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FINITE ELEMENT METHOD ANALYSIS OF LEAKAGE FLOW IN THE NARROW CLEARANCE BETWEEN THE ROTOR AND SIDE PLATES OF A SLIDING VANE ROTARY COMPRESSOR

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ABSTRACT

The effect of inertia terms (especially centrifugal terms) on the leakage flow through the narrow clearance between the rotor and side plates of a sliding vane rotary compressor is investigated, and it is pointed out that the flow model without inertia terms can be sufficiently accurate in our situation. Then, our problem is solved by Finite Element Method under the non-axisymmetric boundary conditions and the inconsistent clearance configuration. In several cases, pressure maps and velocity distributions of the leakage flow field are obtained. Finally, the vertical equilibrium position of the rotor in the cylinder is estimated, considering the thrust loading of our vertical type rotary compressor. It is found out that almost all leakage flow occurs in the upper clearance.

INTRODUCTION

Recently, detailed analyses of compressors have caught attention because of the growing interest in possible developments of more efficient and reliable compressors. Concerning the internal leakage loss, it is found out to be a significant factor of loss especially in rotary compressors. Reed and Hamilton analyzed the internal leakage effects in sliding vane rotary compressors, and concluded that the most significant internal leakage occurs through the lubricating oil system in rotary compressors [1]. However, few studies discuss the detailed analysis of the lubricating system in a rotary compressor. Because the lubricating system affects also its reliability, it is essential to establish a complete model for detailed analysis and design.

The lubricating system of the rotary compressor usually comprises two major components, that is, journal bearings and rotor face clearances which bear the thrust loading. Concerning the rotor face clearances, there are two points to be considered. One of which is the effect of the centrifugal term caused by rotation of the face. This effect is examined in an axisymmetric model by reducing the governing equations to a simple form through order-of-magnitude arguments. Another point which complicates the flow problem is that parallel plates, one of which is rotating while the other is stationary, cannot bear any thrust loading theoretically, or in other words, the clearance height between two plates cannot be determined by the thrust load. Besides, the leakage flow rate cannot be calculated unless the clearance height is known.

In order to establish a model for the lubricating system, the leakage flow between the rotor and side plates of a sliding vane rotary compressor should be studied analytically under the non-axisymmetric boundary conditions.

Bein et al. solved the problem by applying the perturbation technique [2]. Their analysis is restricted to a consistent clearance configuration, and their method is difficult to apply to our compressors which have circular grooves on the rotor faces or side plates. On the other hand, since Reddi established the application of the Finite Element Method (denoted as FEM) to lubrication problems [3,4], many spiral groove thrust bearings have been analyzed by FEM. In those cases, the boundary conditions were usually homogeneous (i.e., the pressure on the boundary was uniform).

In this work, a FEM model for the lubricating system including journal bearings and the narrow clearances between the rotor and side plates is established, and solved to obtain the vertical equilibrium position of the rotor in the cylinder.
DEFINITION OF PROBLEM

Figure 1 and 2 show our problem configuration. The compressor involved is a hermetic rotary compressor with four vanes sliding in a circular cylinder. Figure 1 is a cut-away view of the rotary compressor, and Figure 2 shows a detailed view of the leakage flow field investigated. The leakage flow field is defined as being bounded by two concentric circles, one of which has a radius of $R_n$ (inner boundary) while the other has a radius of $R_o$ (outer boundary). On the inner boundary the flow field is connected to a journal bearing which has an oil feeding pressure of $P_n$ (which equals the discharge pressure of the compressor). The outer boundary is non-axisymmetric, on which a steady-state pressure distribution is assumed though the exact nature is not steady. Constant temperature, density, and viscosity properties are assumed for the leakage fluid which is lubricating oil diluted with an equilibrium concentration of gaseous refrigerant. The flow is considered to be laminar, because the clearance height is sufficiently small as compared with the rotor outer radius.

The dimensions and the conditions of the compressor studied are indicated in the Nomenclature.

EXAMINATION OF THE CENTRIFUGAL EFFECT

There are three non-dimensional parameters characterizing the flow in the narrow clearance between the rotating surface and the stationary surface [2], which are stated below:

The Euler number defined as the ratio of the external pressure difference to the centrifugal pressure build up during rotation,

$$ E = \frac{P_d - P_s}{\rho \omega^2 R_0} $$

The Reynold's number based on the clearance height $h$,

$$ Re = \frac{\omega h^2}{v} $$

The aspect ratio defined as the ratio of rotor outer radius to the clearance height,

$$ s = \frac{R_o}{h} $$

Bein et. al. analyzed the problem under the following ranges of these parameters [2]:

$$ Re \ll 1, \ s \gg 1, \ E \geq \frac{1}{Re} $$

and expanded around the asymptotic solution of $Re \rightarrow 0$.

