Current Pulsation Calculations of an Induction Motor Connected to a Reciprocating Compressor

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Cyclic vibratory torque requirement of a Reciprocating Compressor sets in torsional vibrations in compressor motor system. Explanation of development of pulsating currents, due to vibratory motion of motor rotor is given. The pulsating currents are superimposed on steady state currents. Mathematical models for analysing torsional vibrations of the system and net motor current due to combination of steady state and pulsating currents is explained. Step-wise procedure for calculating instantaneous line current over a period for few revolutions of compressor and subsequently current pulsation is given.

INTRODUCTION

Rotor of an Induction Motor, directly coupled to a Reciprocating Compressor, is subjected to periodic angular oscillations i.e. torsional vibrations. The torsional vibrations are induced due to periodic excitation exerted by vibratory nature of cyclic torque requirement of compressor. Frequencies of vibrations are equal to compressor RPM and its integer multiples i.e. higher harmonics. This vibratory motion is superimposed on mean angular movement of motor. Synchronising torques, damping torques and pulsating primary currents are developed due to vibratory motion of rotor. The actual instantaneous line current is sum of instantaneous values of steady state mean load current and pulsating currents. Figure 1 illustrates the typical current (instantaneous) versus time graph for motor subjected to torsional vibrations.

As per NEMA (1) current pulsation is defined as ratio of difference between maximum and minimum amplitude of current and 1.421 times rated motor full load current (rms). The maximum and minimum amplitude is found out by drawing enveloping curve as shown in Figure-1.

Current pulsations can be recorded by oscillographs. However exact evaluation of it is necessary at design stage in order to ensure that current pulsation in proposed system will not exceed stipulated limits.

Mathematical analysis confirms that the performance characteristics of motor such as operating values of current, power factor and efficiency are different than those determined from steady state design graphs. Hence motor performance at operating conditions, particularly when motor rotor is subjected to vibratory conditions should be analysed more critically at design stage considering present emphasis on energy conservation.

CALCULATION PROCEDURE

Riches(2), Middlemiss (3) & Cummings(4) have published papers on calculation methods for current pulsation. Riches method neglects synchronising torque effect and frequency dependence of damping torque. Method for calculating synchronising and damping torques at different vibration frequencies for an induction motor is developed by Concordia (5) same has been used by Middlemiss (3) and Cummings (4). Proposed method of analysis is an extension of method published by Cummings (4).
The basic difference is in assumption of equivalent torsional system. The Figure 2 illustrates the difference, whereas cummings (4) assumes single mass torsional system, it is proposed to use multimass torsion system for the analysis. Multimass torsional system is more exact representation of actual system. With this analysis effect of compressor cylinder damping, torsional stiffness of coupling and its damping (in case of Flexible couplings), torque excitation of individual cylinder and effect of intermass shaft stiffnesses are taken into account.

Procedure in steps to calculate current pulsation, which is shown in figure 3, is as follows:-

1. Torque - Crank angle diagrams at crank for every compressor cylinder is calculated and harmonic components (i.e. Amplitude and phase angle) of Fourier series for these graphs are obtained for predetermined number of harmonics (Ref.6).

2. Equivalent multimass torsional system is obtained from geometric dimensions of crankshaft, motor shaft, coupling and details such as various rotating & reciprocating masses in compressor, coupling and motor (Ref.6). Effect of cylinder damping is based on previous experience. Coupling dynamic torsional stiffness, its frequency dependent characteristic, damping coefficient are based on data obtained from coupling manufacturer. If compressor and motor are rigidly coupled, inertia of both halves of coupling and compressor flywheel are added together to represent one single mass without damping in equivalent torsional system.

3. Motor side end shaft stiffness K5 and D5 (figure 2) are frequency dependent and are calculated for all harmonic frequencies under consideration. Refer Appendix A. For method of calculation of these values which is reproduced from ref (4).

4. Equation of motion for each mass of torsional system is now defined. Each equation is second order differential equation which is split into two linear equations. These equations are solved simultaneously to obtain angular displacement vector of each mass for every harmonic excitation frequency.

