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COMPUTER SIMULATION FOR A SMALL HERMATIC COMPRESSOR

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ABSTRACT

This paper presents a simplified analysis for the estimation of reciprocating hermetic compressor performance using personal computer that can be easily handled. The type of compressor is scotch-yoke(1/10-1/4 HP) and uses R-12 as a refrigerant. The thermodynamic properties are represented by ideal gas equation of state. A comparison is made between the results of this simulation and experimental data.

The results of this simulation are mass flow rate, cycle work, various losses, heat transfer rate, volumetric efficiency, p-v diagram etc. And this simulation can be easily handled in the factory.

INTRODUCTION

The purpose of this simulation is to predict compressor's performance easily by anyone working in the factory. So much attention was made to reduce the relative formulas so that this program can be run by personal computer.

Assumptions used in this paper are followings,

(1) Refrigerant is regarded as an ideal gas.
(2) Adiabatic process during suction and discharge process.
(3) Polytropic process during expansion and compression process.
(4) Temperature at discharge chamber is constant.
(5) Temperature at suction chamber is same as inner side of case.
(6) Pressure at suction chamber is same as suction pressure at evaporator.
(7) The efficiency and rpm of motor are constant.

Fig. 1. Compressor Thermodynamics

MATHEMATICAL MODELING

Analyzing the process in the cylinder, there would be 4 steps as expansion, suction, compression and delivery process.

Cylinder volume

The volume in the cylinder $V_c$ is given as

$$V_c = V_0 + D^2 r (1 - \cos \theta)/4 \quad (1)$$

where $V_0$ is the clearance volume, $D$ and $r$ is the piston diameter and crank radius.

Expansion and compression process

From assumption, pressure and temperature in the cylinder can be written as

$$V_{ef} = V_{ei} \left( \frac{P_{ei}}{P_{ef}} \right)^{1/n} \quad (2)$$

$$T_{ef} = T_{ei} \left( \frac{P_{ef}}{P_{ei}} \right)^{1-1/n} \quad (3)$$

$$V_{cf} = V_{ci} \left( \frac{P_{ci}}{P_{cf}} \right)^{1/n} \quad (4)$$
\[ T_{cf} = T_{ci} \left( \frac{P_{cf}}{P_{ci}} \right)^{1-1/n} \]  

where suffix c, e, i, f mean compression, expansion, initial and final state.

### Suction process

It is assumed that pressure and temperature in the suction chamber and specific heat capacity \((C_v, C_p)\) are constant. Refrigerant gas is treated as an ideal gas and follows adiabatic process.

**Mass flow equation** is given as

\[
\frac{d\dot{m}_s}{dt} = \frac{dP}{dt} \tag{6}
\]

**Energy equation** is given as

\[
\frac{d\dot{m}_s}{dt} \cdot h_{sc} = \frac{d}{dt} \left( \dot{m}_c u_c \right) + P_c \frac{dV_c}{dt} \tag{7}
\]

where kinetic energy term is neglected. Applying the definition of enthalpy,

\[ H = U + PV \]  

mass flow equation (6), and the equation of ideal gas, then

\[
P_c \frac{V_c}{R} \frac{dT_c}{dt} = C_p \left( T_{sc} - T_c \right) \frac{dV_c}{dt}
\]

\[
- V_c \frac{dP}{dt} = 0 \tag{7b}
\]

The equation for one dimensional adiabatic flow through a valve may be written as

\[ h_{sc} = h_c + \left( \frac{v_{ev}}{2} \right) \]  

mass flow equation (6), and the equation of ideal gas, then

\[
P_c \frac{V_c}{R} \frac{dT_c}{dt} = C_p \left( T_{sc} - T_c \right) \frac{dV_c}{dt}
\]

\[
- V_c \frac{dP}{dt} = 0 \]  

So, mass flow equation can be rewritten as

\[
\frac{d\dot{m}_c}{dt} = \rho_c \left( \frac{As_v}{A_{sc}} \right) \frac{v_{sc}}{2}
\]

where \(\rho\) is the density, \(\frac{As_v}{A_{sc}}\) is the effective area.

Substituting equation (8a) into equation (7b) gives

\[ \frac{d\dot{m}_c}{dt} = -(A_{sc}) \frac{C_p \left( T_{sc} - T_c \right)}{R} \frac{dV_c}{dt} \]  

### Ideal gas equation

\[ P_c \frac{V_c}{R} \frac{dT_c}{dt} = \left( \frac{v_{ev}}{2} \right) + C_p \frac{dT_c}{R} \]  

Differentiating equation (10) gives

\[
\frac{dP}{dt} = \frac{R}{V_c} \frac{dV_c}{dt} + \frac{\dot{m}_c}{V_c} \frac{dV_c}{dt} - \frac{\dot{m}_c}{V_c} \frac{dV_c}{dt} \tag{10a}
\]

