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Compressor Roller Bearing Dynamics Analysis

C.T. R. Slayton
E. M. Hall

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ABSTRACT

An analytical model of roller dynamics in a stationary vane type rotary compressor is presented. With the roller considered as a free-body, equations are derived for frictional and viscous drag forces and a second order non-linear differential equation developed which governs roller motion. Solutions are presented for both hydrodynamic roller to shaft bearing lubrication and simple friction contact. Use of the model for sensitivity analysis is discussed and several observations made to improve bearing performance.

INTRODUCTION

In a stationary vane rotary compressor the roller serves to divide the energy input to the eccentric vane nose bearing system into levels which can be reliably tolerated by the design materials and lubrication system under all normally expected operating conditions.

The relative velocity between the roller and the shaft eccentric determine for any load the energy levels imparted to both the roller I.D./eccentric bearing and the roller O.D./vane nose bearing. Limits can be established by considering the roller fixed rigidly to the shaft eccentric and rotating at shaft speed, in which case no energy is expended in the roller I.D./eccentric bearing, and maximum frictional wear occurs at the vane nose to roller O.D. contact. Maximum energy can be imparted to the eccentric bearing by allowing the roller to roll around the cylinder wall in the direction opposite to shaft rotation. In this case, maximum relative velocity between roller I.D. and shaft eccentric occurs and, with periods of boundary lubrication, can result in eccentric bearing seizure or wear.

Many parameters influence roller velocity such as gas pressure loads, friction coefficients, bearing clearances, oil effective viscosity, etc. To do an experimental sensitivity analysis to evaluate the relative effects of all possible combinations of critical parameters is an insurmountable task.

The mathematical model developed and presented in this paper can easily be used to predict roller velocity and acceleration as each parameter is independently varied.

The analysis serves as a tool to understanding bearing failures and provides design guidance for new designs.

ASSUMPTIONS

1. Gas compression begins with crank angle, ε, past zero degrees.
2. The deflection of the vane due to pressure loads is not significant.
3. Shaft speed, O, will be considered to be constant at 3400 RPM.
4. Deflection of the shaft between its support bearings is negligible.
5. Contact between vane and roller is frictional. There is no hydrodynamic film lubrication.
6. Inertia in the vane springs is negligible.
7. The radial clearance between the roller and compressor cylinder will be considered constant with respect to ε.
8. Intake and discharge ports are sufficiently close to the vane that compression starts immediately and gas pressure will not exceed discharge, or case, pressure.
9. The compressor cylinder is considered infinitely stiff.
10. Gaseous drag effects at the inlet and discharge ports are negligible.
11. The viscous drag between the roller ends and the cylinder end plates produces a force normal to the roller which is negligible.
12. During the first $180^\circ$ of shaft rotation, vane inertia and slot friction forces are not sufficient to lift the vane from the roller.

13. Gas compression is polytropic with the compression coefficient equal to 1.2.

The following assumptions are involved in the application of Hydrodynamic Theory to lubrication problems. These assumptions will apply at all places except the vane-roller interface.

14. Lubricant is Newtonian, i.e. shear stress is proportional to the rate of shear.

15. Film is so thin compared with the ratio of kinematic viscosity to linear velocity, that motion of the fluid is laminar.

16. No slip occurs between fluid and bearing surfaces.

17. Fluid inertia terms are negligible.

18. Weight of fluid is negligible.

19. Fluid is incompressible.

20. Fluid film is so thin that the pressure remains constant across its depth.

21. Fluid film is so thin that any curvature of the bearing surfaces may be ignored and the films unwrapped for analysis.

22. The viscosity of the fluid is uniform throughout the film.

23. There is no end leakage from the bearing (this is equivalent to assuming the bearing to have an infinite length).

MODEL DEVELOPMENT

Figure 1 is a cross-sectional view of a stationary vane rotary compressor design showing the tangential forces acting on the roller which determine its angular acceleration, $\dot{\psi}$, with respect to the shaft.

$F_5$ is the frictional force on the roller produced by the roller motion with respect to the vane nose.

$F_{15}$ is the combination of viscous and frictional forces which occur between the ends of the roller and the compressor end plates.

