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RETRIEVAL OF COASTAL WATER INFORMATION FROM LANDSAT MSS DATA

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ABSTRACT

In recent years an inverse problem of determining optical parameters of air-water system from the remotely sensed spectral radiance of water has deserved much attention in the field of environmental monitoring. Particularly, optimization approach to the inference of optical properties of sub-surface water from the measurements of multispectral scanner data of the sea has been developed analytically and computationally.

In the present paper, making use of the invariant imbedding, an initial-value solution of the transfer equation in air-water system with allowance for the interaction of radiation at the system interface is provided. The Cauchy system of the scattering function under consideration requires much computational works. In order to make tractable the inverse solution, we introduce the effective water surface albedo into the Cauchy system of the scattering function for the specular reflection problem, allowing for the multiple scattering effect of radiation in the water. Then, the least squares estimation of the effective water surface albedo from the noisy measurements of the total spectral radiance of water from space is made via the quasilinearization. The numerical experiments showed rapid convergence of the desired results. Based on these data, aqueous optical properties such as chlorophyll concentration and turbidity will be discussed in our subsequent papers.

I. INTRODUCTION

In a series of our preceding papers (cf. Ueno, Haba, and et. al. 1978; Haba, Kawata, and et. al. 1979; Ueno, Kawata, and et. al. 1980), the least squares estimation of the ground albedo from the LANDSAT MSS data has been made.

The multiple scattering problem in an atmosphere bounded by a specularly reflecting surface has been reduced to the determination of the scattering and transmission functions (cf. Sobolev 1963; Casti, Kalaba and Ueno 1969; Ueno and Mukai 1973). Another problem of the specular reflection is the determination of the radiation field within the sea under quiet condition. Let the fraction of specular reflection be denoted by $r(u)$, where u is

the cosine of incident angle. Fraction $1-r(u)$ of these photons being incident on the interface penetrates into the sea and experiences both scattering and absorption processes.

In a rigorous manner the spectral radiance emanating from the air-water system with allowance for both reflection and refraction at the system interface has been dealt by several authors (cf. Ambarzumian 1964; Sobolev 1963, 1975; Gutshabash 1963; Raschke 1972; Tanaka and Nakajima 1977; Mukai and Ueno 1978). In recent years approximate solution approach to the above problem has been developed by several authors (Duntley 1963; Plass and Kattawar 1969; Jain and Miller 1976).

On the other hand, an inverse problem of determining the spectral surface albedo from the noisy spectral radiance measurements has been solved via the quasilinearization and invariant imbedding (cf. Bellman, Kagiwada, Kalaba and Ueno 1965; Kagiwada 1974; Kagiwada, Kalaba and Ueno 1975; Ueno 1981). In recent years an estimation of unknown parameters in distributed parameter system from the noisy measurements of the input and output has been interested by researchers in science and technology. A need for approximate solution of conditionally well-posed problem with inaccurate data gave rise to an innovation with a regularization by which the identification problems are reduced to the minimization of performance (quadratic) function.

In the present paper, the second section deals with the initial-value solution of the transfer equation for the air-water system, allowing for the interaction of radiation at the system interface. On keeping in mind the interaction of radiation at the air-water system surface, the simultaneous determination of the radiation field in the air and in the water is reduced approximately to the determination of the radiation field in the atmosphere bounded by the effective surface albedo of the specular reflector. In the third section, it is shown how to use the quasilinearization for the least squares estimation of the effective water surface albedo from the noisy measurements of the spectral radiance emergent from the air-water system. The numerical experiments for the successive approximation of the effective water surface albedo in the simulation model showed rapid convergence

towards the true value. In our later papers, we shall discuss the subsurface water parameters such as the chlorophyll concentration, turbidity and others, making use of the effective water surface albedo thus obtained.

