Optimization of Shell-and-Tube Intercooler in Multistage Compressor System

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ABSTRACT

Multi-staging with intercoolers is an effective method for reducing the necessary energy to drive gas compressors. Increases of efficiency can result in large energy savings for large compressor systems. Optimization of the intercooler is highly desirable as the size of compressor system increases.

This paper presents an optimum design procedure for the intercooler where the objective function includes not only the reduction of compressor power but the reduction of pumping power for the intercooler water and the initial cost of the intercooler. The procedure permits relative weighting of the importance of the combined power reduction compared to the intercooler cost. The multi-stage compressor system is optimized by optimizing each stage independently assuming no coupling effect due to temperature.

INTRODUCTION

Compressor consumes energy during the gas compressive process. Some large compressors require several kilowatt power to run. Even for a small air compressor with displacement of 10 cm³/min, an electric motor of power 50–100 kW is required. Therefore, it is important to increase the efficiency of a compressor for the saving of energy.

For large or middle power compressors, one of the important ways to save energy is the use of multistage compression [1]. The gas compressed part way to the discharge pressure, passed through an intercooler and then compressed further.

The energy saved by using an intercooler in a two-stage compressor can be shown on a Pressure-Enthalpy diagram, Fig.1, where \( p_1 \) is the suction pressure of gas, \( p_3 \) is the discharge pressure of gas. The specific adiabatic indicated work of the compressor equals the specific enthalpy difference between point 3 and point 1,

\[
W_i = h_3 - h_1
\]

If two-stage compression is used instead of one-stage compression, and the assumption is made that there is no pressure drop in...
the intercooler, the overall compressive process curve is 1-2-4-5. The specific adiabatic indicated work of compressor equals

$$W' = (h_2 - h_1) + (h_5 - h_6)$$

Since the slope of the process curve 4-5 is larger than the slope of the curve 1-3 and the pressure limits are equal, then

$$(h_5 - h_6) < (h_3 - h_2)$$

The specific work saved by using this ideal intercooler is

$$\Delta W' = (h_3 - h_2) - (h_5 - h_6)$$

In fact, there is pressure drop in a nonideal intercooler, so the pressure of gas at the outlet of the intercooler is $p_4$, less than $p_4$. The specific adiabatic indicated work of compressor then equals

$$W_2 = (h_2 - h_1) + (h_7 - h_6)$$

The actual specific work saved is

$$\Delta W = (h_3 - h_2) - (h_7 - h_1)$$

$\Delta W$ is less than $\Delta W'$ because $h_4 \approx h_6$ and $h_7 \approx h_5$.

Theoretically, the higher the heat-transfer rate of the intercooler, the lower the temperature $t_6$ will be and the greater the quantity of energy can be saved. But it is impossible to remove too much heat from gas to water because of the reason given below:

From a general heat-transfer rate equation

$$Q = UA \Delta t$$

it can be seen that the heat-transfer rate $Q$ will increase with an increase of the heat-transfer area $A$ and the overall heat-transfer coefficient $U$. But the increase of area $A$ results in a larger size and a higher first cost of the intercooler. Therefore, there is a cost effectiveness limitation on the heat-transfer area $A$. The overall heat-transfer coefficient $U$ also has it's cost effectiveness limitation, because $U$ depends on both the velocity of gas and the velocity of water. By forcing the fluids through the intercooler at higher velocity, the overall heat-transfer coefficient can be increased, but this higher velocity will result in a large pressure drop through the intercooler and correspondingly larger pumping power of fluid. There will also be a drop in the specific work saved by the intercooling process.

From this discussion, it can be seen that an optimum intercooler exists and, therefore, the purpose of this intercooler optimization is to determine the optimal heat-transfer area, the velocity of fluid, and the dimensions of the intercooler.

Generally, the intercooler is designed after the number of the compressor stages and the normal pressures of each stage are determined. Therefore, it is assumed in this paper that the number of stages and the normal pressures are fixed before the intercooler optimization.

THE OBJECTIVE FUNCTION

Many papers have discussed optimum design of heat exchangers as single unit. Shah, R.K. et al. [2] discussed the problem of heat-exchanger optimization for a direct-transfer crossflow intercooler where the objective function was the total fluid pumping power for the exchanger.