$F_{14}$ is the viscous drag force on the roller produced by the oil film between the roller outside diameter and the compressor cylinder wall. In this analysis, the clearance is assumed constant with crank angle, $\theta$, however in actual practice the clearance is set to a minimum at the crank angle where discharge pressure is reached at AHAM rating conditions to minimize leakage past the roller to the suction side.

$F_{16}$ is the viscous drag on the roller inside diameter determined by analyzing the roller bearing as a heavily loaded journal bearing using Sommerfield's equations. The analysis of $F_{16}$ considering the condition of extreme boundary lubrication and resulting frictional contact between the shaft and idler will also be considered.

The above described forces all produce torques accelerating the roller such that

$$\tau T = I_r \ddot{\psi},$$

where $I_r$ is the roller mass moment of inertia.

Figure 1

$F_5$ may be calculated using figure 2 and determining the forces acting on the vane as a function of the crank angle, $\theta$. From figure 2 and algebraic analysis,

$$F_5 = \sigma \left( \frac{V_{\theta}}{\theta} \right) K \left( \left( c + r + R_c \right) - \left( c \cos \theta + \left( r + R_c \right) \cos \alpha \right) \right) h, \text{(2)}$$

where $V_\theta = \frac{R_c R^2 (2\pi - \theta)}{2} - \frac{R_c R^2 (2\pi - \theta)}{2} + \frac{\sqrt{R_c R^2 - 2 \sin^2 \theta + \cos \theta} (c \sin \theta)}{2} h, \text{ and } V_{\theta} = \pi (R_c - R_R^2) h.
The reaction forces $F_2$, $F_3$, and $F_6$ must be solved from equilibrium equations. Some control is required over the signs of the frictional forces $F_1$ and $F_2$ to assure they oppose vane motion. The angle $\alpha$ is used as a key.

$$-F_4 - F_7 + F_{10} + F_3 + F_8 + \frac{\alpha}{\text{ABS}} \cdot \mu_1 (\text{ABS} F_3)$$

$$+ F_6 \cos \alpha + F_5 \sin \alpha = 0. \quad (13)$$

$$F_3 + F_{18} - F_7 + F_2 + F_1 + F_5 \cos \alpha - F_6 \sin \alpha + F_{20} - F_{19} = 0. \quad (14)$$

$$-F_{18}(1/2) + F_{17}(1/2) - F_{21} - F_1 - F_5(\cos \alpha)$$

$$+ F_6(\sin \alpha) - F_{20}(1/3) + F_{19}(1/3) = 0. \quad (15)$$

Then $F_5 = \mu_2 F_6$. \quad (16)

Using figure 3, the viscous drag at the roller ends can be calculated as a function of fluid viscosity and the velocity gradient between the roller ends.
and the compressor end plates. In one case we consider viscous drag on the top face of the roller and frictional drag on the lower face which supports the roller weight and the unbalanced pressure load at the roller top face.

The total $F_{15}$ force then is the sum of viscous drag force and the frictional drag force and can be expressed as

$$F_{15} = \frac{vR e}{2C_e} [2\pi Re (2\pi - 2\pi Re) + \pi (R^2 - R_s^2)] h \mu_3 \tag{17}$$

$$+ Re v \left( \frac{P_r + P_s}{2} \right) \mu_3 + Re (2\pi - e) \left( \frac{P_r + P_s}{2} \right) \mu_3.$$ 

Assuming viscous drag at both ends of the roller, $F_{15}$ is

$$F_{15} = \frac{2vR e}{C_e} [2\pi Re (2\pi - 2\pi Re)]. \tag{18}$$

The parameters for the solution of $F_{14}$ are shown in figure 4. With the constraints of the assumptions previously discussed, the shear force can be expressed as

$$F_{14} = \frac{2R e^2 \sin \theta}{C_{av}} - \frac{2R e R v \sin \theta}{C_{av}}. \tag{19}$$

where $C_{av} = (C_r + h_r)/2$.