II DIRECT MULTIPLE SCATTERING PROBLEM

A. ATMOSPHERE-OCEAN SYSTEM WITH INTERACTING SEA SURFACE

1. The Equation of Transfer

Suppose that the top of a plane-parallel, inhomogeneous, anisotropically scattering atmosphere-ocean system of the overall optical thickness L is illuminated in a direction Ω_0 by parallel rays of net flux πF per unit area normal to the direction of the propagation. Let the upwelling intensity of radiation in the direction Ω at any level t ($0 \leq t \leq L$) be denoted by $I(t, +\Omega)$; similarly, let the downwelling intensity of radiation in the direction Ω at level t be denoted by $I(t, -\Omega)$. In the above the direction Ω (or Ω_0) stands for (v, ϕ) (or u, ϕ_0), where v (or u) is the cosine of inclination to the normal and ϕ (or ϕ_0) is the azimuth referred to a suitably chosen horizontal axis. Furthermore, let the albedo for single scattering be denoted by λ ($0 \leq \lambda \leq 1$), which depends on the optical height t . The atmosphere-ocean system consists of the atmospheric layer and the ocean layer, whose anisotropic scattering property is described by the normalized phase function P . The interacting interface is assumed to reflect specularly the radiation.

The equation of transfer appropriate to this case takes the form

$$\begin{aligned} v \frac{dI(t, \Omega)}{dt} + I(t, \Omega) &= \frac{\lambda(t)}{4\pi} \int_{-1}^{+1} \int_0^{2\pi} P(t; \Omega, \Omega') \\ &\times I(t, \Omega') d\Omega' + \frac{\lambda(t)}{4} P(t; \Omega, \Omega_0) F \exp \\ &\times [-(L-t)/u], \end{aligned} \quad (2.1)$$

where $d\Omega = dv' d\phi'$, together with the boundary conditions

$$I(L, -\Omega) = 0, \quad (2.2)$$

$$I(0, +\Omega) = 0. \quad (2.3)$$

Eq. (2.3) refers to the boundary condition at the non-reentrant surface $t=0$. Furthermore, we should take into account the redistribution in direction of radiation at the interface $t=y$, whose y is the optical thickness of the aqueous layer.

In the case of specular reflection from the water surface, the angle of incidence is equal to that of reflection from the surface, and moreover the incident ray, the normal, and the reflecting ray are in the same plane. Generally speaking, a fraction $r(u)$ of the radiation incident on the interacting surface will be reflected, whereas the

remainder $(1-r(u))$ goes through the water.

In other words, both reflection and refraction take place at the interacting surface, where the redistribution of radiation in various directions occurs. Let the upwelling and downwelling intensities of radiation just above the air-water interface be denoted by $I(y, +\bar{\Omega}; A)$ and $I(y, -\bar{\Omega}; A)$, respectively. Similarly, let the upwelling and downwelling intensities of radiation just below the interface be denoted by $I(y, +\bar{\Omega}; W)$ and $I(y, -\bar{\Omega}; W)$, respectively. In the above A (or W) represents the abbreviation of Air (or Water), and each components layer is assumed to be free from the reflecting boundary. Furthermore, let the scattering and transmission functions of the air (or water) be denoted by $S(x(\text{or } y); \Omega, \Omega_0)$ and $T(x(\text{or } y); \Omega, \Omega_0)$, respectively, where x represents the optical thickness of the atmosphere.

Based on the invariance principles (cf. Ambarzumian 1958; Chandrasekhar 1960), we get the redistribution laws of radiation in various directions at the interaction surface as follows:

$$\begin{aligned} I(y, -\bar{\Omega}; A) &= \frac{F}{4v} T(x; \Omega, \Omega_0) + \frac{F}{4v} e^{-x/u} r(u) \\ &\times S^*(x; \Omega, \Omega_0) + \frac{1}{4\pi v} \int \frac{1}{2\pi} S^*(x; \Omega, \bar{\Omega}') \\ &\times \left[\frac{1-r(w')}{n^2} \right] I(y, +\bar{\Omega}'; W) d\Omega', \end{aligned} \quad (2.4)$$

$$I(y, +\bar{\Omega}; A) = \frac{(1-r(\bar{v}))}{n^2} I(y, +\bar{\Omega}; W) + I(y, -\bar{\Omega}; A) r(\bar{v}), \quad (2.5)$$

$$I(y, -\bar{\Omega}; W) = (1-r(v)) n^2 I(y, -\bar{\Omega}; A) + r(\bar{v}) I(y, +\bar{\Omega}; W), \quad (2.6)$$

$$\begin{aligned} I(y, +\bar{\Omega}; W) &= \frac{1}{4\pi v} \int S(y; \bar{\Omega}, \Omega') I(y, -\bar{\Omega}'; W) d\Omega' \\ &+ \frac{F}{4v} e^{-x/u} (1-r(u)) n^2 S(y; \bar{\Omega}, \bar{\Omega}_0), \end{aligned} \quad (2.7)$$

