 24 hour period and that the temperature was constant during each time period. Therefore, the number of load repetitions for a time period was calculated by multiplying the total
number of load repetitions by the ratio of the time period over the 24 hours as shown in Table 7-9.

Table 7-9 The Associated Time and the Percent of Occurrence for each Specified Time

<table>
<thead>
<tr>
<th>Time</th>
<th>Associated time</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>2:00 AM</td>
<td>3.5</td>
<td>14.6</td>
</tr>
<tr>
<td>6:00 AM</td>
<td>4</td>
<td>16.7</td>
</tr>
<tr>
<td>10:00 AM</td>
<td>4.5</td>
<td>18.8</td>
</tr>
<tr>
<td>3:00 PM</td>
<td>4.5</td>
<td>18.8</td>
</tr>
<tr>
<td>7:00 PM</td>
<td>4</td>
<td>16.7</td>
</tr>
<tr>
<td>11:00 PM</td>
<td>3.5</td>
<td>14.6</td>
</tr>
</tbody>
</table>

For each of the six times, temperature gradients were calculated for the whole year using the ICM program. The collected results were summarized and the average and standard deviation were calculated. Knowing the average and standard deviation, the probability of the temperature being within a certain limit was calculated and the results are summarized in Table 7-10. By multiplying this probability to the number of load repetitions in each time period, the number of load repetitions in a certain time period and within a specific temperature range is evaluated. It must be noticed that nonlinear variation of the temperature can change the stresses, but since it is not possible to consider all temperature gradients, the average temperature gradient and average temperature gradient plus and minus standard deviations were considered. The associated maximum and minimum stresses for this loading were calculated. If the maximum stress was less than zero it was set to zero, since concrete cracking can only occur under tensile stresses.

Stress sequence is an important factor in nonlinear damage accumulation for a pavement. Consequently, in addition to the amount of stresses the sequence of the stresses must be determined. In this study, it was assumed that the temperature increases and then decreases in a year. Thus, the minimum temperatures were used in the beginning and as the days passed by, a higher temperature for each time period was used. For each temperature, different random traffic loadings were applied.
Table 7-10 The Probability of Certain Temperature Gradient

<table>
<thead>
<tr>
<th>Probability %</th>
<th>2:00 am</th>
<th>6:00 am</th>
<th>10:00 am</th>
<th>3:00 pm</th>
<th>7:00 pm</th>
<th>11:00 pm</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.28</td>
<td>T &gt; 1.2</td>
<td>&gt;1.9</td>
<td>&gt;18.8</td>
<td>&gt;23.5</td>
<td>&gt;9.1</td>
<td>&gt;2.1</td>
</tr>
<tr>
<td>13.59</td>
<td>-2.5&lt;T&lt;1.2</td>
<td>-1.6&lt;T&lt;1.9</td>
<td>12.3&lt;T&lt;18.8</td>
<td>16.5&lt;T&lt;23.5</td>
<td>4.8&lt;T&lt;9.1</td>
<td>-1.3&lt;T&lt;2.1</td>
</tr>
<tr>
<td>34.13</td>
<td>-6.2&lt;T&lt;-2.5</td>
<td>-5.1&lt;T&lt;-1.6</td>
<td>5.8&lt;T&lt;12.3</td>
<td>9.5&lt;T&lt;16.5</td>
<td>0.6&lt;T&lt;4.8</td>
<td>-4.7&lt;T&lt;-1.3</td>
</tr>
<tr>
<td>34.13</td>
<td>-9.9&lt;T&lt;-6.2</td>
<td>-8.6&lt;T&lt;-5.1</td>
<td>-0.7&lt;T&lt;5.8</td>
<td>2.6&lt;T&lt;9.5</td>
<td>-3.7&lt;T&lt;0.6</td>
<td>-8.1&lt;T&lt;-4.7</td>
</tr>
<tr>
<td>13.59</td>
<td>-13.6&lt;T&lt;-9.9</td>
<td>-12.0&lt;T&lt;-8.6</td>
<td>-7.2&lt;T&lt;-0.7</td>
<td>-4.4&lt;T&lt;2.6</td>
<td>-8.0&lt;T&lt;-3.7</td>
<td>-11.5&lt;T&lt;-8.1</td>
</tr>
<tr>
<td>2.28</td>
<td>T&lt;-13.6</td>
<td>T&lt;-12.1</td>
<td>T&lt;-7.2</td>
<td>T&lt;-4.4</td>
<td>T&lt;-8.0</td>
<td>T&lt;11.5</td>
</tr>
</tbody>
</table>

7.6.5 Cumulative Damage Program

A Visual Basic program was developed to perform the calculations associated with damage accumulation. Both linear and nonlinear procedures were implemented. Several fatigue equations were implemented in the developed program. This enables the evaluation and comparison of the results obtained from the different methods.

7.6.5.1 Program Application

When the program is run, the first window appears on the screen. This window allows the user to enter the Input and Output file names and specify their locations (Figure 7-11-a). When the “Run” button is selected, the program is executed. Clicking on the “End” button ends the program. Selecting the “Fatigue Equation” button opens another window (Figure 7-11-b). In this window the modulus of rupture and the strength of concrete can be defined. The predefined values for the modulus of rupture and concrete strength are 650 psi and 4,600 psi, respectively.
Darter (1977) proposed the following equation, which is based on the fatigue data for plain concrete beams from three studies:
\[ \log_{10} N = \tilde{a} - \tilde{b} \left( S \right) \]  

Equation 7-6

If \( \tilde{a} \) and \( \tilde{b} \) are both 17.61 then the cracking probability \( (P) \) is 50% and if \( \tilde{a} \) is equal to 16.71 and \( \tilde{b} \) is equal to 17.61 then \( P \) is equal to 24%, which means that for the certain level of stress \( S \), 24% of the slabs would fail after \( N \) number of load repetitions. The coefficients \( \tilde{a} \) and \( \tilde{b} \) can be specified, however, their predefined values are both 17.61. This fatigue equation also assumes that the minimum stress is equal to zero.

Murdock and Kesler (1958) have developed different equation for different rate of loading. The failure probability in this method is 50%. The program selects the appropriate equation based on the stress ratio \( (r) \).

\[
\begin{align*}
r &\approx 0.15 \Rightarrow \log_{10} N = 20.11 - 21.28 \left( S \right) \\
r &= 0.25 \Rightarrow \log_{10} N = 21.00 - 22.22 \left( S \right) \\
r &= 0.50 \Rightarrow \log_{10} N = 30.64 - 32.25 \left( S \right) \\
r &= 0.75 \Rightarrow \log_{10} N = 60.00 - 62.50 \left( S \right)
\end{align*}
\]

Equation 7-7

The new AASHTO method has been developed based on field data. The fatigue equation adopted by AASHTO assumes that the minimum stress is equal to zero. In this equation the crack probability, \( P \), can be specified. The predefined \( P \) is 50 percent. A lower \( P \) results in a lower number of load repetitions to failure. The ASSHTO equation is as follows:

\[
\log N = (-S^{5.367} \log (1-P)/0.0032)^{0.2276}
\]

Equation 7-8

The method by Shi et al. (1993) considers the effect of both cracking probability and stress ratio in the calculation of the number of load repetitions to failure. In this method \( P \) can be selected between given numbers (Figure 7-11-b). In this program, \( P \) was predefined as 50%. A lower \( P \) results in lower number of loads to failure. Their fatigue equation is given by:

\[
N = \left( \frac{S}{A} \right)^{-23.6967 / (1-R)}
\]

Equation 7-9

where \( A \) is a coefficient which varies with the cracking probability \( (P) \).

This equation was improved to consider the effect of load frequency, which was achieved by introducing the coefficient \( c_f \) in the equation. It is referred in the software as
“New Shi”. The user can select values for the coefficients $\bar{a}$, $\bar{b}$, $\bar{c}$, and load frequency ($f$). The predefined amount are $\bar{a} = 0.249$, $\bar{b} = 0.920$, $\bar{c} = 0.796$ and $f = 1$ Hz. If the frequency is set to 20 the New Shi method becomes the original Shi (1993) method.

$$ S = 0.9315 c_f A N^{0.0422(1-R)} , c_f = (\bar{a} \bar{b}^{\log f} + \bar{c}) $$

Equation 7-10

The method by Zhang et al (1996) considers the effect of stress ratio and frequency, but it sets a damage probability of 50%. In this method a material property referred to as $\beta$ can be specified. Its predefined value in the software is 0.0807. The frequency coefficient $c_f$ is discussed in the New Shi method.

$$ \log N = \frac{1}{(1-r)\beta} (1-\frac{S}{c_f}) $$

Equation 7-11

The method by Cornelissen and Reinhardt (1984) considers the effect of stress ratio. Two different equations are used. The first equation is for the case that the minimum stress is tension (tension-tension loading) and the second equation is for the cases that the minimum stress is compression (tensile-compression loading). The user can specify the values for the coefficients $\bar{a}$, $\bar{b}$, and $\bar{c}$ in these equations. The predefined coefficients for tension-tension stresses were selected as $\bar{a}=14.81$, $\bar{b}=14.52$, $\bar{c}=2.79$ and for tension-compression stresses as $\bar{a}=14.81$, $\bar{b}=14.52$, $\bar{c}=2.79$. Their fatigue equation is given by:

$$ \log N = \bar{a} - \bar{b} \frac{\sigma_{\text{max}}}{f_{ctm}} + \bar{c} \frac{\sigma_{\text{min}}}{f_{ctm}} $$

Equation 7-12

7.6.5.2 Input and Output

The input file for the developed program must be a text file. Each line of the input file must contain three numbers. The first number is the number of load repetitions for a specified stress level, and the second and third numbers are the maximum and minimum stress, respectively. The results are printed in an output text file. The name of the fatigue method is automatically added to the end of the output file name. The output file is a text file with ten columns. Following is a description of each these columns:
1) The number of load repetitions.
2) The maximum stress.
3) The ratio of maximum stress to modulus of rupture.
4) The minimum stress.
5) The ratio of the minimum stress to the modulus of rupture for tensile minimum stress and ratio of the minimum stress to the concrete strength for compressive minimum stress.
6) The number of load repetitions to failure for the specific fatigue equation based on the maximum and minimum stresses. For each maximum and minimum stress, a damage path can be developed.
7) The initial $n/N$ (number of repeating loads to number of repeating loads to failure) for a specified damage and a certain path.
8) The increase in the ratio $n/N$ at this level.
9) The amount of nonlinear damage after this level.
10) The amount of linear damage after this level.

7.6.5.3 Program Theory

Initially the program assumes the time and damage to be zero. As the time passes and the loadings are applied to the slab, the program considers a certain amount of damage for the pavement. The program reads the new number of load repetitions, the maximum and the minimum stresses for this number of repetitions from the input file. Knowing the minimum and maximum stress and using Molinas (1992) nonlinear damage equations, the ratio $n/N$ is calculated. This ratio is for certain values of minimum and maximum stresses and, thus, it differs for other stress values. Using the selected fatigue equation, the number of loads to failure can then be determined for the values of minimum and maximum stresses.

With the knowledge of the number of applied loads at this level, the initial $n/N$, and number of loads to failure, a new $n/N$ ratio can be determined. Finally, using Molinas (1992) nonlinear damage equation the new level of damage can be calculated. The
program repeats this process until the data has ended or the nonlinear damage is equal to one. The program also linearly adds the ratios $n/N$ and calculates the linear cumulative damage for comparison purposes. The maximum permissible number of load repetitions is set to $10^{12}$ to avoid numerical errors. In the methods by Zhang (1996) and Shi (1993), if the positive stress is small and the negative stress is large, the fatigue equations predict very small numbers of load repetition, which is not acceptable. To prevent this, the number of load repetitions for the maximum stresses lower than 0.4 of the modulus of rupture was considered to be unlimited (i.e. $10^{12}$).

7.6.6 Stresses

Two FE model were developed. In the first, the soil was simulated using nonlinear springs, which were able to model separation. In the second, solid elements were used to model the soil and contact elements were adopted to model separation and friction between slab and soil. The developed stresses under various loading conditions are summarized in Table 7-11 and Table 7-12 for the first and second model, respectively. The maximum developed stresses in the case of combined negative temperature gradient plus shrinkage and positive temperature gradient were 2,810 kPa (407 psi) and 2,310 kPa (335 psi), respectively. The maximum developed stress in the case of wheel loading over the weak soil was 1,648 kPa (239 psi). Therefore, premature cracking was not expected to occur when temperature and wheel loading act separately (if the soil is extremely weak or experiences pumping or if the concrete is under significant tension due to shrinkage there would be a possibility of cracking under either temperature or tire loading acting alone).

A stiffer subbase/subgrade can greatly reduce the stresses developed by wheel loading. However, when thermal gradient are present, the contact area between subbase and slab decreases when the subbase/subgrade stiffness increases, thus, increasing the stresses. In the positive combined case, when thermal gradient is negligible, tire loading was the major contributor to the stress. Therefore, in this case, a slab over the stiffer soil developed lower stresses. As the thermal gradient increased the thermal stresses became
more significant and the contact area reduced. Thus, higher stresses developed in the slab over stiffer subbase/subgrade. Stresses due to wheel loading did not exceed half of the MR, and therefore they could not have caused fatigue cracking. Combined high positive thermal loading and wheel loading developed stresses high enough to cause fatigue cracking. A stiffer subbase, in this case, produced even higher stresses. The thermal gradient was the main contributor to the stresses produced under the negative combined case, while the tire loading created a minor portion of the stresses. Therefore, in this case a slab over the stiffer soil created higher stresses.

Table 7-11 Stresses of the Slab under Various Loading (Liquid Foundation Model)

<table>
<thead>
<tr>
<th>Spring</th>
<th>Loading</th>
<th>Temperature</th>
<th>Stress (kPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Soil (1)</td>
</tr>
<tr>
<td>Nonlinear</td>
<td>Neg. Temp. (2:00)</td>
<td>Edge wheel</td>
<td>Average 1254</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Ave.-SD 1763</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Ave.-2SD 2244</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(Ave.)+Shr. 1751</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(Ave-SD)+Shr. 2175</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(Ave-2SD)+Shr. 2570</td>
</tr>
<tr>
<td>Linear</td>
<td>Neg. Temp. (2:00)</td>
<td>(Ave-2SD)+Shr.</td>
<td>-------</td>
</tr>
<tr>
<td></td>
<td>Neg. Temp. (2:00) + Edge wheel</td>
<td>(Ave-2SD)+Shr.</td>
<td>2981</td>
</tr>
<tr>
<td>Nonlinear</td>
<td>Pos. Temp. (15:00)</td>
<td>Ave.-SD 1583</td>
<td>1424</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Average 1811</td>
<td>1868</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Ave.+SD 1994</td>
<td>2257</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Ave.+2SD 2109</td>
<td>2451</td>
</tr>
<tr>
<td>Pos. Temp. (15:00)</td>
<td>Ave.+2SD</td>
<td>-------</td>
<td>-------</td>
</tr>
<tr>
<td>Middle wheel</td>
<td>No temperature</td>
<td>1648</td>
<td>-------</td>
</tr>
<tr>
<td>Linear</td>
<td>Pos. Temp. (15:00) + Middle wheel</td>
<td>Ave.+2SD 2200</td>
<td>2789</td>
</tr>
</tbody>
</table>

* K = 875 N/mm, ** K = 1750 N/mm, *** K = 8750 N/mm

The addition of a shrinkage equivalent thermal gradient tended to increase the stresses. The stresses developed by negative thermal gradient are sensitive to the
variation of the temperature with the depth of the slab, i.e., the nonlinear part of the thermal gradient can have a significant effect on the results. This effect can be observed by comparing the results of two cases, a) the case with average negative temperature plus shrinkage and b) the case with the negative temperature plus standard division. Although the total temperature difference is higher in the first case but the stresses are lower. This is because the temperature gradient in the first case is mainly linear. Consequently, it can be concluded that nonlinear temperature loading, increased the stresses in negative case.

Table 7-12 Stresses of the Slab under Various Loading (Elastic Soil Model)

<table>
<thead>
<tr>
<th>Loading</th>
<th>Temperature</th>
<th>Stress (kPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Soil (1) Wheel (1)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Soil (2) Wheel (1)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Soil (1) Wheel (2)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Soil (2) Wheel (2)</td>
</tr>
<tr>
<td>Neg. Temp. (2:00)</td>
<td>Average</td>
<td>1656</td>
</tr>
<tr>
<td>Edge wheel</td>
<td>Ave.-SD</td>
<td>2220</td>
</tr>
<tr>
<td></td>
<td>Ave.-2SD</td>
<td>2729</td>
</tr>
<tr>
<td></td>
<td>(Ave.-2SD)+Shr.</td>
<td>3102</td>
</tr>
<tr>
<td>Neg. Temp. (2:00)</td>
<td>(Ave.-2SD)+Shr.</td>
<td>2723</td>
</tr>
<tr>
<td>Pos. Temp. (15:00)</td>
<td>Average</td>
<td>1473</td>
</tr>
<tr>
<td>+ Middle wheel</td>
<td></td>
<td>1174</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1416</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1083</td>
</tr>
<tr>
<td>Pos. Temp. (15:00)</td>
<td>Average</td>
<td>462</td>
</tr>
<tr>
<td>Middle wheel</td>
<td>No temp</td>
<td>1012</td>
</tr>
</tbody>
</table>

Soil (1): $E_{\text{subbase}} = 241$ MPa, $E_{\text{subgrade}} = 69$ Mpa,
Soil (2): $E_{\text{subbase}} = 69$ MPa, $E_{\text{subgrade}} = 7$ Mpa
Wheel (1): Loading close to large shoulder
Wheel (2): Loading close to small shoulder

If the effect of shrinkage is small, only in cases where a very stiff soil is present and thermal gradient is high for a short period of time, the developed stresses were more than half of the MR. As the effect of shrinkage increased, the stresses remained larger than half of the MR for a larger period of time in the year and this might have lead to top to bottom premature cracking.
It should be noticed that all conclusions related to top to bottom cracking could only be reached because a three dimensional model was adopted, since the nonlinear portion of temperature loading has the greatest effect in the developed stresses. Comparing the linear and nonlinear spring models shows that since linear springs cannot develop separation from the slab, they develop higher stresses (Table 7-11). Although the stresses are slightly lower in the model with the ES subgrade, similar results are expected.

7.6.7 Cumulative Damage

Unless the soil under the slabs was very soft or significant pumping had occurred, the edge stresses developed by the traffic loading over a 14 in slab are not high enough to have caused pavement cracking. Therefore, it can be ascertained that the main reason for the observed cracking was due to the combination of the temperature and traffic loading. As opposed to traffic loading that increases for softer subbases, temperature stresses increase for stiffer subbases.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>P %</td>
<td>Time (day)</td>
<td>Non. Damage</td>
<td>Lin. Damage</td>
</tr>
<tr>
<td>AASHTO</td>
<td>5%</td>
<td>29</td>
<td>1.000</td>
<td>0.869</td>
</tr>
<tr>
<td>AASHTO*</td>
<td>50%</td>
<td>2158</td>
<td>1.000</td>
<td>0.585</td>
</tr>
<tr>
<td>Darter</td>
<td>24%</td>
<td>3650</td>
<td>0.209</td>
<td>0.122</td>
</tr>
<tr>
<td>Darter*</td>
<td>50%</td>
<td>3650</td>
<td>0.050</td>
<td>0.012</td>
</tr>
<tr>
<td>Cornelissen*</td>
<td>50%</td>
<td>3650</td>
<td>0.033</td>
<td>0.010</td>
</tr>
<tr>
<td>Murdock*</td>
<td>50%</td>
<td>3650</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>NewShi*</td>
<td>50%</td>
<td>3650</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>PCA*</td>
<td>5%</td>
<td>638</td>
<td>1.000</td>
<td>0.629</td>
</tr>
<tr>
<td>Shi*</td>
<td>50%</td>
<td>3650</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>Shi</td>
<td>5%</td>
<td>3650</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>Zhang* (f=20)</td>
<td>50%</td>
<td>3650</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>Zhang* (f=1)</td>
<td>50%</td>
<td>3650</td>
<td>0.000</td>
<td>0.000</td>
</tr>
</tbody>
</table>

In this study, three random selections of traffic loads were used. The loading was applied considering the worst case scenario, i.e., the case in which the subbase is stiff and...
the stresses are the maximum. Different fatigue equations were applied and both nonlinear and linear damage accumulations were calculated and compared. The results are provided in Table 7-13, Figure 7-11, and Figure 7-13. It should be noted that in Figure 7-11 the 45° line shown indicates the case for which linear and nonlinear damage accumulation produce the same results.