Procedure for forming equations and their solutions is explained in Appendix B. It is general procedure for torsional vibration analysis of multimass system whose one end is free and other end is fixed. Further it is assumed that each mass of the system is subjected to damping, excitation torque and similarly each shaft provides shaft damping of the system. The actual system shown in figure 2 is a particular case of generalised system. In this case each mass and shaft is not subjected to damping and excitation, wherever they are not shown in figure, their value are assumed equal to zero. Amplitude and phase angle of angular vibration of motor rotor mass for each harmonic frequency is finally obtained from this analysis.

5. Oscillating component of stator current is obtained from hunting frequency network as explained in Appendix A. Oscillating & steady state stator currents are added together to obtain equation for instantaneous stator current as a function of time. This equation is solved at regular increment of time and graph is plotted for current verses time. Finally current pulsation is calculated as per definition of NEMA (1).

RESULTS

Analysis of 150kW, 6pole squirrel cage induction motor rigidly coupled to two cylinder balanced opposed piston air compressor was carried out with above procedure and also as per cummings (4) method.

Instantaneous current graphs similar to Figure 1 were drawn and results were compared.

<table>
<thead>
<tr>
<th>Method</th>
<th>Current Pulsations %</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cummings</td>
<td>11.65</td>
</tr>
<tr>
<td>Proposed procedure</td>
<td>5.5</td>
</tr>
</tbody>
</table>

Comparison confirms variation in results which can be attributed to refined mathematical model of the system.
CONCLUSION

Methods available for calculation of current pulsations in an induction motor use very coarse mathematical model as far as mechanical system is concerned. This may lead to very large error in estimation of current pulsation particularly in systems which are flexibly coupled. The suggested method takes into account system flexibilities and torsional damping available at other locations in system viz cylinder, coupling, etc. In general it can be concluded that the current pulsation calculations is a part of torsional vibration analysis of complete system.

APPENDIX 'A'

DETERMINATION OF SYNCHRONISING AND DAMPING TORQUES OF MOTOR AND INSTANTANEOUS CURRENTS

Steady state and hunting frequency equivalent circuits of an induction motor are shown in Figure A-1. Let steady state stator current vector \( I' \) be defined as follows.

\[
I' = G - jH
\]  

Then its instantaneous value is given by equation

\[
I' = G \cos(2\pi ft) + H \sin(2\pi ft)
\]  

Similarly instantaneous stator current in hunting frequency circuits can be written as

\[
\Delta I_{jn} = Cn - jDn = Cn \cos(1+hn)2\pi ft + Dn \sin(1+hn)2\pi ft.
\]  

\[
\Delta I_{2n} = En - jF n = En \cos(1-hn)2\pi ft + Fn \sin(1-hn)2\pi ft.
\]  

Let

\[
I, \Delta E_1^* = W_1 + jQ_1
\]  
\[
\Delta I_1, \Delta E_2^* = W_2 + jQ_2
\]  
\[
I, \Delta E_3^* = W_3 + jQ_3
\]  
\[
\Delta I_2, \Delta E_4^* = W_4 + jQ_4
\]  

Then

\[
Tse(n) = \frac{W_1(n) + W_2(n) + W_3(n) + W_4(n)}{hn}
\]  

\[
Tde(n) = \left[ \frac{Q_2(n) + Q_3(n)}{hn} \right] - \left[ \frac{Q_1(n) + Q_4(n)}{hn} \right]
\]  

Where

\[
h_n = \text{per unit pulsation frequency of the nth harmonic.}
\]

\[
h_m = \frac{n x N_s x (1-S)}{60 \times f}
\]

The \( Tse(n) \) and \( Tde(n) \) are synchronising and damping torque coefficients in per unit electrical units \( Ts(n) \) and \( Td(n) \) in mechanical units are obtained by following relations.

\[
Ts(n) = Tse(n) \times TB \times \frac{P}{2}
\]  

\[
Td(n) = Tde(n) \times TB \times \frac{P}{2}
\]  

Where \( TB \) is base motor torque defined as

\[
TB = \frac{974.07 \times \text{kVA}}{N_s}
\]  

The above values of \( Ts(n) \) & \( Td(n) \) are used in finalising equivalent torsional system as shown in figures-2.

