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Quality-of-Service Routing in Heterogeneous Networks with Optimal Buffer and Bandwidth Allocation

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***Abstract*—We present an interdomain routing protocol for heterogeneous networks employing different queuing service disciplines. Our routing protocol finds optimal interdomain paths with maximum reliability while satisfying the end-to-end jitter and bandwidth constraints in networks employing heterogeneous queuing service disciplines. The quality-of-service (QoS) metrics are represented as functions of link bandwidth, node buffers and the queuing service disciplines employed in the routers along the path. Our scheme allows smart tuning of buffer-space and bandwidth during the routing process to adjust the QoS of the interdomain path. We formulate and solve the bandwidth and buffer allocation problem for a path over heterogeneous networks consisting of different queuing services disciplines such as generalized processor sharing (GPS), packet by packet generalized processor sharing (PGPS) and self-clocked fair queuing (SCFQ).**

***Keywords*—Quality-of-service (QoS), resource allocation, inter-domain routing, queuing service disciplines.**

I. Introduction

With the evolution of the Internet into a global communications medium consisting of heterogeneous networks, the need for providing quality-of-service (QoS) guaranteed multimedia data over heterogeneous networks has increased tremendously. To provide end-to-end QoS guarantees, the interdomain path routing protocol must take into account the resources such as bandwidth and buffer, and heterogeneity of the networks comprising the Internet. In this paper we consider the heterogeneity arising from the different queuing service disciplines used in the routers in a domain. Little work has been done to incorporate the information about resources and queuing service disciplines into QoS routing. Present interdomain routing protocols such as border gateway protocol (BGP) do not provide such mechanism [1]. We present an interdomain routing protocol which represents QoS metrics as functions of buffers, bandwidth and queuing service disciplines rather than static metrics. Our protocol uses this functional representation of QoS metrics to find optimal interdomain paths with maximum reliability while satisfying end-to-end jitter and bandwidth constraints. Our routing protocol uses the knowledge of different queuing service disciplines in multiple domains to find the optimal interdomain path. Our interdomain routing protocol has the following advantages over existing interdomain routing protocols:

First, our routing protocol is aware of the relationship among QoS metrics and resources belonging to heterogeneous networks and finds an optimal interdomain path by keeping this relationship into consideration.

Second, it models the QoS metrics as functions of resources such as buffer and bandwidth.

Third, it captures the interdependency between various QoS metrics such as reliability and jitter delay.

This paper is organized as follows. In the next section we provide an overview of the related work in the area of QoS routing. In Section III, we formulate and provide an optimal solution to the *interdomain path resource allocation problem* and present an algorithm for *interdomain QoS routing with resource allocation problem*. This algorithm uses the solution to the *interdomain path resource allocation problem* as a subroutine. Finally, in Section IV we conclude the paper.

II. Related Work

The research in the area of QoS routing has mainly focused on two approaches: QoS routing without resource allocation [2], [3] and QoS routing with resource allocation [4], [5], [6], [7], [8].

In QoS routing without resource allocation the QoS metrics are modeled as non-negative integers. In determining these metrics the interaction among resources is ignored. In addition the metrics along a path are simply added [2], [3]. The QoS routing problem formulated in this manner is a constrained shortest path problem.

In QoS routing with resource allocation there are two approaches. In [4], [5], [6] resources are defined as numerical values associated with each edge of the graph. The QoS metrics associated with each edge are computed by using the available resources, traffic description and the queuing service discipline. In this approach the QoS metrics and resources are static and can not be fine-tuned during the routing process. In [7], [8] the QoS metrics are represented as functions of the bandwidth, buffer and the queuing service discipline. This definition incorporates the relationship between QoS metrics and resources. However the authors do not consider interdomain routing problem over heterogeneous networks which captures the relationship between different queuing service disciplines. In this paper we formulate and solve this problem.

III. Problem Formulation and Solution

The network consisting of multiple domains (autonomous systems) is modeled by a directed graph $G=(V,E)$, where V is the set of nodes and E is the set of edges in the graph. In each AS (autonomous system) a particular queuing service discipline is implemented in the output queue of every router belonging to that AS. Multiple $AS's$ may have different queuing service disciplines implemented in them. As a special case of the above network, we consider a graph G consisting of three $AS's$ as shown in Fig. 1. The queuing service disciplines that are used in the three $AS's$ are generalized processor sharing (GPS) [9], packet by packet generalized processor sharing (PGPS) [10] and self-clocked fair queuing (SCFQ) [11]. We formulate and solve the *interdomain QoS routing with resource allocation* problem for the special case of the three $AS's$ shown in Fig. 1. This result can be generalized to more than three $AS's$ employing various queuing service disciplines.

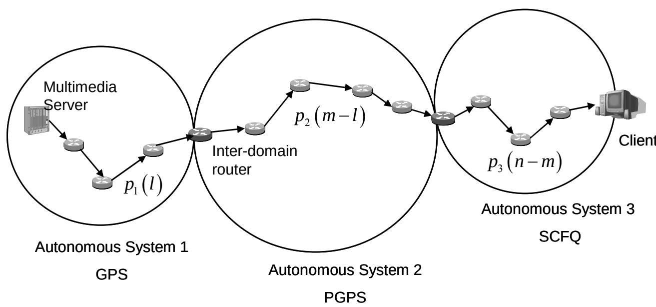

Fig. 1. An interdomain path over heterogeneous networks employing GPS, PGPS and SCFQ queuing service disciplines.

