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Rigid body modeling issue in acoustical topology optimization

Jin Woo Lee, Yoon Young Kim

ABSTRACT

For gradient-based acoustical topology optimization, the acoustical properties of an acoustic medium are interpolated. Although the lower limit values of the density and sound speed simulating a rigid body affect the accuracy of the final results significantly, an in-depth study on the selection of these limit values is rare. In the first part of this work, the importance of the proper choice of the limit values was demonstrated with an example to show that improperly-selected limit values can result in erroneous eigenvalues and eigenmodes of an optimized acoustic cavity. Then, by using two theoretical models representing typical air-rigid body configurations, the effects of the density and sound speed values of a simulated rigid body were investigated by the power transmission coefficient. From the findings of the theoretical investigation, a guideline for properly selecting the low limit values of the density and sound speed of a simulated rigid body is suggested. The validity of the guideline will be checked in numerical case studies.

1. Introduction

This work is motivated by the fact that inappropriate modeling of acoustical material used for complete acoustical insulation in acoustical topology optimization problems can yield erroneous numerical results. Theoretically, a complete acoustical insulation material should fully reflect an incident acoustic wave, and its specific acoustical impedance should be infinite [1]. If a rigid body is simulated as a complete acoustical insulation material to facilitate the topology optimization process, improper selection of the acoustical properties of the rigid body could lead to an erroneous prediction of the acoustical behavior of an acoustic system. After this issue is investigated with simple theoretical models, a guideline for properly selecting the acoustical properties will be given.

Since a structural topological layout design problem was formulated as an optimal material distribution problem by Bendsøe and Kikuchi [2], the numerical optimization method has been applied to various design problems as explained in [3]. For topology optimization, a design variable is assigned to each and every finite element used to discretize a design domain and the material properties of each element are varied as an interpolated function of the design variable. Then an optimal distribution of design variables maximizing (or minimizing) the design objective function and satisfying constraints is found by an optimization algorithm. Because the material properties are varied continuously as the functions of the design variables, a special attention must be paid to choose interpolation functions to obtain two limiting values of the design variable. In the present problem, two limiting values (usually 0 and 1) correspond to the states of air and a rigid body. In topology optimization, the rigid body is modeled as an acoustic medium having a sufficiently large acoustic impedance. As discussed above, a special attention must be paid to the rigid-body modeling in order to avoid erroneous results in acoustical topology optimization.

Recall the governing Helmholtz equation [4] for an acoustic system,

\[
\nabla \cdot \left( \frac{1}{\rho} \nabla p \right) + \frac{(2\pi f)^2}{K} p = 0, \quad K = \rho c^2, \tag{1}
\]

where the acoustic pressure \( p \) is influenced by the density \( \rho \) and the sound speed \( c \), the frequency \( f \), and the boundary and geometry of an acoustic system. The bulk modulus \( K \) of an acoustic medium is defined by \( K = \rho c^2 \). In gradient-based acoustical topology optimization, it is common practice to interpolate \( \rho \) and \( c \) or \( \rho \) and \( K \) as functions of a design variable. A rigid body is simulated as an acoustic medium with finite limit values of acoustical properties. This technique is used because streamlined optimization without re-meshing at every iteration step becomes difficult if a rigid body and air are modeled separately. Several papers on acoustical topology optimization have been reported [4-9], but it appears that no paper has theoretically investigated the effect of selected values of \( \rho \) and \( c \) of a simulated rigid body on the optimized solutions. Naka et al. [10]

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derived the limits and approximations of acoustic eigenvalues of rectangular rooms with arbitrary wall impedances to improve calculation efficiency. Though their work was concerned with good initial guesses for finding all theoretical eigenvalues, the rigid body modeling issue appearing in the present acoustical topology optimization problems should be investigated in a different context. The specific acoustic impedance \( r = \rho c \), the multiplication of density and sound speed, of a rigid body on the order of \( 10^{-2} - 10^4 \) relative to that of air was used to obtain converging results in earlier works. However, improper selections of the acoustical properties of a rigid body can yield spurious results in eigenvalues and eigenmodes.

In order to investigate the effects of the density and sound speed of a simulated rigid body on optimized results, two typical acoustical configurations connected to rigid bodies are considered in Fig. 1. Simplified acoustic models representing the configurations are also suggested. The topological configuration in Fig. 1a represents a cavity divided into two sub-cavities by rigid bodies. The acoustical configuration in Fig. 1b is used to describe the case in which the size of a cavity is reduced by the insertion of a rigid body. For subsequent discussions, the following definition of a rigid body and a rigid wall will be used:

A rigid wall: has semi-infinite or finite length and infinite specific acoustic impedance with total reflection on its boundary (no transmission).

A rigid body: has finite length and finite but very high specific acoustic impedance with approximately total reflection on its boundary (the rigid body will appear as a result of topology optimization).

Let us demonstrate erroneous numerical results due to improper selection of the acoustical properties of a rigid body. A two-dimensional rectangular rigid-walled acoustic cavity of size \( l_1 \times h_1 \) in Fig. 2a is modeled by two acoustic systems with rigid bodies: Model 2-A (Fig. 2b) and Model 2-B (Fig. 2c). The zero particle velocity condition is imposed on the interface between the acoustic medium and a rigid wall. The following values were used to simulate the rigid bodies in Models 2-A and 2-B:

Model 2-A: \( \rho_{\text{rigid}} = 10^5 \rho_{\text{air}}, \quad c_{\text{rigid}} = 10^1 c_{\text{air}} \) and \( r_{\text{rigid}} = 10^5 r_{\text{air}} \).
Model 2-B: \( \rho_{\text{rigid}} = 10^5 \rho_{\text{air}}, \quad c_{\text{rigid}} = 10^1 c_{\text{air}} \) and \( r_{\text{rigid}} = 10^5 r_{\text{air}} \).

\( \rho_{\text{air}} = 1.21 \text{ kg/m}^3, \quad c_{\text{air}} = 343 \text{ m/s}, \quad l_1 = 1.50 \text{ m}, \quad l_2 = 0.75 \text{ m}, \quad h_1 = 0.20 \text{ m}. \)

where the subscripts 'rigid' and 'air' denote rigid body and air, respectively.

Note that although the simulated rigid bodies in Model 2-A and Model 2-B have exactly the same specific acoustic impedance \( r_{\text{rigid}} \), the two models can exhibit considerably different acoustical behavior.

In Fig. 2, the first two eigenfrequencies and eigenmodes of Models 2-A and 2-B are compared with those of the original acoustic cavity, Model 1. Black and white in the topological layouts (on the left-hand side) represent a 'rigid body' and 'air', respectively. On the other hand, the gray scale in the acoustic modes is proportional to the absolute value of the acoustic pressure; as the gray scale approaches black, the acoustic pressure becomes higher. While the eigenfrequencies and eigenmodes of Model 2-A are virtually the same as those of Model 1, Model 2-B yields erroneous results, in particular, the eigenmodes. Although the simulated rigid bodies in Model 2-A and 2-B have the same specific acoustic impedance, very different results were obtained. Therefore, the proper selection of the values of density and sound speed for simulating a rigid body can be an important issue in acoustical topology optimization.

