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REGISTRATION OF DIGITAL IMAGERIES USING OPTIMIZATION TECHNIQUE

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I. ABSTRACT

Registration is an important aspect of any image processing system. Registration may be interpreted as automatic determination of local similarity between two structured data sets. Several digital techniques have been used for registration. Principal among these are cross-correlation, normalized cross-correlation and minimum distance criteria. Fast algorithms like sequential similarity detection for translational differences have been developed. In all the above methods, a portion of one image is taken as reference and similar portion in the other image is located by carrying out a search over the other image. There may be translational, rotational and scale differences between the two imageries.

Here, in this paper, coordinates of the two imageries have been related by affine transformation and these transformation coefficients are then evaluated by carrying out a search for similarity between the two imageries. In order to carry out this search in an optimized way, an algorithm using sequential simplex method (Box's Method) has been developed and implemented. The method is essentially a gradient type and searches the optimum with the steepest route. The method has been applied to register two Landsat scenes. Although, the method is general, in the illustrative example, only the translational differences have been considered. The results of the same are shown in the plates.

Also, because of the presence of noise, there exists the limit on the accuracy of the registration. Here, the two imageries are considered to be the sample functions of homogenous random field with known auto-correlation function. Also, the non-similarity between the two imageries is considered as noise

in one of the imagery. With these assumptions, the upper bound on the accuracy of the registration has been evaluated.

II. INTRODUCTION

Mis-registration results from inability of the sensing system to produce congruenced data due to the fact that the sensors are separated in space and time such that spatial alignment of the sensors is impractical or impossible. Geometric distortions, scale differences and look angle effects can all be combined to produce misregistration. Thus registration is an important aspect of any image processing system.

Several digital techniques have been used for registration of remotely sensed imageries. Ahuta¹ discussed the spatial registration problems from cross-correlation point of view and implemented the algorithm using Fourier transform technique. Barnea and Silverman² described a sequential similarity detection algorithm for translational registration. Essentially, it is distance approach implemented in a fast way not requiring all the terms in the summation to be computed for each translation. Webber³ combined an affine transformation with sequential similarity detection algorithm to determine all the parameters of the affine transformation.⁴

In all of the above methods, a portion of one image, or, window 'W' is taken as a reference image and similar portion in the other image is located by carrying out a search over a search area 'S'. There may be translational, rotational or scale differences between the two imageries. Here in this paper, an algorithm for carrying out this search, in optimized way, has been developed and implemented. The search technique used

is Sequential Simplex Method (Box Method). The search method is essentially a gradient type and searches the optimum point of the objective function, in 'n' dimensional space with a steepest route. In this case, the two imageries are assumed to be related by affine transformation. The algorithm has been developed and implemented to evaluate these transformation coefficients by carrying out a search in six dimensional space. 'Normalized cross-correlation' as well as 'distance' have been used as objective functions in the above algorithm.

Also, the non-similarity between the two imageries is considered as noise in one of the imageries and is assumed to be arbitrary except some restriction on its magnitude. It is obvious that because of the presence of noise, the accuracy of registration is finite. Here, two imageries are assumed to be sample functions of homogenous random field with known auto-correlation function. With these assumptions, the upper bound on registration accuracy has been determined.

The algorithm developed above, as an illustration, has been applied for registration of two Landsat scenes, although the algorithm is general, in the illustrated example, only the translational differences have been considered and an optimized search is carried out in two dimensional space and the results of the same are shown.

III. METHOD

Let $F_1(x, y)$ and $F_2(x, y)$ be the two images to be registered. Let S be the search area in image $F_1(x, y)$ and W be the window from image $F_2(x, y)$ as shown in Fig. 1. S is taken as $L \times L$ array of digital picture elements which assume one of the K grey levels such that,

$$0 \leq S(x, y) \leq K-1 \quad (1)$$

where

$$1 \leq x, y \leq L$$

W is considered to be $M \times M$ array of digital picture elements with same grey levels and M is chosen so as $M \leq L$, as above we can write