Let $p(n)=\langle v_0,v_1,v_{2,}...,v_{l-1},v_l,v_{l+1},v_{l+2},...,v_{m-1},v_m,v_{m+1},v_{m+2},...,v_n\rangle$ be a path in the graph G in Fig. 1, where $v_i\in G$, l,m,n are positive integers such that $l>0$ and $l<m<n$. The path $p(n)$ can be written as $p(n)=p_1(l)\oplus p_2(m-l)\oplus p_3(n-m)$ where $p_1(l)=\langle v_0,v_1,v_2,...,v_{l-1},v_l\rangle$, $p_2(m-l)=\langle v_l,v_{l+1},v_{l+2},...,v_{m-1},v_m\rangle$ and $p_3(n-m)=\langle v_m,v_{m+1},v_{m+2},...,v_n\rangle$ and the operator $\oplus$ acting on $p_1(n)$ and $p_2(m)$ as $p_1(n)\oplus p_2(m)$ concatenates $p_1(n)$ and $p_2(m)$ such that the last node of $p_1(n)$ and the first node of $p_2(m)$ are common. In our analysis we consider that $p_1(l)$ belongs to AS_1, $p_2(m-l)$ belongs to AS_2 and $p_3(n-m)$ belongs to AS_3 as shown in Fig. 1. We consider that the queuing service disciplines implemented in AS_1, AS_2 and AS_3 are GPS, PGPS and SCFQ respectively. Notice that the node v_l is common to AS_1 and AS_2 and node v_m is common to AS_2 and AS_3. The nodes v_l and v_m represent the interdomain routers. Without loss of generality we assume that PGPS is implemented in the output queue of v_l and SCFQ is implemented in the output queue of v_m. Let the data traffic arrival function at the output queue of node v be $A_{out}^{(i)}(t,v)$. We use *Leaky Bucket* constrained sources to model the data sources. $A_{out}^{(i)}(t,v)$ is said to conform to (σ_k,ρ_k) if for any interval $(\tau,t]$, $A_{out}^{i}(t,v)-A_{out}^{i}(\tau,u)\le\sigma_k+\rho_k(t-\tau)$. In our analysis, we assume that the traffic entering AS_1 conforms to (σ_1,ρ_1), the traffic entering AS_2 conforms to (σ_2,ρ_2) and the traffic entering AS_3 conforms to (σ_3,ρ_3). The traffic shaping mechanism is implemented in the interdomain routers, in our case in the nodes v_0, v_l and v_m.

We define the *interdomain path resource allocation problem* for the path $p(n)$ as follows:

$$Q_p^{(i)*}\left(p(n)\right) = \max_{\{b(u,v),r(u,v);\ \forall(u,v)\in p(n)\}} \left\{Q_p^{(i)}\left(p(n)\right)\right\} \quad (P1)$$

subject to:

$$\begin{aligned}
&J_p^{(i)}\left(p(n)\right) \le J_{req} && (C1)\\
&0 \le b^{(i)}(u,v) \le B(u,v) \quad ,\forall (u,v)\in p(n) && (C2)\\
&R_{req} \le r^{(i)}(u,v) \le C(u,v)^{\max} \quad ,\forall (u,v)\in p(n) && (C3)
\end{aligned} \qquad (1)$$

The symbols used in (1) are defined in TABLE I. $Q_p^*\left(p(n)\right)$ is the maximum reliability of the path $p(n)$. The decision variables in the optimization problem (1) are the link buffer size $b^{(i)}(u,v)$ and the link bandwidth $r^{(i)}(u,v)$ for all links (u,v) belonging to the path $p(n)$. $(C1)$ imposes the constraint that the jitter for path $p(n)$ should be less than or equal to the maximum allowable jitter value, J_{req}. Constraint $(C2)$ imposes the upper bound on the maximum physical buffer size available. $(C3)$ imposes the constraint that the link bandwidth allocated, $r^{(i)}(u,v)$, should be greater than or equal to the minimum bandwidth requested, R_{req}, and can not be greater than the maximum possible link bandwidth, $C(u,v)^{\max}$. Note that the inequality $R_{req} \le r^{(i)}(u,v)$ results from the fact that $r_p^{(i)}\left(p(n)\right) = \min_{\{(u,v)\,|\,(u,v)\in p(n)\}} \left\{r^{(i)}(u,v)\right\}$.

We define the *interdomain QoS routing with resource allocation* problem as follows:

$$Q^{**}(s,d) = \max_{\{p|p\in P(s,d)\}} Q_p^*(p)$$

where $P(s,d)$ is the set of all paths from node s to d.

Specifically for the interdomain path $p(n)$ described above and shown in Fig.1, the optimization problem (1) can be reformulated using TABLE II, TABLE III and TABLE IV. The values of parameters in TABLE II, TABLE III and TABLE IV were derived in [7]. Following is the reformulated optimization problem:

$$\begin{aligned}
&Q_p^{(i)*}\left(p(n)\right) = \\
&\max\left\{\min\left\{\left(Q_p^{(i)}\left(p_1(l)\right)Q_p^{(i)}\left(p_2(m-l)\right)Q_p^{(i)}\left(p_3(n-m)\right)\right),1\right\}\right\} \\
&= \max\left\{\min\left\{\left(\min\left\{\frac{\min_{1\le j\le l}\left\{b^{(i)}\left(v_{j-1},v_j\right)\right\}}{\sigma_1^{(i)}},1\right\}\times\right.\right.\right. \\
&\min\left\{\min_{l+1\le j\le m}\left\{\frac{b^{(i)}\left(v_{j-1},v_j\right)}{\sigma_2^{(i)}+(j-l)L_2}\right\},1\right\}\times \\
&\left.\left.\left.\min\left\{\min_{m+1\le j\le n}\left\{\frac{b^{(i)}\left(v_{j-1},v_j\right)}{\sigma_3^{(i)}+(j-m)L_3}\right\},1\right\}\right),\ 1\right\}\right\}
\end{aligned}$$

$$(P1')$$

subject to:

$$
\begin{aligned}
&J_p^{(i)}\left(p(n)\right)=J_p^{(i)}\left(p_1(l)\right)+J_p^{(i)}\left(p_2(m-l)\right)+J_p^{(i)}\left(p_3(n-m)\right)\\
&=\left(\frac{\min\left\{\sum_{j=1}^{l}b^{(i)}\left(v_{j-1},v_j\right),\sigma_1^{(i)}\right\}}{r_p^{(i)}\left(p_1(l)\right)}+\frac{q^{(i)*}\left(p_2(m-l)\right)}{r_p^{(i)}\left(p_2(m-l)\right)}+\right.\\
&\left.\frac{q^{(i)*}\left(p_3(n-m)\right)}{r_p^{(i)}\left(p_3(n-m)\right)}+\sum_{j=m+1}^{n}\frac{\left(K\left(v_{j-1,}v_j\right)-1\right)L_3}{C\left(v_{j-1},v_j\right)}\right)\le J_{req} \qquad (C1')\\
&0\le b^{(i)}\left(v_{j-1},v_j\right)\le B\left(v_{j-1},v_j\right) \quad ,\forall j\in\{1,2,...,n\} \qquad (C2')\\
&\max\left(\rho_1^{(i)},\rho_2^{(i)},\rho_3^{(i)}\right)\le r_p^{(i)}\left(p(n)\right)\le \min_{1\le j\le n}C\left(v_{j-1},v_j\right)^{\max} \qquad (C3')
\end{aligned}
\tag{2}
$$

where $q^{(i)*}\left(p(n)\right)$ is defined according to the following recursive relationship [7]:

$$
\begin{aligned}
&q^{(i)*}\left(p(j)\right)=\min\left\{q^{(i)*}\left(p(j-1)\right)+b^{(i)}\left(v_{j-1},v_j\right),\sigma^{(i)}+jL\right\}\\
&q^{(i)*}\left(p(0)\right)+b^{(i)}\left(v_0,v_1\right)=\min\left\{b^{(i)}\left(v_0,v_1\right),\sigma^{(i)}+L\right\}
\end{aligned}
$$

where the path $p(j)$ is obtained by adding the vertex v_j through link $\left(v_{j-1},v_j\right)$ to the path $p(j-1)$. Note that since the bandwidth of a path is the bottleneck function of the bandwidth of the subpaths, therefore we can replace $r_p^{(i)}\left(p_1(l)\right)$, $r_p^{(i)}\left(p_2(l-m)\right)$ and $r_p^{(i)}\left(p_3(n-m)\right)$ in (2) by $r_p^{(i)}\left(p(n)\right)$.

A. *Solution to the Interdomain Path Resource Allocation Problem*

In this sub-section we solve the *interdomain path resource allocation problem*, defined in (2), and obtain the optimal solution. The optimal solution consists of the values of the resource variables i.e., the link buffer sizes $b^{(i)}\left(v_{j-1},v_j\right)$ for all links $\left(v_{j-1,}v_j\right)$ belonging to the path $p(n)$ and the path bandwidth $r_p^{(i)}\left(p(n)\right)$. We denote the optimal solution values of bandwidth and buffer by $r_p^{(i)*}\left(p(n)\right)$ and $b^{(i)*}\left(v_{j-1},v_j\right)$ respectively.

Lemma III.1: The optimal solution value for $r_p^{(i)}\left(p(n)\right)$ in the optimization problem (2) is given by $\min_{1\le j\le n}C\left(v_{j-1},v_j\right)^{\max}$.

Proof: Let $r_p^{(i)**}\left(p(n)\right)$ and $b^{(i)**}\left(v_{j-1},v_j\right)$ be the optimal solution values for the optimization problem (2). Let the corresponding optimal objective function value be $Q_p^{(i)**}\left(p(n)\right)$. Suppose $r_p^{(i)**}\left(p(n)\right)<r_p^{(i)*}\left(p(n)\right)$ where $r_p^{(i)*}\left(p(n)\right)$ is a feasible solution of (2). Since $r_p^{(i)*}\left(p(n)\right)$ and $r_p^{(i)**}\left(p(n)\right)$ are both feasible solutions, therefore we can replace $r_p^{(i)**}\left(p(n)\right)$ by $r_p^{(i)*}\left(p(n)\right)$ in constraint $(C1')$ in optimization problem (2) and increase the value of the buffers $b^{(i)}\left(v_{j-1},v_j\right)$ allocated along the path $p(n)$ such that $b^{(i)*}\left(v_{j-1},v_j\right)>b^{(i)**}\left(v_{j-1},v_j\right)$ while keeping constraint $(C1')$ feasible. This implies that we can get another feasible solution $r_p^{(i)*}\left(p(n)\right)$ and $b^{(i)*}\left(v_{j-1},v_j\right)$, and corresponding objective function value $Q_p^{(i)*}\left(p(n)\right)$ for (2) such that $Q_p^{(i)*}\left(p(n)\right)>Q_p^{(i)**}\left(p(n)\right)$. This leads to a contradiction, since $Q_p^{(i)**}\left(p(n)\right)$ was the optimal objective function value. Since the bandwidth is the bottleneck function of all the links along the path, therefore, it is not possible to have the following inequality $r_p^{(i)*}\left(p(n)\right)>\min_{1\le j\le n}C\left(v_{j-1},v_j\right)^{\max}$. Hence the optimal solution value for $r_p^{(i)}\left(p(n)\right)$ is $\min_{1\le j\le n}C\left(v_{j-1},v_j\right)^{\max}$. ■

Once the optimal solution value for $r_p^{(i)}\left(p(n)\right)$ is determined, we are left with determining the optimal value for the buffer space i.e., $b^{(i)*}\left(v_{j-1},v_j\right)$ allocated along the path $p(n)$. For convenience we define $J'_{req}=J_{req}-\sum_{j=m+1}^{n}\frac{\left(K\left(v_{j-1,}v_j\right)-1\right)L_3}{C\left(v_{j-1},v_j\right)}$. Hence we can rewrite constraint $(C1')$ as below:

$$\left(\frac{\min\left\{\sum_{j=1}^{l}b^{(i)}\left(v_{j-1},v_j\right),\sigma_1^{(i)}\right\}}{r_p^{(i)*}\left(p(n)\right)}+\frac{q^{(i)*}\left(p_2(m-l)\right)}{r_p^{(i)*}\left(p(n)\right)}+\frac{q^{(i)*}\left(p_3(n-m)\right)}{r_p^{(i)*}\left(p(n)\right)}\right)\leq J'_{req} \tag{3}$$

or equivalently,

$$\min\left\{\sum_{j=1}^{l}b^{(i)}\left(v_{j-1},v_j\right),\sigma_1^{(i)}\right\}+q^{(i)*}\left(p_2(m-l)\right)+q^{(i)*}\left(p_3(n-m)\right)\leq J'_{req}r_p^{(i)*}\left(p(n)\right) \tag{4}$$