Equations for calculating current pulsation:

In an induction motor whose rotor is subjected to vibratory motion, line current will be equal to sum of currents flowing in the stator winding of three circuits of Figure A-1.

The currents \( \Delta I_1' \) and \( \Delta I_2' \) have frequencies \((1+h)\) and \((1-h)\) respectively. The sum of these two currents is as follows:-

\[
\Delta I_1' + \Delta I_2' = Cn \cos (1+hn)2\pi ft + Dn \sin (1+hn)2\pi ft + En \cos (1-hn)2\pi ft + Fn \sin (1-hn)2\pi ft
\]  

\[
\Delta I_1' + \Delta I_2' = \left[ \sqrt{(Cn+En)^2+(Dn-Fn)^2} \right] \cos (hn2\pi ft - Q_1(n)) \cos 2\pi ft + \left[ \sqrt{(Dn+Fn)^2+(-Cn+En)^2} \right] \cos (hn2\pi ft - Q_2(n)) \sin 2\pi ft.
\]
Where
\[ Q_1(n) = \tan^{-1}\left( \frac{Dn-Fn}{Cn+En} \right) \] (19)
\[ Q_2(n) = \tan^{-1}\left( \frac{-Cn+En}{Dn+Fn} \right) \] (20)

The equation 18 gives sum of oscillating component of current for one electrical radian amplitude of rotor oscillation. Therefore, it is multiplied by actual amplitude of rotor oscillation \( \delta \) to give true value of oscillating current. \( \delta \) for the different values of \( n \) have relative displacement expressed as phase angles \( \theta n \). The values of \( \delta \) and \( \theta n \) are obtained by simultaneously solving equations of motions for all masses in equivalent torsional system as explained in Appendix - B.

Then total oscillating current due to all harmonics is:
\[ \Delta I = \sum_{n=1}^{n} \delta(n) \left[ \cos(hn 2\pi ft - \theta n + \beta n) \right] \cdot \cos(2\pi ft) + \sum_{n=1}^{n} \delta(n) \left( \frac{\sqrt{(Cn+En)^2+(Dn-Fn)^2}}{\cos(hn 2\pi ft - Q_2(n) + \beta n)} \right) \cdot \sin(2\pi ft) \] (21)

If terms in square bracket are designated \( L & M \). Hence
\[ \Delta I = L \cos(2 \pi ft) + M \sin(2 \pi ft) \] (22)

To this pulsating component of current steady state current \( I' \) as per equation (2) is added - Hence total current \( I_t \) is as follows:
\[ I_t = (G+L) \cos(2 \pi ft) + (H+M) \sin(2 \pi ft) \] (23)

Assume \[ A = G + L \]
\[ B = H + M \]

Then
\[ I_t = A \cos(2 \pi ft) + B \sin(2 \pi ft) \] (24)

This equation is solved at regular interval of time \( t \) to give instantaneous line current. Graph shown in figure 1 is drawn using these values. Difference of maximum and minimum of peaks of current during consecutive electrical cycles for one revolution of crankshaft gives current pulsations as a ratio.

APPENDIX B
TORSIONAL VIBRATION ANALYSIS OF MULTI MASS TORSIONAL SYSTEM

Figure 1-B shows most general torsional system having \( N \) inertia masses. One end of system is free and other end is fixed. All inertia masses are subjected to excitation of harmonic torques of \( n \) frequencies. Each mass and interconnecting shaft provides damping proportional to vibration velocity. All these parameters are known from physical data of system. Let \( \alpha_{ij} \) be amplitude of torsional vibration of \( i \)th mass due to \( j \)th harmonic excitation torques. The equation of motion for \( i \)th mass can now be written.