$$0 \leq W(x', y') \leq K-1 \quad (2)$$

where

$$1 \leq x', y' \leq M$$

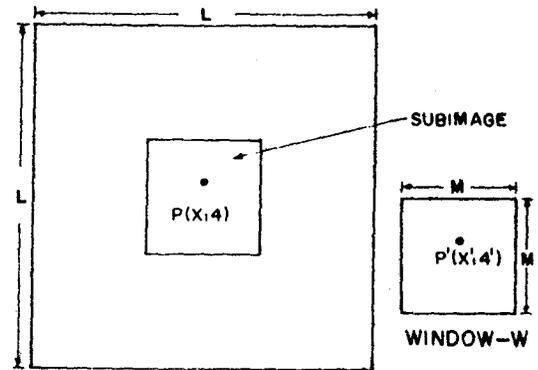


Fig. 1. SEARCH AREAS — S

It will be assumed that enough a priori information is known about the dislocation between the window and the search area such that parameters M and L selected with the virtual guarantee that at registration a complete subimage is contained in the search area.

If we assume, there exists small rotational, translational and scale differences between the two imageries, then the coordinates x, y of the search area S and coordinates x', y' of the corresponding point in window W can be related as,

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} a_{10} \\ a_{20} \end{bmatrix} \quad (3)$$

Here, since we are considering small portion of the image that is represented by window W of size $M \times M$, the first order transformation given by Eqn.(3) is adequate enough to represent geometric differences between the two imageries.

In the presence of noise two imageries can be represented as,

$$S(x, y) = W(x', y') + n(x, y) \quad (4)$$

In Eqn.(4), $n(x, y)$ represents non-similarity between the two imageries and it can be considered as noise in one of the imagery. It can be seen that the problem of registration is to evaluate transformation coefficients in Eqn.(3). If we consider normalized cross-correlation between the two imageries as a criterion for registration, then problem reduces to that of finding out transformation coefficients such that function F given by Eqn.(5) is maximum

$$F = \left[\sum_{y'=1}^M \sum_{x'=1}^M S(x, y) W(x', y') \right]^2$$

$$\left[\sum_{x'=1}^M \sum_{y'=1}^M W(x', y') \right] \left[\sum_{x'=1}^M \sum_{y'=1}^M S^2(x, y) \right] \quad (5)$$

If we consider distance as criterion for registration then the problem reduces to that of finding out transformation coefficients such that function F given by Eqn.(6) is minimum.

$$F = \sum_{x'=1}^M \sum_{y'=1}^M \left[S(x, y) - W(x', y') \right]^2 \quad (6)$$

In Eqn.(5) and Eqn.(6), the double summations are over window W and corresponding subimages in search area S.

In order to determine values of transformation coefficients such that function F is optimum, it is necessary to carry out a search in six dimensional space that corresponds to six transformation coefficients. Here, the algorithm has been developed to carry out optimized search using sequential simplex method or Box's Method 5,6,7. The method is essentially a gradient type. Here to start with the function 'F' is evaluated at few random points chosen in six dimensional space that corresponds to six transformation coefficients. These points are chosen such that they are vertices of some geometric figure called as Simplex. In the case of two dimensional space, the simplex can be a triangle. One of the vertices of the simplex is then rejected as being inferior to other. The direction of the search is chosen as away from the worst point. The new point is then chosen such that movement passes through the centre of gravity of the simplex and the shape of the simplex is retained. In the case of two dimensional space points 1,2,3 represent initial simplex as shown in Fig.2. Point 1 is then rejected as being inferior to other points, the new simplex is then formed by points 2, 3 and 4. The search thus proceeds by the process of vertices rejection and regeneration until the figure saddles around the optimum. Then no subsequent moves would lead to further improvement since last few geometric figures are essentially repeated. The search is finally stopped, when the simplex size is small enough to locate the optimum adequately.