In this work, two simplified acoustic models in Fig. 1 are used to investigate the effects of the acoustical properties of a simulated rigid body on the acoustical characteristics of the total acoustic system. To this end, we will investigate the frequency characteristics of the power transmission coefficient of the simulated rigid body according to the values of the acoustical properties such as density and sound speed. In evaluating the transmission characteristics of the simulated rigid body in the simplified models, the analysis techniques used in [11–17] are employed.

Based on the findings from the theoretical analysis with the simplified models, a guideline for selecting the values of the acoustical properties of a simulated rigid body for acoustical topology
optimization is suggested. Three benchmark-type acoustical topology optimization problems are considered to check the validity of the present suggestions.

2. Transmission characteristics of theoretical models

Two theoretical models in Fig. 3 will be used to investigate the effects of the density, sound speed and characteristic length of a simulated rigid body on the solution accuracy in a numerical calculation for an acoustical analysis. These models may be viewed as the theoretical counterparts of the simplified models in Fig. 1. To check if the acoustical properties of the simulated rigid body represent complete acoustical insulation, the power transmission coefficients of the simulated rigid body in the theoretical models are derived and investigated.

2.1. Three-layer model

The three-layer acoustic system in Fig. 3a is introduced to check if a simulated rigid body can represent complete acoustical insulation in a topological layout configuration depicted in Fig. 1a. The condition of anechoic termination is imposed at the right end. The power transmission coefficient \( T_p \) is defined as:

\[
T_p = \frac{r_1}{r_3} \frac{|\tau|^2}{r_1}, \quad \tau = \frac{p_i}{p_i},
\]

where the transmission coefficient \( \tau \) is defined by the ratio of the transmitted acoustic pressure \( p_t \) and the incident acoustic wave \( p_i \). The symbols \( r_1 \) and \( r_3 \) denote the specific acoustic impedances (i.e., characteristic impedances) of acoustic medium 1 (AM1) and acoustic medium 3 (AM3), respectively.

On the assumption that \( r_1 = r_3 = r_{air} \), \( T_p \) can be represented as a function of the frequency \( f \), the density \( \rho_2 \), the sound speed \( c_2 \) and the characteristic length \( l \) of acoustic medium 2 (AM2):

\[
T_p = \frac{4}{4 \cos^2 k_d l + (r_2/r_1 + r_1/r_2)^2 \cdot \sin^2 k_d l}, \quad r_1 = \rho_1 c_1,
\]

where \( \rho_1 \) and \( c_1 \) are the density and the sound speed of the acoustic medium 1 (AM1), respectively. The value of \( k_d \) is defined as \( k_d = 2\pi f / c_2 \). Depending on the values of \( k_d \) in Eq. (3), the transmission characteristics of AM2 in the frequency domain can be categorized into three cases:

![Fig. 2. Effects of the acoustical properties of a simulated rigid-body. (a) All sides are surrounded by rigid walls, (b) a model interfaced with an acoustical material with \( \rho_{rigid} = \rho_{air} \cdot 10^4 \) and \( c_{rigid} = c_{air} \cdot 10^4 \) simulating a rigid body, and (c) a model interfaced with an acoustical material with \( \rho_{rigid} = \rho_{air} \cdot 10^5 \) and \( c_{rigid} = c_{air} \cdot 10^5 \) simulating a rigid body.](image)

![Fig. 3. (a) Three-layer and (b) two-layer theoretical models used to investigate the effect of acoustical properties of a rigid body on its acoustical behavior.](image)
• Case 1: \( k_2l = nx \), i.e. \( f_{n}^{(l)} = n/(2l) \cdot c_2 \):
  \[ T_{\pi} = 1 \] for any \( r_2 \).

• Case 2: \( k_2l = (n - 1/2)x \), i.e. \( f = (2n - 1)/(4l) \cdot c_2 \):
  \[ T_{\pi} = \left( \frac{2}{r_2/r_1 + r_1/r_2} \right)^2, \]
  - limiting behavior
  \[ T_{\pi} \rightarrow 0 \] for \( r_2/r_1 \rightarrow \infty. \]

• Case 3-1: \( k_2l \neq nx, k_2l \neq (n - 1/2)x \) and \( \rho_2 = \rho_1 \)
  \[ T_{\pi} = \frac{4}{4 \cdot \cos^2(k_2l) + (c_2/c_1 + c_1/c_2) \cdot \sin^2(k_2l)}, \]
  - limiting behavior
  \[ T_{\pi} \equiv \frac{1}{1 + (1/c_1 \cdot \pi \cdot l / f)^2} \approx \frac{1}{f^2} \] for \( c_2 \gg c_1. \]

• Case 3-2: \( k_2l \neq nx, k_2l \neq (n - 1/2)x \) and \( c_2 = c_1 \)
  \[ T_{\pi} = \frac{4}{4 \cdot \cos^2(k_2l) + (\rho_2/\rho_1 + \rho_1/\rho_2) \cdot \sin^2(k_2l)}, \]
  - limiting behavior
  \[ T_{\pi} \equiv \frac{4}{4 \cdot \cos^2(k_2l) + (\rho_2/\rho_1)^2 \cdot \sin^2(k_2l)} \equiv 4 \left( \frac{\rho_1}{\rho_2} \right)^2 \rightarrow 0 \]
  for \( \rho_2 \gg \rho_1. \]

where \( n \) is a non-negative integer.

The frequencies, \( f_{n}^{(l)} = n/(2l) \cdot c_2 \), defined in Case 1 are known as the coincidence frequencies [18]. When \( f = f_{n}^{(l)} \), an incident acoustic wave is totally transmitted to AM\(_2\), and the acoustic pressure near both ends of AM\(_2\) becomes approximately zero regardless of the acoustical properties of AM\(_2\). Also, an incident acoustic wave would partially penetrate AM\(_2\) at a frequency around the coincidence frequency. Therefore, if the frequency of interest is equal or very close to the coincidence frequency, AM\(_2\) cannot simulate a rigid body for any choice of \( \rho_2, c_2 \) and \( l \).

In Case 2, a rigid body can be simulated properly by an acoustical material having a large value of \( r_2 \).

In Case 3, a rigid body can be simulated properly by an acoustical material when \( r_2 = r_1 = \infty \). As Eq. (6b) shows, \( T_{\pi} \) can be close to zero only when the frequency of interest becomes large. In other words, the use of a large sound speed for the simulated rigid body alone cannot represent complete acoustical insulation unless \( f \) is very large. On the contrary, the simulated rigid body becomes a complete acoustical insulation material as \( \rho_2/\rho_1 \rightarrow \infty \) for \( c_2 = c_1 \). However, the simulated rigid body does not become a complete acoustical insulation when \( f = f_{n}^{(l)} \).