Figure 7-11 Nonlinear Versus Linear Damage Accumulation, a) AASHTO Positive Temperature Gradient, b) AASHTO Negative Temperature Gradient, c) PCA (1984)
Comparing the results for nonlinear and linear damage accumulation shown in Figure 7-11 and Figure 7-13, respectively, one can observe that the initial damage for the nonlinear case is higher than that for the linear case. After a number of load repetitions the rate of increase for the nonlinear damage accumulation decreases, while the rate of increase for linear damage accumulation remains constant. That is, the linear damage accumulation increases faster up to a point and becomes higher than the nonlinear damage. In the final period of load application, the nonlinear damage rate increases sharply and the final nonlinear damage may or not be higher than linear damage. For the cases with lower probability of cracking the linear fatigue accumulation provided more conservative results.
Comparing the damage against time (Figure 7-13), it can be seen that the initial nonlinear accumulation of damage for all three different loading cases are very close to each other. However, the final damage accumulation occurs at different times. As expected the diagrams show that the time that the damage associated with 5% probability of cracking, occurs much sooner than the damage with 50% probability of cracking. The rate of increase in damage for the lower probability of cracking is higher.

It should be noticed that the horizontal line on the damage-time diagrams indicates the period of the year in which the thermal stresses counteracted the traffic stresses and the resulting stresses were not sufficient to cause any damage. This shows that a pavement’s service life depends on the time of construction.

The results from the program showed that for the positive temperature case only AASHTO and PCA (1984) predicted the cracking. A comparison between the different fatigue methods showed that if the number of load repetitions is more than a certain amount for the same number of load repetitions these equations required a significantly lower stress. The PCA (1984) method predicted a period of about two years for failure while AASHTO predicted almost immediate failure for 5% failure probability and six years for 50% failure probability. The Darter method with 24% probability showed a fatigue accumulation of 0.2 after ten years. It should be mentioned that these results were for a very stiff subbase. A softer subbase would increase the pavement’s service life.

In case of negative thermal gradient the developed stresses were relatively high. In this case, traffic loading did completely counteract the temperature loading. Therefore, the minimum stress was not zero. Consequently, PCA (1984), DARTER (1977), and AASHTO methods, which do not consider the effect of minimum stresses in their calculation, predicted a rapid failure (less than a year). On the other hand, other methods including Shi (1993), Zhang (1996), Cornelisson and Murdock (1984), which consider the effect of minimum stress, predicted a longer life (more than eight years for 50% probability of cracking) for the slabs.

The current design methods of concrete pavement do not consider the effect of various factors including nonlinear temperature gradient through the depth and nonlinear damage accumulation. This was justified in the previous decades, since including these
factors in the calculation of damage requires sophisticated software tools, which demand a great amount of computer memory. As access to new computers and software have become more widespread, a more comprehensive design procedure that considers various factors including environmental effects, shrinkage, and nonlinear damage accumulation should be developed.

Using the 3D developed FE model with the nonlinear damage accumulation provided a better understanding of the main reasons behind crack development and the rate of crack development. However, the currently available fatigue equations have been calibrated with the old assumptions. Thus, using these equations with the new methods, which considers the effect of temperature and minimum stresses and nonlinear damage, may not be entirely appropriate. A more accurate pavement service life predictor can be achieved by developing and calibrating new fatigue equations and using them in conjunction with the more sophisticated modeling techniques discussed throughout this report.
CHAPTER 8 CONCLUSIONS

8.1 Introduction

This chapter provides concluding remarks and recommendations resulting from the present investigation. The conclusions have been organized into sections related to the different phases of the study. The final section of this chapter is devoted to providing some recommendations concerning improved pavement design and pavement service life prediction.

8.2 Finite Element Model

• Decreasing the mesh size near the joints reduced the effect of shape function incompatibility between the dowel bars and slabs. Refining the mesh by four times, near the joints improved the performance of the finite element model.

• With respect of subgrade modeling, a mesh that used infinite elements converged faster than the one that used ordinary elements. However, in the latter model, when enough depth for the subgrade was modeled, the error remained bounded. It was also found that for the same level of accuracy a significant larger depth of subgrade is needed to accurately calculate deflections than to compute stresses.
• The comparison between two commercial finite element software programs revealed that thermal gradients were handled differently in each of them. For the same level of accuracy, ANSYS uses elements with a lower number of degrees of freedom than ABAQUS. For eight-node brick elements, ABAQUS assumes constant stress throughout the element while ANSYS assumes linear stress distribution. Thus, applying nonlinear temperature gradient in the middle nodes of ABAQUS may lead to inaccurate data because of the theoretical incompatibility between the shape functions and the resulting thermal stresses.

• Both mesh size and element type had a significant effect on stresses due to traffic loading. Refining the mesh and applying higher order elements increased the stresses. Thus, a sufficiently refined mesh must be used to avoid unconservative results. The footprint shape, on the other hand, plays a less important role in the resulting stresses in concrete pavements.

8.3 Parametric Studies

• The nonlinearity in the temperature distribution increases the stresses in the negative temperature gradient case, but it has the opposite effect in positive temperature gradient case.

• For all loading conditions the deflections increased for softer soils. Large deflections may lead to crushing of concrete around the dowel bars and consequently poor performance of the joints. A thick subbase can help reduce these deflections. Using a thick stiff layer of subbase over a soft subgrade can also decrease the traffic stresses developed in the slab, especially for thinner slabs.

• The stresses resulting from the traffic loadings increased for softer soils, while those resulting from thermal loading decreased. This is mainly because concrete slabs curl due to thermal gradient and lose full contact with the sub-base. The stiffer the soil and the larger the slab length, the smaller the contact area, which increases the developed stresses.
• The shorter slab the lower the generated thermal stresses. However, the stresses resulting from traffic loading were insensitive to slab length. Field observations revealed that fewer cracks were present in shorter slabs. Thus, this shows the importance of the effect of thermal stresses concrete pavement cracking. Thus, in order to better predict a pavement’s service life it is imperative that thermal gradients be accurately modeled.

• Under positive thermal gradient, when traffic loading was applied in the middle of the slab, tensile stresses develop at the bottom of the slab. In this case, the stresses produced by the traffic loading are higher than those resulting from positive temperature gradient. Thus, for soft soil there is an increased propensity for bottom to top cracking. This is especially true for thinner slabs.

• Under negative thermal gradient, when traffic loading was applied at the edge of the slab, tensile stresses resulted at the top of the slab. In this case, the thermal stresses were significantly higher than traffic stresses. Thus, a long slab over a stiff subbase has the propensity for the top to bottom cracking. This type of cracking tends to develop faster if concrete is not properly cured against shrinkage.

• Both the stresses and the stress ratios were greatly affected by shrinkage. Since shrinkage has similar effect as a negative temperature gradient, it can be concluded that it contributes to top to bottom cracking.

• When high strength concrete was used in the model, it was observed that the stresses increased, but the stress ratio (stress/modulus of rupture) decreased. This is because as the rate of increase in modulus of rupture is higher than that in the developed stresses.

• For a given slab length, although increasing the thickness of the slabs reduced the stresses for positive combined loading, it increased or did not affect the stresses for negative combined loading. Consequently, increasing the depth of the slab beyond a certain limit did not improve slab performance.

• For a given slab thickness, although decreasing the length of the slabs reduced the stresses for thermal loading, it did not affect the stresses produced by traffic loading.
Consequently, decreasing the slab length beyond a certain limit did not improve slab performance.

- For an optimum design, both top to bottom and bottom to top cracking must be considered. The indiscriminate increase of thickness may cause top to bottom cracking, while reduction in slab length beyond a certain limit might not be justified. Such an optimum design is achieved when the maximum stresses at top and bottom of the slab are of similar magnitude.

- If pumping of soil occurs in the corners of the slab, it has little to no effect on the stresses in the middle of the concrete slab. However, if it occurs in the middle of the slab, mid-panel stresses increase substantially.

- Negative thermal gradients, which are responsible for bottom to top cracking in concrete pavements, have the tendency to cause inclined cracking in skewed pavement sections. However, perpendicular cracking, i.e., cracking perpendicular to traffic direction, is due to traffic loading, which is the leading cause of top to bottom cracking. Field inspection of the skewed pavement in the case study revealed a significant larger number of perpendicular cracks than of inclined cracks. This indicates that cracking due to thermal gradients are of secondary concern in this case.

### 8.4 Fatigue and Damage

- From the comparison of the fatigue equations currently used in design to predict pavement’s service life are based on bottom to top cracking, the following conclusions can be drawn:
  - The AASHTO equation is the most conservative, i.e., it predicts the shortest pavement service life. This is mainly because this equation is calibrated using field experiments and considers additional factors.
  - The PCA method is the second most conservative equation. This is because the PCA equation conservatively assumes 5% for cracking probability.
  - The other equations, which are not calibrated from field data, are less conservative, and thus predict longer pavement service life.
The PCA, DARTER, and AASHTO methods assume that the minimum stress is equal to zero. Thus, they are acceptable when traffic stresses are dominant. However, the stresses on the top of the pavement are mainly due to thermal loading, and in this case the minimum stress is not zero. Thus, these equations tend to predict very short service life, which may not be accurate. Thus, more accurate fatigue equations need to be developed that consider the minimum stress.

The initial damage predicted by a nonlinear damage accumulation was found to be higher than that predicted by a linear accumulation. After a number of load repetitions the linear damage accumulation increased faster and became higher than the nonlinear damage.

Comparing the damage against time it was seen that the initial nonlinear accumulation of damage for three different loading cases were very close to each other. However, the final damage accumulation occurred at different times.

The damage progress is different throughout the year since the developed thermal stresses vary continuously. For certain times of the year, the combined stresses may be lower than the limit for damage. This will result in horizontal line in the damage-time diagrams.

8.5 Recommendations

The factors affecting pavement service life depend on a number of parameters and on the accurate evaluation of the stresses under a number of different conditions. Thus, improved techniques must be devised to address these issues. With the advent of faster computers and the availability of sophisticated finite element software packages, it is recommended that 3D Finite Element models replace the traditional models based on simplified assumptions, which are adopted in current pavement design software programs.

An optimum pavement design can be achieved by considering the different loadings that cause both top-to-bottom and bottom-to-top cracking. Optimum values of slab
length and thickness can be obtained by equally considering the conditions associated with these two types of cracking of concrete pavements.

- The available design fatigue equations are not suitable for negative thermal gradient loading, since the minimum stresses are not zero in these cases. On the other hand, the available equations that consider nonzero minimum stresses have not been calibrated with field data and, thus, are not appropriate for pavement design. Further studies are necessary to develop and calibrate new fatigue equations that can be used in situations where negative temperature gradients are dominant.
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Final Report

FHWA/IN/JTRP-2004/30

DAMAGE ANALYSIS OF JOINTED PLAIN CONCRETE PAVEMENTS IN INDIANA

PART II

INDISLAB: A SOFTWARE TOOL FOR JOINTED PLAIN CONCRETE PAVEMENT DESIGN

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Project No. C-36-46U
File No: 5-11-21
SPR-2643

Prepared in Cooperation with the
Indiana Department of Transportation and the
U.S. Department of Transportation
Federal Highway Administration

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Purdue University
West Lafayette, Indiana
August 2005
Damage Analysis of Jointed Plain Concrete Pavements in Indiana

Introduction

Current design procedures for concrete pavements do not account for several factors that can influence their service life. Such factors include the consideration of nonlinear thermal gradients through the depth of the slab and nonlinear damage accumulation. Furthermore, current design is based on the assumption that the cause of failure of pavements is top-to-bottom cracking. However, recent research has shown that bottom-to-top crack can also play an important role on the service life of concrete pavements.

The objectives of this study are twofold. First, all factors that negatively affect the service life of concrete pavements are investigated and the findings are integrated into a procedure for better predicting long-term performance of concrete pavements. Second, the developed procedure is implemented into a software tool, termed INDISLAB, for use by concrete pavement designers.

The analysis procedure is developed with the aid of sophisticated finite element techniques and a series of parametric studies. The findings from the analysis module are integrated into a nonlinear procedure for damage accumulation. In the development of the comprehensive 3D Finite Element (FE) model several issues are studied including the geometry of the model, mesh refinement, element selection, interaction between pavement components, and loading simulation. The developed model is then used in a number of parametric studies to investigate the effect of soil conditions, subbase and slab thickness, and slab length and stiffness on the developed stresses.

The software tool, INDISLAB, implements the findings from the analysis phase of the research. The goal of this tool is to help engineers better predict the behavior of jointed plain concrete highway pavements. INDISLAB is created to include a user-friendly Graphical User Interface (GUI) designed for INDOT’s utilization in regards to design of concrete highway pavements. The GUI’s main purpose is to serve as an interface to the finite element method software package chosen for this project, ANSYS. INDISLAB takes user provided information about the concrete slabs to create a finite element model, analyze it with ANSYS, and display the results.

Findings

Among other findings, it is established that for a given slab length, increasing the slab thickness beyond a certain limit is not justifiable. The developed FE model is also used to investigate the behavior of skewed concrete pavement slabs under several loading conditions. In particular, the crack patterns obtained from the FE analyses are compared to those observed in an actual skewed concrete pavement. It is found that the developed FE model is able to successfully predict the cause and orientation of the failure of this pavement section. An investigation of various existing fatigue equations is also carried out and a software tool is developed to perform both linear and nonlinear damage accumulation calculations. A case study of a pavement section on Interstate 70, which has failed prematurely, is created using the previously developed finite element techniques. The resulting stresses from the finite element analyses under various loading conditions are then used in the damage analysis of the pavement section. It is predicted that, irrespective
of how the damage is accumulated, the pavement should have failed at an early age. Nonlinear damage accumulation predicted that the failure would occur at an earlier age than linear damage accumulation, which is consistent with the observed behavior of the pavement section.

INDISLAB is developed as an easy-to-use interface for practicing engineers that makes use of 3D complex finite element techniques. The GUI has been thoroughly tested and has been found to be problem free. The layout of the GUI is easy to understand and aesthetically pleasing. The INDISLAB program provides a simple process for engineers to input information about a slab layout, analyze the model and view results of the solution to determine the durability of the slab(s). Unlike previously developed software, INDISLAB includes the ability to analyze pavements under nonlinear thermal loads and it is capable of accumulating damage nonlinearly.

**Implementation**

The developed software tool, INDISLAB, is expected to aid designers better understand the behavior of concrete pavements and, thus, helping produce designs of longer lasting concrete pavements. INDOT Research Division will readily use the developed software tool to analyze current pavements and influence future concrete pavement designs.

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### Abstract

Current design procedures for concrete pavements do not account for several factors that can influence their service life. In this work, these factors are investigated and the findings are integrated into a procedure for better predicting long-term performance of concrete pavements. To achieve this, sophisticated finite element techniques are employed and parametric studies are performed. The findings are then integrated into a nonlinear procedure for damage accumulation. In the development of the comprehensive 3D Finite Element (FE) model several issues are studied including the geometry of the model, mesh refinement, element selection, interaction between pavement components, and loading simulation. The developed model is then used in a number of parametric studies to investigate the effect of soil conditions, subbase and slab thickness, and slab length and stiffness on the developed stresses. Among other findings, it is established that for a given slab length, increasing the slab thickness beyond a certain limit is not justifiable. The developed FE model is also used to investigate the behavior of skewed concrete pavement slabs under several loading conditions. In particular, the crack patterns obtained from the FE analyses are compared to those observed in an actual skewed concrete pavement. It is found that the developed FE model is able to successfully predict the cause and orientation of the failure of this pavement section. An investigation of various existing fatigue equations is also carried out and a software tool is developed to perform both linear and nonlinear damage accumulation calculations. A case study of an pavement section on Interstate 70, which has failed prematurely, is created using the previously developed finite element techniques. The resulting stresses from the finite element analyses under various loading conditions are then used in the damage analysis of the pavement section. It is predicted that, irrespective of how the damage is accumulated, the pavement should have failed at an early age. Nonlinear damage accumulation predicted that the failure would occur at an earlier age than linear damage accumulation, which is consistent with the observed behavior of the pavement section.
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1 INTRODUCTION
1.1 Overview

The Indiana Department of Transportation (INDOT) has realized the need for a software tool that would help their engineers better predict the behavior of jointed plain concrete highway pavements. An accurate tool that would correctly model and analyze these types of pavement and the conditions to which they are subjected is important in determining what is detrimental to a pavement’s life. The finite element method is currently the best analysis tool to accomplish such needs. Due to restrictions that exist in software packages currently available for pavement analysis, it has been decided that a new program, INDISLAB, needs to be developed in order to accurately model the conditions that plain concrete roadways in Indiana face.

INDISLAB has been developed to incorporate the many features required to accurately perform an analysis on concrete highway pavements. In order to create such a program, many technical and software related choices, such as the type of programming language and finite element software had to be handled. Once the development team agreed upon these decisions, important alternatives in creating the Graphical User Interface, preprocessing, and postprocessing were considered. As a result of the goals set forth by the development team, INDISLAB has made many improvements over other software packages with similar goals.

INDISLAB was created to include a user-friendly Graphical User Interface (GUI) designed for INDOT’s utilization in regards to design of concrete highway pavements. The GUI’s main purpose is to serve as an interface to the finite element method software package chosen for this project, ANSYS. INDISLAB takes user provided information about the concrete slabs to create a finite element model, analyze it with ANSYS, and display the results.

The creation of INDISLAB followed a rigorous process that required many steps of development. The programming of the GUI was the first task undertaken. In order to create a user-friendly program, many decisions needed to be made about its appearance. Creation of the ANSYS input file was the next important step in the development of INDISLAB. Many choices had to be made concerning the complex finite element model.
Finally, the postprocessing of the data, including damage analysis and graphical display of results had to be developed. The three above processes are INDISLAB and they form a powerful software package useful for analyzing concrete highway pavements.

1.2 Existing Pavement Analysis Software

INDISLAB has been designed to take into account features that are not available with similar existing software. KENSLABS and ILLI-SLAB are the most popular pavement analysis software currently available for use. Simple modeling techniques employed in both of the aforementioned software tend to give results that are inaccurate. KENSLABS and ILLI-SLAB have many limitations that restrict the types of situations in which they can be utilized.

1.2.1 KENSLABS

KENSLABS is one of the oldest and readily available pavement software packages that use the finite element method. It was first developed when DOS-based computers were popular but the most recent update of the software runs on the Windows platform. However, in the update from the DOS-based to the Windows-based version most of the changes were in the appearance of the program. This program was first developed when the amount of computer memory limited what could be accurately modeled. As a result many simplifications of an accurate model were adopted to reduce computer-processing time. This creates solutions that are not as precise as can be analyzed with a more realistic pavement model.

KENSLABS makes use of thin plate elements to model slabs. Thin plate theory has been found to be invalid for slabs with a transverse-length/thickness ratio of less than 20 (Ugural 1999). In Indiana, pavements have a slab-dimension to thickness ratio of about 10. Thin plate theory overestimates the maximum stresses that occur in a slab with a centrally located point load as opposed to a moderately thick plate element as shown in
Table 1-1. Maximum Tensile Stresses for Differing Element Types

<table>
<thead>
<tr>
<th>Type of Element</th>
<th>Upper Value</th>
<th>Lower Value</th>
</tr>
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<tbody>
<tr>
<td>Thin Plate</td>
<td>214</td>
<td>151</td>
</tr>
<tr>
<td>Moderately Thick Plate</td>
<td>143</td>
<td>79</td>
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KENSLABS has three types of foundations available for use: liquid, solid, and layer also known as the Winkler, Boussinesq, and Burmister foundations, respectively. The Winkler foundation characterizes the force-deflection relationship using an elastic spring (Huang 1993). Ideally an infinite number of springs would represent the force-deflection relationship, but for the rectangular thin plate elements this is reduced to four springs, one at each node as shown in Figure 1-1. The Winkler foundation greatly oversimplifies the relationship between the slab and the subsurface. The force in each spring depends solely on the deflection at its respective node and is independent of the deflections at the other nodes. This is characterized by the following mathematical expression:

\[ F_{wi} = k a b w_i \]  

(1-1)

where \( F_{wi} \) is the force of the spring, \( w_i \) is the deflection of the spring, \( a \) and \( b \) are one-half the lengths of the element sides, and \( k \) is the modulus of subgrade reaction, “a fictitious property not characteristic of soil behaviors” (Huang 1993). The Winkler
foundation was included in KENSLABS despite its inaccuracies in order to reduce the amount of computer memory needed to evaluate the model.