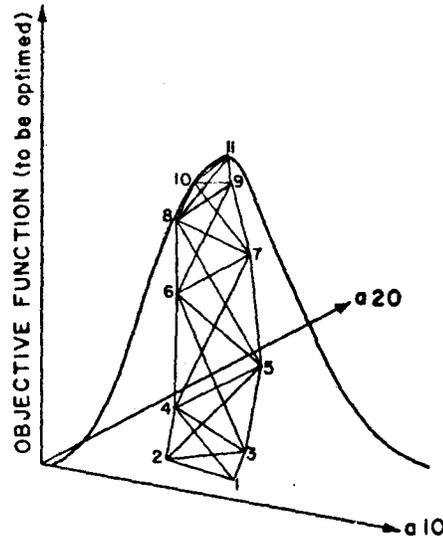


Fig. 2

It can be seen that the direction of the search depends on starting location of the simplex. There exists a possibility that the simplex may saddle around a local maximum and would yield wrong result. In order to avoid such a situation, initial location of the simplex need to be selected in the vicinity of the global optimum. In order to have rough estimate of global optimum, the space can be sampled at coarse grid points and function F can be evaluated at these grid points. With this as initial information, the initial location of the simplex can be determined and the search can be carried out to locate the global optimum.

It can be seen from Eqn.(4), because of the presence of noise $n(x, y)$, the accuracy of the registration is finite. If we assume that noise $n(x, y)$ as an arbitrary except for restriction on its magnitude such as,

$$\sum_x \sum_y n^2(x, y) \leq e^2 \quad (7)$$

when the double summation is over, the subimage that corresponds to window W, then the limit for the accuracy of the registration can be determined as below.

The total error in the registration can be written as,

$$E_{TOT} = E_{mr} + E_n \quad (8)$$

where

E_{mr} is error due to misregistration

E_n is error due to noise.

From Eqn.(7), we can write,

$$E_n \leq e^2 \quad (9)$$

If we assume that the two imageries to be sample functions of homogenous random field $S, 9, 10$ with known auto correlation function, then estimate of E_{mr} can be written as,

$$\hat{E}_{mr} = \sum \sum [S(x, y) - S(x + \Delta x, y + \Delta y)]^2 \quad (10)$$

where the double summation is over a subimage that corresponds to window W and x and y represent registration error in x and y respectively.

If $r_{\Delta x, \Delta y}$ represents auto correlation coefficient at lag $\Delta x, \Delta y$, then Eqn.(10) can be rewritten as,

$$\hat{E}_{mr} = \sum \sum [S^2(x, y)(1 - r_{\Delta x, \Delta y})^2] \quad (11)$$

From Eqn.(8), we note that, the lag points at which the estimate E_{mr} becomes equal to noise E_n , can be considered as the limit for the accuracy of the registration which is given by Eqn.(12)

$$e^2 \geq \sum \sum [S^2(x, y)(1 - r_{\Delta x, \Delta y})^2] \quad (12)$$

where the double summation is over the subimage that corresponds to window W.

The algorithm for the method described above has been developed and implemented. As an illustration, the method has been applied for registration of two Landsat scenes, and the results of the same are discussed in the next section.

IV. RESULTS AND DISCUSSIONS

The above discussed algorithm has been developed and implemented on EC2640 computer system. The algorithm can use both, normalized cross-correlation or distance as the objective function to be optimized. As an illustration, the method has been applied for registration of two Landsat scenes of different dates. The scenes chosen were (i) 155-049 dated 05.01.73, ID # 81166044345 and (ii) 154-049 dated 26.02.75, ID # 82035042645. The overlap portion between the two scenes has been considered for registration. The two scenes are shown in Plate 1 and Plate 2. The search area S is chosen from scene 155-049 and the window W is chosen from scene 154-049.

In order to get rough estimate of the optimum, contours of normalized cross-correlation and distance are plotted. These are shown in Fig.3 and Fig.4 respectively. The optimized search is then carried out in two dimensional space to locate the registration point. The search has been carried out using both cross-correlation and distance as the criterion for registration. Both the methods yield almost same results. The results obtained from optimization program are used to register two scenes and the digital mosaic of the same is shown in Plate 3.

It can be seen that here, since the two scenes were geometrically corrected and both are at the same scale, only the translational differences were considered for registration. The evaluation of cross correlation function at grid points shown in Fig.3 has consumed 25 minutes of CPU. While evaluation of distance at grid points shown in Fig.4 has taken 21 minutes of CPU time. The optimization procedure has consumed in both the cases almost 10 minutes of CPU.

