1982

The Calculating of optimal Pressure Ratios and Cylinder Bore Ratios of Multistage Compressors

X. Qian

Follow this and additional works at: http://docs.lib.purdue.edu/icec

http://docs.lib.purdue.edu/icec/512

This document has been made available through Purdue e-Pubs, a service of the Purdue University Libraries. Please contact epubs@purdue.edu for additional information.
Complete proceedings may be acquired in print and on CD-ROM directly from the Ray W. Herrick Laboratories at https://engineering.purdue.edu/Herrick/Events/orderlit.html
THE CALCULATING OF OPTIMAL PRESSURE RATIOS AND CYLINDER BORE RATIOS OF MULTISTAGE COMPRESSORS

Qian Xinghua, Assistant Engineer, Xian Compressor Works, Xian, Shaanxi, The People's Republic of China

ABSTRACT

This article gives a calculating method of the optimal pressure ratios of multistage compressors. To optimize and compare the compressors easily, this article gives the conception about optimal match of cylinder bores and also the formula calculating the optimal cylinder bore ratios.

INTRODUCTION

It has been 150 years since the first two-stage reciprocating compressor was developed in England. Though the theories about compressors has been approached by many scientists, they have not obtained satisfactory results. The theory about optimal pressure ratios of multistage compressors is one of them. The present theories about that deal with only ideal gas without extracting and condensate, in addition with some assumptions of perfectly back-cooling [1], [2], smaller loss of resistance of incoming and discharge [1], omitting the effect of temperature heating the incoming gas [1], which results in the difference from the actual operating, especially for real gas as a medium with extracting and condensate of the system in chemical technique compressors. So to design a compressor according to this theory, it won't be working in the optimal state. With the tension of energy source in the world it's very important to save energy. So to increase the economy of compressors, especially larger compressors and chemical technique compressors on the basis of reliability have caused caution by scientists in all countries. In this case it is necessary to find a method of determining optimal pressure ratios in accordance with actual conditions in compressors.

This article gives a method to determine the optimal pressure ratios, which can be used in ideal gas, also actual gases and deals with extracting, condensate, back-cooling degree and approaches the actual conditions.

In order to optimize the design of compressors and compare with each other, the writer raises the concept of optimal match between multistage compressor's cylinder bores and derives a set of formulae for determining the optimal ratios of the cylinder bores.

The preceding results were used in the compressors L2-10/8-I and L3.5-20/7 resulting in satisfactory effects.*

DERIVING THE FORMULA FOR DETERMINING THE OPTIMAL PRESSURE RATIOS

The current method for determining the optimal pressure ratios deviate from the actual operating conditions of compressors because of basing on the ideal gas. So it was followed by the two faults:

i. Not dealing with the compressible, condensate, extracting of the gas.

ii. Many factors affecting each other and without an analytical relationship influence the capacity parameter which was included in the function of determining optimal pressure ratios in accordance with actual conditions in compressors.

The current method gives a method to determine the optimal pressure ratios, which can be used in ideal gas, also actual gases and deals with extracting, condensate, back-cooling degree and approaches the actual conditions.

In order to optimize the design of compressors and compare with each other, the writer raises the concept of optimal match between multistage compressor's cylinder bores and derives a set of formulae for determining the optimal ratios of the cylinder bores.

The preceding results were used in the compressors L2-10/8-I and L3.5-20/7 resulting in satisfactory effects.*

* The air compressor of L2-10/8-I had the honour of getting the praises from Scientific Conference of China in 1978, and National First Machine Industry Department in 1980. The air compressor of L3.5-20/7 had the honour of getting the third level of praise from National First Machine Industry Department in 1980.
indicated power. So it cannot be analysed mathematically. In that case, some assumptions were put out to protect from the factors to simplify the deriving.

The two faults caused the formulas of optimal pressure ratios to be inexact.