![Diagram of Liquid Foundation Under a Plate Element.](image)

Figure 1-1. Liquid Foundation Under a Plate Element.

The Boussinesq or solid foundation is an improvement upon the Winkler foundation. The solid foundation still uses the force-deflection relationship; however it does take into account deflections at other nodes by the use of a flexibility matrix (Huang 1993). The following relationship is used:

$$w_{i,j} = \frac{P_j (1 - \nu^2)}{\pi E_f d_{i,j}}$$

where $w_{i,j}$ is the deflection of the spring at node $i$, due to $P_j$, the force at node $j$, $\nu$ is the Poisson ratio of the foundation, $E_f$ is the elastic modulus of the foundation and $d_{i,j}$ is the distance between nodes $i$ and $j$. While the Boussinesq foundation is more accurate than the liquid foundation it still is a simplified elastic relationship.

The Burmister foundation uses Burmister’s layered theory to form the flexibility matrix. It makes use of a more complicated method than the Boussinesq foundation, involving a large formula and interpolation to avoid repeated evaluation of the equation. The simplified version of the equation is as follows:
\[ R = \frac{1}{2\pi H^2} \int_{0}^{\infty} R^* \, dm \] (1-3)

where \( R \) is a dimensionless deflection, \( H \) is the thickness of all slab layers, \( R^* \) is the stress due to loading, and the \( m \) is a parameter for the layered system. While more accurate than the solid foundation this method is just another derivation of the force-deflection relationship.

The Winkler, Boussinesq, and Burmister foundations are simplified representations of the true subsurface conditions. All three use an elastic force-deflection relationship to characterize the slab-soil interaction. In actual pavements, often times there are several different types of subgrade and subbase layers underneath the concrete slab. KENSLABS cannot realistically depict this sort of relationship. Nor can this software determine the stresses, which occur in the subgrade/subbase. All of these issues are very prominent in the design of concrete highway pavements and should be evaluated in a software package.

KENSLABS gives the user the option of using temperature loading. In order to analyze a slab accurately, temperature should be considered since it can cause the slab to curl and affect the slab deflections and stresses. Even when temperature loading is used in KENSLABS, it is not accurate due to the way dowel bars are modeled. The dowel bars in KENSLABS do not restrain the curling that occurs as a result of the temperature variation in the slab. It is assumed that each slab acts independently and Huang argues this is reasonable if all slabs have the same size and thickness (1993). In reality, dowel bars will have an effect on the curling of the slab caused by temperature and the dimensions of adjoining slabs will not always be the same. Furthermore, KENSLABS can only use a linear temperature approximation, since the slab is modeled using thin plate elements. However, in reality, temperature variations vary nonlinearly through the depth of the slab (Richardson and Armaghani, 1987). Thus, KENSLABS cannot realistically model a nonlinear temperature.

The KENSLABS model allows for the input of dowel bars. However, this modeling is not very effective or accurate. The dowel bars do not affect the curling of the
slabs with regards to temperature as mentioned above (Huang 1993). Furthermore, the dowel bars can only be placed in locations in which nodes have been defined. If the user inputs fewer nodes than the number of dowel bars, then multiple bars will appear at a single node. This does not represent the spacing of the dowel bars accurately. KENSLABS simplifies the effect of the dowel bar by only considering the shear transfer through the bar and the effects this has on deflection (Huang 1993). However, dowel bars do more than just transfer shear and effect vertical displacement. Dowel bars also restrain the lateral separation of adjoining slabs. Many highway pavements also include the use of tie bars to help prevent curling and keep slabs from separating in the direction perpendicular to the dowel bars. KENSLABS neglects to take tie bars into account in order to simplify the slab model.

KENSLABS allows the user of the program to decide if the analysis shall include the weight of the slab. This decision is important because it affects the slab-subgrade contact behavior. More specifically, the weight of the slab affects the way the model performs and influences the subsurface conditions beneath it. The weight of the slab causes a precompression of the spring foundation as shown in Figure 1-2. This precompression will have an effect on the total deflection of the nodes in the slab.

![Precompression Due to Weight of the Slab](image.png)

The precompression is the initial deflection of the slab. If a full contact condition is chosen over a partial contact condition, including the weight of the slab is extremely vital to the model. The precompression of the slab makes certain that the slab and subgrade are in full contact (Huang 1993). Applied and thermal loads cause the remaining
deflections and are added to the initial deflections. As can be seen including the weight of the slab is important in correctly analyzing how a concrete slab will behave.

A couple of damage models are employed in KENSLABS to determine the life of the pavement. The first model determines life of the pavement “based on fatigue cracking only” (Huang 1993). The number of repetitions, $N_f$, is expressed as:

$$\log N_f = f_1 - f_2 \left( \frac{\sigma}{S_c} \right)$$

(1-4)

where $\sigma$ is the flexural stress in the slab, $S_c$ is the modulus of rupture of concrete and $f_1$ and $f_2$ are coefficients recommended to be 16.61 and 17.61, respectively by Darter and Barenberg (Huang 1993). The second fatigue model includes the equations recommended by the Portland Cement Association:

$$\begin{align*}
For \quad \frac{\sigma}{S_c} \geq 0.55 : 
\log N_f &= 11.737 - 12.077 \left( \frac{\sigma}{S_c} \right) \\
For \quad 0.45 < \frac{\sigma}{S_c} < 0.55 : 
N_f &= \left( \frac{4.2577}{\sigma / S_c - 0.4325} \right)^{3.268} \\
For \quad \frac{\sigma}{S_c} \leq 0.45 : 
N_f &= \infty
\end{align*}$$

(1-5)

Input of slab dimensions and properties is a complicated and cumbersome procedure in KENSLABS. There are a couple of differences between the Windows and the DOS versions of KENSLABS. The Windows version allows for a maximum of 6 slabs and 15 nodes per slab in each direction. The DOS version allows for a maximum of 9 slabs and an infinite amount of nodes. The KENSLABS interface prompts the user to enter several aspects of the model including the number of slabs, nodes and joints as well as the locations of the nodes and joints. However, the interface does not provide the user with a method of checking if the slab has been entered correctly. This can cause the input of model data to be a difficult and overwhelming task. An example of such a model is shown in Figure 1-3. One can imagine how much more difficult the process would be if there was a desire to add a large number of nodes to a slab in order to make the results
more precise. Entering the node numbers, joint numbers and locations can be a
cumbersome and daunting task that makes KENSLABS difficult to use.

Figure 1-3. Numbering of Slabs, Nodes and Joints.

The postprocessing capabilities of KENSLABS are primitive. From the
KENSLABS interface no link or command exists to access the output text file. The
output text file contains the analysis results, including deflections and stresses of the
slabs. In order to view the file one must go to the directory in which the input file was
saved and open the output file from there. The output file contains a summary of all user-
input variables such as slab dimensions, coordinates of nodes, load summaries and so on.
The deflections and stresses at the nodes are also presented along with the maximum
stress value and the node at which it occurs. However, the maximum deflection is not
provided. Visual representations of the results are minimal. KENSLABS shows a
graphical representation of the slab that includes load locations and the position of the
maximum stress value, as shown below in Figure 1-4. As can be seen it is a basic
representation of the results obtained by KENSLABS. The only positive outcome of
such a figure is to make certain that one has entered the nodes, dimensions and loads correctly. In summary, KENSLABS postprocessing abilities are few and limited.

KENSLABS was a breakthrough piece of software when it was introduced in 1973 and has been continuously updated through the years to reflect better computing abilities. However, KENSLABS has become outdated and oversimplifies the pavement analysis process in almost every area. KENSLABS takes too many shortcuts to cut down processing time, which in turn hampers the ability of the program to produce accurate results. Furthermore, it does not have the postprocessing capabilities that one would expect with the present available computing tools.
1.2.2 **ILLI-SLAB**

ILLI-SLAB is another currently available concrete pavement analysis software. It was developed originally in 1977 and has been continuously updated since. As KENSLABS has done, ILLI-SLAB makes many assumptions and takes shortcuts in order to decrease the processing time needed from the computer in order to calculate results quickly.

There are many similarities between the assumptions made in KENSLABS and ILLI-SLAB. ILLI-SLAB uses thin plate elements, as does KENSLABS. It also makes use of several force-deflection elastic spring foundations. However, ILLI-SLAB gives the user several more choices to pick from including: very soft, soft, medium, stiff, springs, Winkler, Boussinesq and Vlasov but does not contain the Burmister foundation, which KENSLABS does. ILLI-SLAB also includes the effect of temperature curling as well as the option for partial or full contact between the slab and the subgrade. One difference from KENSLABS is that ILLI-SLAB includes the use of dowel bars in both the longitudinal and transverse directions of the slab. ILLI-SLAB does not include any fatigue/damage analysis. It only analyzes the slab(s) to obtain stresses and deflections. ILLI-SLAB has the capability for a maximum of 10 slabs in the longitudinal direction and 3 slabs in the transverse direction. The number of nodes is created automatically by ILLI-SLAB and depends on the slab lengths. However, there is an option to increase the amount of nodes by clicking on a graphic of the mesh as shown in Figure 1-5.

The postprocessing capabilities of ILLI-SLAB are less than desirable. As with KENSLABS, one must locate the output file, as there is no link from the interface program to it. However, ILLI-SLAB does include both the maximum and minimum values for both stress and deflection. ILLI-SLAB provides no graphical output of any results. The only graphical output the program produces is one of the slab mesh as shown in Figure 1-5. ILLI-SLAB postprocessing is an improvement in some respects to that of KENSLABS, particularly in pointing out the maximum and minimum stress and deflection values.
1.2.3 Comparison of ILLI-SLAB with KENSLABS

Since ILLI-SLAB and KENSLABS are very much alike, Huang (1993) used the two to compare and contrast results obtained in three different loading situations. Huang (1993) compared maximum tensile stress at the bottom of the slab as well as maximum deflection and joint efficiency. Joint efficiency is the ratio of deflections on the opposite sides of one of the joints between the loaded and unloaded sections of the slab (Huang 1993). Three unique situations were used to compare results obtained by the two pieces of software: edge, corner, and temperature loading. KENSLABS results were also compared to theoretical solutions. In most cases the results from KENSLABS were found to be conservative (Huang 1993).

The first case considered is edge loading. The model used was a two slab, two-layer pavement with tied shoulders. The wheel load used in this case was a single axle load placed near the edge of the roadway pavement and the shoulder as shown in Figure 1-6. The results are provided in Table 1-2.
As can be seen in Table 1-2 the results are very similar for the aggregate interlock cases. However, the dowel bar modeling did not produce results that relate well. ILLI-SLAB produces results with a higher joint efficiency and smaller stresses and deflections. This could be due to the fact that KENSLABS lacks a tie bar modeling technique.

The second case tested was that of a single axle load placed in the corner where one of the roadway slabs and shoulders meet as shown in Figure 1-7. The results are provided in Table 1-3.
Table 1-3. Comparison of results based on corner loading (Huang 1993)

<table>
<thead>
<tr>
<th>Load Transfer</th>
<th>Type of Interface</th>
<th>Model</th>
<th>Longitudinal Joint Efficiency (%)</th>
<th>Transverse Joint Efficiency (%)</th>
<th>Max. tensile stress of slab (psi)</th>
<th>Max. deflection (10^{-3} in)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aggregate Interlock</td>
<td>Unbonded</td>
<td>KENSLABS</td>
<td>64</td>
<td>85</td>
<td>96.6</td>
<td>11.8</td>
</tr>
<tr>
<td>Aggregate Interlock</td>
<td>Bonded</td>
<td>ILLI-SLAB</td>
<td>65</td>
<td>86</td>
<td>97.9</td>
<td>12.1</td>
</tr>
<tr>
<td></td>
<td>Bonded</td>
<td>KENSLABS</td>
<td>60</td>
<td>84</td>
<td>52.9</td>
<td>9.6</td>
</tr>
<tr>
<td>Dowel Bars</td>
<td>Unbonded</td>
<td>ILLI-SLAB</td>
<td>61</td>
<td>85</td>
<td>51.9</td>
<td>9.7</td>
</tr>
<tr>
<td></td>
<td>Bonded</td>
<td>KENSLABS</td>
<td>34</td>
<td>80</td>
<td>101.1</td>
<td>14.9</td>
</tr>
<tr>
<td></td>
<td>Bonded</td>
<td>ILLI-SLAB</td>
<td>50</td>
<td>88</td>
<td>93.6</td>
<td>13.1</td>
</tr>
<tr>
<td></td>
<td>Bonded</td>
<td>KENSLABS</td>
<td>31</td>
<td>79</td>
<td>67.0</td>
<td>12.1</td>
</tr>
<tr>
<td></td>
<td>Bonded</td>
<td>ILLI-SLAB</td>
<td>47</td>
<td>87</td>
<td>52.2</td>
<td>10.5</td>
</tr>
</tbody>
</table>

Once again the results are very similar for aggregate interlock but vary for the dowel bar cases. ILLI-SLAB reports a higher joint efficiency and smaller maximum stresses and deflections, which makes KENSLABS the more conservative program, but not necessarily the most accurate.

Lastly Huang (1993) compares the two programs with respect to temperature loading. 20 ft by 12 ft slabs with varying thickness, subject to two temperature loadings are tested. One of the temperature gradients is 1.5°F/in (upward curling) and the other is -3°F/in (downward curling). The results are provided in Table 1-4.
Table 1-4. Comparison of curling stress based on temperature loading (Huang 1993)

<table>
<thead>
<tr>
<th>Slab Thickness (in.)</th>
<th>Computer model</th>
<th>Downward Curling (-3°F/in)</th>
<th>Upward Curling (1.5°F/in)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>50</td>
<td>200</td>
</tr>
<tr>
<td>8</td>
<td>KENSLABS</td>
<td>219.6</td>
<td>270.2</td>
</tr>
<tr>
<td></td>
<td>ILLI-SLAB</td>
<td>222.7</td>
<td>283.2</td>
</tr>
<tr>
<td>10</td>
<td>KENSLABS</td>
<td>193.3</td>
<td>258.2</td>
</tr>
<tr>
<td></td>
<td>ILLI-SLAB</td>
<td>198.7</td>
<td>276.6</td>
</tr>
<tr>
<td>14</td>
<td>KENSLABS</td>
<td>139.1</td>
<td>203.3</td>
</tr>
<tr>
<td></td>
<td>ILLI-SLAB</td>
<td>143.2</td>
<td>216.2</td>
</tr>
</tbody>
</table>

As can be seen in Table 1-4, with respect to upward curling, KENSLABS and ILLI-SLAB give results that are very similar. However, the results of the analyses with downward curling differ. This could be due to the difference of the modeling of the contact conditions in the two software packages.

As the above results show there are differences in the ways some of the slabs are modeled in KENSLABS and ILLI-SLAB. Dowel bars and downward curling were two factors, which varied the results of the model. Overall results were similar, showing that both programs approached the simplification process of the model in very much the same way.

1.3 Improvements Made with INDISLAB

The goal with INDISLAB was to develop software that did not take the shortcuts and simplifications KENSLABS and ILLI-SLAB did, thus, producing a more accurate model for jointed plain concrete highway pavements. The INDISLAB program and model were based on research into different components of the model. Based on this research, a precise model for each of these components was developed. As a result INDISLAB uses a better model that can generate reliable results.
INDISLAB makes use of solid brick elements to model the slab such as the one shown below in Figure 1-8. The use of several of these elements through the depth of the slab allows for an accurate input of the temperature distribution through the concrete slab. INDISLAB uses a fixed number of elements in the length, width and depth of the slab. The fixed number of elements is an accurate choice for all slab thickness that can be chosen and is explained further in Chapter 4.

Figure 1-8. Solid 8-node Brick Element.

INDISLAB has also made the modeling of the foundation much more accurate. It includes both a subbase layer and up to two subgrade layers in the model. It uses solid brick elements to model these layers as well. The number of subsurface layers is an option INDISLAB includes due to the fact that ground conditions vary differently with depth at various locations. The subgrade depth is fixed to six feet due to the fact that subsurface conditions below this depth do not affect the slab stress analysis. Any extra depth in the subgrade layer adds analysis time with similar results.

INDISLAB also improves upon the temperature loading scenarios allowed in other types of software. INDISLAB gives the user the option between no thermal loads, a linear temperature load and a nonlinear temperature load.

Dowel and tie bars are modeled more accurately in INDISLAB than in previous software. The bars help to accurately model stress transfer between slabs as well as to more realistically model deflections that occur in the slabs.
Whereas, KENSLABS and ILLI-SLAB developers defined their own finite element modeling within the program, INDISLAB interfaces with an industry recognized finite element method software, ANSYS (ANSYS 5.7.1, 2001), in order to analyze the model. Using such software provides many predefined options as well as the reliability of results expected from a commercial software package.

INDISLAB also provides the user with an easy to use interface program that makes input of preprocessing data simple. The program was developed to be aesthetically pleasing and provide a user-friendly interface to input model data.

The postprocessing portion has also been designed to be easy to use and includes several options for users to view results. Using INDISLAB input files a user can access the results through ANSYS graphs. Damage analysis and the ability to accumulate damage linearly and nonlinearly are also a part of INDISLAB. This last feature is of particular importance when attempting to predict the service life of a pavement.

INDISLAB has made many improvements over existing software. Several tough and important choices were made during the development process in regards to software choices, Graphical User Interface development, formation of the ANSYS input file, interface with ANSYS and the postprocessing of the model data. INDISLAB has been developed as a tool to accurately analyze jointed plain highway concrete pavement slabs in Indiana.
2 SOFTWARE

2.1 Introduction

In the development of INDISLAB one of the most important decisions was that of the software used to develop the program. The creation of INDISLAB was dependent on two types of software packages: a programming language and a commercial finite element analysis program. The programming language had to be capable of developing the three segments of INDISLAB: the Graphical User Interface (GUI), preprocessor and postprocessor. The finite element software had to have the ability to handle the analysis of six jointed plain concrete highway slabs and three jointed plain concrete highway shoulder slabs. The success and failure in creating INDISLAB was, in part, based on the software packages chosen.

There are two main programming languages that fit the profile from which INDISLAB could be developed. Visual Basic and the Java programming languages are widely recognized programming software packages. They are similar in the scope of their capabilities. Both can readily develop the GUI that is the control panel of the INDISLAB software. Either one could have been used to develop INDISLAB. Java can be freely downloaded from Sun Microsystems Java website. This was a major reason why Java was chosen for the present study.

Two commercial finite element analysis software packages, namely ABAQUS and ANSYS, were investigated in this study. ABAQUS is a UNIX-based piece of software that can be run on a Windows-based platform with additional software. However, it is a difficult process to successfully enable ABAQUS to run on a Windows-based computer. Another problem with the ABAQUS software was its ability to reach a convergent solution with contact elements. Contact elements were used in the finite
element model and are an important portion of the present pavement model. ANSYS, a Windows-based software, could successfully converge to a solution when using contact elements. Both software packages enable the creation of the model by a text input file, which contains the commands necessary to develop the model and find a solution. As a result of the above advantages, the ANSYS package was chosen for use in the development of INDISLAB.

2.2 JAVA Programming Language

2.2.1 History of Java

The Java programming language is very powerful, extremely capable and versatile in its ability to develop programs. Sun Microsystems first began development of Java in 1991. Its first application was to be a language used to program home appliances. However, Java did not become popular with appliance makers. In 1994 it was realized that the language was ideal for the creation of a web browser. In the following year Netscape incorporated this technology into their browser. Java is still incorporated highly with the Internet by the way of applets; Java based programs run through a web browser (Savitch, 2001). Subsequent releases of Java have increased the speed of the compiler interpreter as well as added many new features, such as increased graphical capabilities (Flanagan, 1997).

2.2.2 Features of JAVA

Java is primarily an update of the C++ programming language. It incorporates many of the same commands and ideals. However, Java improves on these, as well as introduces new features, to make it a further advanced programming language. Java is a high-level programming language, which means that it is designed to be simple for people to read and write. Most programming languages are high-level languages, such as
FORTAN, C++ and Visual Basic among others. Computer hardware cannot understand such languages and as a result they must be converted into one that the computer can comprehend as shown Figure 2-1. The languages that a computer can understand are called low-level languages or machine language. The compiler is the program that translates the high-level language into a low-level language. A disadvantage of this process is that for each make of computer and each type of operating system a different compiler is required. However, Java uses a different approach to eliminate the need to compile a program on every different computer a program is to be run (Savitch, 2001).

