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CONSTITUTIVE EQUATIONS FOR CAPILLARY TUBE MODELING

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ABSTRACT

This paper presents the results of an experimental study on capillary tubes commonly used as expansion device in household refrigerators and freezers. The experiments were performed with HFC-134a at different condensing pressures and level of subcooling. The pressure and temperature profiles along the capillary tubes were measured in each test run. The data set was then used to evaluate the suitability of some equations previously reported in the literature for the single phase friction factor, the underpressure of vaporization and the entrance contraction loss factor. An equation for the average two-phase friction factor was also developed based on the measured data.

INTRODUCTION

The selection of the proper diameter and length of a capillary tube for a given application is largely a trial-and-error process. In an attempt to overcome this problem, computer models on both the adiabatic and diabatic capillary tubes [1-4] have been developed and validated against specific sets of experimental data [4-6]. Each investigator has achieved certain degree of success in correlating model predictions with experimental data, mainly due to the common practice of accounting for modeling inaccuracies through a calibration variable and then selecting equations from the literature until model predictions fit a limited amount of test data [7].

In view of the need to have reliable and informative system performance data of refrigerant flow through capillary tubes a large research program was established at the Center for Refrigeration, Ventilation and Air Conditioning Research (NRVA) at the Federal University of Santa Catarina in the early 90's. Modeling and experimental studies have been performed both for adiabatic and diabatic capillary tubes. The work effort reported herein focuses only on the analysis of pressure and temperature measurements along adiabatic capillary tubes.

EXPERIMENTATION

The experimental apparatus and the test procedure are fully described in [8]. Type T thermocouples were attached to the outside surface of the capillary tubes to determine the temperature distribution. A special connection tee, designed for a minimal disturbance of the normal flow pattern, was used for the pressure measurements along the liquid region of the capillaries. The pressure profile in the two-phase flow region was inferred from the measured temperatures. The experiments were performed with two capillary tubes, namely capillaries #7 and #8. The geometry of the capillaries were measured with great care [8]. The internal diameter (D), length (L) and wall roughness (Ra) of the capillaries #7 and #8 were 0.606 mm, 2.998 m, 1.08 μm and 0.871 mm, 2.973 m, 0.78 μm, respectively. Both tubes were tested with subcooling ranging from 2 to 14°C and pressures from 9 to 16.5 bar. This gave rise to 228 sets of data like the one shown in Figure 1 for capillary tube #8. The flow characteristics shown in this figure is similar to those presented by Bolstad and Jordan [9], but it is clear that there is a delay of vaporization in these tubes as reported by Cooper et al. [10].

SINGLE-PHASE FLOW FRICTION FACTOR

The measured pressure (P) and temperature (T) were correlated with the distance from the capillary tube entrance (L) along the length of the liquid region, using a least squares linear fit. Since the delay in flashing was considerable and occurred in almost every test run, there was a need to visually examine every data plot to estimate the actual flashing point. The uncertainty associated with the determination of the liquid region length was estimated to be ± 50 mm.
The following equations were proposed:

\[ P = a_p l + b_p \]  \hspace{1cm} (1)

\[ T = a_l l + b_l \]  \hspace{1cm} (2)

The saturation pressure \( P_{sat} \) corresponding to the temperatures measured along the tube were calculated from the HFC-134a equation of state [11], and correlated with the distance from the capillary tube entrance,

\[ P_{sat} = a_{sat} l + b_{sat} \]  \hspace{1cm} (3)

At the intersection point of the two pressure curves shown in Figure 1, the measured pressure is equal to the saturated pressure, therefore the refrigerant is in a non-equilibrium saturated state at this point (point 1), as vaporization does not take place at this intersection.

The intersection point of the two pressure curves \( l_{eq} \) in the experimental capillary tubes was determined by combining equations (1) and (3):

\[ L_{eq} = \frac{(b_{sat} - b_p)}{(a_p - a_{sat})} \]  \hspace{1cm} (4)

The single-phase flow friction factor \( f_{sp} \) was then calculated by the following relationship, where the coefficient \( a_p \) represents the pressure gradient in the liquid region, \( m \) the mass flow rate, and \( \rho \) the liquid density.

\[ f_{sp} = a_p \frac{\pi^2 \rho D^5}{8 m^2} \]  \hspace{1cm} (5)

The refrigerant density and absolute viscosity, for the entire liquid region, were taken at \( L_{eq} \).

Figure 2 presents a comparison of experimental single-phase friction factors with friction factors calculated from Churchill's [12] and Blasius' equations. The experimental value of the single-phase friction factor obtained from the present study, was higher than those calculated using the Blasius' equation for smooth tubes. The results show good agreement with Churchill's equation, which takes into account the relative roughness of the tube inner wall. This confirmed the findings of and Lin et al. [13].
ENTRANCE CONTRACTION LOSS FACTOR

The coefficient $b_p$, in equation (1), represents the value which the inlet pressure would equal if there were no inlet pressure losses. The inlet pressure drop can then be calculated as the difference between this value and the actual inlet pressure ($P_{inlet}$).

Further, the inlet pressure drop is frequently correlated with the velocity head at the tube inlet, by an entrance contraction loss factor ($K$), given by the following equation:

$$K = \frac{\pi^2 D^4 \rho (P_{inlet} - b_p)}{8 \pi^2} - 1 \quad (6)$$

Figure 4 illustrates the entrance loss factor as a function of the Reynolds number for the capillary tube #7. The average value for this tube was found to be 2.7. A similar behavior was observed for capillary tube #8 but an average value of 1.0 was found. In general the measured values are well above the 0.8 - 1.0 values recommended by the literature for re-entrant contractions. For each of the measured values of the entrance loss factor an equivalent liquid length ($L_{eqv} = KD_{inlet}$) was calculated. For the capillary tube #7 the entrance loss factor corresponds to an additional liquid length of 30 to 60 mm. This leads to the conclusion that the pressure drop at the capillary tube inlet has a negligible influence on the mass flow rate through these expansion devices.

