DESIGN METHODOLOGY FOR LOW SPEED VARIABLE RELUCTANCE MOTORS

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TR-EE 92-37
SEPTEMBER 1992

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ABSTRACT

Lowering the gear reduction in actuators by utilizing high-torque low-speed motors enables the use of less expensive and simpler gear systems and decreases the overall system inertia. Variable reluctance machines can produce high torque at low speeds. Their static torque, a critical quantity for determination of low speed operation, is compared for three variable reluctance motor design variations using linear analysis. Saturation effects, which are crucial to the accurate determination of static torque, are modeled using a dual energy technique first proposed by Lord Rayleigh. Dual energy techniques utilizing flux tubes and magnetomotive force slices are developed into a numerical method for predicting nonlinear three-dimensional magnetostatic field parameters. The dual energy method offers a compromise between the accurate but laborious finite element method and the speed of simplified lumped parameter magnetic circuit calculations. A two-dimensional dual energy model of a variable reluctance motor is developed. Results of calculations on a 4 kW Oulton machine are compared to measurements and other calculation methods. Finally, as a demonstration, the model is used to evaluate two competing variable reluctance motors for use as replacements for a DC windshield wiper motor.
CHAPTER 1. INTRODUCTION

Many applications for actuators require the production of high torque at low speeds. This is most commonly accomplished through the use of hydraulics or by geared electric motors. Electric systems offer many advantages over hydraulic systems by eliminating the need for fluid piping and containment. Unfortunately motors generally can not compete in power density with hydraulics unless they operate at very high speed. Thus, they are typically geared down to provide the required torque-speed characteristics.

Gear reductions are bulky, noisy, inefficient, and expensive. They also multiply the effective inertia of the actuator so that it becomes difficult to control. Generally, a lower gear reduction is simpler and less expensive than a high gear ratio, since spur gears may be used. Therefore, a motor with high power density at low speeds using little or no gearing could potentially reduce cost and complexity of a system.

Switched reluctance motors are capable of producing high torque at low speed. Switched reluctance motors are single stack variable reluctance stepper motors coupled with sensors and a converter to permit continuous operation. They operate by electromagnetic attraction between excited stator poles and the nearest salient poles on the rotor. The torque output of variable reluctance motors is roughly proportional to the square of the current.

Many issues related to the construction, operation, and analysis of switched reluctance motors have been studied in the literature. This thesis examines the prospects of utilizing switched reluctance machines for low-speed high-torque applications and presents an approach to modeling and analysis of switched reluctance motors.

Description of Operation

Switched reluctance motors are variable reluctance motors designed to operate efficiently from a controlled source. The fundamentals of switched reluctance motor operation are covered in [1-5]. Shown in Fig. 1 is a basic variable reluctance motor (VRM). The particular machine pictured here has a three phase two pole winding
resulting in six salient stator pole structures. The rotor is made up of four salient poles with no winding. Both the stator and rotor are made of stacks of laminated magnetic steel.

![Diagram of Typical Variable Reluctance Motor](image)

Figure 1.1 Cross-Section of Typical Variable Reluctance Motor

Each phase winding of the conventional two pole switched reluctance motor consists of two serially connected coils for each phase. When excited, this winding excites two poles in the airgap of the machine. Thus, for the machine of Fig. 1, even though there are six salient stator poles, when phase a is excited it produces only two poles in the airgap. A converter applies a DC voltage to each phase in sequence to produce rotation. Only one phase is excited at a given instant in time. The direction of rotation is dependent on the phase sequence. For the example shown, an a-b-c sequence results in clockwise rotation of the rotor. Current flow through each phase is unidirectional.

Torque is produced by the attraction of the rotor poles to the nearest excited stator poles. With fixed current excitation, the torque that is produced by a variation in the reluctance of the magnetic circuit of the excited phase is proportional to the square of the current. Saturation of the iron influences the production of torque since it affects the change in the phase reluctance. References [7,10] in particular discuss the impact of saturation on the availability of torque.
Saturation of the iron manifests itself in two ways [1]: local saturation and bulk saturation. **Local** saturation of the pole tips occurs when the rotor and stator poles are not aligned due to constriction of the flux at the pole tips. Local saturation occurs at low current. On the other hand, bulk saturation of the yoke or poles occurs primarily when the rotor and stator poles are closely aligned.