\[ I_{i} \ddot{\alpha}_{ij} + DM_{i} \dot{\alpha}_{ij} + DS_{i-1} (\dot{\alpha}_{i,j} - \dot{\alpha}_{i-1,j}) + DS_{i} (\dot{\alpha}_{i,j} - \dot{\alpha}_{i+1,j}) + K_{i} (\alpha_{i,j} - \alpha_{i-1,j}) = T_{i,j} \sin(jwt + \Psi_{ij}) \] (25)

Instantaneous values of harmonic torque, of \( i \)th mass can be written as follows
\[ t_{ij} = T_{i,j} \sin(jwt) + TS_{i,j} \cos(jwt) \] (26)

Where \( TC_{i,j} = T_{ij} \cos(Q_{i,j}) \)
\[ TS_{i,j} = T_{ij} \sin(Q_{i,j}) \]

Similarly, if angular displacement of \( i \)th mass is
\[ = \alpha_{ij} \sin(jwt + \Psi_{ij}) \]

where \( \alpha_{ij} \) is amplitude and \( \Psi_{ij} \) is phase angle, then instantaneous value of displacement, velocity and acceleration are
\[ \text{displacement} = \alpha_{Ci,j} \sin(jwt) + \dot{\alpha}_{Si,j} \cos(jwt) \] (27)

Velocity = (jw) \[ \alpha_{Ci,j} \cos(jwt) - (\dot{jw}) \alpha_{Si,j} \sin(jwt) \] (28)
Acceleration = \((j\omega)^2\xi_{i,j} \sin(j\omega t) - j\omega \xi_{i,j} \cos(j\omega t)\)

Where 
\[\xi_{i,j} = \alpha_{i,j} \cos(\psi_{i,j})\]
\[\sin(j\omega t) - (j\omega)^2 \sin(j\omega t) \cos(\psi_{i,j})\]

Substituting equations 26 through 29 in equation 25 and equating sine and cosine terms following two equations are formed.

\[(DS_i)(j\omega)\alpha(S_{i-1},j) = K_i \alpha(C_{i-1},j)\]
\[+ [(C_{j\omega}) (DM_i + DS_i + DS_{i+1}) \alpha(S_i,j)] +
\[+ [(K_i + K_{i+1} - (j\omega)^2 I_i) \alpha(C_i,j)] +
\[+ [(j\omega)(DS_{i+1}) \alpha(C_{i+1},j) - K_{i+1} \alpha(S_{i+1},j)] = T_{C_{i,j}}\]

Thus two linear equations are obtained from an equation of motion for ith mass. Similar equations are written for all N masses. Thus 2 x N number of equations are formed for 2 x N unknowns (i.e. Cos & Sin components of angular displacement (\(\alpha_{C_{i,j}} \) & \(\alpha_{S_{i,j}}\)).

From these equations a matrix equation is constructed which is of the form

\[\begin{bmatrix} A \\ B \end{bmatrix} \begin{bmatrix} X \end{bmatrix} = \begin{bmatrix} E \end{bmatrix}\] (32)

where A is a coefficient matrix of 2N x 2N order. X is column matrix of unknowns \(\alpha_{C_{i,j}} \) & \(\alpha_{S_{i,j}}\) and B is a column matrix of 2N x 1 order of forcing torques. The above matrix equation is solved for \(X\) by Gauss-Jordan’s elimination method. Same procedure is repeated for forcing torques of n different harmonics frequencies and Cos and Sin components of angular vibrations (displacement) at each inertia mass are obtained, for every harmonic frequency. The procedure is based on theory illustrated in References 6 & 7.

Equivalent torsional system shown in Figure 2 is analysed by above procedure to obtain amplitude and phase angle of angular displacements (\(\xi\)) of motor rotor inertia and which are used in equation 21 of Appendix 'A'.