As shown in figure 5, the normal forces acting on the roller which are $F_6$, the vane reaction force, $F_{11}$, the gas pressure force, and $F_{12}$, the roller centrifugal force, can be resolved to an equivalent force, $F_{13}$. The equations for the magnitude of the force vectors not previously shown are (refer to figure 6),

$$F_{11} = (P_r - P_s) \sin \psi/2) h. \tag{20}$$

$$F_{12} = \frac{\pi (R^2 - R_s^2) h \theta^2}{g}. \tag{21}$$

The resultant of the force vectors $F_6$, $F_{11}$, and $F_{12}$ is shown to be

$$F_{13} = \sqrt{[F_6 \cos (\alpha + \frac{\pi}{2}) + F_{12} \cos (3\pi - \theta) + F_{11} \cos (3\pi \psi/2 - \theta)]^2}$$

$$+ [F_6 \sin (\alpha + \frac{\pi}{2}) + F_{12} \sin (3\pi - \theta) + F_{11} \sin (3\pi \psi/2 - \theta)]^2. \tag{22}$$

$$\alpha = \tan^{-1} \left( \frac{F_6 \sin (\alpha + \frac{\pi}{2}) + F_{12} \sin (3\pi - \theta) + F_{11} \sin (3\pi \psi/2 - \theta)}{F_6 \cos (\alpha + \frac{\pi}{2}) + F_{12} \cos (3\pi - \theta) + F_{11} \cos (3\pi \psi/2 - \theta)} \right). \tag{23}$$

From $F_{13}$, the eccentric bearing load as a function of the crank angle can be calculated.

First, consider the eccentric bearing roller I.D. interface as a heavily loaded journal bearing (Ref. 1). From this analysis

$$R_s^2 (H_8) (4\pi) [1 - (K_8 + 1.12)^2]^{1/2} \left[ \frac{1.85v}{2\pi p_4} \right]. \tag{24}$$

where $K_8 = \frac{R_e^2}{C_s}$. \tag{25}

Equation 1 can now be expanded by incorporating equations 16, 18, 19, and 24 such that

$$\psi = K_8 \left( J_2 \right) \frac{1 - (K_8 + 1.12)^2}{[2 + (K_8 + 1.12)^2]} \tag{26}$$

$$= \frac{F_{14} R e^2}{R} + \frac{4\pi v R e^2 R e^2}{C_{av} \mu} \begin{bmatrix} \theta \end{bmatrix}. \tag{27}$$
Equation 26 is non-linear and can be programmed for numerical integration.

Under start-up conditions or conditions of extreme load and/or marginal lubrication from temperature or dilution effects, it is possible that full hydrodynamic lubrication may not exist at the shaft eccentric interface with the roller I.D. In this case, $F_{16}$ may simply be calculated as a frictional force resulting from the force $F_{13}$.

$$F_{16} = \mu_5 F_{13}$$  \hspace{1cm} (30)

In this case equation 26 reduces to

$$\dot{\psi} = K_9 - J_1 \dot{\phi}$$  \hspace{1cm} (31)

The solution of the equation with appropriate values for functional coefficients provides analytical results in good agreement with experimental data.

**DISCUSSION**

An unusual experimental technique was used to verify the model at a single set of geometric clearance parameters. As shown in figure 7, the upper roller end was machined as a circular ramp with a 0.030" step. Four proximity transducers were mounted in the upper plate equally spaced on a circular radius such that the machined roller end would always be sensed by at least one transducer. Because of the eccentric translation motion of the roller, four transducers were necessarily used. To prevent gas leakage across the machined roller end, the ramp was filled with an epoxy and machined to finish end dimensions. The variable reluctance proximity transducer of course, does not sense the nonconducting epoxy.
With this arrangement, the direction of roller rotation is easily determined by the slope of the output voltage, i.e. whether positive or negative. The instantaneous velocity and acceleration is determined by measuring the slope and its rate of change. The output of the transducers was read on a four channel Tektronix oscilloscope in parallel with a high speed oscillograph.

This experimental analysis measures the roller motion with respect to the fixed end plate. Assuming constant shaft speed, we can directly correlate this measurement with the analytical analysis results.

Measurements of roller speed with nominal pump clearance at a standard load point show the average roller velocity to be approximately 30 RPM relative to the plate. At 3400 RPM shaft speed roller speed relative to the shaft is 3370 RPM or 353 rad/sec.

The velocity calculated by use of equation (31) varied from 332 rad/sec at \( \theta = 0^\circ \) to a minimum of 119 rad/sec at \( \theta = 330^\circ \). The average velocity was 260 rad/sec in close agreement with experimental results.