![Figure 2-1. Programming Language Hierarchy (Savitch, 2001).](image)

The Java compiler works differently than other programming language compilers. The Java compiler does not translate a program directly from high-level language to machine language. Instead it translates the high-level language into Java byte-code (JBC). JBC is a language that is similar to what machine language is for many types of computers. As a result it is easy to convert JBC to machine language. JBC is a language that is understood by the Java Virtual Machine (JVM). Through the JVM the interpreter automatically converts the JBC to machine language. The advantage of this is that after the code is translated into JBC it can be run on any computer without recompiling (Savitch, 2001). The process described above is schematically represented in Figure 2-2.
Java takes advantage of the Object-Oriented Programming (OOP) paradigm. OOP "is a programming methodology that views a program as consisting [of] objects that interact with each other by means of actions (known as methods)" (Savitch, 2001). OOP is useful in that it allows for the reuse of code and as a result saves time. It also allows for classes to be tested independently. This makes it much easier to debug code.

Another positive characteristic of Java is that its compiler is available for download for free from the Java website (http://java.sun.com). Since the Java compiler is not distributed with most computers, this provides a way for one to be able to use Java programs for free. The downside to this is that the file that must be downloaded is extremely large and thus high-speed access to the Internet is required.

The graphics capability of Java has vastly improved since its first version was released. Everything required to produce a Windows-based GUI is included in the current version of the Java programming language. A programmer can include menus,
tabs, buttons, input boxes as well as a variety of other graphical components. The programmer can also make use of colors, sizes and fonts for words and drawings in Java. Pictures can be imported and there is even a pre-defined function for save and open windows for a graphical program. The graphics potential of Java is very powerful.

Java like many other programming languages has a vast amount of commands that can be used to perform different operations. In fact it probably has more commands and classes than any other programming language. As a result the developers of Java have developed an entire website (http://java.sun.com/j2se/1.3/docs/api/) where programmers can resource information about all the classes and commands for the most current releases of Java (Figure 2-3). The website allows users many options in ways to access the information they are searching for. This is extremely useful if one only knows so much about the command or class that they are searching for. The website also has a tutorial that helps a programmer learn how to use many of the capabilities of the Java language.

Figure 2-3. Java Help Website (http://java.sun.com/j2se/1.3/docs/api/).
The downside to Java includes the fact that it is dependent on the presence of the JVM. As a result, an executable (.EXE) file cannot be created through Java. However, a Java ARchive (JAR) file can be created that allows the user to access a Java developed program with a double-click of the mouse button. Nonetheless, the system on which a user wishes to perform the above task must configure the computer properly. Also the JAR file must include a manifest file that defines the main class used to run the program. The JAR file has a format very similar to the popular ZIP format. A JAR file allows all of the Java files needed to run a program to be compressed into one file. This is extremely useful for complex programs that require many files.

As can be seen Java is one of the most advanced programming languages currently available. Programs are portable and can be easily developed. Online documentation also provides help for programmers. It is widely used and can be obtained for free. Graphical displays can be easily created and are highly capable. For these reasons Java has been selected for the development of INDISLAB.

2.3 ANSYS Finite Element Software

2.3.1 Introduction to ANSYS

ANSYS is a widely used, powerful finite element analysis program with the capabilities one would expect from commercially available software. Many programs similar to INDISLAB do not make use of a commercial finite element software program. Instead, the developers of the pavement analysis software usually create their own finite element analysis programs. This usually limits the level of sophistication of the models that can be created. Thus, in INDISLAB it was decided that it would be beneficial to use commercially available software for the following several reasons.

ANSYS had all the tools necessary to model concrete highway slabs and shoulders with all the required complexity and detail. INDISLAB takes into consideration details that other similar pavement analysis software leave out or simplify.
As a result, the development of a model with the aid of commercial finite element analysis software saves time. Had the developers of INDISLAB decided upon the creation of their own software to perform the finite element analysis, some of the complexity of the model would have been lost. This is due to the time and monetary restrictions that the creation of a finite element analysis software demands.

2.3.2 ANSYS Capabilities

ANSYS runs on a Windows-platform personal computer. Due to the fact that computer hardware is increasingly more capable and inexpensive, it is now possible to run complex models within a short period of time on a personal computer. Several years ago this was not within the realm of possibility. This is an attractive option due to the familiarity and popularity of Windows-based systems.

ANSYS has an impressive graphics capability as well. It has the ability to model the slabs in three dimensions. The program allows the user to view them at any possible angle using a pan, zoom and rotate tool. ANSYS also displays the results of the solution in a graphical format. For example, the user may view the deflections or stresses of the slab through the program. Figure 2-4 shows the ANSYS interface display.

ANSYS allows for the creation and solution of a model to be performed through an input file. While a model can be developed and run through the ANSYS graphical display, it is the goal of INDISLAB to be able to create a model with as little effort by the user as possible. The user inputs data about the slabs, dowel bars, loading, subbase and subgrade into INDISLAB’s GUI. From this information an input file containing the model is created by INDISLAB. ANSYS then processes the input file to generate the model and form the solution.

ANSYS provides an online help file, which is accessible through the program interface. ANSYS develops the model based on the INDISLAB input file. The help file contains a listing of all commands that can be used in the input file. The help file also lists and explains the parameters associated with each command.
As it can be seen ANSYS has been extremely beneficial in the development of INDISLAB. ANSYS has saved much time in the development of INDISLAB. To develop software of the caliber of ANSYS without the resources of a large corporation would be unlikely. This is the reason many similar pavement analysis software could not include the amount of detail necessary to obtain accurate results. Besides saving time, the reliability of ANSYS has proved invaluable. ANSYS provides a powerful graphics display that is useful for the INDISLAB postprocessor.
3 INDISLAB Development – GUI Creation

3.1 Introduction

The Graphical User Input (GUI) interface was the first step in the development of INDISLAB. The GUI acts as the control panel for INDISLAB. The GUI’s main function is to collect information about the jointed plain concrete pavement model and then to command INDISLAB to formulate an input file defining the finite element model and launch the finite element software, ANSYS. All preprocessing functions and commands are run from within the GUI. The main goal in the creation of the GUI was to make improvements on existing software and create an easy to use pavement software package for jointed plain concrete pavements in Indiana.

One decision made from the outset was to design the program to be Windows-based and dependent. DOS programs are outdated, often more difficult to use and not as visually pleasing. The Java programming language has the capability to develop software for both types of operating systems. Java includes many tools that can develop a GUI that has the same look and feel of many programs created for a Windows-based environment.

Another decision made was to ask the user for the least amount of input variables possible. Many of the existing programs are cluttered with many options and often times the layout of these choices is confusing. By limiting the amount of input variables to a minimum, a less confusing and concise GUI was created. Other values not considered by INDISLAB but considered by existing software were taken into account in a fashion that represents the typical jointed plain concrete pavement in Indiana. For example, the spacing of dowel and tie bars for slabs in Indiana has been predefined in INDISLAB.
However, other options were added to expand the scope of INDISLAB over other software packages.

The ability of the user to have a choice of unit systems is another option included in the GUI. Not knowing if the English system of units will be used for years to come or whether the SI unit system will eventually be preferred was the reason for including this choice in the GUI.

Lastly, extensive testing of the GUI portion of INDISLAB was undertaken to ensure its ease of use. The goal was to discover all potential problems with the GUI and to eliminate them. INDISLAB also checks the validity of the input entered by the user to help ensure the finite element model will run correctly.

Overall, the GUI portion of INDISLAB is programmed to be flexible, easy to use and aesthetically pleasing. This has been accomplished through discussion with future users of INDISLAB and with regards to their requests, as well as by determining what should be improved upon with respect to existing pavement analysis programs.

3.2 INDISLAB’s GUI: Development

With the above decisions made, the development of INDISLAB’s GUI was initiated. Several choices and problems needed to be addressed during the programming of the GUI. Most of these issues developed due to the decision of making it a Windows-based, aesthetically pleasing piece of software. These issues include the decision of whether to make the window resizable, placement of components in the GUI, the addition of pictures to clarify required user-input values, open/save compatibility issues as well as debugging the GUI for potential problems.

3.2.1 Resize Option

Most Windows-based software includes the capability of being resized. This allows the user to make the window in which the program is running larger or smaller
both vertically and horizontally, as well as giving them the option to minimize/maximize the window. The resizable option is usually beneficial to the user, however for the programmer it can be a cumbersome task. As can be seen in Table 3-1 there are pros and cons for including the resize option in INDISLAB.

Table 3-1. Pros/Cons of Including Resize Option for INDISLAB GUI

<table>
<thead>
<tr>
<th>Pros</th>
<th>Cons</th>
</tr>
</thead>
</table>
| • User can size the window in order to view other windows at the same time.  
• Similarity with other programs. | • Distorts the layout of the GUI if advanced programming techniques are not used.  
• Requires inclusion of scroll bars.  
• Requires the knowledge of how to determine a monitor’s size and resolution critical to including a resize option. |

The pros include ease of use and familiarity for the person using the program. Negatives of including the resize option all have to do with the complexity of the program. Including the resize option affects the placement of all components on the screen including text, text boxes and buttons to mention a few. Including the resize option is difficult to include, due to various sizes of computer monitors and varied resolution settings. The INDISLAB program would need to be able to handle each monitor size and resolution with respect to the components of the GUI.

The lack of this knowledge leads to display problems such as the example shown in Figure 3-1. Because of the complication involved with this option, it was decided to not include it with the INDISLAB GUI. Instead of making INDISLAB resizable, it was decided to make INDISLAB compact enough that it would allow space to view other windows on the computer screen. The programmer has the ability to control how large the GUI window is by including two integer values in the code that define the height and width of the window.
3.2.2 INDISLAB GUI Components

After setting the window size of INDISLAB’s GUI it was possible to take the next programming step, which was to add a menu bar. A menu bar is common in most Windows-based applications. Due to familiarity of a menu bar in a Windows-based program it was deemed necessary to include such a feature in the INDISLAB GUI. Java includes a programming feature that makes it very simple to include such an option.

In the Java programming language, menus and menu items are handled in the following way. When the user clicks one of the items present on the menu, an action is fired. When an action is fired this is referred to as an action event. In the code there are items known as action listeners whose sole function is to hear an action event. Within these action listeners is code that tells the program how to respond to the firing of an action. The same methodology works for buttons and pull-down menus.

Four menus were included to handle a majority of the operations INDISLAB would need to perform. Figure 3-2 shows the menu bar in INDISLAB. The menu bar
includes the typical *File* menu found in most Windows programs that provide basic program functions. Items in this menu include *New, Open, Save, Save As* and *Exit* commands. The next menu included is the *Analysis* menu that consists of the option *Run Analysis*. This command is executed after all input data has been correctly entered and the user wants the model to be executed and analyzed. The action listener for this command also checks the validity of the input before generating the ANSYS input file. The third menu is the *Results* menu. Items included on this menu develop short input files to display various plots of the slabs after the analysis has been completed. The final menu is the *About* menu that includes an item that lets the user know where he/she can find the help file.

![Figure 3-2. Menu Bar.](image)

The Save/Open file option is an important menu item in INDISLAB. The saving and opening of a file are very similar. Explained below is the process of saving a file. Once the user clicks the *Save As* item in the *File* menu a Java provided window appears that allows the user to name the file as well as choose the Windows directory and location in which to save it. The Open and Save windows are shown in Figure 3-3. After the user clicks the “Save” button, INDISLAB proceeds to grab all data input to the text boxes and pull-down menus. INDISLAB writes these values into a file. All data is saved as input by the user and is not checked unless the *Run Analysis* menu item has been clicked. This file is saved to the folder that the user has chosen with an .IND extension. The .IND extension identifies the file as an INDISLAB file. The same process occurs when
opening a file except that an existing file is read and places the recorded data into their corresponding text boxes.

![Open/Save Windows](image1.png)

Figure 3-3. Open/Save Windows

The next step included answering the question of how to present all of the input fields required to collect the model information. Since there were more values that needed to be collected from the user than could appropriately fit on one screen this was an important decision to be made. After researching the issue it was established that Java presented three possible solutions to this question: the use of scroll bars, the use of buttons to navigate through a series of screens or the use of tabs to advance from screen to screen. The pros and cons of each tool are presented in Table 3-2.

From a programming perspective scroll bars would be the simplest way to solve the above dilemma in many aspects. Scroll bars allow for most user input variables to be displayed on one screen. To access them one needs to scroll up or down. An example of scroll bars is shown in Figure 3-4. The use of scroll bars permits an easy test of validity due to the fact that most input boxes are present in one window. Despite having some excellent benefits there are some negatives. INDISLAB gives the user the option of choosing the number of wheel loads to be applied on the slab(s). If scroll bars were selected for use in INDISLAB, the wheel loading case would need to be handled by an extra pop-up window. This defeats the benefit of the scroll bar, which is that everything is accessible on one screen. Another problem with scroll bars is that it is the least visually pleasing option. The two negatives associated with the scroll bar outweighed the positives and thus it was decided not to use this option.
Table 3-2. Pros/Cons for INDISLAB Components.

<table>
<thead>
<tr>
<th>Scroll Bars</th>
<th>Buttons</th>
<th>Tabbed Panes</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>PROS</strong></td>
<td><strong>CONS</strong></td>
<td><strong>CONS</strong></td>
</tr>
<tr>
<td>• All data can be</td>
<td>• Cumbersonse for the</td>
<td>• Data cannot be</td>
</tr>
<tr>
<td>accessed within a</td>
<td>user to access data quickly.</td>
<td>checked until</td>
</tr>
<tr>
<td>click of the mouse.</td>
<td>• Menus must be</td>
<td>everything has been</td>
</tr>
<tr>
<td>• Easy to check user-</td>
<td>input data for errors.</td>
<td>input.</td>
</tr>
<tr>
<td>input data for errors.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure 3-4. Example of Scroll Bars in a GUI. ([http://java.sun.com/](http://java.sun.com/))
Another option was to use buttons to advance users through a series of screens. In this scenario the user enters input in one screen, clicks a button that saves the data, INDISLAB closes the existing window and advances to a new window. In the new window the user is prompted to enter more data about the model. An example of buttons is shown in Figure 3-5. One benefit of using buttons is that the programmer can have INDISLAB check for valid data in each window before advancing to the next. When the user clicks a button to advance to another window the program can easily check to make certain that adequate data has been entered into the existing window. The inclusion of buttons in the GUI can be approached in a couple of different fashions. One option is to include one button to advance to the next window and another to the previous window. The other alternative is to include a set of buttons that includes a link to all windows requiring user input data. The problem with the first option is that all windows are not easily accessible to the user. The problem with the choice of several buttons to all windows is aesthetic. Several buttons at the bottom or top of the window adds a cluttered and ill-advised feel to the GUI. Another problem with either alternative is that the menus at the top of the screen need to be reprogrammed for every window. Buttons are a useful tool, but did not provide the complete answer for user navigation of the GUI.

![Figure 3-5. Example of Buttons in a GUI.](http://java.sun.com/)

A third option was the use of tabs instead of a separate window for each category of required data. As shown in Figure 3-6, tabs allow the user to easily access each category of data necessary for the model without closing/opening extra windows. Each
window is within a click of a mouse button. The menus at the top of the GUI need to be programmed only once. Tabbed panes provide an improved aesthetic look over the use of buttons or a scroll bar. The only negative is that the validity of the input data cannot be checked until all necessary information has been entered for each tabbed category. If buttons were used, the user of the GUI would know that there was a problem in the current window thus making it much simpler to discover the error. However, INDISLAB solves this problem through a window that alerts the user where an error exists. Tabbed panes give the flexibility desired that other options cannot provide.

![Figure 3-6. Example of Tabbed Panes in a GUI.](http://java.sun.com/)

After exploring the three above options a decision was made to use a combination of tabbed panes and buttons to help the user navigate through INDISLAB’s GUI. For the main categories it was decided to use tabbed panes. In certain instances, however, it was necessary to include the use of buttons. Through the combined use of the tabbed panes and buttons it was possible to accomplish all goals desired for the GUI.

### 3.2.3 INDISLAB GUI Layout

The INDISLAB GUI is composed of several screens. The most important are the tabbed panes in which the user inputs data regarding the model. The first screen that appears when the INDISLAB program is run is the introduction screen shown in Figure 3-7. As can be seen this screen includes the name of the program, a picture of a finite
element model of three slabs and a button that allows the user to advance further into the program.

![INDISLAB Introduction Screen](image)

Figure 3-7. INDISLAB Introduction Screen

The next screen to appear allows the user to choose to start a new file or to open an existing file, i.e., to create a new application or to open an existing application. Under the option to open a new file the user has the choice of English and SI units. This screen is shown in Figure 3-8. After choosing between a new and existing file as well as units, if necessary, the user must click the continue button to proceed to the main screen of the GUI.

![INDISLAB New/Saved File Chooser](image)

Figure 3-8. INDISLAB New/Saved File Chooser.
If the user decides to open a formerly saved file, the GUI’s main screen opens with an extra window that allows one to find and open the existing file. This performs the same operation as the *Open* command in the *File* menu. At this point the user can find their saved file and perform revisions or run an analysis of the model.

In the main INDISLAB GUI window there are five tabbed panes that are all subsections of user input data (Figure 3-9). The first one is “Slab Properties”, which includes all pertinent required data about the concrete slab(s). This includes the modulus of elasticity and Poisson ratio of concrete as well as the thickness of the concrete slab. Also on this tab is a pull down menu in which the user has the option of choosing one of three available slab layouts: 3x3 (9 slabs), 2x1 (2 slabs), or 1 slab, which were chosen to correspond with the needs of INDOT. After the user makes a decision on the number of slabs to include he/she is prompted to input values for the slab dimensions. This is done by clicking a button, which introduces a new window with text boxes for the lengths and widths of the slab(s). Another button is also included on this screen that allows users to edit dimensions. Initially the “Edit Slab Dimensions” button is unable to be used. This button is grayed out and clicking it will result in no action. However after the “Input Slab Dimensions” button is originally clicked and data is entered into the subsequent screen, the input and edit buttons switch places. The input button becomes grayed out and inactive while the edit button becomes functional. The pull down menu also becomes unusable after the “Input Slab Dimensions” button is initially hit. The reason for two buttons is that initially the pull-down menu in which the user chooses the slab layout must be checked to determine the number of input windows required in the following “Slab Dimension” screen. The text boxes also must be initialized before the “Slab Dimensions” window appears. The “Edit Slab Dimensions” button takes the user to the “Slab Dimensions” screen and allows the user to edit both the dimensions and number of slabs in the model.
The “Slab Dimensions” window appears after the “Input Slab Dimensions” or “Edit Slab Dimensions” button is clicked in the “Slab Properties” tab (Figure 3-10). In this window the user is prompted to enter the width and length dimensions of the slab(s). The pull-down menu in the “Slab Properties” tab determines the amount of user input required. If the user picks either the 2x1 or 1 slab layouts, INDISLAB does not allow the user to input data into boxes such as the third slab length and width boxes. The background color of the boxes automatically changes from white to light gray. If the user decides that he/she wants to change the slab layout it can be done within this window. The user is given the option to change to one of the other two choices by pressing the corresponding button at the bottom of the window. By performing this option the user changes properties of some of the text boxes depending on the switch made. Once the user is finished, the “Done” button must be clicked. INDISLAB checks for valid input in all of the editable text boxes as explained in Section 3.2.4. If all data is valid then
INDISLAB saves the current values, returns to the main GUI screen and closes the “Slab Dimensions” window.