Lemma III.2: If the following inequality holds:

$$\begin{aligned}\Bigg(&\min\left\{\sum_{j=1}^{l}B^{(i)}\left(v_{j-1},v_j\right),\sigma_1^{(i)}\right\}+\\&\min\left\{\sum_{j=l+1}^{m}B^{(i)}\left(v_{j-1},v_j\right),\sigma_2^{(i)}+(m-l)L_2\right\}+\\&\min\left\{\sum_{j=m+1}^{n}B^{(i)}\left(v_{j-1},v_j\right),\sigma_3^{(i)}+(n-m)L_3\right\}\Bigg)\leq J'_{req}r_p^{(i)*}\left(p(n)\right)\end{aligned} \tag{5}$$

then an optimal solution value for $b^{(i)}\left(v_{j-1},v_j\right)$ is given as follows (Note the optimal solution may not be unique in this case):

$$\begin{aligned}&b^{(i)*}\left(v_{j-1},v_j\right)=\\&\begin{cases}\min\left\{B^{(i)}\left(v_{j-1},v_j\right),\sigma_1^{(i)}\right\} & ,j=1,2,\ldots,l\\ \min\left\{B^{(i)}\left(v_{j-1},v_j\right),\sigma_2^{(i)}+(j-l)L_2\right\} & ,j=l+1,l+2,\ldots,m\\ \min\left\{B^{(i)}\left(v_{j-1},v_j\right),\sigma_3^{(i)}+(j-m)L_2\right\} & ,j=m+1,l+2,\ldots,n\end{cases}\end{aligned} \tag{6}$$

Proof: The inequality (5) states that the maximum amount of bits that can be buffered in the nodes along the path $p(n)$ is less than or equal to the buffer space required to violate the maximum jitter value J'_{req}. In other words we can buffer the maximum number of bits in the nodes along the path $p(n)$ without violating the jitter constraint $(C1')$. The optimal objective function value $Q_p^{(i)*}\left(p(n)\right)$ can be obtained by using the maximum value of the buffer space available in each node. Hence an optimal solution for $b^{(i)}\left(v_{j-1},v_j\right)$ is given by (6). The corresponding optimal objective function value is given by the following:

$$Q_p^{(i)}\left(p(n)\right)=\min\left\{\frac{\min\limits_{1\le j\le l}\left\{b^{(i)*}\left(v_{j-1},v_j\right)\right\}}{\sigma_1^{(i)}}\times\min_{l+1\le j\le m}\left\{\frac{b^{(i)*}\left(v_{j-1},v_j\right)}{\sigma_2^{(i)}+(j-l)L_2}\right\}\times\min_{m+1\le j\le n}\left\{\frac{b^{(i)*}\left(v_{j-1},v_j\right)}{\sigma_3^{(i)}+(j-m)L_3}\right\},1\right\}$$

■

Theorem III.1: For the case when the following inequality holds:

$$\left(\begin{array}{l}\min\left\{\sum_{j=1}^{l}B^{(i)}\left(v_{j-1},v_j\right),\sigma_1^{(i)}\right\}+\\ \min\left\{\sum_{j=l+1}^{m}B^{(i)}\left(v_{j-1},v_j\right),\sigma_2^{(i)}+(m-l)L_2\right\}+\\ \min\left\{\sum_{j=m+1}^{n}B^{(i)}\left(v_{j-1},v_j\right),\sigma_3^{(i)}+(n-m)L_3\right\}\end{array}\right)>J'_{req}r_p^{(i)*}\left(p(n)\right) \tag{7}$$

then the optimal solution $\left(r_p^{(i)*}\left(p(n)\right),b^{(i)*}\left(v_{j-1},v_j\right)\right)$ to (2) satisfies the following:

$$r_p^{(i)*}\left(p(n)\right)=\min_{1\le j\le n}C\left(v_{j-1},v_j\right)^{\max} \tag{8}$$

$$\frac{\min\limits_{1\le j\le l}\left\{b^{(i)*}\left(v_{j-1},v_j\right)\right\}}{\sigma_1^{(i)}}=\min_{l+1\le j\le m}\left\{\frac{b^{(i)*}\left(v_{j-1},v_j\right)}{\sigma_2^{(i)}+(j-l)L_2}\right\}=\min_{m+1\le j\le n}\left\{\frac{b^{(i)*}\left(v_{j-1},v_j\right)}{\sigma_3^{(i)}+(j-m)L_3}\right\} \tag{9}$$

Proof: (8) follows from *Lemma III.1*. The inequality (7) means that the number of bits that can be buffered in the nodes along the path $p(n)$ can lead to a jitter value which can violate the jitter constraint. Note that in this case at the optimal solution $b^{(i)*}\left(v_{j-1},v_j\right)$ the inequality (4) will be satisfied as an equality, i.e.,

$$\min\left\{\sum_{j=1}^{l}b^{(i)*}\left(v_{j-1},v_j\right),\sigma_1^{(i)}\right\}+q^{(i)*}\left(p_2(m-l)\right)+q^{(i)*}\left(p_3(n-m)\right)=J'_{req}r_p^{(i)*}\left(p(n)\right) \tag{10}$$