Substituting equation (8a) and ideal gas equation \((\dot{m}_c = P_c V_c / (R T_c))\) into equation (10a) gives

\[
\frac{1}{P_c} \frac{dP}{dt} = (A_{sc}) \frac{(2C_p \left( T_{sc} - T_c \right))^{1/2}}{V_c} + \frac{1}{T_c} \frac{dT_c}{dt} - \frac{1}{V_c} \frac{dV_c}{dt} \tag{10b}
\]

From equation (9) and (10b), the process of suction can be expressed by a first order ordinary differential equation as

\[
\frac{1}{T_c} \frac{dT_c}{dt} = \frac{(C_p \frac{T_{sc}}{T_c} - 1)(A_{sc}) \frac{(2C_p \left( T_{sc} - T_c \right))^{1/2}}{V_c}}{- \frac{R}{C_v V_c} \frac{dV_c}{dt}} \tag{11}
\]

By chain rule,

\[
\frac{d}{dt} = \frac{d}{d\theta} \frac{d\theta}{dt} = \omega \frac{d}{d\theta} \tag{12}
\]

equation (11) can be rewritten as

\[
\frac{1}{T_c} \frac{dT_c}{dt} = \frac{(C_p \frac{T_{sc}}{T_c} - 1)(A_{sc}) \frac{(2C_p \left( T_{sc} - T_c \right))^{1/2}}{V_c}}{- \frac{R}{C_v V_c} \frac{dV_c}{dt}} \tag{13}
\]

where \(\omega\) is the angular velocity, \(\theta\) is the crank angle.

### Delivery process

It is assumed that \(C_p, C_v\) are constant. Refrigerant gas is treated as an ideal gas and follows adiabatic process.

**Mass flow equation** is given as

\[ \frac{d\dot{m}_c}{dt} = - \frac{dA_{sc}}{dt} \tag{14} \]

**Energy equation** is given as

\[ \frac{d}{dt} \left( \dot{m}_c u_c \right) + \frac{\dot{m}_c}{V_c} \frac{dV_c}{dt} \tag{15} \]

The equation for one dimensional adiabatic flow through a valve may be written as

\[ \frac{d\dot{m}_c}{dt} = -(A_{sc}) \frac{P_c}{R} \frac{\left( 2C_p \left( T_c - T_{dc} \right) \right)^{1/2}}{V_c} \]  

By same method as suction process, equation (14), (15), (16), (10), and (10a) give a first order ordinary differential equation as

\[
\frac{dT_c}{dt} = \frac{-(A_{sc}) \frac{P_c}{R} \left( 2C_p \left( T_c - T_{dc} \right) \right)^{1/2}}{V_c} + \frac{1}{V_c} \frac{dV_c}{dt} \tag{17} \]

### Mechanical loss

Mechanical loss of shaft is calculated by a lubrication theory of a journal bearing.

Sommerfeld Number \(S\) is defined as
\[ S = \frac{(\pi N/P_m)}{(C/D)^2} \]  

(18)

where \( S \) is the viscosity of oil, \( N \) is the rotational frequency, \( P_m \) is the bearing pressure, \( D \) and \( C \) are the diameter and the gap of bearing. The friction coefficient is obtained from \( S \) and mechanical loss of shaft is given as

\[ W_1 = K \left( \frac{C}{D} \right) \left( \frac{P_m D L}{6} \right) \]  

(19)

where \( L \) is the length of bearing.

Assuming perfect lubrication without eccentric, the mechanical loss of other moving parts are

\[ W_1 = \sum W_i \]  

Total mechanical loss is given as

\[ W_1 = \sum W_i \]

**Cylinder work, valve losses**

The indicated work is obtained from p-v diagram and effective work is defined by the area shown Fig-5.

1. **Indicated work**
   \[ Wi = \sum \left[ V(\theta_i) - V(\theta_i) \right] P_c N \]  

(21)

2. **Loss during suction**
   \[ W_s = \sum \left[ V(\theta_i) - V(\theta_i) \right] (P_c - P_s) \]  

(22)

3. **Loss during delivery**
   \[ W_d = \sum \left[ V(\theta_i) - V(\theta_i) \right] (P_d - P_c) \]  

(23)

4. **Effective work**
   \[ We = Wi - Js - Wd \]  

(24)

**Heat balance**

Taking a control volume as Fig-2, energy equation is given as

\[ Q_{case} = Welec + \dot{m} (h_d - h_s) \]  

(25)

where \( Q_{case} \) is the heat transfer from compressor case to ambient, \( \dot{m} \) is the mass flow rate. \( Welec \) is power input to motor and can be rewritten as

\[ Welec = \left( Wi + W_i \right) / \eta_m \]  

(26)

where \( \eta_m \) is the motor efficiency.