**Fig. 4** plots the power transmission coefficient of AM\(_2\) in the three-layer acoustic system for different combinations of \( \rho_2/\rho_1 \) and \( c_2/c_1 \). Unless stated otherwise, the values of \( c_1 = 343 \text{ m/s} \) and \( \rho_1 = 1.21 \text{ kg/m}^3 \) will be used to model air. The characteristic length is \( l = 0.007 \text{ m} \). The power transmission coefficient curve has repetitive coincidence frequencies with the period of \( c_2/(2l) \). Also, note that \( T_{\pi} \) decreases as \( \rho_2 \) or \( c_2 \) increases, as shown in Figs. 4a and b. Fig. 4a shows that increasing \( c_2 \) with \( \rho_2 = \rho_1 \) can move \( f_{\pi} \) to a higher frequency range. It also shows that \( T_{\pi} \) cannot be reduced below a certain value especially in the lower frequency range. On the contrary, increasing \( \rho_2 \) with \( c_2 = c_1 \) can make the magnitude of \( T_{\pi} \) vanishingly small but cannot control or shift \( f_{n}^{(l)} \); see Fig. 4b. Therefore, increasing either \( \rho_2 \) or \( c_2 \) alone cannot convert AM\(_2\) to a rigid body in a wide frequency range. If the frequency of interest is a little less than \( f_{n}^{(l)} \), for instance, the use of a large \( \rho_2 \) for AM\(_2\) alone can simulate a rigid body even with \( c_1 = c_2 \). If the frequency range of interest includes \( f_{n}^{(l)} \) for \( c_1 = c_2 \), both \( c_2 \) and \( \rho_2 \) should be sufficiently large to make \( T_{\pi} \) small. Otherwise, erroneous acoustical behavior of the rigid body will be observed near or at \( f = f_{n}^{(l)} \).

**Fig. 5** shows a typical power transmission coefficient curve of AM\(_2\) in the three-layer acoustic system in the frequency domain. The effective frequency range denoted by \( [f_1, f_2] \) is the frequency range where the power transmission coefficient is equal to or less than the threshold power transmission coefficient \( T_{\pi}^{(\text{threshold})} \).

In general, the transmission loss of a door in an anechoic chamber or a reverberant chamber is 40–60 dB, which is equivalent to \( T_{\pi} = 10^{-4} - 10^{-6} \). Thus, \( T_{\pi}^{(\text{threshold})} = 10^{-4} - 10^{-6} \) will be used. From the findings in Fig. 4, the specific acoustic impedance of the simulated rigid body \((\rho c)_{\text{rigid}} = \rho c_{\text{rigid}} \cdot c_{\text{rigid}} \) should be \( 10^5 \) times larger than \( \rho c \) for \( T_{\pi} \leq T_{\pi}^{(\text{threshold})} \). If \( f_{n}^{(l)} \) is much larger than the maximum frequency of interest, one can choose \( \rho c_{\text{rigid}} \gg \rho c \) and \( c_{\text{rigid}} \gg c_{\text{air}} \). This condition means that only the density needs to be interpolated in acoustical topology optimization problems. When \( \rho c_{\text{rigid}} \gg \rho c \) and \( c_{\text{rigid}} \gg c_{\text{air}} \), there are some situations where \( f_{n}^{(l)} \) lies inside the frequency range of interest. In such situations, both of density and sound speed must be interpolated simultaneously, and a larger value of \( c_{\text{rigid}} \) must be used to push \( f_{n}^{(l)} \) away from the frequency range of interest.

The suggested values, based on the findings from Fig. 4, are \( \rho c_{\text{rigid}} \gg \rho c \) and \( c_{\text{rigid}} \gg c_{\text{air}} \). Note that \( f_{\pi} \) depends both on the sound speed \( c_{\text{rigid}} \) and characteristic length \( l \). Unfortunately, the value of \( l \) cannot be known before the acoustical topology optimization is solved, but \( l \) can be estimated conservatively. This estimation will be discussed later with numerical examples.

In the frequency zone marked by \( \Delta f \) near \( f = 0 \) and \( f = f_{n}^{(l)} \) in Fig. 5, \( T_{\pi} \) is larger than \( T_{\pi}^{(\text{threshold})} \). To reduce \( \Delta f \), one must use an extremely large value of \( \rho c_{\text{rigid}} \). However, the use of such a large \( \rho c_{\text{rigid}} \) value causes a numerical singularity in the system matrix for a given acoustical problem. Therefore, when acoustical topology optimization using finite element analysis involves extremely low frequency, some care must be taken in selecting the value of \( \rho c_{\text{rigid}} \).

### 2.2. Two-layer model

The two-layer acoustic system in Fig. 3b is used to investigate the effect of acoustical properties of a simulated rigid body in problems depicted in Fig. 1b on numerical results. The power transmission coefficient \( T_{\pi} \) of AM\(_2\) in the two-layer system in Fig. 3b can be defined as:

\[ T_{\pi} = \frac{r_1}{f_2}, \quad T = \frac{p_2}{p_1}. \]  

(8)

In Eq. (8), a forward-traveling wave \( p_2 \) is assumed to be totally reflected at a rigid wall \((x = l)\). Using a continuity condition at \( x = 0 \) and a boundary condition at \( x = l \), the power transmission coefficient \( T_{\pi} \) in Eq. (8) becomes:

\[ T_{\pi} = \frac{r_1}{f_2} \cdot \frac{1}{\cos^2(k_2l) + (r_1/r_2)^2 \cdot \sin^2(k_2l)}. \]  

(9)

As was done with the three-layer model, the frequency characteristics of \( T_{\pi} \) are investigated for the following three cases:
Case 1: $k_2 l = n p$, i.e. $f = n/(2l) \cdot c_2$:
$$T_p \approx \frac{r_1}{r_2},$$
(10a)

- limiting behavior
$$T_p \rightarrow 0 \text{ for } r_2/r_1 \rightarrow \infty.$$  
(10b)

Case 2: $k_2 l = (n - 1/2)p$, i.e. $f^{(n)}_c = (2n - 1)/(4l) \cdot c_2$:
$$T_p = \frac{r_2}{r_1},$$
(11a)

- limiting behavior
$$T_p \rightarrow \infty \text{ for } r_2/r_1 \rightarrow \infty.$$  
(11b)

Case 3-1: $k_2 l \neq np$, $k_2 l \neq (n - 1/2)p$ and $\rho_2 = \rho_1$:
$$T_p = \frac{c_1}{c_2} \frac{1}{\cos^2(k_2 l) + (c_1/c_2)^2 \cdot \sin^2(k_2 l)},$$
(12a)

- limiting behavior
$$T_p \approx \frac{c_1}{c_2} \left[ 1 + (c_1/c_2)^2 \cdot (2nf/c_2 \cdot l)^2 \right] \approx \frac{c_1}{c_2} \rightarrow 0 \text{ for } c_2 \gg c_1.$$  
(12b)

Fig. 4. (a) Effect of the sound speed and (b) effect of the density of acoustic medium 2 on the power transmission coefficient of AM$_2$ in the three-layer model.