The effects of saturation on the electrical characteristics and torque of variable reluctance motors complicates analysis. Since saturation is an intimate part of the operation of these devices any accurate prediction of torque must include effects from both local and bulk saturation.

**Review of Modeling Techniques**

Many authors have contributed to the modeling and analysis of variable reluctance devices. In reference [6], the authors developed the basic mathematical approach to computing the current and torque of switched reluctance motors assuming idealized switches in the converter. Their method assumes knowledge of the flux linkage of each phase, $\Psi(\theta,i)$, which is a function of the instantaneous phase current $i$ and the angular position of the rotor $\theta$. The torque, $T(\theta,i)$, is the partial derivative of the coenergy, $W'(\theta,i)$, with respect to the rotor angle $\theta$. The coenergy is given by the integral of the flux linkage with respect to current with the angle held constant, that is $W'(\theta,i) = \int_i^f \Psi(i,\theta)dil_{\theta=\text{const}}$. In [6] the authors did not detail methods for finding the flux linkage from machine dimensions, but rather presupposed the existence of such information.

The effects of saturation in switched reluctance drives are discussed in references [7,9]. The approach taken by these authors is to assume sharp breaks between saturated and unsaturated states. The analyses have as their objective to find the impact of iron saturation on the available power rating of the motor and drive and its importance on the steady state behavior of switched reluctance machines. They do not attempt to make highly accurate predictions of the static torque or flux linkages.

Other methods either use linear models of switched reluctance motor operation [8] or **derive** performance equations from curve-fitted experimental data [10]. These too require a detailed description of the phase flux linkage before they can be applied. Reference [12] describes a means of computing losses in switched reluctance machines, and [13] describes various converter circuits for use with these devices.
Computer-aided design (CAD) and simulation of switched reluctance machines are the topics of [14-18]. General design equations for switched reluctance machines based mainly on linear analysis are covered in [19-23]. CAD packages for switched reluctance machines generally rely on numerical methods for determination of the relationship between current and torque for given motor dimensions. These numerical methods use a variety of techniques as described below.

The magnetic circuit in switched reluctance machines is comprised of airgaps as well as iron permeances. For rotor positions where the rotor pole is not aligned closely with the stator pole the airgap permeances predominate the storage and conversion of electromagnetic energy. The analysis of airgap permeance between salient poles is the subject of [24-29]. Approximate fringe path shapes are used to construct the airgap permeances in [27,28] based on work described by Roters [24]. Fourier series expansions of permeance data obtained from finite difference field solutions are utilized in [25]. Carter's coefficients are used to find the permeances in [26]. Hesse [29] describes the use of a truncated Fourier series expression for the gap distance in the derivation of the permeance between salient tooth structures of this type.

The variation in airgap permeances does not account fully for all of the conversion of energy. With the iron saturated, small but nevertheless significant energy exchange occurs in the iron itself. References [30-35] describe various experimental and analytical means of obtaining torque and force between saturable iron structures. An ideal description of the magnetization characteristic of iron is used in [30,34-35] to compute tangential force between contacting iron pieces with extension to systems with airgaps. Finite difference techniques are used to further this work in [31]. An attempt is made in [32] to determine limiting values of torque density for variable reluctance machines taking saturation into account. While their results are encouraging they rely on selecting a value for saturation flux density for which there is some ambiguity. They also do not provide an indication of the torque at other than the fully saturated condition.

There is no satisfactory closed form expression relating the geometry of the variable reluctance machine to its torque when saturation is included. Currently, two commonly used numerical methods are simple magnetic circuit analysis of a lumped parameter model or the finite element method on a distributed model. The magnetic circuit approach [36-41] uses the permeances for various air and iron portions as defined by material boundaries. Airgap and fringe regions can be represented using Roter's method as in [41]. Local saturation effects are difficult to represent this way but some success has been reported [39].
More accurate estimates of motor parameters and torque can be obtained by finite difference [42] or finite element [43-51] methods. Even with powerful workstations these methods require a good deal of time and effort. Moreover, the resulting numerical results of calculated vector potential are difficult to translate into useful direction for design changes.