![Slab Dimensions Window](image)

**Figure 3-10. Slab Dimensions Window.**

The next tabbed pane in INDISLAB is “Reinforcement Properties”, which deals with user-input values regarding the dowel and tie bars connecting the slabs (Figure 3-11). This tabbed pane does not include any editable boxes until the user picks one of the slab layouts. For the one slab case none of the four boxes become editable. For the two-slab case, only boxes concerning dowel bars become editable. The two-slab case includes two slabs in the longitudinal direction and as a result there is no need for tie bars. Lastly, if the nine-slab layout is chosen all text boxes become editable. Required input for this screen is slab layout dependent and includes the elastic modulus and Poisson ratio for steel as well as the diameter for dowel and tie bars.
The “Loads” tabbed pane (Figure 3-12) allows the user to input all data concerning the loading. By using a pull down menu the user can choose -from one up to ten- the number of load cases he/she wants to include in the analysis. Accordingly, a list of load cases is displayed allowing two types of load for every load case: wheel loads and thermal load. Every type of load has a corresponding button with a check box to its left side. After the user makes a decision on the number of load cases they wish to include, he/she must check the type of load to be included for every load case. Any load case can have both wheel loads and thermal load or just one of them. Initially the wheel loads buttons and thermal load buttons are unable to be used. These buttons are grayed out and clicking them will result in no action. However, after the check boxes next to them are checked, these buttons become functional and clicking them allows the user to input loading data for the analysis.
Clicking one of the “Wheel Loads” button prompts the user to input the data related to the wheel loads for the corresponding load case. For example, when the user clicks the “Wheel Loads” button for load case number one the “Loads” tabbed pane becomes the “Wheel Loads” tabbed pane (Figure 3-13) with the text line “Wheel Loads for Load Case #1” indicating that the wheel loads for case number one is to be input.

The “Wheel Loads” tabbed pane (Figure 3-13) asks the user to input the number of wheel loads to be applied on the slab layout for the corresponding load case. This number corresponds to each wheel present on the slab layout. After this box is filled with a value, the “Input New Wheel Load Properties” button must be clicked in order to continue the process. If the number of loads is zero then clicking “Input New Wheel Load Properties” button performs no action. If the number of loads is an integer value greater than zero and less than or equal to ten, INDISLAB opens the “Wheel Loads” window (Figure 3-). If the number of loads is an integer value greater than ten, then
INDISLAB asks the user to enter a number less or equal to ten. Once the “Input New Wheel Load Properties” button has been initially pressed with a valid value it can no longer be pressed again. All input values must be accessed through the “Edit Existing Wheel Load Properties” button. This is due to the fact that pressing the “Input New Wheel Load Properties” clears all existing values and initiates all text boxes for the number of loads input by the user. The “Edit Existing Wheel Load Properties” button does not clear any of the properties but instead reposts the data input earlier. The return button allows the user to go back to the “Loads” tabbed pane (Figure 3-12).

![Figure 3-133. Wheel Loads Tabbed Pane.](image)

The “Wheel Loads” screen that is initiated through the “Wheel Loads” tabbed pane collects data about wheel loads present on the slab layout (Figure 3-4). Items included on this screen are wheel load pressure, tire footprint length and width as well as the longitudinal and transverse position of the load on the slab. Near the bottom of the screen a figure is drawn using Java’s Paint method. The Paint method allows the programmer to develop pictures. It was felt that including a figure of the tire footprint
would be a useful tool for first time users to understand what values need to be input into the text boxes. Below the figure are several buttons that allow users to perform various functions. If more than one load is present the user has the option to progress back and forth between different loads. Other options include the ability to delete/add loads. Once the user is done inputting all load information the “Done” button can be clicked. This saves all data that has been input into the window and closes it, returning the user to the Wheel Loads tabbed pane (Figure 3-13).

![Figure 3-14. Wheel Loads Window.](image)

Clicking a “Thermal Load” button in the “Loads” tabbed pane (Figure 3-12) prompts the user to input the data related to the thermal loading for a given load case. When the user clicks the “Thermal Load” button of a load case, the “Loads” tabbed pane becomes the “Thermal Load” tabbed pane (Figure 3-15). This tab allows the user to pick the type of thermal distribution and input corresponding properties. A pull-down menu appears at the top of the tab that allows the choice of two thermal distributions: linear, and non-linear. All boxes within the tab are not editable and have a light gray background if no thermal distribution has been chosen. If the “linear” option is selected,
one box of input is required: the temperature gradient. The second choice, “nonlinear”, requires input of temperatures at nine nodes as explained in Section 4.6.1. This allows users to define their own nonlinear distribution along the nine nodes present in the depth of the slab.

Figure 3-15. Thermal Load Tabbed Pane.
4 INDISLAB DEVELOPMENT – PREPROCESSING

4.1 Introduction

Once the INDISLAB’s GUI was complete, the next step programming step in the creation of INDISLAB was the development of the ANSYS input file of the jointed plain concrete pavement model. This includes the definition of every aspect of the model so that the finite element software, ANSYS, can analyze it. INDISLAB takes the user input information from the text boxes and uses this data to develop the model. INDISLAB is composed of several Java files to accomplish this task.

The model is defined through an input file that can be understood by ANSYS. This is done in the following way. After all text boxes in the GUI have data in them the user must hit the Run Analysis menu item in the Analysis menu. After this has been performed, INDISLAB commands the Java file, TestValues, to perform a check on the data input by the user. The purpose of this file is to make certain that numerical values have been entered into all text boxes in the GUI and to check if the signs of these values are correct. This is a general check to catch an error before the model begins to run. If this check was not performed and an erroneous value was entered, one could wait several hours for the model to run before the realization that something in the model input is wrong. If INDISLAB finds incorrect value(s) an "Error Box" window appears to inform the user. Otherwise INDISLAB begins the process to create an input file. INDISLAB goes to one of six Java files based on the chosen slab layout and the system of units. These Java files create the input file that uses commands that ANSYS can recognize. The Java file creates a file that uses the saved GUI filename and an extension of .dat. The .dat extension is one that ANSYS recognizes as an input file. After all commands
are written to the .dat file, the file is closed and the finite element program, ANSYS, is launched.

The input file defines every aspect of the model. It must first create the different layers, which include the slab(s) and subsurface. If the model contains more than one slab, dowel and tie bars must be created. Following this contact elements are added to define the behavior between the slab and subbase accurately. Boundary conditions and loads are then added to the model. At this point the model has been completely defined and information with respect to the nonlinear solution that is to be performed on the model is added to the input file. The input file is now complete and the analysis of the model in ANSYS can begin.

4.2 Slab and Subsurface Creation

The first step in the creation of the INDISLAB input file is to define and create the slab(s), subbase and subgrade. This operation is handled in INDISLAB by several Java files. The Java file that handles the creation of the ANSYS input file references Java files to create nodes and then create elements. These Java files generate a new file for the nodes of each layer and another file for the elements of the layers. The ANSYS input file uses these files. The input file includes commands that instruct ANSYS to read the node and element files. Before these nodes and elements can be read, the input file defines the type of elements and the material that make up each layer.

As mentioned earlier the creation of the nodes in the model is one of the first steps in the Java file that creates the input file for ANSYS. The nodes are first created for the concrete slab(s). After the creation of nodes for this layer, the Java file progressively creates nodes for each of the subsurface layers. The nodes in the subbase layer are created first and then the nodes for the subgrade layer(s) are defined next. The user has the option of picking either one or two subgrade layers. Correspondingly one or two layers of subgrade nodes are defined. Each layer of nodes is distinct in several ways and as a result several different Java files are produced in order to create each layer of nodes.
4.2.1 Slab Creation

For the concrete slab layer, the definition of the nodes is determined by several different factors. These factors include the number of slabs in the model and the length of the slab(s). The number of slabs helps determine the type of elements that will be used in the model and thus the number of nodes. For the one- and two-slab cases, 20-node brick elements are used. For the nine-slab case, eight of the nine slabs use 8-node brick elements and one slab uses 20-node brick elements. The difference between an 8-node and a 20-node brick element is the number of nodes and, thus, of degrees of freedom, as can be seen in Figure 4-1 and Figure 4-2.

Figure 4-1. 8-Node Brick Element  Figure 4-2. 20-Node Brick Element

For the nine-slab case it would have been ideal to use 20-node brick elements for every slab. However, by using 20-node brick elements the model became too large for a typical modern computer to run the ANSYS model. As a result it was decided to use 8-node brick elements. It was determined that since the user of INDISLAB would be placing wheel loads on the center slab more than on any other slab, it was beneficial to refine the mesh in this slab using 20-node brick elements. The layout of the nine slabs with the type of elements is shown in Figure 4-3.
For all slab layouts the node numbering for each concrete slab begins from the lower left-hand corner of each slab shown in Figure 4-4. The numbering begins in the positive x-direction, then through the negative z-direction. Once this layer is complete another layer is numbered after the increase of one increment in the positive y-direction. Each individual slab follows the same three steps of numbering shown in Figure 4-4. The nodes for each slab are separate from those of the surrounding slabs. This is because each slab is modeled to be independent of each other as they are when they are poured in the field. The front, left slab in the two- and nine-slab layouts is the first slab to have nodes created for it. For these layouts, the slab(s) to the right of the first slab are then numbered. If the nine-slab layout has been chosen, after the first row of slabs have had nodes defined, the slab behind the first numbered slab has its nodes defined next. After this slab the numbering proceeds to the slabs on the right. After this row is complete, the shoulder slabs have their nodes created in the same order. For the nine-slab case the Java file that creates the nodes that correspond to the 8-node mesh then returns to the center slab to define the extra nodes necessary to create 20-node brick elements.

As mentioned above the other factor that decides how the nodes are defined are the dimensions of the slab(s). Each slab has a pre-defined number of elements in them. An important decision was made with regards on whether to fix the number of elements in the slab or to fix the length, width and height of each element in the slab. Both choices provide the level of accuracy required in the finite element mesh of pavement slabs. Even with the use of the option to fix the number of elements in the slab this is possible with the knowledge that pavement slabs have similar lengths and widths. It was
determined that fixing the number of elements in each slab would avoid complexities in the long run over fixing the width, length and height of each element. The reasoning behind the above statement is discussed next.

![Figure 4-4. Order of Node Definition in One Slab](image)

Fixing the number of elements in each slab provided one benefit for the programmer that made it extremely attractive. This was that the creation of the input file would be much easier due to the fact that the placement of each node would be in a similar place for every model. For example the same node numbers exist on the bottom of the slab for each and every model created. As a result it was much easier to place the reinforcement in the slab, create contact elements, boundary conditions as well as place the loads. The definitions of the above portions of the model do not vary much from model to model. They are created with the knowledge that the same node numbers exist on the bottom of this slab as did in the previous model. The use of fixed element dimensions would cause INDISLAB to determine what node numbers existed in the slabs and subsurface before the creation of many portions of the model. The decision to use a fixed number of elements, however, generated its own set of problems as discussed next.

The main problem is the existence of tie bars within the nine-slab layout. In Indiana pavements tie bars are placed three feet from one another in the slab. This creates a problem with the definition of element increments in the x-direction within the slabs. If the decision had been made to include elements with fixed lengths this would have been a problem that would have been much easier to solve. The fixed lengths could have been made with regard to the three-foot spacing required. However the decision to use a fixed number of elements resulted in the necessity of deriving an algorithm to size
the elements so that the tie bars could be accurately placed. This algorithm is described in the following paragraphs.

Before any nodes have been created, INDISLAB determines the value of the increments at which the nodes will be defined. This is done in the x-, y- and z-directions. The y- and z-direction increments are very simple to define. Since the number of elements has been predefined in each of these directions, the increments are found by simply dividing the user-input dimension by the corresponding number of elements. Dowel bars are placed in the z-direction of the slab and are supposed to be placed every twelve inches. The number of elements in this direction permits their approximate placement in the slab. This is possible since the width of highway pavement slabs in Indiana is most often 12 feet.

The x-direction nodes are defined by several different increments, which enable the model to include tie bars at exactly three-foot spacing. The algorithm first determines the amount of tie bars that should be placed in the slab. Next, the first and last increments of the slab are determined from the lengths outside of the first and last tie bars and the division of this space into eight equal increments. INDISLAB then checks to make sure that the first and last tie bars do not intersect with the dowel bars. If the tie bars do not interfere with the dowel bars INDISLAB goes on to define the other increments, which are determined by the number of tie bars in the particular slab. An example of the different increments and how they are used to define the nodes of the slabs are shown in Figure 4-5. If the tie bars do coincide with the dowel bars, the number of tie bars in the slab is reduced by one. The first and last increments are then revised to include the extra space gained by the reduction of the tie bar. INDISLAB then defines the other increments, which are determined by the number of tie bars within the slab. An example of how the nodes are defined in the slab when the number of tie bars is reduced by one is demonstrated in Figure 4-6. This process to determine the number of nodes is used in both the one- and two-slab layout cases despite the nonexistence of tie bars within these slab(s). The reason for this is to make all slabs in the program uniform. This decision makes it possible to reuse the code programmed for the nine-slab case.
After the creation of the nodes in the slab(s), INDISLAB next calls on a Java file to create the elements in the slab(s). As explained earlier the elements created are either 20-node brick elements or a combination of 8-node and 20-node brick elements for the nine-slab layout. The 8-node element used in ANSYS is the SOLID45 element and was shown above in Figure 4-1. The 20-node element used in the ANSYS input file is the SOLID95 element shown previously in Figure 4-2. The combination of the elements in the nine-slab layout presents no compatibility issues due to the independent nature of the slabs.

Numerical testing and knowledge of typical slab dimensions helped determine the number of elements in each direction. Numerical experimentation with element dimensions in ANSYS revealed that an approximately one-foot dimension for the length and width of the element provided an accurate determination of key results. A finer mesh produced a much longer solution time but produced similar results. Tests showed that a finer mesh near the dowel bars was necessary to produce accurate results. As a result a finer mesh was included at the beginning and end of the x-direction in the slab(s).
this information it was then possible to determine a number of elements sufficient for an accurate analysis in the x- and z-directions. Based on typical slab lengths and widths it was determined that 15 elements be used for each slab in the z-direction and 28 elements be used in the x-direction. In the y-direction, the depth of the slab, a decision was made to include 8 elements. This decision was based on the need for the inclusion of enough elements to allow for an accurate thermal load distribution through the depth of the slab, which can be highly nonlinear. Had any fewer elements been used, such an accurate representation of the thermal gradients could not have been defined. The number of elements in the slab finite element mesh is shown graphically in Figure 4-7.

![Figure 4-7. Number of elements present in each direction of a slab.](image)

INDISLAB creates a file for elements that contains the element number, the nodes that define the element as well as the material and element type. The elements are defined in much the same order as the nodes. Beginning with the front, left-hand slab, elements are defined with the same methodology used for nodes (Figure 4-4). Within each slab as with the nodes, elements are defined starting in the lower, left-hand corner progressing right through the x-direction, back along the negative z-direction and finally upwards through the y-direction. Once all elements have been created for the slabs, the subsurface layers are defined next.

4.2.2 Subsurface Creation

The subsurface layers consist of the subbase and one or two layers of subgrade depending on how the user defined the model. The subsurface layers are defined differently than the slab layer. This is due to the fact that each of these layers is one
complete layer without any separation, whereas the slabs above are separate entities. As a result less nodes are used to define these layers. In these layers 8-node brick elements are used due to the fact that INDISLAB is developed to accurately obtain the stresses in the slab(s). Thus, a refined model of the subgrade/subbase would not be justifiable.

Nodes are defined in a different way than those in the slabs, since no separations exist through the length, width and height of the subsurface layers. As a result nodes are defined across the entire x-direction distance, then the x-direction a row back in the negative z-direction is defined. This process continues until this whole layer has its nodes created. Then INDISLAB proceeds to create more nodes by the decrease of an increment in the negative y-direction and continues to fill this layer until it has been completed as explained above. This is continued until all nodes in every layer are defined. The number of layers in the negative y-direction depends on the particular subsurface layer. The number of elements in the y-direction for the subbase is two. Once again, this number was selected by performing a number of numerical tests. As mentioned above the number of subgrade layers is analysis dependent. As a result if one subgrade layer is chosen there are five elements in this direction. If two are chosen the first subgrade layer contains two elements and the second subgrade layer contains three elements. The distribution of elements in the y-direction is demonstrated in Figure 4-8 and Figure 4-9. The same nodes on the bottom of the subbase layer are also used to define the top of the first subgrade layer. Even though these two layers are made of different material properties they are still connected to one another. As a result it is necessary to define them by use of the same nodes.

<table>
<thead>
<tr>
<th>Slab: 8 Elements</th>
<th>Slab: 8 Elements</th>
</tr>
</thead>
<tbody>
<tr>
<td>Subbase: 2 Elements</td>
<td>Subbase: 2 Elements</td>
</tr>
<tr>
<td>Subgrade: 5 Elements</td>
<td>Subgrade Layer 1: 2 Elements</td>
</tr>
<tr>
<td></td>
<td>Subgrade Layer 2: 3 Elements</td>
</tr>
</tbody>
</table>

Figure 4-8. Elements in y-direction for One Subgrade Layer.  

Figure 4-9. Elements in y-direction for Two Subgrade Layers.
The number of elements in the x- and z-direction remains the same as in the concrete slab layer. This is due to the fact that compatibility between the slab and subbase layer would not exist if different spacing were used, and the contact elements could not be properly defined.

Elements are then defined in a similar way as the nodes were defined for the subsurface layers. Different files are created by INDISLAB to define each of the layers separately. Once these files have been read into ANSYS, the first step of creating the model is complete.

4.3 Dowel and Tie Bars

Dowel and tie bars are the items added to the ANSYS input file after creation of the slab and subsurface layers. These are added to only the nine-slab layout. The dowel bars are also added to the two-slab layout, however no tie bars need to be defined. The dowel bars are placed in the longitudinal direction (x-direction) whereas tie bars are placed in the transverse direction (y-direction) as shown in Figure 4-10.

![Figure 4-10. Placement of dowel and tie bars in 9-slab layout.](image)

The dowel bars are the first reinforcing bars to be added to the model. Before they can be added to the slabs several constants must be defined. A new material, steel, and its properties must be defined. After this takes place constants for the elements must be defined. Two types of elements are used to define dowel bars: the COMBIN14 and
BEAM4 elements. The COMBIN14 element is a spring-damper element that is used to act as a connection between the concrete slabs and the dowel bars. This is intended to model the effect of concrete crushing. For this element a spring constant must be defined for the dowel bars. For most nodes of the dowel bar the spring is only present in the y-direction, however at one end of the dowel bar springs are present in the x-, y- and z-direction. It is present in the x- and z-directions to help restrain the bar from moving in these directions. This task is accomplished by using a high spring stiffness value of 10,000,000 k/in. In the y-direction the spring constant is defined using values obtained from Porter (2001). The value of the spring constant depends on the diameter of the bar.

Shown in Table 4-1 are spring constant values for typical diameters of dowel bars. These values must be multiplied by the tributary area to calculate the true spring constant as shown below:

\[ k = k_o \cdot x_{inc} \cdot d \]  

(4-1)

where \( k \) is the spring constant (lb/in), \( k_o \) is the value from Table 4-1 (lb/in\(^3\)), \( x_{inc} \) is the length of the dowel bar between nodes, and \( d \) is the diameter of the bar. The BEAM4 element represents the actual dowel bar. Constants that must be defined for this element include the area of the bar and moments of inertia. Figure 4-11 represents a typical dowel bar and its connections to slab nodes through the spring elements. It should be noted that the incompatibility between the beam element used for the dowel bars and the spring elements used in the connection with the slab was resolved by refining the mesh sufficiently. In other words, the dowel bar was modeled by a sufficient number of beam elements instead of a single beam element.
INDISLAB creates dowel bars in the following way. When the node file is created for the slabs, several pieces of information are saved into a file for reference when creating commands for the dowel bars in the input file. This file has a .DOW extension and contains information that includes spacing of elements to help define constants and new nodes. Once this information is read and the constants for the dowel bar elements have been defined, new nodes are created for the beam elements. These nodes are created at the same position as existing nodes in the slab. In Indiana the dowel bars are about 20", i.e. 10" inside each of the two slabs that it connects. Since user-input slab dimensions can vary, it was important to determine how many nodes on either side of the slab joint needed to be included in the creation of dowel bars. Fourteen slabs with sizes that vary from 12 to 21.75 feet were created to determine how many nodes gave a proper estimate of the predefined 10" length. The locations of the third, fourth and fifth nodes from the edge of each slab in the x-direction of the slab were compared. After the results were viewed it was determined that four nodes would give the best estimation for

<table>
<thead>
<tr>
<th>Bar Diameter</th>
<th>Stiffness Constant (lb/in^3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.75</td>
<td>1,200,000</td>
</tr>
<tr>
<td>1</td>
<td>1,500,000</td>
</tr>
<tr>
<td>1.25</td>
<td>1,800,000</td>
</tr>
<tr>
<td>1.5</td>
<td>2,100,000</td>
</tr>
<tr>
<td>1.75</td>
<td>2,400,000</td>
</tr>
</tbody>
</table>
two reasons: the average value was closer to 10" and the largest difference value was similar to the 3 nodes as shown in Table 4-2.