Enthalpy terms can be expressed by temperature terms as

\[ h_d - h_s = (Cp)_d T_d - (Cp)_s T_s \]  

(27)

So, equation (25) can be rewritten as

\[ Q_{case} = \left( Wi + W_i \right) / \eta_m - \dot{m} \left( (Cp)_d T_d - (Cp)_s T_s \right) \]  

(28)

Heat transfer also can be represented as

\[ Q_{case} = Acase \lambda \left( T_{g} - T_a \right) \]  

(29)

where \( Acase \) is the surface area of case, \( T_a \) is the ambient temperature, \( T_g \) is the temperature of refrigerant at inner side of case. From equation (28) and (29), temperature \( T_g \) is given as

\[ T_g = T_a + \left( \frac{\left( Wi + W_i \right)}{- \dot{m} \left( (Cp)_d T_d - (Cp)_s T_s \right)} \right) / \left( Acase \lambda \right) \]  

(30)

**Fig. 2. Heat Balance of Compressor**

**Outline of the Computer Program**

It is assumed that pressure and temperature in the suction chamber are same as the inner temperature and pressure. The properties of refrigerant in the cylinder are obtained from equation (2), (3), (4), (5), (13) and (17). The calculation of properties starts from top dead center to next top dead center using small increment of crank angle with assumed temperature and pressure in the cylinder at top dead center. Equation (13) and (17) are solved by Runge-Kutta method over small increment of crank angle. At each crank angle pressure \( P_c \) and temperature \( T_c \) in the cylinder are known, and the other properties such as density, specific heat, enthalpy can be obtained from ideal gas equation.

After completion of the first cycle, the pressure and temperature in the cylinder are compared between the assumptions and calculated results. If deviation is large, the properties are recalculated with new assumptions till deviation is small enough. After the calculation of properties in the cylinder, mechanical loss is obtained from equation (21).

Inner temperature of the case is calculated by equation (30). If deviation between assumed and
calculated temperature at inner side of case is large, iteration is took place with reassumed temperature till convergency is achieved.

Then, the properties of refrigerant at every crank angle, p-v diagram, mechanical loss, indicated work, effective work, volumetric efficiency can be obtained.

RESULTS

The model was applied to a 1/5.5 h.p. scotch yoke type hermatic compressor with cylinder bore 23.5 mm, piston stroke 12.2 mm, speed 3540 rpm.

Simulation has been performed varying pressure ratio (4.4, 5.2, 7.4, 10.2).

The change of mass flow rate is shown Fig-6. Calculated mass flow rate is expressed in the case where there is no reverse flow at valve, and there is no leakage from cylinder, no heat transfer during suction and discharge process. So, calculated mass flow rate is greater than measured mass flow rate. But, tendency can be known by this method.

Motor input power, shaft power, indicated work, effective work are shown Fig-4. There is a good agreement between calculated and measured power. Calculated lost power during discharge and suction process are shown Fig-7. In this calculation, the behaviour of valve was ignored (the mass of valve and spring force of valve were ignored).

Calculated p-v diagram is shown Fig-5.

CONCLUSIONS

Presented method method for the estimations and analysis of small hermatic compressor is suitable for improvement and development. Mathematical model is so simple that it can be run by anyone with personal computer. The properties in the cylinder can be obtained without indicator diagram, etc. at every crank angle. Heat transfer coefficient of case, suction gas and discharge gas temperature at outside of case, and motor efficiency are required as experimental data.

NOMENCLATURE

A: Area
Cp: Isobaric Specific Heat Capacity
Cv: Isochoric Specific Heat Capacity
C: Clearance
D: Diameter
h: Specific Enthalpy
L: Length
m: Mass
ṁ: Mass flow Rate
n: polytropic Index
P: Pressure
Q: Heat Transfer
R: Gas Constant
r: Radious

S: Sommerfeld Number
T: Temperature
t: Time
u: Specific Internal Energy
U: Internal Energy
V: Volume
v: Velocity
N: Rotational Frequency
H: Enthalpy
θ: Crank Angle
α: Heat Transfer Coefficient
ω: Angular Velocity

Suffix
a: Ambient
c: Cylinder
d: Outlet
i: Inlet
g: Gas
dc: Discharge Chamber
cs: Suction Chamber
dv: Discharge Volume
sv: Suction Volume

Read Data
Assume the Temperature at the inner side of Case, Tgi
Assume the Temperature & Pressure in the Cylinder at the beginning of Expansion, Tci, Pci
Calculate the properties in the Cylinder during 1 cycle
Calculate the Temperature and Pressure in the Cylinder at the end of Discharge, Tcf, Pcf
Compare Tci, Pci & Tcf, Pcf
Calculate Indicated Work, Mechanical Loss, Electric Input
Calculate the Temperature at the inner side of Case, Tgf
Compare Tgi and Tgf
Stop

Fig. 3. Flow Chart of the Analysis
Fig. 4. Change of Work with Pressure Condition Change

Fig. 5. P-V Diagram

Fig. 6. Change of Mass Flow Rate with Pressure Condition Change

Fig. 7. Change of Valve and Passage Loss
Fig. 8. Change of Volumetric Efficiency with Pressure Condition Change

Fig. 9. Change of Mass Flow Rate with Piston Stroke Change

REFERENCE


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