Fig. 5. Transmission characteristics of AM$_2$ characterized by $f_c$, $\Delta f$ and $T_{\text{threshold}}$ in the three-layer model.
Case 3-2: \( k_2 l \neq n \pi, k_2 l \neq (n - 1/2) \pi \) and \( c_2 = c_1 \):

\[
T_\pi = \frac{\rho_1}{\rho_2} \cdot \frac{1}{\cos^2(k_2 l) + (\rho_1 / \rho_2)^2 \cdot \sin^2(k_2 l)}, \quad (13a)
\]

- limiting behavior

\[
T_\pi \equiv \frac{\rho_1}{\rho_2} \to 0 \quad \text{for} \quad \rho_2 \gg \rho_1 .
\quad (13b)
\]

In Case 1, \( T_\pi \) becomes zero only when the specific acoustic impedance \( r_2 \) of acoustic medium 2 (AM2) is sufficiently large compared with the specific acoustic impedance \( r_1 \) of acoustic medium 1 (AM1).

Case 2 considers the coincidence frequencies, \( f_c^{(n)} = (2n - 1) \cdot \frac{c_2}{(2l)} \). In this case, the acoustic pressure distribution in AM2 is very similar to that of a resonant mode of a long rectangular cavity having pressure release and rigid wall boundary conditions; the acoustic pressure is approximately zero near the left end and becomes the maximum at the interface with an ideal rigid wall at \( x = l \). Therefore, the power transmission coefficient of AM2 becomes very large at \( f = f_c^{(n)} \).

At frequencies other than \( f = f_c^{(n)} \), increasing either \( \rho_2 / \rho_1 \) or \( c_2 / c_1 \) alone converts AM2 to a rigid body, as shown in Case 3. However, the limiting values of \( T_\pi \) are not the same. Therefore, different rationales should be used in selecting the limit values of \( \rho_{\text{rigid}} \) and \( c_{\text{rigid}} \). Fig. 6 is prepared for this purpose.

Fig. 6a demonstrates the effect of \( c_2 \) on \( T_\pi \) for \( \rho_2 = \rho_1 \) while Fig. 6b, the effect of \( \rho_2 \) on \( T_\pi \) for \( c_2 = c_1 \). The coincidence frequencies \( f_c^{(1)} \) corresponding to the frequencies with the peak values of \( T_\pi \) repeat with the period of \( c_2 / (2l) \) starting at \( f_c^{(1)} = c_2 / (4l) \). The value of \( T_\pi \) decreases rapidly as \( c_2 \) increases for fixed \( \rho_2 = \rho_1 \) except at \( f = f_c^{(1)} \). This phenomenon occurs because \( f_c^{(1)} \) is pushed to a higher frequency range as \( c_2 \) becomes larger. In this limit, \( T_\pi \) becomes very small even near \( f = 0 \), unlike in the three-layer model. On the other hand, increasing \( \rho_2 \) with a fixed \( c_2 = c_1 \) reduces \( T_\pi \) to a small value, but the values of \( f_c^{(n)} \) are not changed. Therefore, the simulation of a rigid body with the limit values of acoustical properties is only valid when the frequency of interest is away from the coincidence frequencies.

Fig. 7 shows a typical power transmission coefficient curve of AM2 in a two-layer acoustic system in the frequency domain. As in the three-layer acoustic system, the effective frequency range \([f_e]\) can be enlarged as the density or the sound speed of AM2 increases. The main difference between the three-layer acoustic system and the two-layer acoustic system is that the effective

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**Fig. 6.** (a) Effect of the sound speed and (b) effect of the density of acoustic medium 2 on the power transmission coefficient of AM2 in the two-layer model.
frequency range starts from 0 Hz in the two-layer system. From the findings in Fig. 6, the specific acoustic impedance \( Z_{\text{air}} \) of the simulated rigid body should be at least 10 times larger than that of air \( Z_{\text{air}} \) to satisfy the condition, \( T_{\pi} < T_{\text{threshold}} \). If \( f_c^{(1)} \) for \( c_{\text{rigid}} = c_{\text{air}} \) is much larger than the maximum frequency of interest, one may select \( r_{\text{rigid}} \geq 10^2 \cdot r_{\text{rigid}} \) and \( c_{\text{rigid}} = c_{\text{air}} \). If the frequency range of interest includes \( f_c^{(1)} \), large \( c_{\text{rigid}} \) values must be used. For example, the values of \( r_{\text{rigid}} > 10^3 \cdot r_{\text{air}} \) and \( c_{\text{rigid}} > 10^1 \cdot c_{\text{air}} \) would be appropriate. Since the effective frequency range starts from 0 Hz in the two-layer system, increasing the sound speed alone is sufficient to convert \( \text{AM}_2 \) to a rigid body in a low-frequency range. Therefore, one can choose \( c_{\text{rigid}} \geq 10^3 \cdot c_{\text{air}} \) and \( r_{\text{rigid}} = r_{\text{air}} \) in a low-frequency range.

3. Selection of acoustical properties of a rigid body for topology optimization

On the basis of the frequency characteristics of the power transmission coefficients of the two theoretical models, a guideline for selecting the lower limit values of \( c_{\text{rigid}} \) and \( r_{\text{rigid}} \) of an acoustic medium simulating a rigid body may be suggested. The key idea of this guideline is to widen the effective frequency range so that \( [f] \) could include the frequency range of interest. The following sequential procedure may be employed for acoustical topology optimization.

- Selection of the threshold value \( T_{\text{threshold}} \)
  The threshold value \( T_{\text{threshold}} \) of the power transmission coefficient may be varied depending on the acoustical problems in consideration. In general, the value of \( 10^{-4} \sim 10^{-6} \) is recommended as a reasonable threshold value of the power transmission coefficient.

- Selection of a characteristic length
  The characteristic length \( l \) is required to evaluate \( f_c^{(1)} = c_z/(2l) \) in the three-layer model and \( f_c^{(1)} = c_z/(4l) \) in the two-layer model. Because it is not possible to exactly calculate \( l \) at the initial iteration step of acoustical topology optimization, one can choose as an initial estimate of \( l \) the largest length scale of a design domain in a topology optimization problem multiplied by the mass or volume ratio appearing in a material usage constant. Then, one must ensure that the largest length of a rigid body region in a final optimized topological layout is equal to or less than the initially estimated characteristic length.

- Selection of sound speed
  The sound speed \( c_{\text{rigid}} \) should be sufficiently large so that \( f_c^{(1)} \) becomes much larger than the maximum frequency of interest. If this condition is satisfied even with \( c_{\text{rigid}} = c_{\text{air}} \), \( c_{\text{rigid}} \) may be set to \( c_{\text{air}} \). In most problems, \( c_{\text{rigid}} \geq 10^1 \cdot c_{\text{air}} \) is recommended.