Knowing this, four new nodes are created on one side of the slab and three in the other slab. This is due to the fact that one node from each slab coincides with each other thus making it necessary to create only one new node in that same location. After each new node is created INDISLAB creates the spring element in the y-direction between the new node and the existing node(s). If the node is at the end of the dowel bar, spring elements are also created for the x- and z-directions. After all spring elements have been created the dowel bar beam elements are created using the newly defined nodes. Once this has been completed the dowel bar for that particular location is complete and is as shown in Figure 4-11.

Table 4-2. Node Analysis Results for Dowel Bars.

<table>
<thead>
<tr>
<th>Slab Length</th>
<th>Nodes</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>12.00</td>
<td>9</td>
<td>13.5</td>
<td>18</td>
<td></td>
</tr>
<tr>
<td>12.75</td>
<td>6.7</td>
<td>10</td>
<td>13.1</td>
<td></td>
</tr>
<tr>
<td>13.50</td>
<td>6.7</td>
<td>10</td>
<td>14.2</td>
<td></td>
</tr>
<tr>
<td>14.25</td>
<td>6.8</td>
<td>10.1</td>
<td>13.5</td>
<td></td>
</tr>
<tr>
<td>15.00</td>
<td>9</td>
<td>13.5</td>
<td>18</td>
<td></td>
</tr>
<tr>
<td>15.75</td>
<td>6.7</td>
<td>10</td>
<td>14.2</td>
<td></td>
</tr>
<tr>
<td>16.50</td>
<td>6.7</td>
<td>10</td>
<td>15.7</td>
<td></td>
</tr>
<tr>
<td>17.25</td>
<td>6.7</td>
<td>10.1</td>
<td>13.4</td>
<td></td>
</tr>
<tr>
<td>18.00</td>
<td>9</td>
<td>13.5</td>
<td>18</td>
<td></td>
</tr>
<tr>
<td>18.75</td>
<td>6.7</td>
<td>10</td>
<td>13.1</td>
<td></td>
</tr>
<tr>
<td>19.50</td>
<td>6.7</td>
<td>10</td>
<td>14.2</td>
<td></td>
</tr>
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<td>13.4</td>
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<td>13.5</td>
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</tr>
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<td>Average</td>
<td>7.4</td>
<td>11.0</td>
<td>15.2</td>
<td></td>
</tr>
<tr>
<td>Largest Difference</td>
<td>3.3</td>
<td>3.5</td>
<td>8</td>
<td></td>
</tr>
</tbody>
</table>
Tie bars are created in much the same way as the dowel bars. Tie bars also use COMBIN14 and BEAM4 elements. Constants are defined in the same way. The only differences may be the diameter and the increment between elements (since tie bars are defined in a direction perpendicular to that of the dowel bars). As before the first step is to create new nodes. Tie bars are 18” (1.5 feet) on either side of the slab or 36” (3 feet) total in length. Once again it was necessary to see how many nodes to include for each slab. Several different slab sizes were tested for both regular and shoulder slabs to determine the correct amount of nodes to use. As seen in Table 4-3 the location of the third node from the edge of the slab in the x-direction in each slab is closer to the correct location than that of the fourth or fifth node. Once the new nodes were created, spring elements are added between the existing and newly created nodes. For tie bars, springs were included in both the x- and y-directions for every node. Springs were included in the z-direction for the end node to fix the tie bar in this direction. After the springs were created the beam elements were added between the new nodes to create the tie bars. An example tie bar is shown in Figure 4-12.

Table 4-3. Node Analysis Results for Tie Bars.

<table>
<thead>
<tr>
<th>Shoulder Slab Width (feet)</th>
<th>Nodes</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>12</td>
<td>18</td>
<td>24</td>
<td></td>
</tr>
<tr>
<td>3.75</td>
<td>15</td>
<td>22.5</td>
<td>30</td>
<td></td>
</tr>
<tr>
<td>4.5</td>
<td>18</td>
<td>21</td>
<td>24</td>
<td></td>
</tr>
<tr>
<td>5.25</td>
<td>21</td>
<td>31.5</td>
<td>42</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>24</td>
<td>36</td>
<td>48</td>
<td></td>
</tr>
<tr>
<td>Average</td>
<td>18</td>
<td>27</td>
<td>36</td>
<td></td>
</tr>
<tr>
<td>Largest Difference</td>
<td>6</td>
<td>18</td>
<td>30</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Regular Slab Width (feet)</th>
<th>Nodes</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.75</td>
<td>14</td>
<td>21</td>
<td>28</td>
<td></td>
</tr>
<tr>
<td>9.5</td>
<td>15.2</td>
<td>22.8</td>
<td>30.4</td>
<td></td>
</tr>
<tr>
<td>10.25</td>
<td>16.4</td>
<td>24.6</td>
<td>32.8</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>17.6</td>
<td>26.4</td>
<td>35.2</td>
<td></td>
</tr>
<tr>
<td>11.75</td>
<td>18.8</td>
<td>28.2</td>
<td>37.6</td>
<td></td>
</tr>
<tr>
<td>12.5</td>
<td>20</td>
<td>30</td>
<td>40</td>
<td></td>
</tr>
<tr>
<td>13.25</td>
<td>21.2</td>
<td>31.8</td>
<td>42.4</td>
<td></td>
</tr>
<tr>
<td></td>
<td>14</td>
<td>22.4</td>
<td>33.6</td>
<td>44.8</td>
</tr>
<tr>
<td>-----</td>
<td>------</td>
<td>------</td>
<td>------</td>
<td>------</td>
</tr>
<tr>
<td></td>
<td>Average</td>
<td>18.2</td>
<td>27.3</td>
<td>36.4</td>
</tr>
<tr>
<td></td>
<td>Largest Difference</td>
<td>4.4</td>
<td>15.6</td>
<td>26.8</td>
</tr>
</tbody>
</table>

It should be noted that whereas tie bars have springs in the x- and y-direction of every node, dowel bars only had springs in the y-direction for every node. This is due to the difference in the functionality of these two types of bars. Dowel bars are smooth bars and have grease applied to them so that friction between the bar and the slab is minimized. As a result the purpose of the bar is to solely transfer shear loads. This prompts the slab to crack underneath the saw joints instead of elsewhere in the slab. After the slab cracks beneath the saw joint, the dowel bars also help to keep the two slabs leveled with one another. Tie bars are not smooth like dowel bars but include grooves. This allows for friction and thus axial load transfer. Therefore, in this case there is a need for springs in the x-direction as well as the y-direction.

![Figure 4-12. Typical Tie Bar.](image)

4.4 Contact Elements

After the dowel and tie bars have been added to the ANSYS input file, contact elements are defined. Contact elements model the interaction between the concrete slab(s) and the first subsurface layer, the subbase. For this analysis rigid-to-flexible contact was used. In this type of contact only the rigid surface can penetrate the flexible surface (ANSYS 5.7.1 Online Help). The rigid surface must have the friction coefficient defined for the material in order for the contact elements to work properly.
A series of several commands must be added to the ANSYS input file to properly define the contact elements. The first step is to create the nodes and elements of the model. This is done previously in the input file where nodes and elements were created for the slab(s) as well as all subsurface layers. The next step is to determine where a contact pair is warranted, which in this case it is between the slab and subbase layers. Due to the fact that the node numbering is similar for every model of the same slab layout it is possible to know every node number that is present on both the slab and subbase contact surfaces. The next step is to determine which layer is the contact surface and which is the target surface. The target surface is the rigid surface and can penetrate the contact surface (ANSYS 5.7.1 Online Help). In this case the rigid surface is the concrete slab(s) surface. The contact surface is the deformable surface and relates to the subbase layer, as shown in Figure 4-13.

![Figure 4-13. Two types of surfaces required for contact elements.](image)

After this preliminary work is completed, definitions of the elements and their respective surfaces can begin. First it is necessary to define the target element and its surface. In ANSYS the target element is TARGE170. Once this element has been introduced and defined, the input file selects all nodes belonging to the proper surface. Next the ANSYS command, ESURF, is used to generate the TARGE170 elements along this surface. The same process is performed for the contact surface. For this surface however the CONTA174 element is introduced and defined. Nodes of the contact surface are then selected. The ESURF command is once again used to generate the contact elements among the selected nodes. At this point the contact elements have been properly defined and are a part of the model.
4.5 Boundary Conditions

Boundary conditions are placed on the model in order to ensure that it behaves properly in the analysis and can represent an actual pavement section accurately. INDISLAB places boundary conditions on the model in several positions in the subsurface. The slab has no boundary conditions so that it can act realistically accordingly to the loads applied to it. The slab has a gravitational weight applied to it and is connected to the subsurface through the contact elements. Without boundary conditions the subsurface is free to move around. As a result it is necessary to add restraints to it. Under a highway slab, the subsurface exists infinitely in all directions and cannot move as a rigid body in any direction. Adding boundary conditions in the ANSYS model helps to represent the actual behavior.

The boundary conditions are added in a couple of places that include the sides of the subsurface layers as well as the bottom layer of nodes located on the underside of the subgrade section. The sides of the subsurface layers are fixed in the direction perpendicular to the particular side of the node. The deepest layer of nodes is fixed in all directions to prevent any movement of the subsurface. This is accomplished in the input file by using a command to restrain a selected set of nodes in a particular direction. Since the same node numbers exist in the same place for every model of the same slab layout this is a simple task. This is represented visually in Figure 4-14.

![Figure 4-14. Boundary Conditions](image-url)
4.6 Loading

The user can choose from several loading options in INDISLAB. The loading is entered by means of load cases. From one up to ten load cases can be included. A load case is set of loads applied to the slab(s), which can include both wheel loads and thermal load or just one of them. As explained in Chapter 3, for each load case, the user is allowed to place any number of wheel loads on the slab(s), and is also allowed to apply a linear or non-linear thermal distribution. The Java file that creates the ANSYS input file reads the number of load cases included, then checks to see which loads the user wants applied in every load case and correspondingly includes them in the input file.

4.6.1 Thermal Loads

First, the thermal load sequence is discussed. In any load case, if the user does not check in the GUI the check box corresponding to thermal load, the Java file that creates the input file does not include any thermal loading commands in the file.

If the linear option is chosen, INDISLAB distributes the thermal distribution defined by the user to the concrete slab. Due to the fact that for each slab layout the nodes will again be present in the same locations, INDISLAB just needs to determine what thermal load to place at each layer. Once this is done a series of commands defines which loads correspond to which layer of nodes. This is all that needs to be done to define the thermal loads for a linear temperature distribution.

For the nonlinear thermal distribution option the user is responsible for defining the temperatures corresponding to each layer of nodes for an 8-node brick element. For 20-node brick elements a linear thermal distribution between the two edge nodes is used to determine the temperature value of the mid-side node. The reason for choosing this linear thermal distribution within the quadratic element is that strains (stresses) are directly related to the thermal gradient. Quadratic elements approximate the strain (stress) field linearly. So for compatibility the thermal gradient should vary linearly.
Once all values have been determined, commands are then written to the input file assigning the values to the corresponding layer of nodes. Once this is complete the nonlinear thermal loading distribution has been assigned to the slab and is complete.

4.6.2 Wheel Loads

Defining wheel loads involves a more intricate process than that of thermal loads. If the user specifies that wheel loads are present on the slab(s) and should be analyzed with the model, INDISLAB calls a Java file to determine where to place such loads. Since the slab dimensions are variable as are the placement of the wheel loads on the slab, it must be determined on which elements and nodes that the wheel load has been placed on. Once this is accomplished the load is appropriately distributed to the element nodes using work equivalency, as discussed below.

The WheelFinder java file first determines the locations of the boundaries of the tire footprint. These locations are stored as temporary variables in the INDISLAB program. These variables are then sent to an algorithm that determines the slab(s) in which the load is located on. Once this has been determined these variables are sent to another algorithm to determine which element(s) the load lies on top of. This is done by checking the tire footprint boundaries against the boundaries of the elements. If the tire footprint is found to lie on top of the particular element, this element's top four or eight node numbers, depending on if it is an 8- or 20-node brick element, are recorded by INDISLAB. Also recorded is the element's position and sizes, as are the tire footprint's position, size and load. This information is then forwarded to yet another algorithm. This algorithm computes the work equivalent nodal loads from the distributed pressure acting on the particular element. An example of this can be seen in Figure 4-15. The equations derived to determine the work-equivalent nodal loads for both the 8- and 20-node brick elements are provided in Appendix A and B.

Once this transformation is complete, WheelFinder writes the node number as well as the value of the nodal force into a file with a .ntxt extension. After this information has been recorded for a particular element, INDISLAB searches for other
elements that the tire footprint may touch. The above process is repeated for any other elements found to be present underneath the tire pressure. After this tire footprint has been completed INDISLAB repeats the process for other tire footprints, if the user has entered more than one.

The .ntxt file is later read by the Java file creating the ANSYS input file. This Java file writes the command that assigns the nodal force to the node number. Once this has been completed all input commands concerning loading aspects of the model have been finished.

![Transformation of Pressure to Nodal Loads](image)

Figure 4-15. Transformation of Pressure to Nodal Loads.

4.7 Solution Commands

After the loads have been applied to the model, all the pertinent information has been included in the ANSYS input file. After this the only commands left to include in the input file are those that concern the type of solution to be performed on the model. In INDISLAB a static, nonlinear solution is used. The number of nonlinear step increments is also pre-defined as ten steps. However, within ANSYS the number of steps may be increased or decreased automatically during runtime. This would depend on how quickly the solution is converging. The minimum number of steps is defined as five and the maximum is 100. These options allow for accurate results to be determined.
After these options have been defined and included in the input file, INDISLAB closes the file and launches the ANSYS software. An extra window pops up from INDISLAB that alerts the user of the command that needs to be entered into ANSYS for the program to read the input file created by INDISLAB.

![Figure 4-16. ANSYS Opening Window.](image)

The first window that appears after ANSYS starts asks the user to input the directory in which the input file is included as well as to define the memory requirements that ANSYS may use as shown in Figure 4-16. After this information has been entered the user hits the "Run" button. ANSYS displays all information in the boxes from the previous time ANSYS was run. ANSYS then enters the main screen of the program as shown in Figure 4-17 and enters the input command specified by the INDISLAB pop-up window. Once the ANSYS command has been entered the model is read into ANSYS and analysis of the model begins. From this point on the user does not interact with the program until the analysis is completed. Analysis time varies and depends on the slab layout chosen and the type and number of loads applied. For a nine-slab layout the analysis takes several hours. For the one- and two-slab layouts, the analysis can take
from one to four hours to complete. After completion of the analysis, the user can view the results through the ANSYS postprocessing tools. The detailed discussion on the postprocessing capabilities of INDISLAB is provided in Chapter 5.

Figure 4-17. ANSYS Main Screen and INDISLAB Input File Command Box.
5 INDISLAB DEVELOPMENT – POSTPROCESSING

5.1 Introduction

The third and final step in the development of the INDISLAB program is the addition of postprocessing capabilities. Postprocessing is the portion of the program that analyzes the model results. After the solution in the finite element program, ANSYS, is complete, INDISLAB can start to perform some of the postprocessing functions. Most of the postprocessing functions are handled by ANSYS with a little help from the INDISLAB software.

There are two main parts of postprocessing: display of results and damage model analysis. Display of results can be done either graphically or numerically in ANSYS. INDISLAB gives the user the option of creating an input file that creates the graphical results in the form of 3-D contour plots and path plots. The user can display numerical results in ANSYS through the input of several commands. Damage analysis is done through INDISLAB after ANSYS has analyzed the model and the output is available. These are the main postprocessing commands embedded in INDISLAB.
5.2 Numerical Results in ANSYS

One capability of ANSYS is the ability to view numerical results in the postprocessing phase of the model. Numerical results include the stress/strain/deflection results for each or a selected number of nodes/elements. The maximum absolute value of the response is also presented at the conclusion of the list of values. This option enables the user to find particular values anywhere in the slab.

To display the numerical results the user must proceed through several menus in ANSYS to determine the type of result desired and whether a nodal or elemental solution is desired. From the Main Menu on the leftmost side of the ANSYS screens, the user must begin by clicking the General Postprocessor item. In the General Postprocessor menu, the List Results item must be clicked and from here the user must either click the Nodal Solution or Element Solution menu item as shown in Figure 5-1. Clicking either

![Figure 5-1. Menu Selections to Retrieve Nodal Solutions.](image)
the *Nodal* or *Element Solution* items causes a new window to appear. This window allows the user to pick the type of result to be presented as shown in Figure 5-2. A sample window is shown in Figure 5-3.

Another option from the *List Results* menu that can be chosen is the *Sort Nodes* and *Sort Elements* menu items. These two items permit the user to sort the results in either ascending or descending numerical order. This can be helpful in that it allows one
to find the locations with the most critical stress values. After one of the two menu items is clicked a window appears similar to the one in Figure 5-4. One of the only differences is the choice of having the results appear in ascending or descending order as shown in Figure 5-4.

Figure 5-4. Sort Nodes Window

5.3 Graphical Postprocessing Options

INDISLAB develops input files that the user can enter into ANSYS to display the results graphically. The user can pick from a multitude of options including 3-D contour plots and path plots. The 3-D contour plots show the result desired in relation to the whole model. Path plots make a cut through a cross-section of the slab(s). The results are plotted on two axes.

Several deflection and stress 3-D contour plots can be developed through INDISLAB. INDISLAB creates an input file that consists of several lines that perform the same operations as clicking several menu items in ANSYS. In order to create a
similar graph in ANSYS the user must first click the *General Postprocessor* item from the *Main Menu*. From the new menu that appears the *Plot Results* menu item must be clicked. From this menu the *Nodal* or *Element Solution* can be chosen as shown in Figure 5-5. After this menu item is clicked a window appears that presents the user with an option of the type of result desired such as stress, strain, deflections, etc. This window is shown in Figure 5-6. INDISLAB creates input files for stress and deflection results. Deflection input files can be generated for results in the x-, y- and z-directions. Stress input files can be generated for x-, y- and z-directions as well as for the first principal stress.

![Menu Selections to Retrieve Graphical Results.](image)

Figure 5-5. Menu Selections to Retrieve Graphical Results.
The INDISLAB menus with these options are presented in Figure 5-7. A 3-D contour plot of deflections in the y-direction can be seen in Figure 5-8.
The other type of graphical postprocessing available through an input file developed by INDISLAB is a path plot. Path plots take a cross-section of the slab layout in either the transverse or longitudinal direction as shown in Figure 5-9. Once the user decides upon the direction they must next choose where in the depth of the slab they want the path plot to be. Three choices are given in INDISLAB: the top, center, and bottom of the slab(s). These positions within the depth of the slab are shown in Figure 5-10. These plots are much more difficult to create in ANSYS due to the fact that they are position-dependent. The INDISLAB input file can easily create the necessary commands due to the fact that the nodes are in the same position for each type of layout. As a result the input file defines the path using the node numbers. The menu in INDISLAB in which the path plot input files for ANSYS are created is shown in Figure 5-11. An example path plot is shown in Figure 5-12.
Figure 5-9. Longitudinal and Transverse Directions for each Slab Layout.

Figure 5-10. Path Locations through the Depth of the Slab.
After the user decides the type of graphical postprocessing desired and a menu item is clicked, INDISLAB generates the ANSYS input file. After this is complete, an "Input File Command" box appears as shown in Figure 5-13 and alerts the user to the
command that must be entered into ANSYS in order to view the plot. Within ANSYS the user can change the view in order to look at specific areas of a 3-D contour plot. This can be done through a click of the PlotCtrls menu at the menu bar at the top of the ANSYS screens and then click the Pan, Zoom, Rotate menu item that results in the “Pan-Zoom-Rotate” window shown in Figure 5-14. From within this window the user can choose to view the 3-D contour plot in any way desired. The user can rotate, zoom in or out and pick several pre-defined viewing options in order to look at the model.
5.4 Damage Analysis in INDISLAB

INDISLAB includes two options for doing damage analysis and help to predict the service life of a jointed plain concrete pavement through damage accumulation. One of them allows the user to model the damage and accumulate it both linearly and nonlinearly as explained in Chapter 6 of Part 1 of this report. In the second option the damage is computed by using two damage models: the Darter model and the Rigid Pavement Performance (RPPR) model. The Darter model determines the number of stress applications to failure whereas the RPPR model determines the percentage of cracked slabs. The Darter model is much more simple and straightforward than the RPPR model, and it is described first.

