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A STUDY ON RESONANT STATE OF PRESSURE PULSATION IN A SIMPLE SUCTION PIPELINE OF RECIPROCATING COMPRESSOR

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ABSTRACT

Analysis and experiment show that to supercharge the suction volume of a cylinder by means of pressure pulsation is not certain to result in delay of the valve closing. To arrange a proper suction pipeline and valve cavity, we can not only get supercharge of the cylinder, but also get economical specific power consumption, while valve working condition is good too. In this paper the methods of calculating pipe length and amplitude of pressure pulsation as well as loss of pressure at resonant state are recommended. Experiment of verification was done on a double-acting air compressor type L 2-10/7.

INTRODUCTION

It is the nature that a reciprocating compressor sucks and discharges intermittently, thereby it generates pressure pulsation in the piping system. Generally, the pressure pulsation results in vibration of the piping system and must be limited within a permitted value. But sometimes it can also be made use of supercharging in the suction volume of the cylinder or reducing specific power consumption, while it doesn't make the valve working condition worsen. In the reference [1], [6], [8], [9], [10], these cases were studied. In this paper we studied some rules of resonance and its influence on movement of the valve plate.

ANALYSIS

The pressure pulsation comes from intermittence of the suction and variable motion of the piston during suction process. Excitement to be stirred up by the piston varies with time as following

\[
\text{Single-acting (cycle } 2\pi) \quad \begin{cases} 
    u(t) = 0, & \theta \leq \omega t < 0 \\
    u(t) = r \omega_b (\sin \omega t + \frac{r}{2} \sin 2\omega t), & \pi \leq \omega t > \theta \\
    u(t) = 0, & \frac{3\pi}{2} \leq \omega t \geq \pi
\end{cases} \quad (1)
\]

\[
\text{double-acting (cycle } \pi) \quad \begin{cases} 
    u(t) = 0, & \theta \leq \omega t \leq 0 \\
    u(t) = r \omega_b (\sin \omega t + \frac{r}{2} \sin 2\omega t), & \pi \leq \omega t \geq \theta
\end{cases} \quad (2)
\]

The excitation function can be expanded in the Fourier series

\[
u(t) = \frac{U}{2} + \sum U_m \sin(m \omega t + \varphi_m), \quad m = 1, 2, 3, \ldots
\]

for single-acting \( n = 1 \)
for double-acting \( n = 2 \)

Excitation source of each order harmonic component should be regarded at the top of the piston, so that both single and double acting cylinders become ordinary volume which is separated by valve, and there is only a concentrated pressure loss like orifice at the place of valve. The volume of cylinder should be replaced by an equivalent volume. The equivalent volume of
the cylinder equals the steady component of harmonic analysis formula as follows:

\[ V(t) = A_0 \left[ 1 - \cos \omega_0 t + \frac{\omega_0}{2} (1 - \cos 2 \omega_0 t) \right] + V_r, \]

\[ 0 < \omega_0 t < \pi \]

The cylinder is considered as a steady component of harmonic analysis. The actual value of the L 2 - 10/7 compressor, expanded in a Fourier series, has proved the first order harmonic amplitude is 103.33% of the general amplitude. The wave composed of the first and the second order harmonics shown in fig. 3 shows that the general amplitude is almost not affected by the second order harmonic. The actual value of the L 2 - 10/7 compressor, expanded in a Fourier series, has proved the first order harmonic amplitude is 103.33% of the general amplitude. The wave composed of the first and the second order harmonics shown in fig. 3 shows that the general amplitude is almost not affected by the second order harmonic.

In double-acting compressor first order harmonic takes a greater portion. The method of harmonic analysis can be applied to measured pressure curve. The rate of first order harmonic amplitude to the general amplitude is larger than 95%. The pressure curve measured at resonant region is shown in fig. 4. These curves are similar.
to sine wave. It illustrates that the first order harmonic is the main part and other orders may be negligible. It also explains that the damping makes first order harmonic component smaller distortion.