In order to prevent the faults, the writer raises the concept of indicated power ratio about actual gas, which means indicated power consumed by a unit capacity. It is given by:

\[
N = \frac{L}{V_m} \quad (1)
\]

It is more rigid to appreciate the power consumption in compressors using the indicated power ratio than indicated power, because of the shortage of the gas amount for the latter. But there is not the factor of gas amount within indicated power ratio, so the factors which interfere with the gas amount are removed. In this case, there is no need to make such assumptions: perfectly back-cooling, smaller resistance of incoming and discharge, temperature heating the incoming air little. The deriving of the optimal pressure ratio formulae will be based on the function of indicated power for actual gas.

First, the formulae required are introduced from material [3] [1], the formulae from [3] <2-65>, <2-53>:

\[
\begin{align*}
\epsilon_i &= \frac{P_i^c V_i^c \lambda_i m_i}{m-1} \left[ \left( \frac{P_i^{c+}}{P_i^{c-}} \right)^{m_i} - 1 \right] \frac{\xi_{S_i}^c + \xi_{D_i}^c}{2 \xi_{S_i}^c} \\
\lambda_i &= \frac{\mu_{di} \mu_{ci}}{\xi_i} \frac{P_i^c}{P_i^{c+}} \frac{S_i^c}{S_i^d} \frac{V_i}{V_m} \quad (2)
\end{align*}
\]

the formulae from [2] <11-28>:

\[
\begin{align*}
\frac{P_i^c}{P_i^{c+}} &= (1 + \delta_i) \frac{P_i^c}{P_i^{c+}} \\
\delta_i &= \delta_{S_i}^c + \delta_{D_i}^c
\end{align*}
\]

\( \delta_{S_i}^c, \delta_{D_i}^c \) can be got from the curve [1]. See Fig. 1. If the resistances in valves and pipe system are smaller, they can be found by the dotted line; if greater, by the full line. In the case that the piston mean velocity \( C_m \neq 3.5 \text{ m/s} \), it will be corrected by \( (C_m/3.5)^2 \). When the density of the gas is far different from air, it will be corrected by \( (\gamma'/\gamma)^{2/3} \), \( \gamma \) is the air density.

Substituting (2) (3) into (1) and dealing with (4), the indicated power ratios of i-stage compressors can be obtained:

\[
N = \frac{L_i}{V_m} = \frac{\mu_{di} \mu_{ci} P_i^c}{\xi_i} \frac{m_i}{m-1} \left\{ \left( \frac{P_i^{c+}}{P_i^{c-}} \right)^{m_i} - 1 \right\} \frac{\xi_{S_i}^c + \xi_{D_i}^c}{2 \xi_{S_i}^c}
\]

\[
+ \frac{\mu_{di} \mu_{ci} P_i^c}{\xi_i} \frac{m_i}{m-1} \left\{ \left( \frac{P_i^{c+}}{P_i^{c+}} \right)^{m_i} - 1 \right\} \frac{\xi_{S_i}^c + \xi_{D_i}^c}{2 \xi_{S_i}^c}
\]

\[
+ \frac{\mu_{di} \mu_{ci} P_i^c}{\xi_i} \frac{m_i}{m-1} \left\{ \left( \frac{P_i^{c+}}{P_i^{c+}} \right)^{m_i} - 1 \right\} \frac{\xi_{S_i}^c + \xi_{D_i}^c}{2 \xi_{S_i}^c}
\]

\( (5) \)

Because of the discharge pressure of the i - 1 stage equal to the suction pressure of the i-stage, i.e., \( P_i = P_{di-1} \), therefore the indicated power ratio can be treated as a function of dischage pressure of every stage:

\[
N = N(P_{d1}, P_{d2}, \ldots, P_{di-1})
\]