For a compact-in-fin heat exchanger, an objective function $f$ was given by Mandel, S.W. et al. [3].

$$f = \frac{1}{N_u} \left( \frac{h_4 - h_6}{p_0} \right)$$

where $p_0$ is the local static pressure, $p_0$ is the total pressure at inlet, and $N_u$ is the Nusselt number. The objective function $f$ was minimized while maintaining Reynolds number similarity.

Radhakrishnan, V.R. et al. [4] made an optimum design of shell-and-tube heat exchangers using four terms in his objective function:

a. Fixed charges on the heat exchanger
b. Cost of utility fluid
c. Pumping cost of process fluid
d. Pumping cost of utility fluid

The objective functions listed above could not be used for the optimization of compressor intercooler, because one of the most important purposes of using intercooler in compressor system is to reduce the work of the compressor and therefore a term which directly connects with the adiabatic indicated power of compressor have to be included in the objective function. From this point of view, it is reasonable to use the following objective function for the compressor system

$$f = f\left( \text{Cost of compressor adiabatic indicated power, Pumping cost of cooling water, cost of intercooler} \right)$$

(1)
By dividing the compressor system into \( n \) units, Fig. 2, and letting \( f_i \) be the partial objective function for the \( i \)th unit, the objective function of compressor system \( f \) can be expressed as follows:

\[
f = \sum_{i=1}^{n} f_i
\]

Assuming that the cost of intercooler is proportional to the heat-transfer area, the partial objective function of the \( i \)th unit can be written as follows:

\[
f_i = c_1 P_{c,i} + c_2 P_{w,i} + c_3 A_i\]  \quad (2)

where \( c_1 \) and \( c_2 \) are coefficients, \( P_{c,i} \) is the compressor adiabatic indicated power of the \( i \)th unit, \( P_{w,i} \) is the pumping power of cooling water in the \( i \)th unit, and \( A_i \) is the heat-transfer area of the \( i \)th unit.

Rearranging equation (2) gives

\[
f_i = c_1 \left[ (P_{c,i} + P_{w,i}) + m A_i \right]
\]

where \( m = c_2 / c_1 \), the power-cost weighting parameter.

For optimization \( c_1 \) can be omitted and we have

\[
f_i = (P_{c,i} + P_{w,i}) + m A_i\]  \quad (3)

It is assumed that during the minimization of the partial objective function of the \( i \)th unit, the partial objective functions of other units do not change. Therefore, the minimization of partial objective function of each unit can be treated respectively and the optimized objective function \( f \) will be obtained after each partial objective function \( f_i \) is minimized.

The assumption mentioned above is only approximately correct because during the minimization of \( f_i \), the exit temperature \( T_i \) of the \( i \)th unit does change. For example, for an intercooler with one shell pass and two of tube pass which is widely used type of intercooler in the compressor system, the change of the temperature \( T_i \) causes changes of the exit temperatures of other units and changes of the compressor powers \( P_{c,i+1} \), \( P_{c,i+2}, \ldots \). The partial objective functions \( f_{i+1}, f_{i+2}, \ldots \) will change also because the compressor power is a part of the partial objective function. However, the change of temperature \( T_i \) has a reduced influence on the compressor indicated powers \( P_{c,i+1}, P_{c,i+2}, P_{c,i+3}, \ldots \). For example 10 K change of temperature \( T_i \) during the minimization of \( f_i \) only causes 0.9% change of \( P_{c,i+1} \) (see appendix 1) and even more less change of the \( P_{c,i+2}, P_{c,i+3}, \ldots \). Therefore, the above assumption is assumed to be acceptable especially when the small error caused by treating the partial objective functions separately is compared to the major simplification of the optimization procedure.

The compressor adiabatic indicated power \( P_{c,i} \) for the \( i \)th unit is given by
The pressure drops $\Delta p_g$ and $\Delta p_d$ of gas passing through the compressor valves are assumed to be fixed, so they have no influence on the intercooler optimization and do not appear in equation (4).

To determine the exit temperature $T_e$, the following equations have been used:

1. Equation for heat flow rate $\dot{Q}$

$$\dot{Q} = (h_{in} - h_{out}) \cdot \dot{M}$$

$\dot{Q}$ is also the heat-transfer rate.