With experimental verification complete, the model can now be used to predict roller motion in a sensitivity analysis.

Analytical and experimental data suggests gas pressure loading, oil viscosity, and development of a hydrodynamic oil film at the eccentric bearing to be primary drivers that govern roller motion.

Even without doing a sensitivity analysis, two areas in the compressor deserve further design attention. First, if the roller to shaft interface is to be treated as a journal bearing, full oil film development is difficult due to a low (.6) length to diameter ratio. With a value this low, high oil leakage at the ends of the bearing will occur, violating an assumption. It is possible that a stable oil film will not exist due to the rapidly changing bearing load and load direction. Secondly, the vane to roller reaction load may be considerably reduced by the simple expedient of reducing the vane width.

REFERENCE 1

Machine Design
McGraw-Hill Series in Mechanical Engineering
Richard G. Folsom, Consulting Editor
Chapter 9, pgs 270-312
### IDENTIFICATION OF ANALYSIS SYMBOLS

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_1$</td>
<td>Net side gas pressure force on vane</td>
<td>lb</td>
<td></td>
</tr>
<tr>
<td>$F_2$</td>
<td>Reaction force between vane and cylinder housing</td>
<td>lb</td>
<td></td>
</tr>
<tr>
<td>$F_3$</td>
<td>Reaction force between vane and cylinder housing</td>
<td>lb</td>
<td></td>
</tr>
<tr>
<td>$F_4$</td>
<td>Gas pressure force on back of vane</td>
<td>lb</td>
<td></td>
</tr>
<tr>
<td>$F_5$</td>
<td>Friction force between vane and roller</td>
<td>lb</td>
<td></td>
</tr>
<tr>
<td>$F_6$</td>
<td>Normal force between vane nose and roller</td>
<td>lb</td>
<td></td>
</tr>
<tr>
<td>$F_7$</td>
<td>Spring force on back of vane</td>
<td>lb</td>
<td></td>
</tr>
<tr>
<td>$F_8$</td>
<td>Suction gas pressure force on vane nose</td>
<td>lb</td>
<td></td>
</tr>
<tr>
<td>$F_9$</td>
<td>Compressed gas pressure force on vane nose</td>
<td>lb</td>
<td></td>
</tr>
<tr>
<td>$F_{10}$</td>
<td>Vane inertia force</td>
<td>lb</td>
<td></td>
</tr>
<tr>
<td>$F_{11}$</td>
<td>Net gas pressure force on roller</td>
<td>lb</td>
<td></td>
</tr>
<tr>
<td>$F_{12}$</td>
<td>Roller centrifugal force</td>
<td>lb</td>
<td></td>
</tr>
<tr>
<td>$F_{13}$</td>
<td>Resultant of $F_6$, $F_{11}$, and $F_{12}$</td>
<td>lb</td>
<td></td>
</tr>
<tr>
<td>$F_{14}$</td>
<td>Viscous drag force at roller O.D.</td>
<td>lb</td>
<td></td>
</tr>
<tr>
<td>$F_{15}$</td>
<td>Viscous and friction drag force between roller O.D. and cylinder end plate</td>
<td>lb</td>
<td></td>
</tr>
<tr>
<td>$F_{16}$</td>
<td>Viscous drag force between roller I.D. and shaft</td>
<td>lb</td>
<td></td>
</tr>
<tr>
<td>$F_{17}$</td>
<td>Gas pressure forces on vane from gas in vane slot</td>
<td>lb</td>
<td></td>
</tr>
<tr>
<td>$F_{18}$</td>
<td>Gas pressure forces on vane from gas in vane slot</td>
<td>lb</td>
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</tr>
<tr>
<td>$F_{19}$</td>
<td>Gas pressure forces on vane from gas in vane slot</td>
<td>lb</td>
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</tr>
<tr>
<td>$F_{20}$</td>
<td>Gas pressure forces on vane from gas in vane slot</td>
<td>lb</td>
<td></td>
</tr>
<tr>
<td>$\phi$</td>
<td>Crank angle</td>
<td>radians</td>
<td>Used in the geometric derivation of $\beta$</td>
</tr>
<tr>
<td>$\tau$</td>
<td>One half arc angle over which $F_{14}$ is distributed</td>
<td>radians</td>
<td></td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Angle between vane centerline and the vane/roller reaction force line</td>
<td>radians</td>
<td></td>
</tr>
<tr>
<td>$\psi$</td>
<td>With roller center as the vertex, the angle between line to vane nose and roller/cylinder contact point</td>
<td>radians</td>
<td></td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Line of action of resultant force $F_{13}$</td>
<td>radians</td>
<td></td>
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</table>