where

$$\bar{\Omega}' = ([1-n^2(1-w'^2)]^{1/2}, \phi'), \quad (2.8)$$

$$\bar{\Omega} = ([n^2-(1-v^2)]^{1/2}/n, \phi), \quad (2.9)$$

$$\bar{\Omega}_0 = ([n^2-(1-u^2)]^{1/2}/n, \phi_0). \quad (2.10)$$

In Eqs. (2.4) through (2.7) $r(v)$ is given by Fresnel's formula

$$r(v) = \frac{1}{2} \left[\frac{\sin^2(\theta_2 - \theta_1)}{\sin^2(\theta_2 + \theta_1)} + \frac{\tan^2(\theta_2 - \theta_1)}{\tan^2(\theta_2 + \theta_1)} \right], \quad (2.11)$$

where $v = \cos\theta_1$, $u = \cos\theta_2$, and furthermore,

$$n \sin \theta_2 = \sin \theta_1. \quad (2.12)$$

In Eq. (2.12) n is the refractive index of the second medium to the first one.

It should be mentioned that, allowing for the polarity of multiple scattering of radiation (cf. Ueno 1960), $S^*(x; \Omega, \Omega_0)$ is the scattering function of the atmospheric layer for the upwelling intensity of incident radiation at the interface $t=y$, whereas $T(x; \Omega, \Omega_0)$ - and $S(y; \Omega, \Omega_0)$ -functions are the transmission function of the atmospheric layer and the scattering function of the aqueous layer for the downwelling intensity of incident radiation at each top, respectively.

In the case of LANDSAT MSS data the effect of sun glint and wind wave on the ocean imagery should be taken into account (cf. Cox and Munk 1954; Plass, Kattawar, and Guinn 1977).

2. Total Spectral Radiance at the Top

In Eq. (2.4) through (2.7) the scattering and transmission functions $S^*(x; \Omega, \Omega_0)$ and $T(x; \Omega, \Omega_0)$ fulfill the Riccati type of integrodifferential equations (cf. Ueno 1960). These expressions in the conservative and homogeneous case have been given by Chandrasekhar (1960, p.169). In Eq. (2.7) $S(y; \Omega, \Omega_0)$ is governed by the above Riccati-type of initial-value solution, whose parameter x in S - and p -functions is replaced by y . These initial value solutions should be solved subject to the initial conditions, which vanish as the optical thickness reduces to zero.

Then, the required total spectral radiance emerging from the top is expressed in terms of S -, T^* - and I -functions as follows:

$$\begin{aligned} I(L, +\Omega) &= \frac{F}{4v} S(x; \Omega, \Omega_0) + I(y, +\Omega; A) e^{-x/v} \\ &+ \frac{1}{4\pi v} \int_{2\pi} T^*(x; \Omega, \Omega') I(y, +\Omega'; A) d\Omega' \\ &+ \frac{F}{4v} T^*(x; \Omega, \Omega_0) e^{-x/u} r(u), \quad (2.13) \end{aligned}$$

where $I(y, +\Omega'; A)$ is given by Eq. (2.5), and $T^*(x; \Omega, \Omega_0)$ is the transmission function of the atmospheric layer for the upwelling intensity of incident radiation at $t=y$.

B. ATMOSPHERE-OCEAN SYSTEM WITH EFFECTIVE SPECULAR REFLECTOR

1. The Equation of Transfer

Exact solution of the total spectral radiance $I(L, +\Omega)$ with allowance for the redistribution in direction of radiation energy at the air-water system in surface is not so readily found (cf. Mukai and Ueno 1978). Then, in order to make tractable the inverse problem, we shall take into account approximately the contribution of the up-

welling intensity of radiation just below the interface to the upwelling intensity of radiation just above the interface. In other words, in place of the surface albedo $r(v)$, we deal with the effective surface albedo $R(v)$, allowing for the interaction of radiation at the air-water interface. The quantity R corresponds to the effective specular reflector albedo. The equation of transfer appropriate to this case takes the form Eq. (2.1), where the overall optical thickness L is replaced by the atmospheric optical thickness X . Furthermore, it should be solved subject to the boundary conditions

$$I(x, -\Omega) = 0, \quad (2.14)$$

and

$$I(0, +\bar{\Omega}) = R(v) I(0, -\Omega), \quad (2.15)$$

where $\bar{\Omega}$ stands for $(v, \phi + \pi)$. In this case allowing for Eqs. (2.5) and (2.7), the effective surface albedo $R(v)$ is evaluated as below,

$$R(v) \cong [r(v) + \frac{(1-r(\bar{v}))}{n^2} \frac{I(y, +\bar{\Omega}; W)}{I(y, -\Omega; A)}], \quad (2.16)$$

where $R(v)$ includes the contribution due to the $I(y, +\bar{\Omega}; W)$ component.