2. Equation for overall heat-transfer coefficient $U$

$$\frac{1}{U} = \frac{1}{\alpha_m} + \frac{d_0}{d_m} \cdot \frac{L}{\lambda} + \frac{1}{\alpha_w} \cdot \frac{d_0}{d_w} + R_f$$

c. Equation for heat-transfer coefficient $\alpha_m$ in gas side

$$\alpha_m = \beta \cdot j_m \cdot (C_m \cdot \rho_c) \cdot (\frac{\mu_m}{\mu_{wm}})^{\frac{3}{2}} \cdot (\frac{h_m}{h_{wm}})^{\alpha_{1/4}}$$

d. Equation for heat-transfer coefficient $\alpha_w$ in water side

$$\alpha_w = \gamma \cdot j_w \cdot (\frac{\mu_w}{\mu_{ww}})^{\frac{3}{2}} \cdot (\frac{h_{wm}}{h_w})^{\alpha_{1/4}}$$

To determine the pressure drop $\Delta p_c$, the following equation has been used:

$$\Delta p_c = 2 \frac{\dot{M}^2}{d_0} + \eta (N_i - 1) \frac{\dot{M}^2}{d_0} + N_i \frac{\dot{M}^2}{d_w}$$

AN EXAMPLE

An intercooler listed in literature [5] has been used as an example. This is an intercooler with mass flow of gas of 6480 kg/hr. The intercooler receives gas at a pressure of $175 \times 10^4$ N/m$^2$ and cools it to a temperature of 325K. The cooled gas can then be compressed in a compressor up to $525 \times 10^4$ N/m$^2$. The parameters of the intercooler before the optimization are (see Fig. 3)

$d_0$ - The diameter of tube, $d_0 = 0.025$ m

$n_2$ - Number of columns of tube, $n_2 = 21$

$n_3$ - Number of rows of tube, $n_3 = 18$

$L$ - Length of tube, $L = 2.28$m

$N_i$ - Number of baffles, $N_i = 13$

$V_w$ - The velocity of the cooling water, $V_w = 0.16$m/sec

The optimization problem of this intercooler can be restated in the following form:

Minimize: $f = f(V_w, d_0, n_2, n_3, L, N_i)$

Subject to the inequality constraints

$$0.7 \leq \frac{n_2}{N_i} \leq 0.9$$

$$\frac{L - 0.05D}{N_i} \geq 0.06$$

$$2 \leq \frac{L}{D} \leq 4$$

and the variable boundary constraints

$$0.14 < V_w < 0.20$$

$$0.02 < d_0 < 0.03$$

$$20 < n_2 < 35$$

$$15 < n_3 < 25$$

$$2 < L < 4$$

$$10 < N_i < 30$$

The values of the contraints are given arbitrary in accord with the author's experience.

The optimization problem is solved by using the Generalized Reduced Gradient Method which is one of the most efficient optimization method and has been coded as OPT in Purdue University [6] [7].

The optimization is made under different values of power-cost weighting parameter $m$, so that we can evaluate the effect caused by different costs of the intercooler.

When coefficient $m = 0$, equation (3) reduces to

$$f = P_c + P_w$$

where only the thermodynamic performance is chosen as the objective function. The total power consumed before and after the optimization are 323.5kw and 310.0 kw respectively, i.e., 13.5 kw has been saved after the optimization.
Figures 4 through 6 show the optimal values of the compressor adiabatic indicated power p, the heat-transfer area A and the velocity of water v, plotted against the weighting coefficient m.

From Fig. 4&5, it can be seen that the power p increases with the increase of the coefficient m, while the heat-transfer area A reduces with the increase of coefficient m. The shape of the curves can be explained as follows:

With the increase of coefficient m, the cost of the intercooler plays a more and more important role in the objective function. Therefore, the heat-transfer area has to be reduced with the increase of coefficient m even though it will cause the increase of the power. The optimal area A is equal to the original value given before the optimization when m = 0.31, and A must be lower than the original value when m > 0.31. The minimum objective function can be obtained by the increase of the area A only when m = 0.31.

From Fig. 7&8, it can be seen that the optimal length and the optimal diameter of the tube are reduced with an increase of the coefficient m, because the heat-transfer area reduces.