If (4) is a strict inequality. i.e.,

$$\min\left\{\sum_{j=1}^{l}b^{(i)*}\left(v_{j-1},v_j\right),\sigma_1^{(i)}\right\}+q^{(i)*}\left(p_2(m-l)\right)+q^{(i)*}\left(p_3(n-m)\right)<J'_{req}r_p^{(i)*}\left(p(n)\right) \tag{11}$$

then there exists $b^{(i)**}\left(v_{j-1},v_j\right)$ for at least one $j\in\{1,2,...,n\}$ such that $b^{(i)**}\left(v_{j-1},v_j\right)>b^{(i)*}\left(v_{j-1},v_j\right)$ and $b^{(i)**}\left(v_{j-1},v_j\right)$ is a feasible solution. Let the objective function value corresponding to $b^{(i)**}\left(v_{j-1},v_j\right)$ be $Q_p^{(i)**}\left(p(n)\right)$, then $Q_p^{(i)**}\left(p(n)\right)>Q_p^{(i)*}\left(p(n)\right)$, which is a contradiction. Hence (10) must hold at optimality. Now we show that (9) must hold at optimality. Suppose by way of contradiction that (9) does not hold. Without loss of generality we can assume that the path $p(n)=p_1(l)\oplus p_2(m-l)$. The result can be extended to a path consisting of more than two subpaths by induction.

Assume that at optimality the following holds:

$$\frac{\min_{1\le j\le l}\left\{b^{(i)*}\left(v_{j-1},v_j\right)\right\}}{\sigma_1^{(i)}} < \min_{l+1\le j\le m}\left\{\frac{b^{(i)*}\left(v_{j-1},v_j\right)}{\sigma_2^{(i)}+\left(j-l\right)L_2}\right\} \tag{12}$$

$$Q_p^{(i)*}\left(p(n)\right)=\frac{\min_{1\le j\le l}\left\{b^{(i)*}\left(v_{j-1},v_j\right)\right\}}{\sigma_1^{(i)}}\times\min_{l+1\le j\le m}\left\{\frac{b^{(i)*}\left(v_{j-1},v_j\right)}{\sigma_2^{(i)}+\left(j-l\right)L_2}\right\}$$

where $Q_p^{(i)*}\left(p(n)\right)$ is the optimal objective function value. From (12) it implies that there exists $\varepsilon>0,\ \varepsilon\in\mathbb{R}$ such that:

$$\frac{\min_{1\le j\le l}\left\{b^{(i)*}\left(v_{j-1},v_j\right)\right\}}{\sigma_1^{(i)}}+\varepsilon=\min_{l+1\le j\le m}\left\{\frac{b^{(i)*}\left(v_{j-1},v_j\right)}{\sigma_2^{(i)}+\left(j-l\right)L_2}\right\}$$

It is possible to reallocate the buffers along the path $p(n)$ such that (10) holds and the new variables $b^{(i)**}\left(v_{j-1},v_j\right)$ lead to the following objective function:

$$\begin{aligned}Q_p^{(i)**}&=\left(\frac{\min_{1\le j\le l}\left\{b^{(i)*}\left(v_{j-1},v_j\right)\right\}}{\sigma_1^{(i)}}+\frac{\varepsilon}{2}\right)\left(\min_{l+1\le j\le m}\left\{\frac{b^{(i)*}\left(v_{j-1},v_j\right)}{\sigma_2^{(i)}+\left(j-l\right)L_2}\right\}-\frac{\varepsilon}{2}\right)\\&=Q_p^{(i)*}+\frac{\varepsilon^2}{4}\end{aligned} \tag{13}$$

i.e., $Q_p^{(i)**}\left(p(n)\right)>Q_p^{(i)*}\left(p(n)\right)$ which leads to a contradiction since we started of by assuming that $Q_p^{(i)*}\left(p(n)\right)$ is the optimal objective function value. ■

To find the optimal values of buffers $b^{(i)*}\left(v_{j-1},v_j\right)$, we proceed as follows. Let $J_{req}^*\left(p_1(l)\right)$, $J_{req}^*\left(p_2(m-l)\right)$ and $J_{req}^*\left(p_3(n-m)\right)$ be the jitter values corresponding to the paths $p_1(l)$, $p_2(m-l)$ and $p_3(n-m)$ respectively when optimal values of the decision variables $\left(b^{(i)*}\left(v_{j-1},v_j\right),r^{(i)*}\left(p(n)\right)\right)$ are used along the path $p(n)$ where $p(n)=p_1(l)\oplus p_2(m-l)\oplus p_3(n-m)$. Then (10) can be rewritten as follows:

$$J_{req}^*\left(p_1(l)\right)+J_{req}^*\left(p_2(m-l)\right)+J_{req}^*\left(p_3(n-m)\right)=J'_{req} \tag{14}$$

In [7] it was proved that if $J_{req}^*\left(p_1(l)\right)$ is the required jitter value for a GPS domain then the optimal buffer values $b^{(i)*}\left(v_{j-1},v_j\right)$ for $j=1,2,\ldots,l$ and the optimal objective function value $Q_p^{(i)*}\left(p_1(l)\right)$ are given as follows:

$$\begin{aligned}b^{(i)*}\left(v_{j-1},v_j\right)&=\min\left\{B^{(i)}\left(v_{j-1},v_j\right),\frac{J_{req}^*\left(p_1(l)\right)r_p^{(i)*}\left(p(n)\right)}{l}\right\}\quad,j\in\{1,2,\ldots,l\}\\Q_p^{(i)*}&=\frac{\min_{1\le j\le l}\left\{\min\left\{B^{(i)}\left(v_{j-1},v_j\right),\frac{J_{req}^*\left(p_1(l)\right)r_p^{(i)*}\left(p(n)\right)}{l}\right\}\right\}}{\sigma_1^{(i)}}\end{aligned} \tag{15}$$