Although in the above example, the search has been carried out in two dimensional space, the optimization program developed is general and can be used to carryout search in six dimensional space to evaluate six transformation coefficients. However, for this rough estimate of optimum or approximate values of the transformation coefficients should be known for initiating the search. This is to ensure that optimization procedure results in saddling global optimum.

It is also possible to use optimization technique developed above, alongwith sequential similarity detection algorithm for fast registration system.

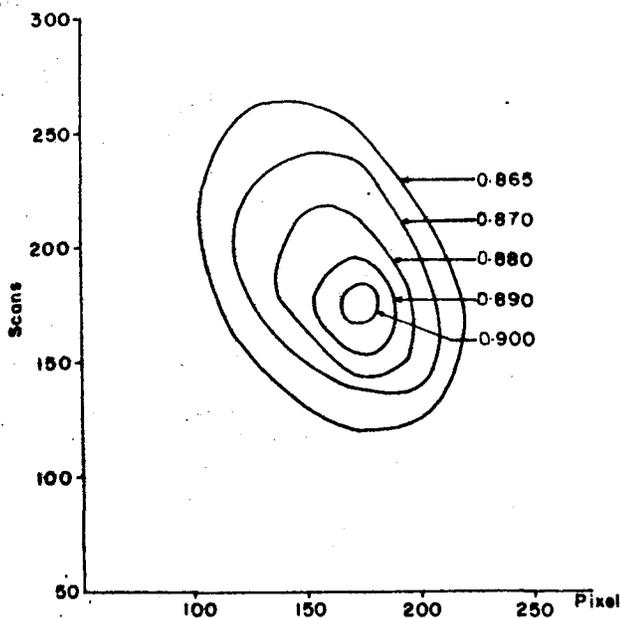


Fig. 3. CONTOURS OF CORRELATION

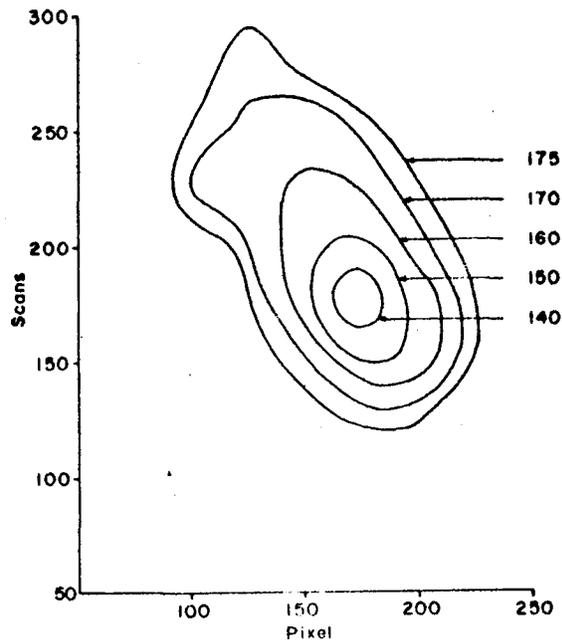


Fig.4. CONTOURS OF $\sum [S(X_i, Y_j) - W(X_i, Y_j)]^2$

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Plate 1



Plate 2

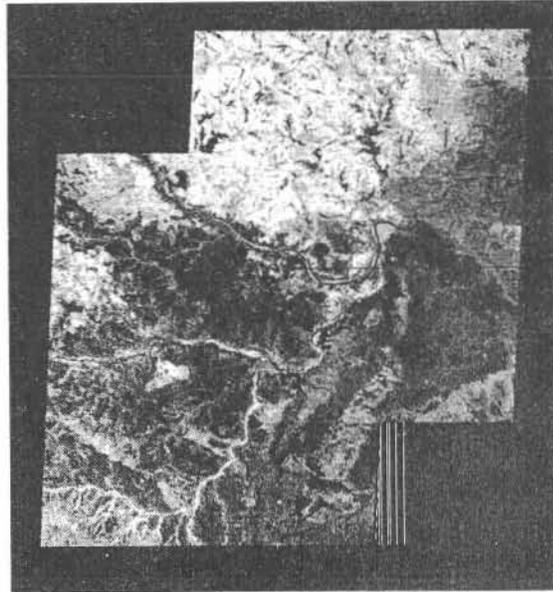


Plate 3

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