In Fig. 9, the number of baffles N is reduced when the coefficient m is increased because the length of the tube also decreased and there is a constraints to keep the distance between two baffles from becoming too small.

Fig. 10&11 show the changes of n and n with the coefficient m. Since the change are quite small, the values of n and n can be taken as constants after ma > 0.1.

CONCLUSION

Additional terms as well as the compressor adiabatic indicated power should be included in the objective function used for the intercooler optimization in a compressor system. Therefore, an objective function f including the cost of the compressor adiabatic indicated power, the pumping cost of cooling water and the cost of intercooler has been presented.

The objective function of the compressor system f can be obtained by dividing the compressor system into multiple units and then minimizing the partial objective functions of each unit.

APPENDIX

For the (i+1)th unit (see Fig.12), the heat flow rate Q_i+1 equals

\[ \dot{Q}_{i+1} = \dot{M} \cdot C_p \cdot (T_{out,i} - T_{in,i}) \] (a)

![Fig. 12](image-url) The Schematic of unit i and unit(i+1)
On the other hand, \( Q_{i+1} \) can be determined by heat-transfer rate equation

\[
Q_{i+1} = UA \frac{d \theta}{dt} = UA \frac{\sqrt{\Delta T_g + \Delta T_w}}{\ln \left( \frac{\theta_1 + \theta_2 + \sqrt{\Delta T_g + \Delta T_w}}{\theta_1 + \theta_2 - \sqrt{\Delta T_g + \Delta T_w}} \right)}
\]

where

\[
\begin{align*}
\Delta T_g &= T_{ci+1} - T_{ci} \\
\Delta T_w &= t_{out,i+1} - t_{in,i+1} \\
\theta_1 &= T_{ci} - t_{in,i+1} \\
\theta_2 &= T_{ci} - t_{out,i+1}
\end{align*}
\]

Since \( \Delta T_g \gg \Delta T_w, \frac{\Delta T_g - \Delta T_w}{\Delta T_g} = \Delta T_g \), and

\[
Q_{ci+1} = UA \frac{T_{ci+1} - T_{ci}}{2T_{ci} - 2T_{ci+1}}
\]

Letting \( t_{out,i+1} = t_{in,i+1} = t_{i+1} \) and comparing equation (c) with equation (a), we have

\[
\frac{2T_{ci+1} - 2T_{ci+1}}{2T_{ci} - 2T_{ci+1}} = C
\]

Therefore

\[
2T_{ci+1} - 2T_{ci+1} = C
\]

where \( C = \frac{U \cdot A}{C_{pm}} \). C is a constant because \( U \) of the \( (i+1) \)th unit nearly have no change during the optimization of the \( i \)th unit. Rearranging equation (e) and derivating the both sides, we have

\[
\frac{dT_{ci+1}}{dT_{ci+1}} = \frac{C}{2T_{ci+1} - 2T_{ci+1}}
\]

and then

\[
\frac{dT_{ci+1}}{dT_{ci+1}} = \frac{2}{2T_{ci+1} - 2T_{ci+1}}
\]

Since the temperature change of water is quite small than the temperature change of gas, \( \frac{dT_{ci+1}}{dT_{ci+1}} \approx 2 \), we have

\[
\frac{dT_{ci+1}}{dT_{ci+1}} = \frac{2T_{ci+1} - T_{ci+1}}{2T_{ci+1} - T_{ci+1}} + \frac{2T_{ci+1} - T_{ci+1}}{2T_{ci+1} - T_{ci+1}}
\]

According to the engineering practice, the temperatures usually can be approximately taken as

\[
T_{ci+1} = 323 K, T_{ci} = 423 K, T_{ci+1} = 616 K, \text{ and } \frac{dT_{ci+1}}{dT_{ci+1}} = 0.19
\]

Therefore,

\[
\frac{dT_{ci+1}}{dT_{ci+1}} = 1.36
\]

For isentropic process, we have

\[
\frac{dT_{ci+1}}{dT_{ci+1}} = \frac{1}{k-1}
\]

where \( E \) is the ratio of pressure in the \( i \)th unit, and \( k \) is the ratio of specific heats. Usually, \( E = 3 \).