Unfortunately while the magnetic circuit method is expedient it lacks the accuracy and geometric descriptiveness found in the finite element method. An intermediate approach using tubes of flux and slices of magnetomotive force [89] can produce improved accuracy over the magnetic circuit technique and more convenient handling of geometrical dimensions at less computational cost than the finite element approach. This alternate approach is used in [16] for calculation of airgap and fringe permeance. Based on work done by Hammond [74], it provides an accurate calculation of the permeance very rapidly while using a detailed geometric description of the structure. This method is extended and applied to the modeling of the saturation effects in variable reluctance machines in Chapters 3-5.

**Preview**

The objective of this thesis is to present analysis pertinent to the design of low speed variable reluctance motors. Both motor structure and analysis techniques are discussed. In Chapter 2, three different motor structures are compared based on applicability to low-speed high-torque operation. In Chapter 3 a method for computing permeance of structures containing nonlinear materials is developed. This method, which incorporates Hammond's dual energy techniques, is then applied to computation of operating characteristics of variable reluctance machines. The iterative refinement of dual energy machines is demonstrated in Chapter 4 by successively changing the tubes and slices to improve the accuracy of calculations. Results obtained from the model are compared to those calculated using a lumped magnetic circuit model, a finite element model, and measurements. Using the same model of Chapter 4, a design example is considered in Chapter 5 with reference to the structures discussed in Chapter 2. Finally, conclusions are discussed in Chapter 6.
CHAPTER 2. VARIABLE RELUCTANCE MOTOR STRUCTURES FOR LOW SPEED OPERATION

Electric machines generally do not convert energy efficiently at low speed. Induction machines, for example, are notorious for producing low torque at low speeds. Some types of motors, such as DC machines can produce large torque at low speed. Variable reluctance machines have a nonlinear steady state speed-torque characteristic in which torque increases dramatically as speed decreases. Further improvement in the torque at low speed is possible by implementing structural or winding changes. Many applications have stringent power density requirements and fixed volt-ampere converter capability. This chapter examines the merits of three variable reluctance motors in light of these conditions.

Review of Modifications for High Torque

A means of increasing the torque output of switched reluctance motors is to increase the number of teeth per pole. This technique is described in [52-57]. Increasing the number of stator teeth per pole decreases the step size and reduces the rotational period over which the energy conversion process occurs. In the standard switched reluctance motor each stator pole is comprised of only one tooth. Doubling the number of teeth on each stator pole and subsequently increasing the number of rotor teeth can be accomplished without seriously reducing the maximum inductance when the rotor teeth and stator teeth are aligned. Thus the torque can be nearly doubled by using two teeth per stator pole. Articles [56-57] describe the successful implementation of this concept to produce a high torque direct drive switched reluctance motor.

Unfortunately, an increase in the number of teeth requires a commensurate increase in the resolution of the position sensing and an increase in switching frequency. Besides higher switching losses, higher switching frequency produces higher core losses over an equivalent motor with fewer teeth per pole operating at the same speed.

A large number of teeth per pole is used in vernier reluctance motors [25,58-59]. In the vernier motor a three phase winding is placed over a salient stator structure similar to
that found in a switched reluctance machine. The three phase winding is excited by a three phase sinusoidal source to produce a rotating MMF which moves around the interior of the stator. As in a switched reluctance machine, the rotor of a vernier reluctance motor has a different number of salient poles than that of the stator. The rotor teeth align with the stator teeth as the MMF rotates. The speed of rotation is a function of the ratio of the number of stator to rotor teeth. High torque is produced because large variations in the phase inductances result from small rotor displacement.

The phase windings of the vernier motor span multiple salient stator poles. In contrast, the phase windings of a switched reluctance machine are wound around individual salient stator poles. Furthermore, multiple phases are excited simultaneously in the vernier motor as opposed to the single phase excitation of a conventional switched reluctance machine.