TWO-PHASE FLOW FRICTION FACTOR

Capillary tube models based on the integral solution of the governing momentum equation through the two-phase region [14,15] require an average value for the two-phase friction factor ($\bar{f}_{tp}$). Combining the momentum and continuity equations and integrating the resulting equation between points 2 and 3, in Figure 1, yields:

$$\bar{f}_{tp} = 2 \cdot \frac{D}{L_{tp}} \cdot \left[ \ln \left( \frac{\rho_2}{\rho_1} \right) - \frac{1}{G^2} \cdot \int_2^3 \frac{dp}{\rho} \right]$$

where $p$, $G$ and $L_{tp}$ are the pressure, mass flux and length of the two-phase flow region, respectively.

The length of the two-phase flow region can be taken from the experimental data. The measured ($p_2$) and saturated ($p_1$) pressures at point 2 are easily found using equations (1) and (3), together with the liquid region length. The saturated pressure at point 2 is greater than the measured pressure, characterizing a thermodynamic non-equilibrium condition. Taking the temperature at point 2 as the saturated temperature corresponding to pressure $p_2$ and applying the energy and continuity equations between points 1 and 2, yields:

$$h_{l12} + (h_{v2} - h_{l12}) \cdot x_2 + \frac{G^2}{2} [v_{l12} + (v_{v2} - v_{l12}) \cdot x_2]^2 = h_{l11} + \frac{G^2}{2} v_{l11}^2 \quad (8)$$

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where \( h_l \) and \( h_v \) are the specific enthalpies of the liquid and vapor and \( v_l \) and \( v_v \) are the specific volumes of the liquid and vapor, respectively. The refrigerant quality at point 2, \( x_2 \), is obtained by solving equation (8).

Point 2 corresponds to the position of the inception of vaporization and therefore the refrigerant quality at this point is actually zero. Assuming a different value for the quality at this point, as given by equation (8), was an approach to consider the metastable flow effects. The range of variation of this parameter in the present investigation was 0.0006 - 0.06. The refrigerant density at point 2 can then be expressed as:

\[
\rho_2 = x_2 \cdot \rho_{v,2} + (1 - x_2) \cdot \rho_{l,2}
\]  

(9)

The integral term of equation (7) depends only on the thermodynamic path and this is given by the Fanno line. In other words, for an adiabatic flow, the entropy change in the direction of the flow can never decrease. The critical flow corresponds to zero entropy change (\( \left( \delta s/\delta T \right)_{3} = 0 \)).

The plot of measured friction factor vs. Reynolds number at point 1 for the capillary #7 is shown in Figure 5. Calculations with Erth’s equation [14], considering point 2 as the capillary tube inlet state, is also shown in this figure. It can be seen that Erth’s equation overpredicts the experimental data. A new correlation, based on Whitesel’s work [16] and fitted to the present experimental data was then derived:

\[
\tilde{f}_{ip} = 0.59 \left( \text{Re}_{l,in} \right)^{-0.284} \left( \frac{T_{in}}{T_{crit}} \right)^{1.28} \left( \frac{\rho_{l,2}}{\rho_{v,2}} \right)^{-0.0173}
\]  

(10)

where \( T_{crit} \) and \( \text{Re}_{l} \) are the critical temperature of the refrigerant and the liquid Reynolds number, respectively. The subscript \((in)\) represents the two phase region inlet state (point 2 of Figure 1).

Predicted two-phase flow friction factors are compared with the current experimental data in Figure 6. It can be seen that the calculated values are in good agreement with the measured ones.

**UNDERPRESSURE OF VAPORIZATION**

The flow behavior, shown in Figure 1, was observed in almost every case. The metastability can be characterized by the persistence of the liquid state to pressures less than the saturation pressure corresponding to its temperature. This pressure difference is designated as underpressure of vaporization [17]. The question of a delay in flashing is an important and basic one, since it increases the liquid length in the capillary tube and thus the mass flow rate.

Figure 7 is a representative plot of the observed underpressure of vaporization for capillary #7. It is shown that the correlation proposed by Chen et al. [17] generates almost a constant value for the underpressure of vaporization for all runs. The average values of the measured underpressure of vaporization for the capillary tubes #7 and #8 were 0.50 ± 0.34 bar and 0.52 ± 0.29 bar, respectively.

Figure 8 shows the dimensionless underpressure of vaporization for capillary #7 compared with experimental data. The expressions along the horizontal and vertical axes correspond to the right and left hand side terms of Chen et al.’s equation, respectively. Clearly, even considering a relative error of 26% the proposed relationship [17], that was originally developed for CFC-12, was unable to properly predict the experimental data. The deviation between predictions and experiments, observed in Figure 8, is not related to the refrigerant type, but reflects the variation of the metastable liquid length for the same operating conditions.
CONCLUSIONS

The presented experimental data confirmed that Churchill's equation [12] for the determination of the single-phase friction factor is applicable to the flow in capillary tubes.

The calculated entrance contraction loss factor were well above the values recommended by the literature for re-entrant contractions. The pressure drop associated with a sudden contraction from a 4.75 mm cross-sectional diameter to the capillary tube diameter has a negligible influence on the capillary flow.

A correlation for the average two-phase friction factor has been derived based on the measured data. A reasonable agreement with the measured data has been found.
The equation proposed by Chen et al. [17] seems to be inappropriate for the calculation of the underpressure of vaporization in capillary tubes. This is so, because the length of the non-equilibrium region, in a given capillary tube, may vary from test to test even if the operating conditions are unchanged.

REFERENCES


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