4. Numerical test

In this section, six numerical examples will be considered using three acoustical topology optimization problems illustrated in Fig. 8. Design Problem A in Fig. 8a is concerned with the optimal multiple-hole distribution in the partition of a double cavity. The motivation and formulation of the design problem were given in [19,20]. However, the topology optimization problem is revisited to check the rigid body modeling issue related to the three-layer acoustic system. Design Problem B in Fig. 8b is an idealized cavity design problem to check the rigid body modeling issue related to the two-layer system. A muffler design problem [21] by topology optimization in Fig. 8c is revisited as Design Problem C, whose objective function is a function of acoustic power transmission coefficient. Two acoustical properties \( \rho \) and \( K \) are interpolated,
instead of $\rho$ and $c$, because these acoustical properties are better for interpolation. This interpolation strategy has been widely used in acoustical topology optimization problems [4,6,19–21].

The specific issues related to the rigid body modeling in each acoustical design problem are as follows.

- **Design Problem A** (partition hole design of a double cavity)
  - Investigation A-1: low limit value of the specific acoustic impedance of a simulated rigid body.
  - Investigation A-2: low limit values of the density and sound speed of a simulated rigid body.
- **Design Problem B** (rectangular cavity design with harmonics)
  - Investigation B-1: low limit value of the specific acoustic impedance of a simulated rigid body.
  - Investigation B-2: low limit values of the density and sound speed of a simulated rigid body.
- **Design Problem C** (muffler design)
  - Investigation C-1: low limit value of the specific acoustic impedance of a simulated rigid body.
  - Investigation C-2: low limit values of the density and sound speed of a simulated rigid body.

To solve Design Problems A, B and C, a two-dimensional acoustical finite element analysis was carried out. The design variables were updated during topology optimization by the method of moving asymptotes [22]. Because the topology optimization procedure for these acoustical problems can be found elsewhere [4–7,19–21], the details of the procedure will not be given here. All calculation results were obtained without any numerical treatment such as filtering.

Additional acoustical analysis was also performed for post-processed exact layouts; in the layouts, the simulated rigid bodies were replaced by rigid walls so that no error is associated with the adjusted $\rho$ and $c$ values of the rigid body. To compare the accuracy of the eigenmodes of the layouts before and after the post-processing in Design Problems A and B, the notion of **NMD** (Normalized Modal Difference), a single measure representing the difference between two eigenmodes ($\Phi_1$, $\Phi_2$) [23]. It is expressed by

$$\text{NMD} = \sqrt{1 - \frac{\text{MAC}}{\text{MAC}}}$$

where MAC (Modal Assurance Criterion) represents the relationship between two eigenmodes ($\Phi_1$, $\Phi_2$) [23]. It is expressed by

$$\text{MAC} = \frac{\Phi_1^T \Phi_2}{\sqrt{\Phi_1^T \Phi_1 \Phi_2^T \Phi_2}}$$

Using the single parameters, NMD and MAC, one can qualitatively compare the difference and resemblance of two eigenmodes.

In Design Problem C, the power transmission coefficients of an optimal muffler are compared for the layouts before and after the post-processing.

### 4.1. Design Problem A

The design objective is to minimize the first eigenfrequency of a two-dimensional double cavity shown in Fig. 8a for a given volume of a rigid body:

$$\min_{\rho_e, c_e, X_e} f_1,$$

subject to a volume constraint

$$\sum_{c=1}^N X_c/N \leq V_r,$$

where $N$ denotes the number of total sub-partitions and $V_r$, the permitted volume ratio of the total rigid body region in the partition ($N = 40$ and $V_r = 0.9$ were used).

The double cavity consists of two two-dimensional rectangular cavities and a holey partition between them. The acoustical characteristics of the double cavity are strongly affected by the distribution of the holes in the partition [19,20]. The partition is divided into sub-partitions that can be filled with air or rigid body elements. One design variable $\chi_e$ ($0 \leq \chi_e \leq 1$) is assigned to each sub-partition. The values of $\chi_e = 1$ and $\chi_e = 0$ correspond to a rigid body and air, respectively.

To obtain a distinct air-rigid body state in the final optimized topology, two acoustical properties (density $\rho_e$ and bulk modulus $K_e$) are interpolated [19–21]:

$$1/\rho_e(\chi_e) = 1/\rho_{air} + \chi_e(1/\rho_{rapid} - 1/\rho_{air}).$$

$$1/K_e(\chi_e) = 1/K_{air} + \chi_e(1/K_{rapid} - 1/K_{air}).$$

Although the effects of the density $\rho$ and sound speed $c$ on the solution behavior can be investigated more easily, the density $\rho$ and bulk modulus $K$ are interpolated for acoustic topology optimization. (In this case, the sound speed is calculated by $c = \rho^2$.) The main reason to use the $\rho - K$ combination, instead of the $\rho - c$ combination, for interpolation is that the former is shown to yield more stable solution convergence in acoustic topology optimization problems (see, e.g., [4,6,19–21]). It is also noted that the low-density related problems such as artificial local modes [24] typically occurring in structural eigenvalue topology problems do not occur here because two real media (air and a rigid body), not the void-real medium combination in structures, are involved in the interpolation.

The eigenvalue switching problem [24] was not observed in the present problem because the analysis model is a long cavity
and the difference between the first and second eigenvalues is considerably large.

A finite element model is constructed to calculate the fundamental eigenfrequency of the double cavity for the specified dimension:

\[ l_1 = 2.0 \text{ m}, \quad l_p = 0.02 \text{ m}, \quad l_2 = 0.5 \text{ m}, \quad \text{and} \quad h = 0.2 \text{ m}. \]

The cavity model consists of 5040 two-dimensional acoustical four-node elements and 5207 nodes, which are distributed uniformly at a spacing of 0.02 m along the x-direction and at a spacing of 0.01 m along the z-direction.

4.1.1. Investigation A-1

The appropriate specific acoustic impedance \( r_{\text{rigid}} \) of a simulated rigid body can be checked by examining the first eigenfrequencies, effective frequency ranges and NMD values. Because the fundamental coincidence frequency for the given characteristic length \( l = l_p = 0.02 \) and \( c_2 = c_{\text{air}} \) is much larger than the frequency range of interest, only \( r_{\text{rigid}} \) is needed to be taken a large value to simulate a rigid body \( (c_{\text{rigid}} = c_{\text{air}} \text{ was used}) \).