Taking partial differentials and letting them be equal to zero and then based on

\[
\epsilon_i = \frac{P_{d1}}{P_{d1}}, \quad \epsilon_i = \frac{P_{d1}}{P_{d1}}
\]

we have

\[
\begin{align*}
\xi_i &= \xi_i (\gamma', \gamma') \left( \frac{\xi_{S_i}^c + \xi_{D_i}^c}{\xi_{S_i}^d + \xi_{D_i}^d} \right)^{m_i} \frac{\mu_{di} \mu_{ci} P_i^c}{\xi_i} \frac{m_i}{m-1} \\
\epsilon_i &= \epsilon_i (\gamma', \gamma') \left( \frac{\xi_{S_i}^c + \xi_{D_i}^c}{\xi_{S_i}^d + \xi_{D_i}^d} \right)^{m_i} \frac{\mu_{di} \mu_{ci} P_i^c}{\xi_i} \frac{m_i}{m-1}
\end{align*}
\]

\( (6) \)

\[
\begin{align*}
\epsilon_i &= \epsilon_i (\gamma', \gamma') \left( \frac{\xi_{S_i}^c + \xi_{D_i}^c}{\xi_{S_i}^d + \xi_{D_i}^d} \right)^{m_i} \frac{\mu_{di} \mu_{ci} P_i^c}{\xi_i} \frac{m_i}{m-1} \\
\epsilon_i &= \epsilon_i (\gamma', \gamma') \left( \frac{\xi_{S_i}^c + \xi_{D_i}^c}{\xi_{S_i}^d + \xi_{D_i}^d} \right)^{m_i} \frac{\mu_{di} \mu_{ci} P_i^c}{\xi_i} \frac{m_i}{m-1}
\end{align*}
\]

\( (7) \)

\[
\begin{align*}
\epsilon_i &= \epsilon_i (\gamma', \gamma') \left( \frac{\xi_{S_i}^c + \xi_{D_i}^c}{\xi_{S_i}^d + \xi_{D_i}^d} \right)^{m_i} \frac{\mu_{di} \mu_{ci} P_i^c}{\xi_i} \frac{m_i}{m-1} \\
\epsilon_i &= \epsilon_i (\gamma', \gamma') \left( \frac{\xi_{S_i}^c + \xi_{D_i}^c}{\xi_{S_i}^d + \xi_{D_i}^d} \right)^{m_i} \frac{\mu_{di} \mu_{ci} P_i^c}{\xi_i} \frac{m_i}{m-1}
\end{align*}
\]

\( (8) \)
The preceding pressure ratio should obey the relationship:

\[
E_1 \cdot E_2 \cdots \cdot E_i = \xi_i
\]

(9)

The optimal pressure ratios can be obtained by solving (9) and the formulae above. For a two-stage compressor, solving (6) and (9), the optimal pressure ratio of stage 1 and 2 can be obtained:

\[
\begin{align*}
E_1 &= \left[ \xi \left( \frac{1 + \delta_1}{1 + \delta_2} \right) \right] \left( \frac{T_{i_2}}{T_{i_1}} \right) \frac{m}{(m-1)} \\
E_2 &= \left[ \xi \left( \frac{1 + \delta_2}{1 + \delta_3} \right) \right] \left( \frac{T_{i_2}}{T_{i_1}} \right) \frac{m}{(m-1)}
\end{align*}
\]

(10)

For a three-stage compressor, solving (6), (7) and (9), the optimal pressure ratios of stage 1, 2 and 3 can be obtained:

\[
\begin{align*}
E_1 &= \left[ \xi \left( \frac{1 + \delta_1}{1 + \delta_2} \right) \right] \left( \frac{T_{i_2} T_{i_3}}{T_{i_1} T_{i_3}} \right) \frac{m}{(m-1)} \\
E_2 &= \left[ \xi \left( \frac{1 + \delta_2}{1 + \delta_3} \right) \right] \left( \frac{T_{i_2} T_{i_3}}{T_{i_1} T_{i_3}} \right) \frac{m}{(m-1)} \\
E_3 &= \left[ \xi \left( \frac{1 + \delta_3}{1 + \delta_4} \right) \right] \left( \frac{T_{i_2} T_{i_3}}{T_{i_1} T_{i_3}} \right) \frac{m}{(m-1)}
\end{align*}
\]

(11)