**NOTATION**

- \(DM\): Damping coefficient at inertia mass.
- \(DS\): Damping coefficient of shaft.
- \(E\): Per unit voltage across rotor resistance, = steady state circuit.
- \(f\): Electric supply frequency.
- \(h\): Per unit pulsation frequency.
- \(I\): Per unit current through rotor = steady state circuit.
- \(I^*\): Per unit current through stator = steady state circuit.
- \(I_i\): Mass moment of inertia of ith mass.
- \(K_i\): Torsional stiffness of ith shaft.
- \(KVA\): KVA input to the motor at its rated load.
- \(N\): Number of masses in equivalent system.
- \(n\): Number of harmonic frequencies.
- \(N_s\): Synchronous speed of motor.
- \(r\): Resistance.
- \(S\): Motor slip.
- \(T\): Steady state mean load torque.
- \(T_{B}\): Base motor torque.
- \(T_{i,j}\): Amplitude of excitation torque of jth harmonic at ith mass.
- \(T_{de}\): Damping torque coefficient, per unit kw/electrical rad/per unit time.
- \(T_{se}\): Synchronising torque coefficient, per unit kw/electrical rad.
- \(T_s\): Synchronising torque coefficient of the motor in kg-m per mechanical radian displacement of the rotor per second.
- \(\alpha_{i,j}\): Amplitude of angular displacement of ith mass due to j harmonic torques.
- \(\beta_n\): Phase angle of angular displacement of motor rotor of nth harmonic frequency.
- \(\xi_n\): Amplitude of angular displacement of motor rotor of nth frequency in radian.
- \(\delta_{en}\): Amplitude of electrical radians.
SUBSCRIPT

1 1th inertia mass.
j jth harmonic frequency.
n nth harmonic frequency.
1 hunting frequency circuit one
2 hunting frequency circuit two
* Conjugate of vector.

REFERENCES

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(5) Concordia, C., Induction motor
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(7) Thomson, W.T., Theory of Vibration
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A - ACTUAL SYSTEM

B - EQUIVALENT TORSIONAL SYSTEM (PROPOSED).

C - TORSIONAL SYSTEM (CUMMINGS [4]).

FIGURE 2 COMPARISON OF EQUIVALENT TORSIONAL SYSTEM ASSUMED IN PROPOSED ANALYSIS & CUMMINGS [4] ANALYSIS.
CALCULATE TORQUE CRANK ANGLE DIAGRAM FOR EACH CYL. & ANALYSE IT FOR HARMONIC COMPONENTS OF FOURIER SERIES.

ESTABLISH EQUIVALENT TORSIONAL SYSTEM.

SOLVE STEADY STATE AND HUNTING FREQUENCY CIRCUITS OF MOTOR FOR DIFFERENT FREQUENCIES.

FORM EQUATION OF MOTION FOR EACH MASS AT PARTICULAR HARMONIC FREQUENCY.

DETERMINE DAMPING AND SYNCHRONISING TORQUES.

FORM MATRIX EQUATION \([A] [x]=[B]\) AND SOLVE FOR \([x]\) WHICH IS DISPLACEMENT VECTOR.

IF ALL HARMONICS ARE OVER:

YES

SOLVE HUNTING FREQUENCY AND STEADY STATE EQUIVALENT CIRCUITS TO OBTAIN OSCILLATING COMPONENT AND STEADY STATE STATOR CURRENT RESPECTIVELY.

ADD STEADY STATE AND OSCILLATING COMPONENTS OF CURRENTS AND OBTAIN INSTANTANEOUS VALUES OF CURRENTS AT REGULAR INTERVAL OF TIME.

FIND MAXIMUM AND MINIMUM COMPONENT OF INSTANTANEOUS CURRENT AND CALCULATE CURRENT PULSATIONS.

FIGURE: STEPWISE PROCEDURE FOR CURRENT PULSATION CALCULATIONS.
A STEADY-STATE CIRCUIT

B HUNTING FREQUENCY NETWORKS

FIGURE A-1 EQUIVALENT CIRCUITS FOR STEADY STATE AND HUNTING FREQUENCY OPERATING CONDITIONS OF MOTOR.
FIGURE - B-1. EQUIVALENT TORSIONAL SYSTEM OF FREE FIXED SYSTEM FOR FORCED AND DAMPED TORSIONAL VIBRATION ANALYSIS.