The Darter model consists of a single formula with four different variables, which has the following form:

\[
\log N_f = f_1 - f_2 \left( \frac{\sigma}{S_c} \right)
\]

(5-1)

where \(N_f\) is the number of repetitions to failure, \(f_1\) and \(f_2\) are the Darter coefficients, \(\sigma\) is the flexural stress and \(S_c\) is the Modulus of Rupture of concrete. The constants, \(f_1\) and \(f_2\) have recommended values of 16.61 and 17.61 respectively (Huang, 1993).

In INDISLAB this result can be found by clicking on the "Damage Models" button of the “Damage Analysis” tabbed pane. The top portion of this pane is dedicated to the computation of the Darter model. The user needs to input values regarding the modulus of rupture of the concrete. The flexural stress is provided from the model results in ANSYS. The user also has the option of changing the Darter coefficients. Initial values of 16.61 and 17.61 are predefined in the text boxes. After all input has been entered into the text boxes the user hits the "Compute Darter Model" button. At this point INDISLAB checks for valid input and if this check is passed, the result is computed. The result is displayed in a "Results" window that appears after the “Compute Darter Model” button is clicked.
The RPPR model involves a much lengthier process to determine the result than the Darter model. Several steps must be followed beginning with the first, which is to determine the effective thickness of the slab using the following formula:

\[ h_e = \sqrt[3]{h_{PCC}^3 + \frac{E_{base} h_{base}^3}{E_{PCC}}} \]  

(5-2)

where \( h_e \) is effective thickness, \( h_{PCC} \) is the Portland Cement Concrete (PCC) slab thickness, \( h_{base} \) is the subbase thickness, \( E_{PCC} \) is the elastic modulus of PCC and \( E_{base} \) is the elastic modulus of the subbase material.

The second step is to calculate the load stresses in the slab with the following equation:

\[ \sigma_{LOAD} = f_{LTE} \sigma_{fe} \]  

(5-3)

where \( \sigma_{LOAD} \) is the computed stress due to the load, \( f_{LTE} \) is the edge support reduction factor computed in Equation 5-7, 5-8 or 5-9 and \( \sigma_{fe} \) is the free edge stress calculated by Equation 5-10.

Before \( f_{LTE} \) and \( \sigma_{fe} \) can be computed a few parameters need to be determined. The radius of relative stiffness, \( l \), is calculated by the following formula:

\[ l = 4 \left[ \frac{E_{PCC} h_e^3}{12(l - \mu^2_{PCC})} \right] \]  

(5-4)

where \( h_e \) is effective thickness, \( \mu_{PCC} \) is Poisson’s ratio for PCC and \( k \) is the modulus of subgrade reaction.

The effective wheel radius, \( a_e \), can be calculated as follows:

\[ a_e = a \left[ 0.909 + 0.339485 \left( \frac{s}{a} \right) + 0.103946 \left( \frac{a}{l} \right) - 0.017881 \left( \frac{s}{a} \right)^2 - 0.045229 \left( \frac{a}{l} \right) \left( \frac{s}{a} \right)^2 \right] \]  

(5-5)

\[ + a \left[ 0.000436 \left( \frac{s}{a} \right)^3 - 0.301805 \left( \frac{s}{a} \right) \left( \frac{a}{l} \right)^3 + 0.034664 \left( \frac{s}{l} \right)^2 + 0.001 \left( \frac{s}{a} \right)^3 \left( \frac{a}{l} \right) \right] \]

where \( a=3.883019 \) in., \( s \) is the tire spacing and \( l \) is the radius of relative stiffness computed in Equation 5-4.

The nondimensional wheel radius, \( a_i \), can be computed as:
where \( a_e \) is the effective wheel radius determined by Equation 5-5 and \( l \) is the radius of relative stiffness computed in Equation 5-4.

The edge support factor, \( f_{LTE} \) depends on the type of edge support of the slab. If no edge support is present, then:

\[
f_{LTE} = 1
\]  

(5-7)

Otherwise, if the slab has a tied PCC shoulder, then:

\[
f_{LTE} = \frac{1}{1 + LTE_{\sigma}}
\]  

(5-8)

where \( LTE_{\sigma} \) is the stress load transfer efficiency that has a value between 0.05 and 0.30. If the edge support present is a widened slab, then:

\[
f_{LTE} = 0.454147 + \frac{0.013211}{D_i} + 0.386201 \left( \frac{a_i}{D_i} \right) - 0.24565 \left( \frac{a_i}{D_i} \right)^2 + 0.053891 \left( \frac{a_i}{D_i} \right)^3
\]  

(5-9)

where \( a_i \) is the nondimensional radius calculated by Equation 5-5 and \( D_i \) is equal to 42/1.

The free edge stress, \( \sigma_{fe} \), is calculated as follows:

\[
\sigma_{fe} = \frac{3(1 + \mu_{PCC})^P}{\pi(3 + \mu_{PCC})h_e^3} \left[ \ln \left( \frac{E_{PCC} h_e^3}{100k a_e^4} \right) + 1.84 - \frac{4\mu_{PCC}}{3} + \frac{1 - \mu_{PCC}}{2} + 1.18(1 + 2\mu_{PCC})a_i \right]
\]  

(5-10)

where \( P \) is one half of an axle load.

The third step is to compute an array of temperature stresses. In this step, 22 temperature gradients defined by Khazanovich and Yu, which range from -8°F to 34°F in 2°F increments are used (2001). The curling stress due to temperature due to each temperature gradient is as follows:

\[
(\sigma_{\text{TEMPERATURE}})_i = \frac{CE_{PCC} \alpha_{PCC} (\Delta T - T_s)}{2}
\]  

(5-11)

where \( (\sigma_{\text{TEMPERATURE}})_i \) is the computed curling stress, \( i \) is the array index number corresponding to the 22 temperature gradients, \( \Delta T \) is the actual temperature gradient through the slab, \( T_s \) is the temperature shift factor computed in Equation 5-12, and \( PCC \) is the
PCC coefficient of thermal expansion, and $C$ is the curling stress coefficient computed in Equation 5-13.

The temperature shift factor is computed as follows:

$$T_s = \xi \frac{h - 2}{h^3} + \psi$$ \hspace{1cm} (5-12)

where $T_s$ is the temperature shift factor, $h$ is the slab thickness and $\xi$ and $\psi$ are climatic specific parameters, 373.0878 and 5.734 respectively for a Wet-Freeze climatic zone.

The curling stress coefficient, $C$, is determined as follows:

$$C = 1 - \frac{2 \cos \lambda \cosh \lambda}{\sin 2\lambda + 2 \sinh \lambda \cosh \lambda} (\tan \lambda + \tanh \lambda)$$ \hspace{1cm} (5-13)

where $\lambda$ is:

$$\lambda = \frac{L}{l\sqrt{8}}$$ \hspace{1cm} (5-14)

where $L$ is the joint spacing.

The fourth step is to compute an array of combined temperature and load stresses as follows:

$$\left(\sigma_{\text{COMB}}\right)_i = \frac{h_{\text{PCC}}}{h_e} \left(\sigma_{\text{COMB-NOADJUST}}\right)_i$$ \hspace{1cm} (5-15)

where $\left(\sigma_{\text{COMB}}\right)_i$ is the array of combined temperature and load stresses and $\left(\sigma_{\text{COMB-NOADJUST}}\right)_i$ is the unadjusted array of combined load and temperature stresses as calculated in Equation 5-16:

$$\left(\sigma_{\text{COMB-NOADJUST}}\right)_i = \sigma_{\text{LOAD}} + R_i \left(\sigma_{\text{TEMPERATURE}}\right)_i$$ \hspace{1cm} (5-16)

where $\sigma_{\text{LOAD}}$ was computed in Equation 5-3 and $R_i$ is the curling stress reduction factor computed in Equation 5-17:
\[ R_i = 1.062 - 1575.68 \alpha_{PCC} \left( \Delta T - T_s \right) - 0.0000876 k - 0.01068 \frac{L}{l} + \\
387.317 \alpha_{PCC} \left( \Delta T - T_s \right) \frac{L}{l} + 1.17 \times 10^{-6} E_{PCC} k \alpha_{PCC} \left( \Delta T - T_s \right), \\
- 0.0181 E_{PCC} k \left[ \alpha_{PCC} \left( \Delta T - T_s \right) \right]^2 - \left[ 1.051 \times 10^{-8} E_{PCC} k \alpha_{PCC} \left( \Delta T - T_s \right) \left( \frac{L}{l} \right)^2 \right] \\
+ 1.84 \times 10^{-3} E_{PCC} k \left[ \alpha_{PCC} \left( \Delta T - T_s \right) \right]^3 - 17.487 \left( \frac{L}{l} \right)^2 \alpha_{PCC} \left( \Delta T - T_s \right), \\
+ 3.4351 \times 10^9 \left[ \alpha_{PCC} \left( \Delta T - T_s \right) \right]^3 + 8.697 \times 10^9 \left( \frac{L}{l} \right)^3 - 816396 \left( \frac{L}{l} \right) \left[ \alpha_{PCC} \left( \Delta T - T_s \right) \right]^3 \] (5-17)

If the above equation results in a value less than 0.42, 0.42 should be used instead.

The fifth step is to compute an array of the allowable load repetitions as follows:

\[(\log N)_i = 2.13 \left( \frac{\sigma_{COMB}}{MR_{28}} \right)^{-1.2} \] (5-18)

where \((\log N)_i\) is the allowable load repetitions and \(MR_{28}\) is the PCC modulus of rupture at 28 days.

The sixth step is the computation of an array of pass-to-coverage ratios. The pass-to-coverage ratio depends on the computed stress ratio:

\[ SR_i = \frac{MR_{28}}{\left( \sigma_{COMB} \right)_i} \] (5-19)

Then to calculate the pass-to-coverage ratio:

If \(SR_i < 0\), then:

\[ pc_i = 10,000 \] (5-20)

If \(SR_i > 1\), then

\[ pc_i = \left[ 4163 - 11486 \left( \frac{\sigma_{COMB}}{MR_{28}} \right) + 12599 \left( \frac{\sigma_{COMB}}{MR_{28}} \right)^2 - 49155 \left( \frac{\sigma_{COMB}}{MR_{28}} \right)^3 \right] (1-WL) + 2.6 \] (5-21)

Else, if \(0 \leq SR_i \leq 1\), then

\[ pc_i = \left[ 1402 - 1723 \left( \frac{\sigma_{COMB}}{MR_{28}} \right) + 77.94 \left( \frac{\sigma_{COMB}}{MR_{28}} \right)^2 - 12.04 \left( \frac{\sigma_{COMB}}{MR_{28}} \right)^3 \right] (1-WL) + 2.6 \] (5-22)
where $pc_i$ is the computed pc ratio and WL is a value representing the presence of a widened lane (WL=1 if widened lanes are present, otherwise WL=0).

The seventh step is to select an appropriate temperature gradient frequency distribution for each of the 22 gradients. This value is determined by rounding $h_e$, the effective thickness, to the nearest 0.25 in. Then based on the climatic region and the gradient these values can be found on the frequency distribution table in Khazanovich and Yu(2001).

The eighth step comprises of a computation of an array of actual traffic coverage values as follows:

$$n_i = \frac{f_ixCESAL}{pc_i}$$

(5-23)

where $n_i$ is the actual traffic value, $f_i$ is the frequency of the $i$th $\Delta T$ and CESAL is the cumulative ESALs on the PCC slab.

The ninth step is the computation of the cumulative damage as shown in the following equation:

$$FD = \sum_{i=1}^{22} \frac{n_i}{N_i}$$

(5-24)

where $n_i$ was calculated in Equation 5-23 and $N_i$ was computed in Equation 5-18.

The final step is to compute the percent of slabs cracked as follows:

$$Percent\ Slabs\ Cracked = \frac{100}{1+1.16FD^{-1.3}}$$

(5-25)

This process was developed by Lev Khazanovich and H. Thomas Yu as outlined in their journal paper “Modeling of Jointed Plain Concrete Pavement Fatigue Cracking in PaveSpec 3.0” (2001).

5.5 Conclusions

INDISLAB includes many powerful tools to help the user analyze and determine the results of the jointed plain concrete pavement model. INDISLAB makes use of the excellent postprocessing tools available in ANSYS. Through ANSYS one can view the
results numerically or graphically. This enables the user to gain a broad perspective of the results or to determine results within a specific portion of the model.

INDISLAB also includes damage analysis that allows the user to analyze the longevity of the pavement. The user has two options: (1) to compute the damage by using the Darter and RPRP models; and (2) to compute the linear and nonlinear cumulative damage, which is of particular importance when attempting to predict the service life of a pavement.
6 Summary and Conclusions

6.1 Summary

A need was recognized for a software tool that could accurately predict the behavior of jointed plain concrete pavements in Indiana. INDISLAB, a software tool developed to analyze highway plain concrete pavements, was designed specifically to address this need. It uses sophisticated three-dimensional finite element analysis techniques. Current software, KENSLABS and ILLI-SLAB, that perform this task are outdated and too simplified to correctly evaluate the behavior of JPC pavements.

KENSLABS is one of the oldest and most readily available pavement software that uses the finite element method. It was developed when DOS-based computers were popular but recent updates to the software have transformed it into a Windows-based program. However, a major update on the modeling techniques have not been introduced to the program since it was first developed. Currently it is possible for computers to handle more complex models than those used in KENSLABS.

ILLI-SLAB was developed a few years after KENSLABS and has also been continuously updated. As with the KNESLABS software, ILLI-SLAB takes many shortcuts in order to reduce computer processing time. ILLI-SLAB, however, does include slightly superior postprocessing tools. Yet it still does not take full advantage of computer capabilities available.

A comparison of the maximum and minimum deflections was made between KENSLABS, ILLI-SLAB and INDISLAB. A 9-slab layout was chosen with thermal and wheel loads applied to it. The results are summarized below in Table 6-1. The slabs in
the KENSLABS model lift completely off of the subbase. The most likely explanation for the poor results is the unrefined mesh in the slabs. The computer, which KENSLABS ran on, could not handle the maximum nine slabs with fifteen nodes in both the x- and y-directions. Instead nine slabs with five nodes in each direction was the largest model that could be run. As can be seen maximum deflection values for KENSLABS and ILLI-SLAB are both conservative. However, the ILLI-SLAB solution is much closer than the KENSLABS solution. Minimum deflection values are also much different. A percent error calculation was performed between the KENSLABS and ILLI-SLAB results, and the INDISLAB results.

<table>
<thead>
<tr>
<th></th>
<th>Minimum Deflection (in.)</th>
<th>Percent Error (%)</th>
<th>Maximum Deflection (in.)</th>
<th>Percent Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>KENSLABS</td>
<td>0.007597</td>
<td>139.6</td>
<td>0.1226</td>
<td>753.0</td>
</tr>
<tr>
<td>ILLI-SLAB</td>
<td>-0.004447</td>
<td>76.8</td>
<td>0.015456</td>
<td>7.5</td>
</tr>
<tr>
<td>INDISLAB</td>
<td>-0.019205</td>
<td>-</td>
<td>0.014372</td>
<td>-</td>
</tr>
</tbody>
</table>

The programming language used to develop INDISLAB, was Java. It was chosen for this project because it could be freely downloaded from Sun Microsystems’ Java website by anyone. Other benefits of Java include the ability to develop a GUI as well as a free online tutorial and help guide.

The finite element software chosen for use with INDISLAB was ANSYS, an industry respected finite element program. This has found to be a successful choice, since ANSYS works easily with Windows-based personal computers and includes excellent postprocessing capabilities.

A major goal of INDISLAB was to develop software that did not take the shortcuts current software packages do and produce an accurate model for jointed plain highway concrete pavements in Indiana. There are three parts that INDISLAB is composed of: the GUI, preprocessing, and postprocessing.

The GUI is the visible portion of INDISLAB. It includes text boxes for the user to enter input in regards to the model to be analyzed. This includes information about the concrete material properties and dimensions, dowel and tie bar properties, wheel and
thermal loading, subsurface material properties and dimensions, and postprocessing damage models. The GUI also acts as the control panel for INDISLAB. Included are save/open commands as well as the ability to initiate the preprocessing and control certain aspects of postprocessing in ANSYS.

The preprocessing portion of INDISLAB involves the creation of an input file that defines the model for use in the finite element analysis program, ANSYS. INDISLAB first checks to make certain valid data has been entered into the text boxes. If this check is passed, INDISLAB creates the input file with the user input values. At this point INDISLAB opens ANSYS and the user must enter a command so ANSYS can read the input file that defines the model and begins the analysis.

After the analysis is complete and a solution has been found, INDISLAB lets the user choose from several postprocessing options. This includes plots of the results. Three-dimensional and path plots are available to users of the program in conjunction with ANSYS.

The INDISLAB GUI is flexible enough to allow for several different configurations. The user can choose from three optional slab layouts: nine-slab layout (3 slabs x 3 slabs), two-slab layout (2 slabs x 1 slab) and a one-slab layout (1 slab x 1 slab). If the user chooses the nine- or two-slab layout, INDISLAB allows the user to enter data regarding bar reinforcement in the slabs. Other flexible parts of the program include the ability to choose the number of wheel loads present on the slab configuration and the type of thermal load: none, linear and non-linear. Users also have a choice of either one or two subgrade layers.

6.2 Conclusions

INDISLAB was developed as an easy-to-use interface for practicing engineers that makes use of 3D complex finite element techniques. The GUI was tested thoroughly and has been found to be problem free. The layout of the GUI is easy to understand and aesthetically pleasing. The INDISLAB program provides a simple process for engineers
to input information about a slab layout, analyze the model and view results of the solution to determine the durability of the slab(s).

Unlike KENSLABS and ILLI-SLAB, INDISLAB includes the ability to analyze pavements under nonlinear thermal loads. This is extremely attractive since it has been shown that temperature does vary nonlinearly throughout the depth of a jointed plain concrete slab. Also, unlike the previously developed software packages, INDISLAB is capable of accumulating damage nonlinearly. This is also quite important, since it has been shown that nonlinear accumulation of damage leads to a more accurate prediction of the service life of a pavement.

The main goal in the development of INDISLAB was to provide an easy to use, accurate tool for pavement analysis. The INDISLAB GUI is easy to use because it has the look and feel of typical Windows-based applications. The creation of a realistic model in ANSYS accomplished the task of making INDISLAB an accurate tool. The definition of the finite element model considers all the aspects that affect the most jointed plain concrete pavements and makes INDISLAB a software tool that can accurately predict the behavior and the service life of jointed plain concrete pavements in Indiana.
LIST OF REFERENCES

ANSYS 5.7.1 Help (2001), ANSYS, Inc.


“Java 2 Platform SE v1.3.1” (March 25, 2003), http://java.sun.com/j2se/1.3/docs/api, Sun Microsystems, Inc.