2. Scattering Function

The total spectral radiance at top in the diffuse radiation field is expressed in terms of the scattering function $S(x, R; \Omega, \Omega_0)$

$$I(x, +\Omega) = \frac{F}{4v} S(x, R; \Omega, \Omega_0), \quad (2.17)$$

where the interaction effect of radiation at the calm sea surface is included approximately in the effective surface albedo $R(v)$. Now, the phase function $p(t; \Omega, \Omega_0)$ is expanded in Legendre polynomials

$$\begin{aligned} P(t; \Omega, \Omega') &= \sum_{m=0}^M (2-\delta_{0m}) \left\{ \sum_{k=m}^M C_k^m(t) P_k^m(v) P_k^m(v') \right\} \\ &\times \cos m(\phi - \phi'), \quad (2.18) \end{aligned}$$

where $P_k^m(t)$ is the associated Legendre function of degree k and order m , and

$$C_k^m(t) = C_k(t) \frac{(k-m)!}{(k+m)!} \quad (k=m, \dots, M, 0 \leq m \leq M), \quad (2.19)$$

$$\delta_{0m} = 1 \quad \text{for } m=0,$$

$$= 0 \quad \text{otherwise.} \quad (2.20)$$

Then, the scattering function is expressed in terms of the Fourier components

$$S(x, R; \Omega, \Omega_0) = \sum_{m=0}^M S^{(m)}(x, R; v, u) \cos m(\phi - \phi_0). \quad (2.21)$$

In a manner similar to our preceding paper (cf. Bellman and Ueno 1972), it is shown that the Fourier component of the scattering function $S^{(m)}(x, R; v, u)$ fulfills

$$\frac{dS^{(m)}}{dx} + \left(\frac{1}{v} + \frac{1}{u}\right) S^{(m)} = \lambda(x) (2 - \delta_{0m}) \times \sum_{k=m}^M (-1)^{k+m} C_k^m(x) \Psi_k^m(x, v) \Psi_k^m(x, u), \quad (2.22)$$

where

$$\Psi_k^m(x, v) = P_k^m(v) + P_k^m(v) R(v) e^{-2x/v} + \frac{(1)^{k+m}}{2(2-\delta_{0m})} \times \int_0^1 S^{(m)}(x, R; v, w) P_k^m(w) \frac{dw}{w}, \quad (2.23)$$

for $m=0, 1, 2, \dots, M$. The initial condition of Eq. (2.22) is given by

$$S^{(m)}(0, R; v, u) = 0. \quad (2.24)$$

Furthermore, the $S^{(m)}$ -function has angular symmetric property, i.e., the reciprocity principle,

$$S^{(m)}(x, R; v, u) = S^{(m)}(x, R; u, v). \quad (2.25)$$

III. INVERSION OF THE TOTAL SPECTRAL RADIANCE FOR EFFECTIVE SURFACE ALBEDO

A. LEAST SQUARES ESTIMATION

In a series of papers (cf. Bellman et al 1965), the least squares estimation of such optical properties as the optical thickness, phase function, and others has been done with the aid of quasilinearization and imbedding, assuming an atmosphere bounded by a completely absorbing background. In what follows, we shall consider the estimation of the albedo of the reflecting underlying surface.

The inverse problem which we wish to solve is to estimate the effective surface albedo of the air-water system interface discussed in the preceding section, in the least squares sense, using the noisy total spectral radiance measurements. Mathematically speaking, we wish to minimize the expression

$$\sum_{i,j,k} [I(x, R; v_i, \phi_k; v_j, \phi_0) - b_{ijk}]^2, \quad (3.1)$$

over all choices of the unknown parameter R , provided that the phase function of multiple scattering, the optical thickness, and others are together known. In Eq. (3.1) b_{ijk} represents the intensity of the diffusely reflected light, $b_{ijk} \approx I_{ijk}(x, R)$, for incident directions (v_j, ϕ_0) , $j=1, 2, \dots, N$.