Alternate winding configurations for variable reluctance actuators have been proposed [60-67]. These devices, often referred to as Law's actuators, are two-pole two-phase devices with only two rotor poles. They are capable of only limited rotation, and hence have been restricted in use. Fully rotational Law's actuators are possible, as discussed in the following sections. The Law's actuator operates by bipolar excitation of both phases simultaneously. A comparison of the Law's actuator to a switched reluctance actuator is shown in Fig. 2.1. In the example shown, both phase a and phase b of the Law's actuator are shown excited in Fig. 2.1(a). In Fig. 2.1(b) the equivalent excitation of a switched reluctance actuator is shown. The interpolar space available for each phase coil in the Law's actuator is twice that of the switched reluctance actuator. The Law's actuator can be wound with more turns to produce more torque than the standard switched reluctance actuator. The phases of the Law's actuator are mutually coupled, unlike those of the conventional switched reluctance motor, thus it will be referred to as the mutually coupled reluctance motor (MCR).

A third structure considered is that described in [68-70], shown in Fig. 2.2. Referred to as a current regulated reluctance motor (CRR), it is comprised of a stator similar to that of the switched reluctance motor but with saliency removed. The rotor is made of axial laminations which produce an anisotropic permeance resulting in higher permeability in the shaded direction. The rotor poles aligned with phase a are shown in Fig. 2.2(a), while Fig. 2.2(b) shows the rotor poles unaligned with phase a. Flux does not easily traverse the gaps between laminations, so the unaligned inductance of phase a is lower than the aligned inductance.
With a two-pole two-phase construction, the CRR motor is capable of self starting and fully rotational operation [69]. The windings of the two pole CRR motor span two salient stator poles for the example shown. Only one is excited at a time as distinguished from the Law’s actuator where both phases are excited simultaneously.

![Figure 2.1 Comparison Between Law’s and Switched Reluctance Actuator](image)

![Figure 2.2 CRR Motor with Anisotropic Rotor Aligned (a) and Unaligned (b) With Phase a](image)

**Candidate Structures**

Much controversy has arisen over a comparison between the conventional switched reluctance machine (SRM) and the CRR motor [69-70]. Confusion has arisen due to a comparison made in [68] between a two-pole four-phase SRM and a six-pole two-phase CRR motor. The comparison is complicated by the differences in the number of phases.
and excited poles of each type. It is unclear from [69-70] in what aspects the CRR motor is superior or inferior to the SRM. It is the intent here, however, to evaluate the CRR, SRM and Law's structures on the basis of their low-speed high-torque capability subject to fixed converter capability and equal size constraints. For many vehicular applications these are appropriate constraints.

In order to evaluate the relative merits of the SRM, CRR and MCR structures for the purposes of this study they are first be placed on an common basis and then examined in accordance with appropriate design criteria. Fig. 2.3 shows fully rotational versions of the SRM, CRR and MCR motors. Each motor shown has a three-phase two-pole winding. The number of salient stator poles in each motor is six, being the product of the number of excited poles per phase and the number of phases. Operation of each motor utilizes a converter to supply current from a DC source.

In the SRM motor shown in Fig. 2.3(a), each phase is excited separately. Marked are the coil sides for the top and bottom coils. When a DC current flows in the phase a winding, two opposite stator poles are excited, producing a two-pole spatial MMF distribution in the airgap. The phases are excited for 30° intervals of rotor movement. For simplicity, an ideal rectangular current wave shape with no overlap is assumed. The right column of Fig. 2.3 shows the excitation waveforms for the three phases over a 90° rotation of the rotor. For one revolution of the rotor, there will be 12 switchings; four in each phase.

The MCR motor is shown in Fig. 2.3(b). Two phases are excited simultaneously to produce a two-pole MMF in the airgap. The excitation of both phases a and b, for example, magnetizes two salient stator poles producing a MMF pattern corresponding to that produced by the excitation of phase a in the SRM. Each phase is turned on for 60° and consecutive excitation current pulses overlap with each other for 30° periods. In one rotor revolution, there are 12 switchings, each phase conducting four times. As with a SRM, rotation can be reversed by merely reversing the excitation sequence.