For the mixture of \( H_2 \) and \( N_2 \), \( k = 1.4 \), hence

\[
\frac{dT_{ci+1}}{dT_{ci+1}} = 1.36
\]

It means that the 10K change of temperature \( T_i \) will cause 2.6K change of temperature \( T_{i+1} \).

From equation (4)

\[
P_{ci+1} = \frac{C_i}{k-1}
\]

\( P_{ci+1} \) and \( P_i \) are normal pressure of the \( (i+1) \)th stage. They are fixed before the optimization. \( P_i \) is the pressure drop of the \( (i+1) \)th unit. Since the intercooler size of the \( (i+1) \)th unit has no change during the optimization of the \( i \)th unit, the velocity change of gas in the \( (i+1) \)th unit caused by the change of the temperature \( T_i \) is also very small. Therefore, the change of the pressure drop \( P_{i+1} \) is negletable, and we have

\[
P_{ci+1} = \frac{C_i}{k-1}
\]
\[ P_{c,i+1} = Z T_{c,i+1} \]
\[ \frac{\Delta P_{c,i+1}}{P_{c,i+1}} = \frac{\Delta T_{c,i+1}}{T_{c,i+1}} \]

where \( Z \) is a constant, \( \Delta T_{c,i+1} \) is the change of temperature \( T_{i+1} \), \( \Delta P_{c,i+1} \) is the change of compressor power of the \((i+1)\)th unit.

When \( \Delta T_{i+1} = 2.6 \text{K} \), \( T_{i+1} = 323 \text{K} \)
\[ \frac{\Delta P_{c,i+1}}{P_{c,i+1}} = 0.009 \]

NOMENCLATURE

- \( h \): Specific enthalpy of gas
- \( w \): Specific adiabatic indicated work
- \( \dot{Q} \): Heat-transfer rate
- \( U \): Overall heat-transfer coefficient
- \( \Delta t \): Mean temperature difference
- \( t_{in} \): Inlet temperature of water
- \( t_{out} \): Outlet temperature of water
- \( T_{i} \): Inlet temperature of gas in the \( i \)th unit
- \( T_{i} \): Outlet temperature of gas in the \( i \)th unit
- \( P_{s,i} \): Suction pressure of gas in the \( i \)th unit
- \( P_{d,i} \): Discharge pressure of gas in the \( i \)th unit
- \( M \): Mass flow rate of gas
- \( R \): Gas constant
- \( \Delta P_{i} \): Pressure drop of gas through the \( i \)th intercooler
- \( Nu \): Nusselt number
- \( \alpha_{m} \): Heat-transfer coefficient at gas side
- \( \alpha_{w} \): Heat-transfer coefficient at water side
- \( N_{b} \): Number of baffles
- \( P_{w,i} \): Power for pumping water in the \( i \)th unit
- \( P_{c,i} \): Compressor adiabatic indicated power in the \( i \)th unit
- \( d_{o} \): Outside diameter of tube
- \( d_{i} \): Inside diameter of tube
- \( d_{m} \): Mean diameter of tube
- \( c_{p,g} \): Specific heat of gas
- \( c_{p,w} \): Specific heat of water
- \( \mu_{g} \): Viscosity of gas
- \( \mu_{w} \): Viscosity of water
- \( \lambda \): Conductivity of tube
- \( \lambda_{m} \): Conductivity of gas
- \( \lambda_{w} \): Conductivity of water
- \( \Delta P_{B} \): Pressure drop of gas vertically passing through the tubes
- \( \Delta P_{B} \): Pressure drop of gas in two ends of the intercooler
- \( \Delta P_{w} \): Pressure drop of gas passing through the breach of the baffle

REFERENCES

Fig. 3 Schematic of a Shell-and-Tube Intercooler

Fig. 4 The Optimal Values of Power $P_c$

Fig. 5 The Optimal Heat-Transfer Area $A$
Fig. 6 The Optimal Velocity of Water

Fig. 7 The Optimal Length of Tube

Fig. 8 The Optimal Outside Diameter of Tube
Fig. 9 The Optimal Number of Baffles

Fig. 10 The Optimal Number of Columns of Tube

Fig. 11 The Optimal Number of Rows of Tube