- $\beta$: Bearing eccentricity ratio used for the derivation of $F_{16}$
- $\varepsilon$: Shaft eccentricity
- $\zeta$: Eccentricity between roller center line and shaft eccentric center
- $\mu_1$: Coefficient of friction between vane and slot walls
- $\mu_2$: Coefficient of friction between vane and roller
- $\mu_3$: Coefficient of friction between roller end and end plate
- $R_C$: Radius of cylinder
- $R_R$: Roller outside radius
- $R_S$: Shaft eccentric radius
- $R_e$: Roller mid-radius between O.D. and I.D.
- $r$: Vane nose radius
- $P_S$: Suction pressure
- $P_g$: Compressed gas pressure
- $P_c$: Case (or discharge) pressure
- $P_4$: Roller bearing unit load
- $K$: Polytropic compression coefficient
- $K_S$: Vane spring constant
- $K_8$: "Convenience" constant used in roller bearing journal analysis
- $C_e$: Roller end clearance to cylinder
- $C_r$: Roller radial clearance to cylinder
- $C_s$: Roller radial clearance to shaft
- $C_{avg}$: Average roller radial clearance to cylinder across oil film arc
- $h_r$: Minimum oil film thickness at roller O.D.
- $l_1$: Characteristic vane dimensions
- $l_2$: Characteristic vane dimensions

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<thead>
<tr>
<th>Symbol</th>
<th>Unit</th>
<th>Description</th>
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<tr>
<td>l_2</td>
<td>Inches</td>
<td>Characteristic vane dimensions</td>
</tr>
<tr>
<td>h</td>
<td>Inches</td>
<td>Height of cylinder, roller and vane</td>
</tr>
<tr>
<td>w</td>
<td>Inches</td>
<td>Width of vane</td>
</tr>
<tr>
<td>A_θ</td>
<td>in²</td>
<td>Discharge area</td>
</tr>
<tr>
<td>V_θ</td>
<td>in³</td>
<td>Discharge volume</td>
</tr>
<tr>
<td>V_θ₀</td>
<td>in³</td>
<td>Displacement volume</td>
</tr>
<tr>
<td>²θ</td>
<td>Rad/sec</td>
<td>Shaft angular velocity</td>
</tr>
<tr>
<td>²ψ</td>
<td>Rad/sec</td>
<td>Roller angular velocity with respect to shaft</td>
</tr>
<tr>
<td>³ψ</td>
<td>Rad/sec²</td>
<td>Roller angular acceleration</td>
</tr>
<tr>
<td>³α</td>
<td>Rad/sec</td>
<td>Rate of change of α</td>
</tr>
<tr>
<td>ρ</td>
<td>lb/in³</td>
<td>Roller material density</td>
</tr>
<tr>
<td>m_V</td>
<td>Slugs</td>
<td>Vane mass</td>
</tr>
<tr>
<td>m_R</td>
<td>Slugs</td>
<td>Roller mass</td>
</tr>
<tr>
<td>I_R</td>
<td>Slugs</td>
<td>Roller mass moment of inertia</td>
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<tr>
<td>g</td>
<td>in/sec²</td>
<td>Gravitational constant</td>
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<tr>
<td>t</td>
<td>Inches</td>
<td>Roller radial thickness (R_R-R_s)</td>
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<tr>
<td>H_8</td>
<td>Inches</td>
<td>Height of shaft eccentric lobe</td>
</tr>
<tr>
<td>μ_5</td>
<td></td>
<td>Coefficient of friction between shaft and roller</td>
</tr>
<tr>
<td>K_9</td>
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<td>&quot;Convenience&quot; constant</td>
</tr>
<tr>
<td>J_1</td>
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<tr>
<td>J_2</td>
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<td>&quot;Convenience&quot; constant</td>
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APPENDIX B

Paper Arrived After Completion of Editing