For a PGPS domain it was shown in [7] that the values for $b^{(i)*}\left(v_{j-1},v_j\right)$, $j\in\{l+1,l+2,...,m\}$ and $Q_p^{(i)*}\left(p_2\left(m-l\right)\right)$ are given as follows:

$$b^{(i)*}\left(v_{j-1},v_j\right)=\min\left\{B^{(i)}\left(v_{j-1},v_j\right),\frac{J_{req}^{*}\left(p_2\left(m-l\right)\right)r_p^{(i)*}\left(p\left(n\right)\right)}{\left(m-l\right)\left[\sigma_2^{(i)}+\frac{m-l+1}{2}L_2\right]}\left(\sigma_2^{(i)}+\left(j-l\right)L_2\right)\right\}$$

$$Q_p^{(i)*}\left(p_2\left(m-l\right)\right)=\min\left\{\min_{l+1\le j\le m}\frac{B^{(i)}\left(v_{j-1},v_j\right)}{\sigma_2^{(i)}+\left(j-l\right)L_2},\frac{J_{req}^{*}\left(p_2\left(m-l\right)\right)r_p^{(i)*}\left(p\left(n\right)\right)}{\left(m-l\right)\left[\sigma_2^{(i)}+\frac{m-l+1}{2}L_2\right]}\right\}$$

$$(16)$$

Similarly, for SCFQ domain it was shown in [7] that for $j\in m+1,m+2,...,n$:

$$b^{(i)*}\left(v_{j-1},v_j\right)=\min\left\{B^{(i)}\left(v_{j-1},v_j\right),\frac{J_{req}^{*}\left(p_3\left(n-m\right)\right)r_p^{(i)*}\left(p\left(n\right)\right)}{\left(n-m\right)\left[\sigma_3^{(i)}+\frac{n-m+1}{2}L_3\right]}\left(\sigma_3^{(i)}+\left(j-m\right)L_3\right)\right\}$$

$$Q_p^{(i)*}\left(p_3\left(n-m\right)\right)=\min\left\{\min_{m+1\le j\le n}\frac{B^{(i)}\left(v_{j-1},v_j\right)}{\sigma_3^{(i)}+\left(j-m\right)L_3},\frac{J_{req}^{*}\left(p_3\left(n-m\right)\right)r_p^{(i)*}\left(p\left(n\right)\right)}{\left(n-m\right)\left[\sigma_3^{(i)}+\frac{n-m+1}{2}L_3\right]}\right\}$$

$$(17)$$

Hence finding the optimal solution $b^{(i)*}\left(v_{j-1},v_j\right)$ for $j\in\{1,2,...,n\}$ reduces to finding $J_{req}^{*}\left(p_1\left(l\right)\right)$, $J_{req}^{*}\left(p_2\left(m-l\right)\right)$ and $J_{req}^{*}\left(p_3\left(n-m\right)\right)$. Using Theorem III.1, the values of $J_{req}^{*}\left(p_1\left(l\right)\right)$, $J_{req}^{*}\left(p_2\left(m-l\right)\right)$ and $J_{req}^{*}\left(p_3\left(n-m\right)\right)$ can be found by solving the following two equations:

$$J_{req}^{*}\left(p_1\left(l\right)\right)+J_{req}^{*}\left(p_2\left(m-l\right)\right)+J_{req}^{*}\left(p_3\left(n-m\right)\right)=J'_{req}$$

$$(18)$$

$$\frac{\min\left\{\min_{1\le j\le l}\left\{B^{(i)}\left(v_{j-1},v_j\right),\frac{J_{req}^{*}\left(p_1\left(l\right)\right)r_p^{(i)*}\left(p\left(n\right)\right)}{l}\right\}\right\}}{\sigma_1^{(i)}}=$$

$$=\min\left\{\min_{l+1\le j\le m}\frac{B^{(i)}\left(v_{j-1},v_j\right)}{\sigma_2^{(i)}+\left(j-l\right)L_2},\frac{J_{req}^{*}\left(p_2\left(m-l\right)\right)r_p^{(i)*}\left(p\left(n\right)\right)}{\left(m-l\right)\left[\sigma_2^{(i)}+\frac{m-l+1}{2}L_2\right]}\right\}$$

$$=\min\left\{\min_{m+1\le j\le n}\frac{B^{(i)}\left(v_{j-1},v_j\right)}{\sigma_3^{(i)}+\left(j-m\right)L_3},\frac{J_{req}^{*}\left(p_3\left(n-m\right)\right)r_p^{(i)*}\left(p\left(n\right)\right)}{\left(n-m\right)\left[\sigma_3^{(i)}+\frac{n-m+1}{2}L_3\right]}\right\}$$

$$(19)$$

where (19) is obtained by substituting $b^{(i)*}\left(v_{j-1},v_j\right)$ from (15), (16) and (17) into (9). In order to solve (18) and (19) simultaneously we proceed as follows:

Assume first that,

$$\min_{1\le j\le l}\left\{B^{(i)}\left(v_{j-1},v_j\right)\right\} > \frac{J^*_{req}\left(p_1(l)\right)r_p^{(i)*}\left(p(n)\right)}{l} \text{ and}$$
$$\min_{l+1\le j\le m}\left\{\frac{B^{(i)}\left(v_{j-1},v_j\right)}{\sigma_2^{(i)}+\left(j-l\right)L_2}\right\} > \frac{J^*_{req}\left(p_2(m-l)\right)r_p^{(i)*}\left(p(n)\right)}{\left(m-l\right)\left[\sigma_2^{(i)}+\frac{m-l+1}{2}L_2\right]} \text{ and}$$
$$\min_{m+1\le j\le n}\left\{\frac{B^{(i)}\left(v_{j-1},v_j\right)}{\sigma_3^{(i)}+\left(j-m\right)L_3}\right\} > \frac{J^*_{req}\left(p_3(n-m)\right)r_p^{(i)*}\left(p(n)\right)}{\left(n-m\right)\left[\sigma_3^{(i)}+\frac{n-m+1}{2}L_3\right]} \tag{20}$$