Five different values of \( r_{\text{rigid}} \) are tested. For all values of \( r_{\text{rigid}} \), the same topological layout shown in Fig. 9 is obtained. The post-processed exact eigenfrequency is 56.61 Hz. For each \( r_{\text{rigid}} \), the effective frequency range \( f_{[j]} \), the first eigenfrequency \( f_1 \) of the optimized acoustic system, NMD at \( f = f_1 \) are summarized in Table 1. As the density of a rigid body decreases, \( f_1 \) for \( r_{\text{rigid}} \) becomes narrower and the error in the first eigenfrequency increases. When \( r_{\text{rigid}} \) becomes smaller than \( 10^4 \), \( r_{\text{air}} \) starts to lie outside of \( f_{[j]} \) and thus, the errors in the first eigenfrequency and the NMD increase. At \( r_{\text{rigid}}/r_{\text{air}} = 10^2 \), considerable errors in the first eigenfrequency range does not include the frequency range of interest. From the numerical results in Table 1, one can conclude that an acoustic medium having \( r_{\text{rigid}} \geq 10^4 r_{\text{air}} \) can indeed simulate a complete acoustical insulation material when the medium is surrounded by air.

4.1.2. Investigation A-2

Table 2 compares the effective frequency ranges, calculated eigenfrequencies and NMD for four different combinations of density and sound speed of a simulated rigid body. The specific acoustic impedance is fixed at \( r_{\text{rigid}}/r_{\text{air}} = 10^5 \). As the sound speed increases, the effective frequency range broadens but the lower bound of the effective frequency also increases. Therefore, the first eigenfrequency is usually unlikely to lie within \( f_{[j]} \) for high sound speed and low density for a fixed specific acoustic impedance value, and significant errors occur in the eigenmodes and/or eigenfrequencies of the final optimized topological layout.

Fig. 10 compares two fundamental acoustic modes of the optimized results with different combinations of \( r_{\text{rigid}} \) and \( c_{\text{rigid}} \): \( (r_{\text{rigid}}/r_{\text{air}} = 10^5 \) and \( c_{\text{rigid}}/c_{\text{air}} = 10^4 \) and \( (r_{\text{rigid}}/r_{\text{air}} = 10^2 \) and \( c_{\text{rigid}}/c_{\text{air}} = 10^4 \)). Because \( f_1 \) lies outside of \( f_{[j]} \) for \( r_{\text{rigid}}/r_{\text{air}} = 10^4 \) and \( c_{\text{rigid}}/c_{\text{air}} = 10^4 \), serious errors occur in acoustic mode prediction; the nodal line location is not correct and the NMD value in Table 2 is very large. This example demonstrates the importance of using a proper combination of \( r_{\text{rigid}} \) and \( c_{\text{rigid}} \) as well as using a sufficiently large specific acoustic impedance of a rigid body to avoid erroneous results in acoustical topological optimization problems.

4.2. Design Problem B

The cavity design domain in Fig. 8b interfaces with an air-filled non-design domain. The non-design domain is introduced to obtain a topological layout as illustrated in Fig. 1b. The design domain is divided by \( N \) thin rectangular layers. Thus no two-dimensional geometrical variation is allowed in this problem, although two-dimensional rectangular acoustic elements are used for acoustical analysis. Using the design model in Fig. 8b, the following problem is solved:

\[
\begin{align*}
\text{max} & \quad f_1, \quad \text{equivalently, max } \log f_1, \\
\text{subject to a volume constraint } & \\
\sum_{e=1}^{N} x_e/N &= V_r, \\
\text{where } V_r &\text{ is the permitted volume ratio of a rigid body in the design domain. The density } \rho_s \text{ and the bulk modulus } K_e \text{ of each thin rectangular layer are the interpolated functions of the } e\text{-th design variable } x_e \text{ given by Eq. (18).}
\end{align*}
\]

An exact solution to Design Problem B is illustrated in Fig. 11: \( f_1 = 171.54 \) Hz and \( f_2 = 345.35 \) Hz. The exact eigenfrequencies are independent of \( V_r \) because the length of an acoustic medium filled with air is constant \( (1.0 \text{ m}) \) in all final topologies although the number of thin rectangular layers filled with a rigid body changes with \( V_r \). Considering the topological layout in Fig. 11a, one can see that this problem is suitable for the investigation of the effects of \( r_{\text{rigid}} \) and \( c_{\text{rigid}} \) values on the solution accuracy related to the simplified acoustical model in Fig. 1b or the two-layer theoretical model in Fig. 3b. Note that the eigenfrequencies of the acoustic modes in Figs. 11b and 11c form harmonics: \( f_2/f_1 \approx 2 \). Thus, one can check the solution accuracy of the optimized results obtained with various values of \( r_{\text{rigid}} \) and \( c_{\text{rigid}} \) by comparing the first two eigenfrequencies of final optimized layouts and investigating their acoustic pressure distributions.

4.2.1. Investigation B-1

To estimate the low limit value of the specific acoustic impedance of a simulated rigid body, Design Problem B is solved with the following numerical data:

\[ l_1 = 0.50 \text{ m}, \quad l_2 = 0.70 \text{ m}, \quad h = 0.10 \text{ m}, \quad V_r = 2/7, \quad N = 28. \]

The finite element model used for the eigenvalue analysis consists of 480 four-node rectangular elements, each of which has the dimensions of \( 0.025 \text{ m} \times 0.020 \text{ m} \).

Table 3 lists \( f_1 \), \( f_2 \), \( NMD_{E1} \), \( NMD_{E2} \) and \( NMD_{E3} \) of the optimal results obtained with various values of \( r_{\text{rigid}} \). Because the estimated coincidence frequency \( f_{(1)} = c/(4l) = 428.75 \) Hz is much higher than the frequency range of interest, one can simply choose \( c_{\text{rigid}} = c_{\text{air}} \) in varying \( r_{\text{rigid}} = c_{\text{rigid}}/c_{\text{air}} \). Note that the characteristic length is \( l = 0.20 \text{ m}, \) determined by the multiplication of \( V_r \) and \( l_2 \).

### Table 1

<table>
<thead>
<tr>
<th>( r_{\text{rigid}}/r_{\text{air}} )</th>
<th>( c_{\text{rigid}}/c_{\text{air}} )</th>
<th>( f_1 ) (Hz)</th>
<th>( f_1 ) (Hz)</th>
<th>NMD_{E1} (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 10^0 )</td>
<td>( 10^0 )</td>
<td>( 10^0 )</td>
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<tr>
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</tr>
<tr>
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<td>( 10^0 )</td>
<td>( 10^0 )</td>
<td>55–8520</td>
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</tr>
<tr>
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<td>( 10^0 )</td>
<td>( 10^0 )</td>
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</tr>
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<td>( 10^2 )</td>
<td>( 10^2 )</td>
<td>Not exist</td>
<td>61.27</td>
</tr>
</tbody>
</table>

### Table 2

The effects of the combination of \( r_{\text{rigid}} \) and \( c_{\text{rigid}} \) on the optimized topology in terms of \( f_1 \), \( f_2 \), NMD_{E1} for Investigation A-2.