The subscript k denotes the k -th component of the azimuth of the view angle. In Eq. (3.1) $I_{ijk}(x, R)$ is given by

$$I(x, R; v_i, \phi_k; v_j, \phi_0) = \frac{F}{4v_i} \sum_{m=0}^M S^{(m)}(x, R; v_i, v_j) \times \cos m(\phi - \phi_0), \quad (3.2)$$

On making use of quasilinearization, the problem will be successively solved. Putting the function $S_{ij}^{(m)}$ as S_{mij} and similarly, writing Ψ_{ki}^m as Ψ_{mki} , $P_k^m(v_i)$ as P_{mki} we get a system of linear differential equations for the $(n+1)$ st approximation of S_{mij} and R . A Cauchy system of the S -component takes the form

$$\frac{dS_{mij}^{n+1}}{dx} = f(S_{mij}^n, R^n) + \sum_{i,j} [S_{mij}^{n+1} - S_{mij}^n] \frac{\partial f}{\partial S_{mij}^n} + (R^{n+1} - R^n) \frac{\partial f}{\partial R^n}, \quad (3.3)$$

$$\frac{dR^{n+1}}{dx} = 0, \quad (3.4)$$

where

$$f(S_{mij}^n, R^n) = -\left(\frac{1}{v_i} - \frac{1}{v_j}\right) S_{mij}^n + \lambda(x) (2 - \delta_{0m}) \times \sum_{k=m}^M (-1)^{k+m} C_k^m(x) \Psi_{mki} \Psi_{mkj}, \quad (3.5)$$

and S^0 is a known set of first approximation to the solution.

In Eq. (3.5) Ψ_{mki} is given by

$$\Psi_{mki} = P_{mki} + \frac{(-1)^{k+m}}{2(2-\delta_{0m})} \sum_{j=1}^M S_{mij}^n P_{mkj} \frac{w_j}{v_j} + P_{mki} R(v_i) e^{-2x/v_i}. \quad (3.6)$$

In Eq. (3.6) $\{v_i\}$ is the set of N roots of the shifted Legendre polynomial of degree N , $P_N^*(v) = P_N(1-2v)$. Furthermore, w_j is the Christoffel weight corresponding to the value of v_j . The initial condition of S_{mij} is such that it vanishes as x tends to zero.

In Eq. (3.3) the Fourier component S_{mij}^n satisfies the principle of reciprocity with respect to the angular argument.

$$S_{mij}^n = S_{mji}^n. \quad (3.7)$$

There are basically $(MN^2 + 1)$ differential equations, which reduce to $\{MN(N+1)/2 + 1\}$ differential equations with the aid of the reciprocity relation Eq. (3.7), whereas the full set of values S_{mij}^n representing MN^2 matrix is always available. The solution is subject to the initial condition (2.24)

and the boundary condition (3.1) with Eq. (3.2).

Since S_{mij}^{n+1} is a solution of a system of linear differential equations, we express it in terms of a linear combination of a particular solution q_{mij} and a homogeneous solution h_{mij}

$$S_{mij}^{n+1} = q_{mij}^{n+1}(x) + R^{n+1} h_{mij}^{n+1}(x). \quad (3.8)$$

The particular solution is defined by the equation

$$\frac{dq_{mij}^{n+1}}{dx} = f(S_{mij}^n, R^n) + \sum_{i,j} [q_{mij}^{n+1} - S_{mij}^n] \frac{\partial f}{\partial S_{mij}^n} + (R^{n+1} - R^n) \frac{\partial f}{\partial R^n}, \quad (3.9)$$

$$q_{mij}^{n+1}(0) = 0, \quad (3.10)$$

while the homogenous solution is defined through the differential equations

$$\frac{dh_{mij}^{n+1}}{dx} = \sum_{i,j} h_{mij}^{n+1} \frac{\partial f}{\partial S_{mij}^n} + R^{n+1} \frac{\partial f}{\partial R^n}, \quad (3.11)$$

$$h_{mij}^{n+1}(0) = \delta. \quad (3.12)$$

where δ is the Kronecker delta function, which vanishes except for the last component of unity. From the definition of the initial condition in Eqs. (3.10) and (3.11), the coefficient R in Eq. (3.8) is identified with initial conditions of the system at x .