For an i-stage compressor, the optimal pressure ratios are:

\[
\begin{align*}
E_1 &= \left[ \xi \left( \frac{1 + \delta_1}{1 + \delta_2} \right) \cdots \left( \frac{1 + \delta_{i-1}}{1 + \delta_i} \right) \right] \left( \frac{T_{i_2} T_{i_3} \cdots T_{i_i}}{T_{i_1}} \right) \frac{m}{(m-1)} \\
E_2 &= \left[ \xi \left( \frac{1 + \delta_2}{1 + \delta_3} \right) \cdots \left( \frac{1 + \delta_{i-1}}{1 + \delta_i} \right) \right] \left( \frac{T_{i_2} T_{i_3} \cdots T_{i_i}}{T_{i_1}} \right) \frac{m}{(m-1)} \\
E_3 &= \left[ \xi \left( \frac{1 + \delta_3}{1 + \delta_4} \right) \cdots \left( \frac{1 + \delta_{i-1}}{1 + \delta_i} \right) \right] \left( \frac{T_{i_2} T_{i_3} \cdots T_{i_i}}{T_{i_1}} \right) \frac{m}{(m-1)} \\
& \vdots \\
E_i &= \left[ \xi \left( \frac{1 + \delta_i}{1 + \delta_{i+1}} \right) \right] \left( \frac{T_{i_2} T_{i_3} \cdots T_{i_i}}{T_{i_1}} \right) \frac{m}{(m-1)}
\end{align*}
\]

(12)

Generally,

\[
\nu_{d1} = 1, \quad \nu_{o1} = 1, \quad \xi_{s1} = 1, \quad \xi_{d1} = 1.
\]

For ideal gas without extracting, condense

\[
\xi_d = 1, \quad \xi_s = 1, \quad \nu_d = 1, \quad \nu_o = 1
\]

in every stage.

It is easy to use the formulae for calculating optimal pressure ratios. When designing, substituting the known parameters \(m, \xi, \delta, T_{i_1}, \xi_i, \delta_i, \nu_i, \nu_o\) into the above formulae the optimal pressure ratios would be obtained.

We come to the conclusions from the above:

i. The optimal pressure ratios of a compressor have something to do with \(\delta, T_{i_1}, \xi, \delta_i, \nu_i, \nu_o\) of not only this stage, but also other stages (feedback function).

ii. It depends on the process index \(m\) and the kind of mediums for every suction temperature to affect the optimal pressure ratio. The smaller the \(m\), the greater the function of the temperature. The greater the \(m\), the smaller the function. Therefore if the compressible medium is made of multiatomic gas or is a gas with smaller K value, more attention must be paid to the influence of temperature on optimal pressure ratio.

iii. When perfectly back-cooling, the optimal pressure ratios do not distribute uniformly, because the resistance loss of the low pressure stage is larger than the high, i.e., \(\delta_1 > \delta_2 > \cdots > \delta_i\), so the pressure ratio
of a following stage is larger than that of the preceding stage, i.e.,
\[ f_1 < c_2 < \cdots < c_i. \]

**DERIVING THE FORMULAE OF THE OPTIMAL CYLINDER BORE RATIOS**

First of all, the concept about cylinder bore ratio, which is the ratio between the equivalent diameters of the cylinders, is recommended. The equivalent bore represents a diameter of the circle, the area of which is equal to that of the cylinder's cross-section. To adopt the equivalent bore is for removing the influence of the piston rod, differential piston and single or double acting. Only when the diameter of the piston rod is much less (<<) than the cylinder diameter may we regard the equivalent diameter to be equal to the cylinder bore. It is necessary to introduce this conception, because it can show the relationship (how many times) among the parameters of these stage in the same single compressor. The distribution of cylinder bores of a compressor is regular, so the parameters, such as piston area, piston displacement, pressure ratio, reciprocating inertia force, piston force, moment, power and so forth distribute regularly. Therefore it should be treated as a parameter of the compressor. Since the pressure ratio has its optimal value, the cylinder bore should have its own optimal value too, and they are uniform. The pressure ratio of the compressor which is built in accordance with the optimal cylinder bore ratio must be in optimum. Instancing some two-stage compressors to illustrate that it is true to exist the optimal cylinder bore ratio which has its regularity. See Table 1 which lists some two-stage reciprocating compressors which have capacities of 10-100 M3/min and excellent economic index. They are of various types such as fixed, crosshead, double-acting two-stage reciprocating compressors.