Then (19) reduces to the following:

$$\frac{J^*_{req}\left(p_1(l)\right)r_p^{(i)*}\left(p(n)\right)}{l} = \frac{J^*_{req}\left(p_2(m-l)\right)r_p^{(i)*}\left(p(n)\right)}{\left(m-l\right)\left[\sigma_2^{(i)}+\frac{m-l+1}{2}L_2\right]} = \frac{J^*_{req}\left(p_3(n-m)\right)r_p^{(i)*}\left(p(n)\right)}{\left(n-m\right)\left[\sigma_3^{(i)}+\frac{n-m+1}{2}L_3\right]} \tag{21}$$

By solving (18) and (21) simultaneously we can obtain the optimal values of $J^*_{req}\left(p_1(l)\right)$, $J^*_{req}\left(p_2(m-l)\right)$ and $J^*_{req}\left(p_3(n-m)\right)$ as follows:

$$J^*_{req}\left(p_1(l)\right) = \frac{J'_{req}}{\left[1+\left(\frac{m-l}{l}\right)\left(\sigma_2^{(i)}+\frac{m-l+1}{2}L_2\right)+\left(\frac{n-m}{l}\right)\left(\sigma_3^{(i)}+\frac{n-m+1}{2}L_3\right)\right]}$$
$$J^*_{req}\left(p_2(m-l)\right) = \frac{J'_{req}\left(\frac{m-l}{l}\right)\left[\sigma_2^{(i)}+\frac{m-l+1}{2}L_2\right]}{\left[1+\left(\frac{m-l}{l}\right)\left(\sigma_2^{(i)}+\frac{m-l+1}{2}L_2\right)+\left(\frac{n-m}{l}\right)\left(\sigma_3^{(i)}+\frac{n-m+1}{2}L_3\right)\right]}$$
$$J^*_{req}\left(p_3(n-m)\right) = \frac{J'_{req}\left(\frac{n-m}{l}\right)\left[\sigma_3^{(i)}+\frac{n-m+1}{2}L_3\right]}{\left[1+\left(\frac{m-l}{l}\right)\left(\sigma_2^{(i)}+\frac{m-l+1}{2}L_2\right)+\left(\frac{n-m}{l}\right)\left(\sigma_3^{(i)}+\frac{n-m+1}{2}L_3\right)\right]} \tag{22}$$

Now put the values of $J^*_{req}\left(p_1(l)\right)$, $J^*_{req}\left(p_3(n-m)\right)$ and $J^*_{req}\left(p_2(m-l)\right)$ into the inequalities (20). If the inequalities (20) hold then $J^*_{req}\left(p_1(l)\right)$, $J^*_{req}\left(p_2(m-l)\right)$ and $J^*_{req}\left(p_3(n-m)\right)$ are the optimal jitter values for subpaths $p_1(l)$, $p_2(m-l)$ and $p_3(n-m)$ respectively. In this case the optimal solution $b^{(i)*}\left(v_{j-1},v_j\right)$ and the optimal objective function value $Q_p^{(i)*}\left(p(n)\right)$ can be obtained by using the optimal solutions in (15), (16) and (17) . Now we consider the case when $J^*_{req}\left(p_1(l)\right)$, $J^*_{req}\left(p_3(n-m)\right)$ and $J^*_{req}\left(p_2(m-l)\right)$, as given by (22), do not satisfy (20). Without loss of generality assume that the first inequality in (20) is not satisfied and the rest of the two inequalities are satisfied. This implies that there exists k, $1\le k\le l$, such that $B^{(i)}\left(v_{k-1},v_k\right) < \frac{J^*_{req}\left(p_1(l)\right)r_p^{(i)*}\left(p(n)\right)}{l}$. In this case the maximum reliability value can not be increased beyond $Q_p^{(i)*}\left(p(n)\right) = \frac{B^{(i)}\left(v_{k-1},v_k\right)}{\sigma_1^{(i)}}$. This is because increasing the values of $b^{(i)*}\left(v_{j-1},v_j\right)$ for $j\ne k$ will not change the value of maximum reliability $Q_p^{(i)*}\left(p(n)\right)$ since the buffer value $b^{(i)*}\left(v_{k-1},v_k\right)$ is physically bounded from above by $B^{(i)}\left(v_{k-1},v_k\right)$. In this case again the optimal solution $b^{(i)*}\left(v_{j-1},v_j\right)$ and the optimal objective function value $Q_p^{(i)*}\left(p(n)\right)$ can be obtained form equations (15), (16) and (17).

B. Interdomain QoS Routing with Resource Allocation Algorithm

In this sub-section, we present a dynamic programming algorithm to solve the *interdomain QoS routing with resource allocation problem*. This algorithm uses the solution to *interdomain path resource allocation problem* presented in sub-section A. The algorithm is specified in Algorithm III.1.

The subroutine **Pre-Process** deletes edges in the graph G which have insufficient resources. Algorithm III.1 relies on the sub-routine **Int-Path-Opt** which solves the optimization problem (1). The solution to the optimization problem (2), which is a special case of (1), can be used with minor modifications to solve the **Int-Path-Opt** in the general case. At the termination of algorithm III.1, $Q^{|V|-1}(s,x) = Q^{**}(s,x)$ for every node $x \in V$ such that there exists a path from the source node s to the destination node v and $Q^{|V|-1}(s,x) = -\infty$ if no path exists from s to x. The optimal path can be easily constructed using the parent nodes $\pi^{|V|-1}(x)$. The computational complexity of the algorithm III.1 is given by $O\left(K|V||E|\right)$ where it takes K computational steps to solve **Int-Path-Opt.**

Algorithm III.1: Interdomain QoS Routing $(G(V,E), s, J_{req}, Q_{req}, R_{req})$