Appendix A: Work Equivalent Nodal Load for an 8-Node Brick Element

Shape Functions for a Quadrilateral Element:

\[ N_1 = \frac{(a - x) (b - y)}{4 \cdot a \cdot b} \]
\[ N_2 = \frac{(a + x) (b - y)}{4 \cdot a \cdot b} \]
\[ N_3 = \frac{(a + x) (b + y)}{4 \cdot a \cdot b} \]
\[ N_4 = \frac{(a - x) (b + y)}{4 \cdot a \cdot b} \]

Derivation of Work Equivalent Nodal Forces:
Basic Equation:

\[ p \left( \int \int N_x \, dx \, dy \right) \]

Nodal Forces:

Node 1

\[ F_i = - \frac{1}{16 a b} \cdot p \left( -4 x_2 y_2 b + 2 y_2 a x_2^2 + 2 x_2 y_2^2 b - y_2^2 x_2^2 + 4 x_2 a y_1 b - 2 y_1 a x_2^2 - 2 x_2 y_1^2 b + y_1^2 x_2^2 + 4 x_1 a y_2 b - 2 y_2 a x_1^2 - 2 x_1 y_2^2 b + y_2^2 x_1^2 - 4 x_1 a y_1 b + 2 y_1 a x_1^2 + 2 x_1 y_1^2 b - y_1^2 x_1^2 \right) \]
**Node 2**

\[
F_2 = -\frac{1}{16ab} * p * (-4x_2 ay_2 b + 2y_2 ax_2^2 - 2x_2 y_2^2 b + y_2^2 x_2^2 + 4x_2 ay_1 b - 2y_1 ax_2^2 + 2x_2 y_1^2 b - y_1^2 x_2^2 + 4x_1 ay_2 b - 2y_2 ax_1^2 + 2x_1 y_2^2 b - y_2^2 x_1^2 - 4x_1 ay_1 b + 2y_1 ax_1^2 - 2x_1 y_1^2 b + y_1^2 x_1^2)
\]

**Node 3**

\[
F_3 = -\frac{1}{16ab} * p * (-4x_2 ay_2 b - 2y_2 ax_2^2 - 2x_2 y_2^2 b - y_2^2 x_2^2 + 4x_2 ay_1 b + 2y_1 ax_2^2 + 2x_2 y_1^2 b + y_1^2 x_2^2 + 4x_1 ay_2 b + 2y_2 ax_1^2 + 2x_1 y_2^2 b + y_2^2 x_1^2 - 4x_1 ay_1 b - 2y_1 ax_1^2 - 2x_1 y_1^2 b - y_1^2 x_1^2)
\]

**Node 4**

\[
F_4 = -\frac{1}{16ab} * p * (-4x_2 ay_2 b - 2y_2 ax_2^2 + 2x_2 y_2^2 b + y_2^2 x_2^2 + 4x_2 ay_1 b + 2y_1 ax_2^2 - 2x_2 y_1^2 b - y_1^2 x_2^2 + 4x_1 ay_2 b + 2y_2 ax_1^2 - 2x_1 y_2^2 b - y_2^2 x_1^2 - 4x_1 ay_1 b - 2y_1 ax_1^2 + 2x_1 y_1^2 b + y_1^2 x_1^2)
\]
Appendix B: Work Equivalent Nodal Load for a 20-Node Brick Element

Derivation of Work Equivalent Nodal Forces:

Basic Equation:

\[ P_b = \int_A f^T b dv \]

where \( P_b \) = equivalent nodal forces, \( f \) = displacement shape function matrix, \( b \) = pressure loading matrix

\[
\begin{align*}
 f &= \begin{bmatrix} f_i & 0 & 0 \\ 0 & f_i & 0 \\ 0 & 0 & f_i \end{bmatrix} \quad (i = 1, \ldots, 8) \\

\end{align*}
\]

and

\[
\begin{align*}
 f_i &= \frac{1}{8} (1 + \xi_0)(1 + \eta_0)(1 + \zeta_0)(\xi_0 + \eta_0 + \zeta_0 - 2) \quad (i = 1, \ldots, 8) \\
 f_i &= \frac{1}{4} (1 - \xi^2)(1 + \eta_0)(1 + \zeta_0) \quad (i = 9, 11, 17, 19) \\
 f_i &= \frac{1}{4} (1 - \eta^2)(1 + \xi_0)(1 + \zeta_0) \quad (i = 10, 12, 18, 20) \\
 f_i &= \frac{1}{8} (1 - \zeta^2)(1 + \xi_0)(1 + \eta_0) \quad (i = 13, 14, 15, 16) \\

\end{align*}
\]

and
\[ b = \begin{bmatrix} 0 & 0 & p \end{bmatrix} \]

where \( p \) = wheel pressure.

The integral is evaluated with limits from \(-1\) to \(a\) in the \(x\)-direction and \(-1\) to be in the \(y\)-direction.  \( \gamma \) is set equal to 1.

Nodal Forces:

**Node 1**

\[
F_1 = \frac{pxy}{3} \left( \frac{1}{8} b^2 - \frac{1}{8} a^2 + \frac{1}{4} b - \frac{1}{4} a \right) (d^3 - c^3) + \frac{pxy}{2} \left( -\frac{1}{12} b^3 + \frac{1}{12} a^3 - \frac{1}{8} b^2 - \frac{1}{8} a^2 \right) (d^2 - c^2) + \frac{pxy}{12} b^3 (d - c) - \frac{pxy}{12} a^3 (d - c) - \frac{pxy}{4} b (d - c) + \frac{pxy}{4} a (d - c)
\]

**Node 2**

\[
F_2 = \frac{pxy}{3} \left( \frac{1}{8} b^2 - \frac{1}{8} a^2 + \frac{1}{4} b - \frac{1}{4} a \right) (d^3 - c^3) + \frac{pxy}{2} \left( -\frac{1}{12} b^3 + \frac{1}{12} a^3 - \frac{1}{8} b^2 - \frac{1}{8} a^2 \right) (d^2 - c^2) + \frac{pxy}{12} b^3 (d - c) - \frac{pxy}{12} a^3 (d - c) - \frac{pxy}{4} b (d - c) + \frac{pxy}{4} a (d - c)
\]

**Node 3**

\[
F_3 = \frac{pxy}{3} \left( \frac{1}{8} b^2 - \frac{1}{8} a^2 + \frac{1}{4} b - \frac{1}{4} a \right) (d^3 - c^3) + \frac{pxy}{2} \left( -\frac{1}{12} b^3 + \frac{1}{12} a^3 - \frac{1}{8} b^2 - \frac{1}{8} a^2 \right) (d^2 - c^2) + \frac{pxy}{12} b^3 (d - c) - \frac{pxy}{12} a^3 (d - c) - \frac{pxy}{4} b (d - c) + \frac{pxy}{4} a (d - c)
\]
Node 4

\[ F_4 = \frac{p_{xy}}{3} \left( \frac{-1}{8} b^2 + \frac{1}{8} a^2 + \frac{1}{4} b - \frac{1}{4} a \right) (d^3 - c^3) + \frac{p_{xy}}{2} \left( \frac{1}{12} b^3 - \frac{1}{12} a^3 - \frac{1}{8} b^2 + \frac{1}{8} a^2 \right) (d^2 - c^2) + \frac{p_{xy}}{12} b^3 (d - c) - \frac{p_{xy}}{12} a^3 (d - c) - \frac{p_{xy}}{4} b (d - c) + \frac{p_{xy}}{4} a (d - c) \]

Node 5

\[ F_5 = \frac{p_{xy}}{2} \left( \frac{1}{6} b^3 - \frac{1}{6} a^3 - \frac{1}{2} b + \frac{1}{2} a \right) (d^3 - c^2) - \frac{p_{xy}}{6} b^3 (d - c) + \frac{p_{xy}}{6} a^3 (d - c) + \frac{p_{xy}}{2} b (d - c) - \frac{p_{xy}}{2} a (d - c) \]

Node 6

\[ F_6 = \frac{p_{xy}}{2} \left( \frac{1}{6} b^3 - \frac{1}{6} a^3 + \frac{1}{2} b - \frac{1}{2} a \right) (d^3 - c^2) - \frac{p_{xy}}{6} b^3 (d - c) + \frac{p_{xy}}{6} a^3 (d - c) + \frac{p_{xy}}{2} b (d - c) - \frac{p_{xy}}{2} a (d - c) \]

Node 7

\[ F_7 = \frac{p_{xy}}{3} \left( \frac{1}{4} b^2 - \frac{1}{4} a^2 - \frac{1}{2} b + \frac{1}{2} a \right) (d^3 - c^3) - \frac{p_{xy}}{4} b^2 (d - c) + \frac{p_{xy}}{4} a^2 (d - c) + \frac{p_{xy}}{2} b (d - c) - \frac{p_{xy}}{2} a (d - c) \]

Node 8

\[ F_8 = \frac{p_{xy}}{3} \left( \frac{1}{4} b^2 + \frac{1}{4} a^2 - \frac{1}{2} b + \frac{1}{2} a \right) (d^3 - c^3) + \frac{p_{xy}}{4} b^2 (d - c) - \frac{p_{xy}}{4} a^2 (d - c) + \frac{p_{xy}}{2} b (d - c) - \frac{p_{xy}}{2} a (d - c) \]
Appendix C: INDISLAB User Help Guide

C.1. Installation Instructions

In order to run INDISLAB properly on a personal computer, the JAVA runtime environment must be installed. The installation file can be found on the INDISLAB CD or can be downloaded from Sun Microsystems Java website (http://java.sun.com/j2se/1.3/download.html). Instructions for proper installation are also included on the CD or can be found on the Java website as well (http://java.sun.com/j2se/1.3/install-windows.html). As mentioned in the installation instructions PATH variables can be set in order to run the INDISLAB program by double-clicking the file. Otherwise, the full command line must be entered. This command is as follows:

C:\INDISLAB> /jdk1.3.1_07/bin/java –jar INDISLAB.jar

where INDISLAB is the directory that the INDISLAB.jar file exists.

C.2. INDISLAB Introduction

A step-by-step overview of INDISLAB is presented below. This guide will give users an overview of INDISLAB through an explanation of all screens and commands that appears in INDISLAB. Figure C-15 shows the first screen that appears when the user runs INDISLAB. After the user clicks the “Continue” button the window in Figure C-16 appears. The user must choose between the option of creating or opening a file. If the user decides to create a new file, either the English or SI option must be clicked before the “Continue” button is hit.
The window in Figure C-17 appears next. This is the main INDISLAB program where the user provides all input and all program commands can be found. This screen is made of five tabbed panes and a menu bar at the top of the screen.
C.3. INDISLAB Menus

As can be seen in Figure C-17, the menu bar consists of four main menus: *File*, *Analysis*, *Results* and *About*. Figure C-18 shows the *File* menu and its options. Items in this menu perform basic commands found in most Windows-based software.

The *New* menu item clears all existing input in text boxes, closes the main INDISLAB window and the window in Figure C-16 reappears to allow the user to begin a new file. The *Open* command causes a file chooser window to appear, similar to the one in Figure C-19. From this window one can find the directory in which the saved file is located and
open it. The *Save* command overwrites the file with the same name and location with the existing information in the text boxes. This option is unavailable until the file has been saved with the use of the *Save As* menu item. Figure C-20 shows the save file window that appears when the *Save As* menu item is clicked. The save file window allows one to choose the directory and pick a name for the file. The *Exit* menu item closes the INDISLAB program.

![Open File Window](image1)

*Figure C-19. Open File Window*

![Save File Window](image2)

*Figure C-20. Save File Window*

The *Analysis* menu includes one menu item, *Run Analysis*, as shown in Figure C-21. When this command is clicked INDISLAB checks all user-input boxes to make certain that all essential information has been included and consist of numerical values. As can be seen it is important to include input in all necessary boxes before this command is chosen. If this check fails then INDISLAB alerts the user through an “Error Message” window as shown in Figure C-22. Otherwise, INDISLAB proceeds to create an input file for ANSYS. Once this is completed a window, shown in Figure C-23, alerts the user of the input command to be used within ANSYS.
The \textit{Results} menu contains numerous options for the postprocessing of results in ANSYS. There are three menus within the \textit{Results} menu: \textit{Deflection Plots}, \textit{Stress Plots} and \textit{Path Plots} as seen in Figure C-24. The selection of any of these commands results in the formation of an input file for ANSYS. Figure C-25 shows the window that appears with the ANSYS input command necessary to display the graphical results.
Path plots take a cross-section of the slab layout in either the transverse or longitudinal direction as shown in Figure 5-9. Once the user decides upon the type of direction, he/she must next choose where in the depth of the slab he/she wants the path plot to be. Three choices are given in INDISLAB: the top, center and bottom of the slab(s). These positions within the depth of the slab are shown in Figure 5-10.
Figure C-27. Path Locations through the Depth of the Slab.

Figure C-28 shows the \textit{About} menu with the \textit{Help} menu item. When clicked this item displays information regarding the location of this guide.

![About Menu](image)

Figure C-28. About Menu

C.4. INDISLAB Tabbed Panes

The main portion of the screen is made up of several tabbed panes. Figure C-29 shows the first tabbed pane, “Slab Properties.” The pull-down menu at the top of the screen allows one to chose the slab layout desired: nine-, two-, or one-slab. It is recommended that the user pick the slab layout before clicking the “Input Slab Dimensions” button at the bottom of the screen. Users also need to enter information regarding the slab’s Poisson’s ratio, modulus of elasticity, and thickness. As mentioned previously, after the user chooses the slab layout the “Input Slab Dimensions” button can be clicked to cause the window in Figure C-30 to appear. Depending on the slab layout chosen, all text boxes may not be editable. If it is decided to change the slab layout, it
may be done within this window with a click of a button. For the nine-slab layout, the “Slab 3 width” box refers to the width of the shoulder slabs. Once all input has been entered and the “Done” button is clicked, INDISLAB checks to make sure valid data has been entered in all editable windows. If all data is valid the window in Figure C-30 closes and returns the user to the window in Figure C-29. Otherwise, an error message appears to alert the user that suitable data must be entered.

![Figure C-29. Slab Properties Tabbed Pane](image1)

![Figure C-30. Slab Dimensions Window](image2)
If the user chose a one-slab layout then the tabbed pane in Figure C-31 can be ignored. If the two-slab layout is chosen then the user must enter input for all boxes except the tie bar diameter. For the nine-slab layout all boxes must include a numerical value.

![Figure C-31. Reinforcement Properties Tabbed Pane](image)

The third tabbed pane, shown in Figure C-18, allows the user to input all data concerning the loading. By using a pull down menu the user can choose -from one up to ten- the number of load cases he/she wants to include in the analysis. Accordingly, a list of load cases is displayed allowing two types of load for every load case: wheel loads and thermal load. Every type of load has a corresponding button with a check box to its left side. After the user makes a decision on the number of load cases they wish to include, he/she must check the type of load to be included for every load case. Any load case can have both wheel loads and thermal load or just one of them. Initially the wheel loads buttons and thermal load buttons are unable to be used. These buttons are grayed out and clicking them will result in no action. However, after the check boxes next to them are checked, these buttons become functional and clicking them allows the user to input loading data for the analysis.
Clicking one of the “Wheel Loads” button prompts the user to input the data related to the wheel loads for the corresponding load case. For example, when the user clicks the “Wheel Loads” button for load case number one the “Loads” tabbed pane becomes the “Wheel Loads” tabbed pane (Figure C-19) with the text line “Wheel Loads for Load Case #1” indicating that the wheel loads for case number one is to be input.

The “Wheel Loads” tabbed pane (Figure C-19) asks the user to input the number of wheel loads to be applied on the slab layout for the corresponding load case. This number corresponds to each wheel present on the slab layout. After this box is filled with a value, the “Input New Wheel Load Properties” button must be clicked in order to continue the process. If the number of loads is zero then clicking “Input New Wheel Load Properties” button performs no action. If the number of loads is an integer value greater than zero and less than or equal to ten, INDISLAB opens the “Wheel Loads” window (Figure C-19). If the number of loads is an integer value greater than ten, then INDISLAB asks the user to enter a number less or equal to ten. Once the “Input New Wheel Load Properties” button has been initially pressed with a valid value it can no longer be pressed again. All input values must be accessed through the “Edit Existing Wheel Load Properties” button. This is due to the fact that pressing the “Input New Wheel Load Properties” clears all existing values and initiates all text boxes for the
number of loads input by the user. The “Edit Existing Wheel Load Properties” button does not clear any of the properties but instead reposts the data input earlier. The return button allows the user to go back to the “Loads” tabbed pane (Figure C-18).

![Figure C-19. Wheel Loads Tabbed Pane.](image)

The “Wheel Loads” screen that is initiated through the “Wheel Loads” tabbed pane collects data about wheel loads present on the slab layout (Figure 3-). Items included on this screen are wheel load pressure, tire footprint length and width as well as the longitudinal and transverse position of the load on the slab. Near the bottom of the screen a figure helps the user to understand what values need to be input into the text boxes. Below the figure are several buttons that allow users to perform various functions. If more than one load is present the user has the option to progress back and forth between different loads. Other options include the ability to delete/add loads. Once the user is done inputting all load information the “Done” button can be clicked. This saves all data that has been input into the window and closes it, returning the user to the Wheel Loads tabbed pane (Figure C-19).
Clicking a “Thermal Load” button in the “Loads” tabbed pane (Figure C-18) prompts the user to input the data related to the thermal loading for a given load case. When the user clicks the “Thermal Load” button of a load case, the “Loads” tabbed pane becomes the “Thermal Load” tabbed pane (Figure C-21). This tab allows the user to pick the type of thermal distribution and input corresponding properties. A pull-down menu appears at the top of the tab that allows the choice of two thermal distributions: linear, and non-linear. All boxes within the tab are not editable and have a light gray background if no thermal distribution has been chosen. If the “linear” option is selected, one box of input is required: the temperature gradient. The second choice, “nonlinear”, requires input of temperatures at nine nodes from the top to the bottom of the slab. This allows users to define their own nonlinear distribution along the nine nodes present in the depth of the slab.
The “Subbase/Subgrade Properties” tabbed pane shown in Figure C-32 collects information about all subsurface layers. Here the user can choose from two main options for modeling the soil: (1) by using the simplified technique of axial spring elements or (2) by using advanced finite elements. The user decides for the first one by checking the check box located next to the “Modulus of Subgrade Reaction” label and has to enter that data. Doing so, unable to be used the rest of text boxes data. If this check box is not checked then the layers of soil will be model by using the finite element technique and the Poisson’s ratio, modulus of elasticity and thickness values are required for each layer. The user has the choice of one or two subgrade layers.

The option of modeling the soil by using a spring-based simplified model has been included to give the user the flexibility of analyzing a model in a shorter time span.
The last tabbed pane shown in Figure C- is that of the “Damage Analysis”. INDISLAB includes two options for doing damage analysis and an option to predict the service life of a jointed plain concrete pavement through damage accumulation. Clicking the “Damage Program” button allows the user to model the damage and accumulate it both linearly and nonlinearly. The user has to check which fatigue equation he/she wants to use during the damage computation (Figure C-24).
On the other hand, clicking the “Damage Models” button the damage is computed by using two damage models: the Darter model and the Rigid Pavement Performance (RPPR) model. Users have the choice of using the Darter or RPPR model. This tab can be completed separately from the other tabbed panes. The Darter Model requires input in three text boxes. The RPPR model requires input of several variables. The elastic modulus, Poisson’s ratio, and thickness of both the slab and subbase must be entered in order for the calculation of the RPPR model to work properly. The stress load transfer efficiency must only be input if the tied PCC shoulder edge support button is chosen. The stress load transfer efficiency must be a value between 0.05 and 0.30. After all pertinent information is input the button may be clicked and a window similar to the one in Figure C- displays the results of the damage model.

![Fatigue Equation](image)

Figure C-24. Fatigue Equation for the Cumulative Damage Analysis
C.5. ANSYS Background

After INDISLAB has developed an input file, the ANSYS window in Figure C-34 appears. In this window there are several important pieces of information the user must enter. The working directory that the user should choose is that of the location of the INDISLAB saved file. INDISLAB saves the ANSYS input file in the same directory.
All INDISLAB files should be saved in separate directories. This is due to the fact that ANSYS may overwrite files if more than one analysis is performed in a similar directory. The other information that must be entered is that of the memory requirements. For a computer with 2 GB of RAM it is recommended that the user enter a value of 1200 for the “Total Workspace” and 250 for the “Database” box. It is important to close all other programs before starting the ANSYS analysis. If ANSYS runs out of memory during the formation of the solution, the analysis may not run properly or take much longer to complete. After the above information is input the “Run” button should be clicked.

![ANSYS Opening Screen](image)

Figure C-34. ANSYS Opening Screen

When this button is clicked the ANSYS window in Figure C-35 appears as well as the INDISLAB “Input File Command” window. The user must enter the command in this window into the boxed area shown above in Figure C-35. The model is then read from the input file and the analysis begins.
After the model is complete several options are available to the user. To view plots that display the results the user can create an input file from the Results menu in INDISLAB and input the command line in the same location as before. To change the view of the results or zoom in or out the Pan-Zoom-Rotate tool shown in Figure C-36 should be used. Also shown are the menus used to open this tool.
Another postprocessing tool available in ANSYS is a listing of numerical values. The process shown in Figure C-1 to display numerical results causes the window in Figure C-3131 to appear. From this window the type of result to be displayed can be chosen. After picking the result a window similar to the one in Figure C-322 will appear. At the end of the results list, ANSYS displays the maximum absolute value alongside the node or element number in which it occurs.
Before exiting ANSYS it is a good idea to save the analysis since it is not done automatically. Pressing the “SAVE_DB” button in the ANSYS Toolbar will save the file. The “QUIT” button should be clicked to exit the program. As a default ANSYS automatically gives the user the option to save the file before exiting the program.
To open a saved file the user should input the directory where the ANSYS analysis was saved into the “Working Directory” box shown in Figure C-34. Once the main screen of ANSYS has opened one should follow the process outlined in Figure C-38. After clicking the *Resume from...* menu item a box appears to allow a file to be chosen.

![Figure C-38. Process to Open a Saved File](image)

Figure C-38. Process to Open a Saved File