<table>
<thead>
<tr>
<th>( r_{\text{rigid}}/r_{\text{air}} )</th>
<th>( c_{\text{rigid}}/c_{\text{air}} )</th>
<th>( f_1 ) (Hz)</th>
<th>( f_2 ) (Hz)</th>
<th>NMD_{E1} (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 10^0 )</td>
<td>( 10^0 )</td>
<td>( 10^0 )</td>
<td>1–8574</td>
<td>56.61</td>
</tr>
<tr>
<td>( 10^2 )</td>
<td>( 10^2 )</td>
<td>( 10^2 )</td>
<td>55–8520</td>
<td>56.67</td>
</tr>
<tr>
<td>( 10^3 )</td>
<td>( 10^3 )</td>
<td>( 10^3 )</td>
<td>550–8025</td>
<td>57.29</td>
</tr>
</tbody>
</table>
shows the convergence history for $\frac{\rho_{\text{rigid}}}{\rho_{\text{air}}} = 10^4$ and $\frac{c_{\text{rigid}}}{c_{\text{air}}} = 10^0$ during the optimization process. (The convergence histories of the first eigenfrequency for other values of $r_{\text{rigid}}/r_{\text{air}}$ in Table 3 are almost identical to that shown in Fig. 12.)

For $r_{\text{rigid}}/r_{\text{air}} = 10^4$, $f_i$ is almost exact, $\text{NMD}_{\text{f}}(i = 1, 2)$ becomes almost 0%, and $f_1$ and $f_2$ form harmonics as the eigenfrequencies of the exact solution do. However, the errors in $f_i$ and $\text{NMD}_{\text{f}}(i = 1, 2)$ become large when $r_{\text{rigid}}/r_{\text{air}} \leq 10^2$. Based on this
observation, the use of $r_{\text{rigid}}/r_{\text{air}} \geq 10^3$ is suggested for the two-layer system considered here.

### 4.2.2. Investigation B-2

Design Problem B in Fig. 8b is re-visited to investigate the effect of the $r_{\text{rigid}} - c_{\text{rigid}}$ combination on the solution accuracy for the following numerical data:

$l_1 = 0.50 \text{ m}$, $l_2 = 1.00 \text{ m}$, $h = 0.10 \text{ m}$, $V_r = 5/10$, $N = 40$.

Table 4 compares the first two fundamental frequencies, $f_{1(i)}^{(1)}$ and NMD values of the optimal results obtained with three different combinations of $r_{\text{rigid}}/r_{\text{air}}$ and $c_{\text{rigid}}/c_{\text{air}}$ while $r_{\text{rigid}}/r_{\text{air}}$ is fixed at $10^4$. The second eigenfrequency $f_2$ for $r_{\text{rigid}}/r_{\text{air}} = 10^4$ and $c_{\text{rigid}}/c_{\text{air}} = 10^6$ is far from the exact eigenfrequency, 343.35 Hz. On the contrary, all eigenfrequencies obtained with other values of $r_{\text{rigid}}/r_{\text{air}}$ and $c_{\text{rigid}}/c_{\text{air}}$ are almost identical to the exact eigenfrequencies. The main reason for the big deviation in $f_2$ for $c_{\text{rigid}}/c_{\text{air}} = 10^6$ is that the coincidence frequency $f_c$ controlling $[f_3]$ is very close to the first eigenfrequency (see Fig. 7).

The optimized topological layouts and the corresponding acoustic modes for $(r_{\text{rigid}}/r_{\text{air}} = 10^4$ and $c_{\text{rigid}}/c_{\text{air}} = 10^6$) and $(r_{\text{rigid}}/r_{\text{air}} = 10^3, c_{\text{rigid}}/c_{\text{air}} = 10^6)$ are shown in Figs. 13 and 14, respectively. Although the two topological layouts are almost the same, the acoustic pressure distributions, especially in the second acoustic mode, are considerably different from each other: the acoustic mode in Fig. 14c is the same as that in Fig. 11c, but that in Fig. 13c is very different. The erroneous result for the second mode in Fig. 13c is due to the improper selection of $c_{\text{rigid}}$. Based on the present investigation, the value of $c_{\text{rigid}}$ cannot be chosen arbitrarily even if $r_{\text{rigid}}$ is sufficiently large. The suggested values are $r_{\text{rigid}}/r_{\text{air}} \geq 10^3$ and $c_{\text{rigid}}/c_{\text{air}} \geq 10^6$ in this case.

### 4.3. Design Problem C

The design goal is to maximize the transmission loss of a 2-dimensional concentric expansion chamber muffler show in Fig. 8c around a target frequency $f_t$ for a given volume of a rigid body:

$$\max_{0 \leq c \leq 1} TL(f_t),$$

subject to a volume constraint

$$\sum_{c=1}^N X_c/N \leq V_t,$$

where $N$ and $V_t$ denotes the number of total design variables and the permitted volume ratio of the total rigid body region in the design domain in Fig. 8c, respectively. The non-design domain is assigned for fluid passage from the inlet to the outlet. The transmission loss (TL) is defined by

$$TL = -\log_{10}|T_m|.$$
where $T_m$ is the acoustic power transmission coefficient, which is the ratio of acoustic powers at an outlet and an inlet of the muffler, and can be expressed by acoustic pressures $p_1$ and $p_2$ at two points of an inlet and $p_3$ acoustic pressure at one point of outlet [25]:

$$
T_m = \frac{p_3}{p_1 - p_2} \left(1 - e^{-j\kappa x_{12}}\right)^2
$$

(22)

where $x_{12}$ is the distance between two measurement points of the inlet. The density $\rho$ and the bulk modulus $K$ of each finite element in the design domain are interpolated by the interpolation functions of the $e$-th design variable $\gamma_e$ given by Eq. (18) as used in [21].

Design Problem C is solved for the following values:

- $l_m = 0.40 \text{ m}$, $d = 0.15 \text{ m}$, $d_i = d_o = 0.03 \text{ m}$, $N = 480$, $V_t = 0.1$

The expansion chamber of a concentric muffler in Fig. 8c is discretized by 600 2-dimensional four-node elements of which nodes are distributed at uniform spacing of 0.02 m along the $x$-direction and 0.005 m along the $y$-direction. A velocity of unit magnitude and the characteristic impedance of air are imposed at the face of the inlet and that of the outlet, respectively, and the other boundaries are surrounded by rigid walls. The target frequency is 433.7 Hz in this design problem. Acoustic power transmission coefficient $T_m$ of a reference muffler, which is filled with air only, is almost one (1) at the target frequency. Regardless of the specific values of the density and sound speed of a simulated rigid body, the same final topological layouts are obtained as shown in Fig. 15a. Fig. 15b shows an optimal muffler where simulated rigid bodies are replaced by a rigid wall.

Fig. 14. Optimal results for $\rho_{\text{rigid}}/\rho_{\text{air}} = 10^3$ and $c_{\text{rigid}}/c_{\text{air}} = 10^1$. (a) Topological layout, (b) the 1st acoustic mode (171.54 Hz), and (c) the 2nd acoustic mode (343.35 Hz).