They showed that the zeroth order solution (which does not include the effect of inertia terms, but does satisfy non-axisymmetric boundary conditions) is fairly close to the zeroth plus first order solution (which includes the effect of inertia terms) under the ranges of parameters:

$$ E \geq 20, \ Re \leq 0.3 $$

They also noted, the difference between the zeroth plus first order solution and the zeroth order one grows as Euler number decreases due to larger contribution of the inertia terms. In the study cited above they restricted the range of another parameter, $\beta$ (the ratio of the inner radius to the outer radius):

$$ \beta \leq 0.3 $$

However, in our problem configuration, the range of $\beta$ is quite different from the above, as follows:

$$ \beta \geq 0.35 $$
This means, in our problem, the leakage flow rate due to pressure difference between the inner boundary and outer boundary is more significant than the case investigated by Bein et. al. Consequently, in order to examine the effect of the centrifugal term on the leakage flow, we consider an axisymmetric flow model with the term included.

In general, the steady viscous flow of incompressible fluid between a rotating and a stationary surface is governed by the well-known Navier-Stokes equations:
\[ (v \cdot \nabla)v = -\frac{1}{\rho} \nabla P + \nu \nabla^2 v \]  (1)
and the continuity equation:
\[ \text{div } v = 0 \]  (2)

After further reduction allowed by assuming axial symmetry and restricting the range of parameter as \( h << R_0 \) (i.e. \( S >> 1 \)) upon the Navier-Stokes equations, asymptotic transformations can be made. Finally, more simplified equations in cylindrical coordinate are obtained with a centrifugal term included [5]:
\[ \frac{v_h}{r} = -\frac{1}{\rho} \frac{3P}{\partial r} + \nu \frac{3}{2} v_r \]  (3-1)
\[ 0 = \frac{3}{2} \frac{v_h}{\partial z^2} \]  (3-2)
\[ 0 = \frac{3}{2} \frac{P}{\partial z} \]  (3-3)

And also the continuity equation of axisymmetric form is:
\[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial v_r}{\partial r} \right) + \frac{\partial v_z}{\partial z} + \frac{v_r}{r} = 0 \]  (4)

The boundary conditions are as follows:
\[ v_r = v_\theta = v_z = 0 \]
for \( z = h \) (stationary surface)  (5-1)
\[ v_r = v_z = 0, \quad v_\theta = \omega r \]
for \( z = 0 \) (rotating surface)  (5-2)
\[ P = P_d \]
for \( r = R \ln \) (inner boundary)  (5-3)
\[ P = P_s \]
for \( r = R_0 \) (outer boundary)  (5-4)

The equation (3-3) indicates that pressure distribution depends only on \( r \) coordinate:
\[ P = P(r) \]

Now the equations (3) and (4) can be solved analytically with the boundary conditions (5), and we obtain:
\[ P(r) = \frac{P_d \ln \frac{R_0}{R} + P_s \ln \frac{R}{R_0}}{\ln \frac{R_0}{R_1 \ln}} + 0.15\omega^2 \left( r^2 - R_1 \ln^2 \right) \ln \frac{R_0}{R_1 \ln} - \left( r_0^2 - R_1^{\ln} \right) \ln \frac{R_0}{R_1 \ln} \]  (6-1)
\[ v_r = \frac{z(z-h^2)}{2} \frac{R_d - P_s + 0.15\omega^2 \left( R_0^2 - R_1^{\ln} \right)}{r \ln \frac{R_0}{R_1 \ln}} + \frac{\omega^2}{60v_n h} (h-z) (5z^2 - 15hz + 6h^2) \]  (6-2)
\[ v_\theta = \frac{\omega z}{r} (h-z) \]  (6-3), \quad \[ v_z = -\frac{\omega z^2}{30v_nh} (h-z)^2 (3h-z) \]  (6-4)

The results shown in Figure 3 display two different conditions. Figure 3-a reveals the negligible effect of the centrifugal term in the case of \( R_0 < 0.05, E > 10 \), which is the case for our problem, while Figure 3-b indicates the contribution of the centrifugal term to the pressure distribution in the case of \( R_0 < 0.05, E < 0.2 \). Figure 3-b also shows such a significant effect that the negative pressure region occurs in the outer half of the face clearance.