$$V_v = 4V_c$$
$$d = 150\text{mm}$$
$$l = 1240\text{mm}$$

Fig. 4a The case of largest amplitude 1, 2, 3 — measured points shown by fig. 10

$$V_v = 4V_c$$
$$d = 150\text{mm}$$
$$l = 1750\text{mm}$$

Fig. 4b The case of valve closing is moved up

$$V_v = 4V_c$$
$$d = 150\text{mm}$$
$$l = 810\text{mm}$$

Fig. 4c The case of valve opened delay

Due to the reason above, it is exact enough for calculating in engineering that the general wave is simplified as the sum of a first order harmonic and a steady flow, so that the following formula can express excitation function just as the solid line of the fig. 5.

$$u_v = \bar{u}_v + \bar{u} = U_v \sin(\omega t + \psi_v) + \bar{U}_v \quad (4)$$

CALCULATION OF RESONANT LENGTH OF THE SUCTION PIPELINE

In the reference [2], [3] the calculation of natural frequency of gas column in complicated piping system is recommended, but how to deal with the condition that valve opens intermittently and cylinder volume change continuously, isn’t mentioned. From the experiment by Sodel et., even if the amplitude of pressure pulsation is close to 30% of the nominal pressure, acoustics equation is still applicable. Here the amplitude of pressure pulsation at resonant state in compressor isn’t larger than 30%, so that the equation can be applied at resonance. In actual compressor system the steady flow velocity isn’t zero, in order to investigate the influence of steady flow velocity, the following equations are solved.

$$\rho_v \frac{\partial u}{\partial x} = -\frac{\partial p}{\partial x} - aU_v u \quad (5)$$
$$\frac{\partial p}{\partial t} = -c^2 \rho_v \frac{\partial u}{\partial x} - \bar{U}_v \frac{\partial p}{\partial x} \quad (6)$$

$$u = \bar{U}_v \frac{A_v}{A_1} - \bar{u} = \bar{U}_v + \bar{u}$$

$$\rho_v, \bar{U}_v$$ — average value at any section of suction pipeline. The solution of above equation is

$$\begin{bmatrix} A_1 \\ U_1 \end{bmatrix} = e^{-j\gamma l} \begin{bmatrix} \cos \varphi & j\sin \varphi \\ j \sin \varphi & \cos \varphi \end{bmatrix} \begin{bmatrix} A_2 \\ U_2 \end{bmatrix} \quad (7)$$

$$M_o = \frac{U}{c}, \quad \varphi = \frac{\omega}{c} l, \quad A = \frac{p}{\rho v}$$
The difference between the formula above and the equation without steady flow velocity is only the term $e^{-j\omega_{0}t}$. It illustrates that steady flow velocity makes the phase between pressure and velocity turn round an angle along the pipe, in general, value $\omega_{0}$ is very small, so that the amplitude and velocity almost doesn’t change. Thus acoustics equation can be applied.

How to deal with valve and volume of cylinder while calculating resonant pipeline length is a question which is worth studying. It is evident that place of valve was regarded as a closed end doesn’t accord with the real condition. Because the section of valve in comparison with the section of pipeline isn’t very small and, for a double-acting compressor the valve is closed merely in 30% of a cycle period, that is to say, linking intermittently of two volumes and changing continuously of the cylinder volume must be accounted. The changing cylinder volume should be replaced by an equivalent volume, just as shown by formula (3) and, the top of the piston is regarded as a closed end.

The equation to find out resonant length of pipeline shown by fig.6a is
\[ t = \frac{\omega L}{c} = \frac{CA}{\omega V} \]

Shown by fig.6b is
\[ t = \frac{\omega L}{c} = \frac{A_{1}\cos k_{2}l_{2} - A_{2}\sin k_{2}l_{2}}{A_{2}\sin k_{2}l_{2} + \frac{\omega V}{c}\cos k_{2}l_{2}} \]

where $V = V_{V} + V_{o}$

\[ V_{o} = \frac{4\pi}{\omega} \int_{0}^{\pi} \left( 1 - \cos \omega_{s}t + \frac{e^{j\omega_{c}t}}{2} (1 - \cos 2\omega_{c}t) + V_{i} \right) dt \]

With the assumption of opened end $p = 0$, the calculating method mentioned above is performed. In fact, pressure pulsation could travel out in spherical surface wave at the opened end, thus damping of the inlet of pipeline doesn’t equal zero. From reference (4) damping of opened end as follows
\[ I \cong \rho C \left( 1 - \frac{2}{\omega_{s}}(W) \right) - J \rho C M(\omega) \]  

where
\[ W = \frac{\omega D}{\lambda} = \frac{2\pi D}{\lambda} \]