The CRR motor is shown in Fig. 2.3(c). Each phase of the CRR motor conducts for 60° of rotor rotation with no overlap with the conduction of other phases. In one rotor revolution, there will be six switchings, each phase conducting two times.
Figure 2.3 Variable Reluctance Motor Configurations; (a) SRM, (b) MCR, (c) CRR
Criteria of Evaluation

In order to assure a consistent and fair comparison of the three motor types, assumptions are made regarding commonality of the structures. Among the features which are forced to be consistent between the motors being compared are:

1.) Outside diameter, diameter of the rotor, and axial length of the core
2.) Airgap distance
3.) Number of phases and excited poles per phase
4.) Converter volt-ampere limit
5.) Average copper losses for a given DC link current.

In automotive and aerospace applications where power density is important, design constraints such as maximum current draw and maximum available voltage are practical necessities. For the purposes of this study the motors are assumed to be voltage sourced. There are also physical criteria such as maximum continuous current density and thermal limits to avoid accelerated aging of the winding insulation. In the following analysis, consideration is given to these restrictions by assuming that the maximum current draw from the supply is I Amperes and that the supply voltage is limited to $V_{dc}$ Volts. Maximum attainable static torque at the current limit with full voltage applied is examined as an indicator of the relative merit of each structure. Implied is that higher static torque is translatable into higher torque at low speed by suitable converter control. Each structure is assumed to have the same physical dimensions in length and diameter.

Linear Analysis

As in previous studies [68-70] to compare SRM and CRR motors, it is instructive to look at the three motor structures using a linear analysis. Added complications of iron nonlinearities, although important, are ignored for simplicity. A typical application has an available DC voltage supply with $V_{dc}$ volts and maximum allowable current draw I. For very low speed, high torque operation, the available MMF at I is maximized. The maximum MMF is produced by the winding which fills the available space with the greatest number of conductor turns, using conductors of the appropriate current carrying capability. To facilitate reading, quantities are denoted by a subscript of 1, 2, and 3 for the SRM, MCR and CRR motors respectively.
Equivalence of Motor Copper Losses

As a demonstration of the equivalence of the three motor structures it is instructive to examine the average phase copper losses in each motor. The copper loss in each phase is calculated using the RMS current in the phase. The RMS current is obtained from consideration of the ideal phase excitation period shown in Fig. 2.4.

![Figure 2.4 Single Period of Phase Excitation](image)

The RMS current is computed from the definition of RMS current [96]:

$$I_{\text{RMS}} = \sqrt{\frac{1}{T} \int_{0}^{T} i^2 dt} = \sqrt{\frac{1}{\theta_{\text{cyc}}} \int_{0}^{\theta_{\text{cyc}}} i^2 d\theta} = i \sqrt{\frac{\theta_{\text{on}}}{\theta_{\text{cyc}}}}. \quad (2.1)$$

A comparison of the RMS currents for each of the three motor structures is given in Table 2.1. Each motor structure has a different combination of conduction angles and peak current. Each motor is compared on the basis of A Amperes flowing through the DC link.

<table>
<thead>
<tr>
<th>Motor Type</th>
<th>$i$</th>
<th>$\theta_{\text{on}}$</th>
<th>$\theta_{\text{cyc}}$</th>
<th>$I_{\text{RMS}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>SRM</td>
<td>1</td>
<td>$\pi/6$</td>
<td>$\pi/2$</td>
<td>$I_{\sqrt{3}}/3$</td>
</tr>
<tr>
<td>MCR</td>
<td>1/2</td>
<td>$\pi/3$</td>
<td>$\pi/2$</td>
<td>$I_{\sqrt{6}}/6$</td>
</tr>
<tr>
<td>CRR</td>
<td>1</td>
<td>$\pi/3$</td>
<td>$\pi$</td>
<td>$I_{\sqrt{3}}/3$</td>
</tr>
</tbody>
</table>

To complete the comparison of copper losses, the resistances of the motor phases are needed. Since the DC link current is limited to $I$ Amperes, the phase windings must be
wound so as to limit the current to this value under static conditions. In the case of the SRM with \( P \) excited poles per phase, \( P \) coils connected in series form a phase winding. Each coil would at most require a resistance in a static situation of \( R_1 \), given by

\[
PR_1 = \frac{V_{dc}}{I}. \quad (2.2)
\]