1. Pre-Process(G)
2. For each $x \in adj(s)$
   $Q^1(s,x) := Int-Path-Opt(G,x,s)$
3. For each $x \in adj^{-1}(s)$ $\quad \pi^1(x) := x$
4. For each $x \notin adj(s)$ $\quad Q^1(s,x) := -\infty$
5. for $k := 1 \to |V| - 2$
6.   for each node $x \in V - \{s\}$
7. $Q^{k+1}(s,x) := Q^k(s,x)$
8. $\pi^{k+1}(x) := \pi^k(x)$
9.   for each node $\omega \in adj^{-1}(x)$
10. $\pi_{old} := \pi^{k+1}(x)$
11. $\pi^{k+1}(x) := \omega$
12. $Temp := Int-Path-Opt(G,x,s)$
13.   if $Temp > Q^{k+1}(s,x)$
14. $Q^{k+1}(s,x) := Temp$
15.   else
16. $\pi^{k+1}(x) := \pi_{old}$

IV. Conclusion

In this paper we presented a new formulation and solution of the Interdomain QoS routing problem. Our problem formulation takes into consideration the heterogeneity of the queuing service disciplines employed in various autonomous systems comprising the Internet. Our routing protocol uses QoS metrics which are functions of the buffers in the routers and the link bandwidths. We provide solution to a special case of interdomain QoS routing problem in a network consisting of three autonomous systems employing GPS, PGPS and SCFQ queuing service disciplines. Our solution technique can be generalized to autonomous systems employing other queuing service disciplines in a similar manner. Our proposed formulation can be extended by finding a generalized interdomain QoS routing protocol which can accommodate a large number of queuing service disciplines.

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TABLE I
DEFINITIONS OF SYMBOLS

Symbol	Quantity
Q_{req}	Minimum reliability required by user. Reliability is defined as the ratio of the packets received to packets transmitted along a path
J_{req}	Maximum required end-to-end jitter delay
R_{req}	Minimum required average bandwidth
Q_{uv}	Reliability of the link (u,v)
J_{uv}	Jitter of link (u,v)
$b^{(i)}(u,v)$	Buffer allocated to ith data session in the queue residing in node u and transmitting to node v
$B(u,v)$	Maximum buffer space available to a data session transmitting from node u to v
$r^{(i)}(u,v)$	Bandwidth allocated to ith data session along the link (u,v)
$C(u,v)$	Maximum transmission capacity of the link (u,v)
$C(u,v)^{\max}$	Maximum bandwidth available to a connection along link (u,v), $C(u,v)^{\max} \le C(u,v)$
$Q_p^{(i)}(p(n))$	Reliability of path $p(n) = \langle v_0, v_1, ..., v_n \rangle$ for ith data session
$J_p^{(i)}(p(n))$	Jitter along path $p(n)$ for ith data session
$r_p^{(i)}(p(n))$	Bandwidth of path $p(n)$ for ith data session
$A_{out}^{(i)}(t,u)$	Arrival function of ith data session at the output queue of node u
AS_k	kth Autonomous System
$\sigma_k^{(i)}$	Bucket size for ith data session in kth Autonomous System
$\rho_k^{(i)}$	Average traffic rate for ith session in kth Autonomous System
$\phi^{(i)}(u,v)$	Fraction of the bandwidth along link (u,v) assigned to ith data session
L_k	Maximum packet size in kth Autonomous System
$q^{(i)*}(u,v)$	Maximum number of backlogged bits from ith data session in the output queue of node u
$q^{(i)*}(p(n))$	Maximum number of backlogged bits from ith data session in the path $p(n) = \langle v_0, v_1, ..., v_n \rangle$
$K(u,v)$	Total number of sessions traversing the link (u,v)

TABLE II
BANDWIDTH FUNCTIONS

Queuing Service Discipline	Bandwidth of path $p(n)$, $r_p^{(i)}(p(n))$
GPS	$\min_{1 \le j \le n} \frac{\phi^{(i)}(v_{j-1}, v_j)}{\sum_{l=1}^{K(v_{j-1}, v_j)} \phi^{(l)}(v_{j-1}, v_j)}$
PGPS	$\min_{1 \le j \le n} \frac{\phi^{(i)}(v_{j-1}, v_j)}{\sum_{l=1}^{K(v_{j-1}, v_j)} \phi^{(l)}(v_{j-1}, v_j)}$
SCFQ	$\min_{1 \le j \le n} \frac{\phi^{(i)}(v_{j-1}, v_j)}{\sum_{l=1}^{K(v_{j-1}, v_j)} \phi^{(l)}(v_{j-1}, v_j)}$

TABLE III
RELIABILITY FUNCTIONS

Queuing Service Discipline	Reliability of path $p(n)$, $Q_p^{(i)}(p(n))$
GPS	$\min\left\{\frac{\min_{1\le j\le n}\left\{b^{(i)}(v_{j-1},v_j)\right\}}{\sigma^{(i)}},1\right\}$
PGPS	$\min\left\{\min_{1\le j\le n}\left\{\frac{b^{(i)}(v_{j-1},v_j)}{\sigma^{(i)}+jL}\right\},1\right\}$
SCFQ	$\min\left\{\min_{1\le j\le n}\left\{\frac{b^{(i)}(v_{j-1},v_j)}{\sigma^{(i)}+jL}\right\},1\right\}$

TABLE IV
JITTER FUNCTIONS

Queuing Service Discipline	Jitter along path $p(n)$, $J_p^{(i)}(p(n))$
GPS	$\frac{\min\left\{\sum_{j=1}^{n} b^{(i)}(v_{j-1},v_j),\sigma^{(i)}\right\}}{r_p^{(i)}(p(n))}$
PGPS	$\frac{q^{(i)*}(p(n))}{r_p^{(i)}(p(n))}$
SCFQ	$J_p^i(p(n))=\frac{q^{(i)*}(p(n))}{r_p^{(i)}(p(n))}+\sum_{j=1}^{n}\frac{(K(v_{j-1},v_j)-1)L}{C(v_{j-1},v_j)}$