Given an initial approximation to the solution S , both the particular solution q and the homogeneous solution h can be computed from Eqs. (3.9) and (3.11)

By a simple differentiation of Eq. (3.1) with respect to R , we get the value of R minimizing the performance function

$$R^{n+1} = \sum_{i,j,k} [4v_{ij} b_{ijk} - \sum_{m=0}^M q_{mij} \cos m\phi_k] \times \left[\sum_{m=0}^M h_{mij} \cos m\phi_k \right] \left[\sum_{i,j,k} \left(\sum_{m=0}^M h_{mij} \cos m\phi_k \right)^2 \right]^{-1/2} \quad (3.13)$$

where ϕ_0 is put to be zero. Eq. (3.13) is the required expression, permitting the least squares estimation of the effective water surface albedo from the noisy spectral radiance measurements.

Numerical Experiments

In our preceding papers (cf. Kagiwada 1974; Kagiwada, Kalaba and Ueno 1975), the inverse pro-

blems of inhomogeneous, anisotropically scattering atmospheres bounded by diffuse reflectors have been solved computationally. Initial estimates of the parameters as the optical thickness, phase function, surface albedo and others are sequentially refined, the sequence usually being rapidly convergent.

For the sake of simplicity of presentation, the inverse problem of homogeneous, isotropically scattering atmosphere bounded by quiet water surface is computed for the case in which true values of the parameters are $\lambda=1$, $x=0.3$, $R=0.02$ (or 0.04). The measurements b_{ij} for seven Gaussian division points are made based on the numerical solution of Eq. (2.22) under consideration. On making use of the reciprocity principle, the number of numerical integrations has been reduced. Several numerical experiments are performed. The results of three experiments with initial guesses $R=0$, 0.1, and 0.2, respectively, are given in Table 1.

Table 1. Successive approximations of R , The effective water surface albedo

Approximations	Run 1	Run 2	Run 3
0	0.0	0.1	0.2
1	0.063	0.047	0.037
2	0.0206	0.0203	0.0202
3	0.02021	0.02021	0.02021
4	0.02021	0.02021	0.02021
True Value	0.02	0.02	0.02

The values of R obtained in the first, second, third and fourth experiments are tabulated. The initial guess of R in Run 1 is too low, in Run 2 and 3 too high. Yet the correct values of R are almost accurately found in 3 to 4 iterations.

The time required for each run is less than a few minutes on the IBM 3031 digital computer, using Runge-Kutta fourth order integration scheme with a grid size of $\Delta x=0.01$.

In the case of LANDSAT II MSS data, making use of the Takayama scene in the north side of Japan on May 23, 1979, the test site of ten by ten pixels is fixed in the coastal area of Japan sea in the neighbourhood of Kanazawa. At the local transit time 9^h 45^m the used parameters are $\theta_0=0^\circ$, $\theta=31^\circ 25'$, on making use of the optical parameters as $x=0.35$, $\lambda=1$, in atmospheric model similar to Elterman's one the mean effective water surface albedo $R=0.02$ in 4th channel verified the minimization of the performance function for the inversion of spectral radiance.

IV DISCUSSION

In the present paper, first of all, an initial -value solution of the transfer equation for the air-water system is evaluated with the aid of invariant imbedding, keeping in mind both reflection and refraction at the quiet water surface. Then, allowing for the interaction of radiation at the system interface, an introduction of effective

water surface albedo into a Cauchy system of the scattering function for the composite system made tractable the initial-value solution of the inverse problem. Several numerical experiments of the successive approximations of the effective water surface albedo showed satisfactory rapid convergence towards the expected values. Then, the inference of subsurface water parameters as the chlorophyll concentration and scattering function will be discussed later on, allowing for various kinds of effects as the sun glint, wind wave and others on the quiet water surface. Finally, we express our gratitude to Mr. K. Takemata for his numerical computation.