LUBRICATION FLOW NETWORK MODEL

Lubrication flow system in a rotary compressor comprises several components listed below (the numbers correspond to those of Figure 1):
1. centrifugal pump
2. oil path in the shaft
3. journal bearing (upper & lower)
4. face clearances (upper & lower)
5. circular groove in the cylinder head
6. rotor holes (back chambers for sliding vanes)
7. face clearances (upper & lower)

A lubrication flow network model for our rotary compressor is constructed and shown in Figure 4. Oil flow equations for those components are listed in Table 1. Values pertinent to our compressor are applied to the equations and the magnitude of those contribution to pressure distributions is evaluated.

It is found out that some components have negligible effects on the flow network. So, simplified network model can be constructed for our compressor as shown in Figure 4.

FEM MODEL FOR FACE CLEARANCE FLOW

Lubrication flow through the rotor face and the side plate is a problem with 2-dimensional pressure distribution. As shown in the previous section, the centrifugal term is negligible in our problem. Then, the steady-state lubrication flow is described by the conventional Reynolds's equation.
\[ \frac{1}{12\mu} \left( \frac{3}{r^3} \frac{\partial}{\partial r} \left( r^3 \frac{\partial P}{\partial r} \right) + \frac{3}{r^3} \frac{\partial P}{\partial \theta} \right) = \frac{\omega^2}{30v_n h} (h-z)^2 (3h-z) \]  (7)
The flow field is divided into many finite elements and the elements encircling the inner boundary are connected with one-dimensional elements which represent the journal bearings. Figure 5 shows the finite element layout of upper and lower face clearances.

The inner boundary condition is:

\[ P = P_d \]  
(for all nodes on the inner side of one-dimensional elements).

The outer boundary condition is:

\[ P = P_0(\theta) \]  
(for all nodes on the outer boundary of upper and lower flow fields)

\( P_0(\theta) \) is shown in Figure 6.

The boundary conditions for velocity are:

\[ v_\theta = r\omega, \quad v_r = v_z = 0 \]  
(for \( z = 0 \) (rotating surface))

\[ v_r = v_\theta = v_z = 0 \]  
(for \( z = h \) (stationary surface))

In this analysis, the upper and lower flow fields containing circular groove are expressed respectively by assembly of 648 quadrilateral two-dimensional elements which have a linear interpolation function for pressure. Note that the elements representing the narrow clearance differ in the element thickness from those representing the circular groove. The upper and lower journal bearings are expressed respectively by 72 one-dimensional elements which also have a linear interpolation function. The upper flow field and the lower one are connected to each other at four different nodes located on the circular groove, which represents the existence of rotor holes and corresponds to the dotted lines in Figure 5. Following is a summary of the model.

Total number of nodes: 1584
Total number of elements: 1440
two-dimensional: 1296
one-dimensional: 144
Total unconstrained d.o.f.: 1292

SOLUTION METHOD

Reynold's equation shown above is one of quasi-harmonic equations, which govern several kinds of field problems such as heat conduction and electricity field, etc. and have a scalar potential as the unknown variable. Some analysts suggested solution method of lubrication problem by use of electricity analogy [7]. In view of FEM, Huebner mentioned general treatment for the solution of field problems governed by the quasi-harmonic equations [8].

For the construction of analysis system of complex lubrication problems, because of
The procedure incorporating the analogous solution method was established for our lubrication problem, and satisfactorily functioned with less efforts than coding a special purpose FEM software including the whole processes.

### RESULTS

We calculated at four different values of clearance height and interpolated the results for the estimation of the balance point of thrust loading. Figure 7 shows the relationship of thrust loading v.s. clearance height for our compressor. It is found out that the equilibrium position of the rotor is about 2um for lower clearance ( eccentricity is 0.8 ) with thrust load of 5.1 kgf ( 5.1x9.8 N ). The leakage flow rates through the upper and lower clearances at the equilibrium position are as follows :

<table>
<thead>
<tr>
<th>OUTER BOUNDARY</th>
<th>INNER BOUNDARY</th>
</tr>
</thead>
<tbody>
<tr>
<td>LOWER</td>
<td>INWARD</td>
</tr>
<tr>
<td>OUTWARD</td>
<td>UPPER 944.72</td>
</tr>
<tr>
<td>INWARD</td>
<td>LOWER 1.30</td>
</tr>
<tr>
<td></td>
<td>UPPER 0.79</td>
</tr>
<tr>
<td></td>
<td>LOWER 1.35</td>
</tr>
<tr>
<td></td>
<td>UPPER 366.93</td>
</tr>
</tbody>
</table>

\((\text{mm}^3/\text{sec})\)
It indicates that almost all leakage occurs through the upper clearance. Figure 8 shows the pressure distributions on the upper and lower flow fields. And Figure 9 shows the volume flow velocity distributions in the same way. It can be seen that in the lower clearance the shear flow is more dominant than in the upper clearance.