From this table we can see that the cylinder bore ratios between their stage 1 and 2 are all about 1.6 - 1.64. The development of the compressors of 10 M3, 20 M3, 40M3 of our country have also demonstrated this point. The old model of the 10 M3 compressor, the cylinder bore ratio between stage 1 and stage 2 of which was 300mm/180mm = 1.67, was 3L-10/8, after having been modified, today its model is 12-10/8-I and the bore ratio is 275mm/170mm = 1.62. The old model of the 20 M3 compressor, the cylinder bore ratio of which was 420/250mm = 1.68 was 4L-20/8, its performance was much better. When it had been modified as L3.5-20/8, the cylinder bore ratio of which was 380mm/220mm = 1.78, its performance fell down extremely. Finally, it was developed as L3.5-20/7, the cylinder bore ratio of which is 380mm/235mm = 1.62. For the old 40 M3 compressor 5L-40/8, its cylinder ratio was 580mm/340mm = 1.71. But that of the new one, L5.5-40/8, is 560mm/340mm = 1.65. Their ratios are all about 1.6 - 1.64. So this is another evidence of existing the optimal cylinder bore ratios.

Let us derive the formulae calculating optimum of cylinder bore ratio.

Let the cylinder bore ratio
\[ \beta_{i-1} = \frac{D_{i-1}}{D_i} \]

The ratio of piston displacements is equal to the square of equivalent cylinder bore ratio. From (3) and based on \( P_{si}=P_{di-1} \):
\[ E_{di} = \frac{\lambda i}{\lambda i} \frac{\kappa i}{\kappa i} \frac{\kappa di}{\kappa di} \frac{\kappa d_{di-1}}{\kappa d_{di-1}} \beta_{i-1}^2 \]

Reforming the formula (5) by (14) and (9):
\[ N = \mu d_i \mu d_0 \frac{P_{si}}{P_{di}} \frac{m}{m_i} \left( \frac{\lambda i \kappa di \beta_{i-1}^2 (1+\beta_{i-1})^{m-1}}{\lambda i \kappa d_{si} \beta_{i-1}^2 (1+\beta_{i-1})^{m-1}} \right) \]
\[ + \mu d_i \mu d_c \frac{P_{si}}{P_{di}} \frac{m_i}{m} \left( \frac{\lambda i \kappa di \beta_{i-1}^2 (1+\beta_{i-1})^{m-1}}{\lambda i \kappa d_{si} \beta_{i-1}^2 (1+\beta_{i-1})^{m-1}} \right) \]
\[ + \mu d_i \mu d_0 \frac{P_{si}}{P_{di}} \frac{m}{m_i} \left( \frac{\lambda i \kappa di \beta_{i-1}^2 (1+\beta_{i-1})^{m-1}}{\lambda i \kappa d_{si} \beta_{i-1}^2 (1+\beta_{i-1})^{m-1}} \right) \]

Taking partial differentials to \( \beta_1, \beta_2, \cdots, \beta_{i-1} \) and letting them be zero then using the method like the optimal pressure ratio, the optimal cylinder bore ratios can be obtained.