REFERENCES

1. Ambarzumian, V.A. (Editor), 1958. *Theoretical Astrophysics* (English translation), Pergamon Press.
2. Ambarzumian, V.A. 1864, On the problem of multiple scattering of light in a plane parallel medium with internal reflection at the bounding surface. *Trudy Astron. Obs. Leningrad, Gos. Univ.* Vol.20.
3. Bellman, R., H. Kagiwada, R. Kalaba, and S. Ueno. 1965. Inverse problems in radiative transfer: Layered media. *ICARUS*, vol.4, pp.119-126.
4. Bellman, R., and S. Ueno. 1972. Invariant imbedding and Chandrasekhar's planetary problem of radiative transfer. *Astrophys. Space Sci.*, Vol. 16, pp.241-248.
5. Casti, J.L., R. Kalaba, and S. Ueno. 1969. Reflection and transmission functions for finite isotropically scattering atmospheres with specular reflectors. *J. Quant. Spectrosc. Radiat. Transfer.* Vol.9, pp.537-552.
6. Chandrasekhar, S.. 1950. *Radiative Transfer*, Oxford University Press.
7. Cox, C., and W. Munk. 1954. Measurement of the roughness photographs of the sun's glitter. *J. Opt. Soc. Am.*, Vol.44, pp.838-850.
8. Duntley, S.Q.. 1963. Light in the sea. *J. Opt. Soc. Am.* Vol.53, pp.214-233.
9. Gutshabash, S.D. 1963. The diffusion of radiation in two layer medium with effects at the interface taken into account. *Izv. Geophys. Ser.*, pp.1117-1124.
10. Haba, Y., Y. Kawata, T. Kusaka, and S. Ueno. 1979. The system of correcting remotely sensed Earth imagery for atmospheric effects. In J. Cook (Ed.), *Proc. Intern. Symp. on Remote Sensing of Environment*, April 23-27, Ann Arbor, Michigan, pp.1883-1894.
11. Jain, S.C., and J.R. Miller, 1976. Subsurface water parameters: optimization approach to their determination from remotely sensed water color data. *Appl. Opt.*, Vol.15, pp.886-890.
12. Kagiwada, H. 1974. *System Identification: Methods and Applications*. Addison-Wesley Publ. Co., Inc., Reading, Mass..
13. Kagiwada, H. R. Kalaba, and S. Ueno. 1975. *Multiple Scattering Processes: Inverse and Direct*. Addison-Wesley Publ. Co. Inc., Reading, Mass..
14. Mukai, S., and S. Ueno. 1978. Apparent contrast of an atmosphere-ocean system with an oil polluted sea surface. *App. Math. Modelling*, Vol. 2, pp.254-260.
15. Plass, G.N., and G.W. Kattawar. 1969. Radiative transfer in an atmosphere-ocean system, *Appl. Opt.*, Vol.8, pp.455-466.
16. Plass, G.N., G.W. Kattawar and J.A. Guinn. 1977. Isophotes of sunlight glitter on a wind-ruffled sea. *Appl. Opt.*, Vol.16, pp.643-653.
17. Raschke, E., 1972. Multiple scattering calculation of the transfer of solar radiation in an atmosphere-ocean system. *Beitr. Phys. Atmos.* Vol.45, pp.1-19.
18. Sobolev, V.V., 1963. *A Treatise on Radiative Transfer*, Van Nostrand, Princeton, N. J..
19. Sobolev, V.V.. 1975, *Light Scattering in Planetary Atmospheres*, Pergamon Press, Oxford and New York.
20. Tanaka, M., and T. Nakajima. 1977. Effects of oceanic turbidity and index of refraction of hydrosols on the flux of solar radiation in the atmosphere-ocean system. *J. Quant. Spectrosc. Radiat. Transfer*, Vol.18, pp.93-111.
21. Ueno, S., 1960. The probabilistic method for problems of radiative transfer. X. Diffuse reflection and transmission in a finite inhomogeneous atmosphere. *Astrophys. J.*, Vol.132 pp.729-745.
22. Ueno, S.. 1981., in *Proc. Intern. Workshop on Processes in Marine Remote Sensing* (Edit. by J. Vernberg), Manchester, England, June 26-30, University of North Carolina press.
23. Ueno, S., Y. Haba, Y. Kawata, T. Kusaka, and Y. Terashita. 1978. In *Remote Sensing of Atmosphere: Inversion Methods and Application* (Edit. by Fymat and Zuev), Elsevier Publ. Co. Inc., Amsterdam, pp.305-310.
24. Ueno S., Y. Kawata, T. Kusaka, and Y. Haba, 1980, in *Proceedings of 1980 IFAC Symposium on Water and Related Land Resource System*, Pergamon Press, pp.165-172.
25. Ueno, S., and S. Mukai. 1973. Reduction of Chandrasekhar's planetary problem in the case of a specularly reflecting surface to the standard problem, *J. Quant. Spectrosc. Radiat. Transfer*, Vol.13, pp.1261-1271.