Fig. 15. The optimized topological layout for Design Problem C: (a) an optimal muffler with simulated rigid bodies; and (b) an optimal muffler where simulated rigid bodies are replaced by rigid wall.
4.3.1. Investigation C-1

To check the appropriate specific acoustic impedance of a simulated rigid body in Design Problem C, the acoustic power transmission coefficients, $T_n$, of an optimal muffler in Fig. 15a are compared for different values of $r_{rigid}$. Since the target frequency (433.7 Hz) is much less than the fundamental coincidence frequency for the given characteristic length ($l = 0.04$) and $c_2 = c_{air}$, only $r_{rigid}$ is varied to simulate a rigid body (with $c_{rigid} = c_{air}$).

For five different values of $r_{rigid}$, Table 5 lists the effective frequency range, the acoustic power transmission coefficients of the optimal muffler in Fig. 15a and the corresponding errors. The error is defined as $|T_n(f) - T_n(f_i)|/T_n(f_i) 	imes 100$, where $T_n(f_i)$ and $T_n(f)$ denote the acoustic power transmission coefficients of the optimal muffler (Fig. 15a) for each value of $r_{rigid}$ and the optimal muffler (Fig. 15b) where the simulated rigid bodies are replaced by a rigid wall, respectively. The value of $T_n(f)$ is calculated to be 0.0369. When $r_{rigid}$ becomes smaller than $10^4$, $r_{air}$, the target frequency starts to lie outside of $[f]$ and the errors increases. At $r_{rigid} = 10^2$, one can see considerable errors at other frequencies as well as at the target frequency; see Fig. 16. From Table 5 and Fig. 16, one can also conclude that an acoustic medium having $r_{rigid} > 10^5$, $r_{air}$ can completely simulate a rigid wall in this muffler design problem.

4.3.2. Investigation C-2

Table 6 compares the effective frequency ranges, acoustic power transmission coefficients and errors for five different combinations of density and sound speed of a simulated rigid body with $r_{rigid}$ fixed at $r_{rigid}/r_{air} = 10^6$. As shown in Investigation A-2, the effective frequency range became wider with $c_{rigid}$. However, the lower bound of the effective frequency also increases with $c_{rigid}$. For high sound speed and low density of a simulated rigid body

Table 5

<table>
<thead>
<tr>
<th>$r_{rigid}/r_{air}$</th>
<th>$\rho_{rigid}/\rho_{air}$</th>
<th>$c_{rigid}/c_{air}$</th>
<th>$[f]$ (Hz)</th>
<th>$T_n(f)$</th>
<th>Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
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<td>$10^6$</td>
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<td>16.8</td>
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</table>

Error (%) = $|T_n(f) - T_n(f_i)|/T_n(f_i) 	imes 100$, $T_n(f)$ and $T_n(f_i)$ are the acoustic power transmission coefficients of optimal mufflers with simulated rigid bodies and with rigid wall for the same topological layout at a target frequency $f$, respectively.

Table 6

<table>
<thead>
<tr>
<th>$r_{rigid}/r_{air}$</th>
<th>$\rho_{rigid}/\rho_{air}$</th>
<th>$c_{rigid}/c_{air}$</th>
<th>$[f]$ (Hz)</th>
<th>$T_n(f)$</th>
<th>Error (%)</th>
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<td>0.1231</td>
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</tbody>
</table>

Error (%) = $|T_n(f) - T_n(f_i)|/T_n(f_i) 	imes 100$, $T_n(f)$ and $T_n(f_i)$ are the acoustic power transmission coefficients of optimal mufflers with simulated rigid bodies and with rigid walls for the same topological layout at a target frequency $f$, respectively.

Fig. 17. Comparison of acoustic power transmission coefficients, $T_n$, of the optimized mufflers with different combinations of $\rho_{rigid}$ and $c_{rigid}$: $(\rho_{rigid}/\rho_{air} = 10^5$ and $c_{rigid}/c_{air} = 10^1)$, $(\rho_{rigid}/\rho_{air} = 10^4$ and $c_{rigid}/c_{air} = 10^4)$, $(\rho_{rigid}/\rho_{air} = 10^3$ and $c_{rigid}/c_{air} = 10^4)$, $(\rho_{rigid}/\rho_{air} = 10^2$ and $c_{rigid}/c_{air} = 10^4)$, $(\rho_{rigid}/\rho_{air} = 10^1$ and $c_{rigid}/c_{air} = 10^5)$ for a fixed value of $r_{rigid}$, the target frequency lies outside of $[f]$ and significant errors occur in acoustic power transmission coefficient of an optimal muffler in Fig. 15a.

Fig. 17 compares the acoustic power transmission coefficients of the optimized mufflers with different combinations of $r_{rigid}$ and $c_{rigid}$. The acoustic power transmission coefficient curve of the optimal muffler with rigid wall (Fig. 15b) is also plotted to evaluate the appropriateness of acoustic properties of simulated rigid bodies. One can see serious errors in the acoustic power transmission coefficient curve of the optimized muffler for $r_{rigid}/\rho_{air} = 10^1$ and $c_{rigid}/c_{air} = 10^2$ though the value of $r_{rigid}$ is very high: $r_{rigid} = 10^6$, $T_{air}$. This result explains again that using a proper combination of $\rho_{rigid}$ and $c_{rigid}$ as well as a sufficiently high value of $r_{rigid}$ is very important in order to avoid unexpected simulation errors in acoustical topology optimization problems.

5. Conclusions

This investigation dealt with the issue of selecting the lower limit values of the acoustical properties of a rigid body simulated as a complete acoustical insulation in gradient-based acoustical topology optimization. By using two theoretical models, a three-layer model and a two-layer model consisting of air and an artificial acoustical material simulating a rigid body, the effects of the acoustical properties (density and sound speed) on the transmission characteristics of the simulated rigid body were studied. The study suggested that the combination of $\rho_{rigid} \geq 10^7\cdot \rho_{air}$ and $c_{rigid} \geq 10^1\cdot c_{air}$ was appropriate for most problems. However, one must ensure that the frequency of interest be far away from the coincidence frequencies, at which incident acoustic waves can be transmitted to the other side regardless of the acoustical insulation material. Although $f_{c}^{(n)} = O(n \cdot c_{air}/l)$ is a good estimate
of the coincidence frequency, the characteristic length $l$ of a rigid body is not known in advance before acoustical topology optimization is completed. At the end of the optimization, therefore, one should check either if the frequency of interest is away from a set of $f_k$, or if the eigenfrequencies and eigenmodes of the optimized topological layout are the same as those of the post-processed topological layout having only air and rigid bodies. In the latter cavity model, the rigid bodies should be modeled by rigid walls, not by artificial acoustical materials having limit values.

Acknowledgements

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References