For a two-stage compressor:
\[ \beta_{i-1} = \left[ E_{i-1} \left( \frac{\lambda i \kappa di}{\lambda i \kappa d_{si}} \beta_{i-1} \right) \frac{m}{m_i} \right]^\frac{1}{2} \]
For an i-stage compressor:

\[
\beta_i - 1 = \left( \frac{(1 + \delta_i) (1 + \delta_i) \cdots (1 + \delta_i)}{(1 + \delta_i) (1 + \delta_i) \cdots (1 + \delta_i)} \right)^{\frac{1}{2i}}
\]

\[
\left( \frac{\frac{\delta_i}{1 + \delta_i}}{\frac{\delta_i}{1 + \delta_i}} \right)^{\frac{1}{2i}}
\]

\[
\left( \frac{\theta_i}{1 + \theta_i} \right)^{\frac{1}{2i}}
\]

\[
\left( \frac{\theta_i}{1 + \theta_i} \right)^{\frac{1}{2i}}
\]

When the compressible medium is ideal gas without extracting, condensate

\[
\xi_S = 1, \xi_d = 1, \gamma_d = 1, \gamma_o = 1
\]

in the above formulae. For a two-stage compressor, using the formula (16) we can get 8.1.6 - 1.64.

CONCLUSIONS

There are clear analytical relationships in these formulae given by this article. The method determining optimal pressure ratios are more exact than that of the present one.

The cylinder bore ratio given by this article has practical value. It can join the parameters of every stage.

This method of calculating the optimal cylinder bore ratios given by this article is easy to optimise and compare compressors.

NOMENCLATURE

- \( P_s \): real suction pressure
- \( P_d \): real discharge pressure
- \( \delta_s \): relative pressure loss for suction
- \( \delta_d \): relative pressure loss for discharge
- \( \delta \): relative pressure loss for incoming and outgoing
- \( \lambda \): discharge coefficient
- \( \mu_d \): extract coefficient
- \( \mu_o \): condensate coefficient
- \( \xi_S \): compressible coefficient of gas in suction state
- \( \xi_d \): compressible coefficient of gas in discharge state
- \( T_s \): suction temperature
- \( C_m \): piston mean velocity
- \( \epsilon \): overall pressure ratio
- \( \epsilon_i \): pressure ratio of the i-stage
- \( m \): process index
- \( D \): equivalent diameter of cylinder
- \( b \): equivalent bore (diameter) ratio of a cylinder
- \( K \): \( C_p/C_v \), gas at atmospheric conditions

REFERENCES

Fig. 1 The Relative Pressure Loss Curve

### TABLE 1

<table>
<thead>
<tr>
<th>Country</th>
<th>Company</th>
<th>Model</th>
<th>Bore of Cylinder</th>
<th>Diametral Ratio of Cylinder</th>
</tr>
</thead>
<tbody>
<tr>
<td>U.S.A.</td>
<td>INGERSOLL-RAND CO.</td>
<td>LLE-100</td>
<td>342.9, 209.9</td>
<td>1.64</td>
</tr>
<tr>
<td></td>
<td></td>
<td>XLE</td>
<td>469.9, 292.1</td>
<td>1.61</td>
</tr>
<tr>
<td>West Germany</td>
<td>DEMAG</td>
<td>2PA100E</td>
<td>295, 175</td>
<td>1.63</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2P 170N</td>
<td>400, 250</td>
<td>1.60</td>
</tr>
<tr>
<td>Sweden</td>
<td>ATLAS COPCO</td>
<td>AR 4H</td>
<td>410, 250</td>
<td>1.64</td>
</tr>
<tr>
<td></td>
<td></td>
<td>DT 4</td>
<td>375, 235</td>
<td>1.60</td>
</tr>
<tr>
<td></td>
<td></td>
<td>ER 6</td>
<td>460, 280</td>
<td>1.64</td>
</tr>
<tr>
<td></td>
<td></td>
<td>ER 8</td>
<td>640, 390</td>
<td>1.64</td>
</tr>
<tr>
<td>France</td>
<td>PCB</td>
<td>NGM10</td>
<td>340, 210</td>
<td>1.62</td>
</tr>
<tr>
<td></td>
<td></td>
<td>NGM15</td>
<td>400, 250</td>
<td>1.60</td>
</tr>
<tr>
<td></td>
<td></td>
<td>NGM24</td>
<td>400, 250</td>
<td>1.60</td>
</tr>
<tr>
<td></td>
<td></td>
<td>NGM30</td>
<td>450, 280</td>
<td>1.61</td>
</tr>
</tbody>
</table>