In order to evaluate the effect of circular groove, the same model without the groove was calculated. Figure 10 shows the quite different pressure distributions for the case.

CONCLUSIONS

1. Centrifugal effect on the leakage flow in the narrow clearance between the rotor and side plates of rotary compressor was evaluated and found to be negligible in our problem configuration, that is, in the case of $R_e < 0.05$, $E > 10$.

2. Flow network model comprising journal bearings and rotor face clearances was constructed for the lubrication system of our rotary compressor.

3. Solution method of lubrication problem incorporating heat conduction analysis capability of general purpose FEM code was established.

4. Equilibrium position of the rotor in the cylinder was estimated in a particular thrust loading condition. Pressure and velocity distributions in the clearances were also calculated.

5. It was found out that almost all leakage occurs in the upper clearance. It was also found out that the leakage flow from the discharge chamber to the suction through the upper clearance is significant.

REFERENCES


NOMENCLATURE

Values in parentheses indicate our problem specifications.

- $C$ (cylinder thickness - $R_R$)/2 (10 μm)
- $C_B$ radial clearance of journal bearings (13.5 μm)
- $d$ diameter of oil path (19.3 mm)
- $d_B$ diameter of oil feeding port in journal bearings (19.3 mm)
- $E$ Euler number (see the text)
- $f$ Darcy-Weisbach friction factor [6]
- $g$ gravity (9.8 m/sec)
- $h$ clearance height between rotor face and side plate (200 μm)
- $H_Q$ oil path length (28.5 mm)
- $H_R$ oil feeding port in journal bearings (27 mm)
- $h$ unit outward vector normal to boundary (9 mm)
- $h_B$ journal bearing length (9 mm)
- $h_e$ element interpolation function (vector)
- $h_l$ pressure (27 mm)
- $P$ discharge pressure (19.3 × 10^8 Pa)
- $P_d$ suction pressure (6.8 × 10^8 Pa)
- $Q$ average flow velocity (normal to bound.) (1.9 × 10^8 Pa)
- $Q_l$ heat flux (normal to boundary) (1.9 × 10^8 Pa)
- $Q_r$ volume flow of lubricant (1.9 × 10^8 Pa)
- $Q_{sg}$ rate of heat generation (1.9 × 10^8 Pa)
- $R_e$ Reynolds's number (see the text)
- $R_{in}$ rotor inner radius (10 mm)
- $R_{o}$ rotor outer radius (26.5 mm)
- $S$ aspect ratio (see the text)
- $T$ temperature (100°C)
- $V$ velocity vector of moving surface (100°C)
- $V_r$, $V_o$, $V_z$ components of the velocity vector (100°C)
- $w$ axial coordinate (100°C)
- $\epsilon$ eccentricity ($\epsilon = \frac{C - h_{lower}}{C}$)
- $\lambda$ angular coordinate (100°C)
- $\mu$ thermal conductivity (3.10^2 Pa·sec)
- $\nu$ kinematic viscosity ($\nu = \mu/\rho$)
- $\Pi$ functional of the function (100°C)
- $\rho$ density of lubricant (910 kg/m^3)
- $\omega$ angular velocity of rotor (356 rad/sec)
- $\nabla^2$ Laplacian operator (100°C)
- $L$ (subscript) denotes lubrication problem (100°C)
- $H$ (subscript) denotes heat conduction prob. (100°C)
- $T$ (superscript) denotes transpose of matrix (100°C)
Reynold's equation without squeeze effect is expressed in rectangular coordinate as:

\[ \nabla \cdot \left( \frac{h_l^3}{12\mu} \nabla P \right) = \frac{1}{2} \nabla \cdot (h_l \mathbf{U}) \quad \text{on } \Omega . \quad (A-1) \]