$J(\omega)$ is the Basal Function
\[ M(\omega) = \frac{\pi}{4} \int_{0}^{\frac{\pi}{2}} \sin(w_{0}s) \sin^{2}\alpha da \]

By and large in compressor $w < 0.5$, then limit value of impedance of acoustics at opened end is the following value
\[ I \cong \frac{D^{2}w^{3}}{8c} + \frac{i\omega D^{2}w}{3\pi} \]

When the stimulating frequency resulted from the compressor is lower, damping is smaller, thus impedance to the pipe inlet is $\frac{4PD}{3\pi}$ shown by fig.7. Namely radiation effect of inlet is equal to adding a section pipe as long as $4D/3\pi$

\[ l = l_{o} + \frac{4D}{3\pi} \]

When the width of hinder in inlet shown by fig.7. is far less than wavelength of pressure pulsation, impedance is 0.3

\[ l_{o} = l - 0.3D \]

Fig.7 Construction of inlet

From formula (13), the larger the pipe diameter, the greater the radiation effect. Calculated values in comparison with
experimental values is in table 1.

<table>
<thead>
<tr>
<th>$\omega_0$</th>
<th>$V$</th>
<th>$D$</th>
<th>$l_0$</th>
<th>$l_0'$</th>
<th>error $\frac{4.6}{2.3}$</th>
</tr>
</thead>
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<td>0.04362</td>
<td>100</td>
<td></td>
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<td>4.6</td>
</tr>
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<td>645</td>
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<td>1014</td>
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<td>0.02752</td>
<td>150</td>
<td>1260</td>
<td>1240</td>
<td>1.5</td>
</tr>
</tbody>
</table>

* $l_0$ — calculated, $l_0'$ — measured

CALCULATION OF RESONANT AMPLITUDE OF PRESSURE PULSATION OF SUCTION SYSTEM OF ACTUAL COMPRESSOR

In pipeline the vibration mode is standing wave, then velocity can be written as follows at the first order resonance

$$u = U_0 U(x) Q \sin(\omega t - \frac{\pi}{2}) + \frac{A_i}{A} \tilde{U}$$  \hspace{1cm} (14)

The $U(x)$ is vibration mode of the velocity with given boundary condition and without damping, it can be got from wave equation at $U_1 = l$, $p_1 = 0$.

The Q is a constant depending on piping system and frequency. It expresses what degree a system at resonant state amplifies pulsation to. $Q$ can be got from energy equation, as a result

$$Q = \sqrt{\frac{5 \pi A_0 U_0^2}{8 B, U_0}}$$  \hspace{1cm} (15)

where

$$B_i = \frac{P A_i}{2D} \int_{t_i} U(x_i) \, dx + \frac{P A_i \xi_i}{2} U(x_i)$$

In pulsation flow, concentrated loss can be calculated from

$$p_i - p_i' = \frac{P}{2} \xi_i (\bar{U} \frac{A_0}{A_i} + \tilde{U} \frac{A_0}{A_i})$$  \hspace{1cm} (16)

The sub i shows the order of the pipe sections; the $p_i$ is instantaneous pressure in front of section i, the $p_i'$ is behind that section. Formula (16) is expanded in Fourier series, and the steady term is $(\bar{p}_i - \bar{p}_i')$, first order term is $(\tilde{p}_i - \tilde{p}_i')$.

Let $g_i = \frac{A_i}{A} \frac{U(x_i)}{U}$

when $g_i < 1$, from harmonic analysis

$$\Delta \bar{p}_i = -\frac{P}{2} \xi (\pi + g_i)(\frac{A_0}{A_i} \bar{U})^2$$

$$\Delta \tilde{p}_i = \frac{P}{2} \xi g_i (\frac{A_0}{A_i} \bar{U})^2$$

when $g_i > 1$

$$\Delta \bar{p}_i = -\frac{P}{2} \xi (\frac{7.66 g_i + 0.562 \pi}{\pi})(\frac{A_0}{A_i} \bar{U})^2$$

$$\Delta \tilde{p}_i = \frac{P}{2} \xi (\frac{g_i}{\pi} g_i^2 + 6)(\frac{A_0}{A_i} \bar{U})^2$$

Pressure loss calculated by means of this method is about 87% larger than measured pressure loss, but in the same pipeline the measured pressure loss is about 4~8 times than the $\Sigma \frac{P}{2} \xi (\frac{A_0}{A_i} \bar{U})^2$ as turbulent flow loss in pipeline

EXPERIMENT AND ANALYSIS

Experiments have been made on an L2-10/7 type compressor.