Boundary conditions:

\[ P = P_0(x,y) \quad \text{on } S_p \]
\[ n \cdot \frac{h_l^3}{12\mu} \nabla P = n \cdot \frac{h_l \mathbf{U}}{2} - q_L h_l \quad \text{on } S_q \quad (A-2) \]

Similarly, two-dimensional steady-state heat conduction equation with heat source is:

\[ \nabla \cdot (h_H \nabla T) = -Q_a h_H \quad \text{on } \Omega . \quad (B-1) \]

Boundary conditions:

\[ T = T_0(x,y) \quad \text{on } S_p \]
\[ n \cdot h_H \nabla T = -q_H h_H \quad \text{on } S_q \quad (B-2) \]

The quasi-harmonic equations are classified into linear elliptic self-adjoint partial differential equations, for which existence of variational principle is shown [8]. Following is the functional for Reynold's equation:

\[ \Pi(P) = \int \left[ \frac{h_l^3}{12\mu} \nabla P \cdot \nabla P + \frac{1}{2} \nabla \cdot (h_l \mathbf{U}) \right] dA + \int_{S_q} (q_L h_l - \frac{h_l \mathbf{U}}{2}) d\mathbf{S} . \]

This formulation is somewhat different from Reddi [3,4] and Huebner [8], but also can be proved as follows:

Taking the first variation of the above functional and applying Green's theorem, we get (using \( \delta P = 0 \) on \( S_p \)),

\[ \delta \Pi = \int_{\Omega} \left[ -\nabla \cdot \left( \frac{h_l^3}{12\mu} \nabla P \right) + \frac{1}{2} \nabla \cdot (h_l \mathbf{U}) \right] \delta P dA + \int_{S_q} \left[ n \cdot \frac{h_l^3}{12\mu} \nabla P - n \cdot \frac{h_l \mathbf{U}}{2} + q_L h_l \right] \delta d\mathbf{S} . \]

To stationarize the functional \( \Pi \) with respect to unknown function \( P \), we require that \( \delta \Pi = 0 \).

Since the \( \delta P \) is arbitrary on \( \Omega \) we get the Reynold's equation (A-1) and boundary condition (A-2) simultaneously.

On the other hand, the functional for heat conduction equation is as follows:

\[ \Pi(T) = \int_{\Omega} \left[ \frac{h_l^3}{2} \nabla T \cdot \nabla T - Q_a h_H T \right] dA + \int_{S_q} q_H h_H d\mathbf{S} . \]

Comparing the two matrix equations, we get the equivalent correlation between the two systems as follows:

For Lubrication:

\[ K_p P + F_u + F_s = 0 \]

\[ K_p = \int_{\Omega} \left[ \frac{h_l^3}{12\mu} \left( \frac{\partial}{\partial x} N_{e} \right) \cdot \left( \frac{\partial}{\partial x} N_{e} \right) + \frac{\partial}{\partial y} N_{e} \cdot \frac{\partial}{\partial y} N_{e} \right] dA \]
\[ F_u = \int_{S_q} \left( \nabla \cdot (h_l \mathbf{U}) \right) N_d d\mathbf{S} , \quad \text{where } P \text{ is a nodal pressure vector,} \]
\[ F_s = \int_{S_q} (q_l h_l - \frac{h_l \mathbf{U}}{2}) N_d d\mathbf{S} . \]

For Heat conduction:

\[ K_t T + F_q + F_s = 0 \]

\[ K_t = \int_{\Omega} \left[ h_l \left( \frac{\partial}{\partial x} N_{e} \right) \cdot \left( \frac{\partial}{\partial x} N_{e} \right) + \frac{\partial}{\partial y} N_{e} \cdot \frac{\partial}{\partial y} N_{e} \right] dA \]
\[ F_q = \int_{S_q} (Q_a h_H) N_d d\mathbf{S} , \quad \text{where } T \text{ is a nodal temperature vector,} \]
\[ F_s = \int_{S_q} (q_h h_H) N_d d\mathbf{S} . \]

Assuming constant thickness \( h \) within the element, we can convert the lubrication problem into the heat conduction problem. Note that space derivatives of \( h_l \mathbf{U} \) term must be pre-processed.

4. Assemble whole element equations into a global system matrix.

Fig.7 Thrust Loading Capacity

3. Stationarize the discretized functional for each finite element. Here we get the following matrix equations for each element.
Fig. 8 Pressure Distributions

Fig. 9 Flow Velocity Distributions

Fig. 10 Pressure Distributions without Circular Groove