Pressure pulsation, valve plate movement and pressure loss are recorded. The distribution of measuring point of pressure is shown by fig. 3.

![Fig.3 The place of measured points of pressure](image_url)
Fig. 9  Pressure pulsation of each measured point

Pressur curve recorded in fig. 9 shows the phases of three measured point are almost same. This illustrates that the assumption of standing wave accords with real condition. The curve is like 1st order harmonic. These curves shown by fig.4.

During opening of valve, pressure curve in cylinder is parallel with pressure curve in valve cavity this explains that gas pulsations in cylinder and out of cylinder are synchronous. It brings to light that gas in equivalent volume of cylinder joins in pulsation.

\[ V_a = 6V_c \]
\[ d = 150\text{mm} \]
\[ l = 860\text{mm} \]

Fig.10 The largest supercharge case compares with the largest amplitude

The largest pressure loss measured corresponds to the best supercharge case. The length of pipeline is less than the length corresponded by the largest amplitude case. At the same time pressure amplitude is also less than the largest amplitude. In this case the curve of valve plate movement shows the closing of valve isn't delayed, and valve plate is nestled down valve seat without dashing, valve opening isn't delay too. It can be seen by fig.10.

In case of the largest amplitude closing of valve has been delayed for 12°, the surpercharge isn't the largest, because at the close point pressure isn't maximum. The closing of valve is delayed principally by pulsation of velocity when the design of valve is perfect.

During closing of the valve the direction of velocity of pulsation is towards inside of cylinder, so that impulse of velocity delays the closing of valve, and vice versa closing of valve is moved up.

Though rising of pressure almost doesn't affect closing of valve, it affects opening of valve strongly. During opening of valve pressure in cavity is dropping, then opening of valve could be delayed. The fig.4b shows the case in which closing of valve is moved up. The fig.4c shows the case of valve opened delay. If the pressure is intending to maximum during the process of valve opening, it will help valve plate opening.

If the velocity of pulsation is rushing into cylinder with maximum value, it will help valve plate closing, then power consumption during suction period will be decreased.

Generally the two cases mentioned above don't coincide each other, except damping and clearance volume vanished.

In case of fig.4b corresponding pipe length is \( l = 1.35l_0 \), figure shows pressure pulsation, also affects operation of compressor; in case of fig.4c corresponding pipe length is \( l = 0.628l_0 \) and pressure pulsation amplitude is 81% of the maximum. So that only keeping away from the region of
l=(0.8 ～1.2) l0  to avoid resonance isn't good enough.

CONCLUSION
From the results of experiment and analysis above, following conclusions can be got
1. To make use of the pressure pulsation supercharge, valve working condition isn't certain to become badly
2. A method of calculating pipe length of gas column resonance is recommended.
3. The method of pressure loss calculating coincides with reality basically.
4. The method of calculating resonant amplitude in actual compressor can meet needs of engineering basically.

NOTATION
A Section of pipe
C Adiabatic velocity of sound
D diameter of the suction of pipe
e the ratio of radius of crankshaft to length of connecting rod
l length of pipe
l0 length of pipe at resonant state
Pa steady component of pressure pulsation at the excitation source
p amplitude of first order harmonic pressure pulsation of gas column
p instantaneous valve of pressure of gas column
p instantaneous valve of pressure of first order pulsation of gas column
Q resonant factor of first order
r radius of crankshaft
U steady flow velocity at excitation source
U amplitude of velocity pulsation
U0 amplitude of first order harmonic component of velocity at excitation source
u instantaneous valve of gas column flow velocity
u instantaneous valve of gas column first order pulsation velocity
Vc cyinder volume
Vr clearance volume
Vs volume of valve cavity
Ve equivalent volume of cylinder
a friction factor
a wavelength of first order harmonic
x coefficient of concentrated pressure losses
f gas density
ω0 angular velocity of the machine
ω first order harmonic angular velocity

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