In the MCR motor, two phase coils are excited simultaneously. Assuming equal current draw in each phase, that is \( I/2 \), the corresponding resistance of each phase coil will have to be

\[
\frac{P}{2} R_2 = \frac{V_{dc}}{I/2}, \text{ or } R_2 = 4R_1. \quad (2.3)
\]

Finally, in the CRR motor, with only one phase excited at a time and only one coil per excited pole pair in each phase, the corresponding phase coil resistance will have to be

\[
\frac{P}{2} R_3 = \frac{V_{dc}}{I}, \text{ or } R_3 = 2R_1. \quad (2.4)
\]

Utilizing (2.2)-(2.4) with information from Table 2.1, the average phase copper loss for each of the motors can be expressed as \( P_{RMS1}^2 R_1 \). Thus, with the phase coil resistance in these ratios, it can then be shown that the total DC copper losses in all three machines are equal when the current draw is the same. Hence, even for dynamic operation where inductive drop is no longer negligible, as long as the current draw from the supply is the same and coil resistances are in these ratios, all three motor structures have the same DC copper losses in their windings.

**Turns Ratio of MCR Motor to SRM Motor**

Assuming coil resistance in the ratios given above, the corresponding turns, the MMF, and also the static torque, are maximized when the winding fills all of the available space with the greatest number of conductor turns. Let the turns per coil in the SRM be \( N_1 \), or \( PN_1 \) turns per phase for motors with \( P \) excited poles per phase, and let the coil fill all of the interpolar space. The resistance of a coil in the SRM may be expressed as:

\[
\text{Resistor Coefficient for SRM: } R_{res} = \frac{V_{dc}}{I}. \quad (2.5)
\]

In the MCR motor, the turns ratio is

\[
\frac{P}{2} R_2 = \frac{V_{dc}}{I/2}, \text{ or } R_2 = 4R_1. \quad (2.3)
\]

Finally, in the CRR motor, with only one phase excited at a time and only one coil per excited pole pair in each phase, the corresponding phase coil resistance will have to be

\[
\frac{P}{2} R_3 = \frac{V_{dc}}{I}, \text{ or } R_3 = 2R_1. \quad (2.4)
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Utilizing (2.2)-(2.4) with information from Table 2.1, the average phase copper loss for each of the motors can be expressed as \( P_{RMS1}^2 R_1 \). Thus, with the phase coil resistance in these ratios, it can then be shown that the total DC copper losses in all three machines are equal when the current draw is the same. Hence, even for dynamic operation where inductive drop is no longer negligible, as long as the current draw from the supply is the same and coil resistances are in these ratios, all three motor structures have the same DC copper losses in their windings.
\[ R_1 = \frac{\rho_w N_1 l_1}{a_w l_1} \quad (2.5) \]

where \( \rho_w \) is the resistivity of the wire, \( A_1 \) is the available slot area for the coil, \( l_1 \) is the mean turn length, and \( a_w \) is the cross-sectional area of the wire. The number of turns is determined by

\[ N_1 = \frac{f A_1}{a_w l_1} \quad (2.6) \]

where \( f \) is a maximum slot fill factor. Combining (2.5) and (2.6) gives

\[ R_1 = \frac{\rho_w f A_1 l_1}{a_w l_1} \quad (2.7) \]

Similarly for the MCR,

\[ R_2 = \frac{\rho_w N_2 l_2}{a_w 2 l_2} = \frac{\rho_w f A_2 l_2}{a_w 2 l_2} \quad (2.8) \]

Since \( R_2 = 4R_1 \),

\[ \frac{\rho_w f A_2 l_2}{a_w 2 l_2} = 4 \frac{\rho_w f A_1 l_1}{a_w 2 l_1} \quad (2.9) \]

so that

\[ \frac{a_w l_1}{a_w 2} = 2 \sqrt{\frac{A_1 l_1}{A_2 l_2}} \quad (2.10) \]

Using (2.6) and (2.10), the ratio of turns between the MCR and the SRM motor is

\[ \frac{N_2}{N_1} = \frac{\left( \frac{f A_2}{a_w 2} \right)}{\left( \frac{f A_1}{a_w l_1} \right)} = \frac{A_2 a_w l_1}{A_1 a_w 2} = 2 \frac{A_2}{A_1} \sqrt{\frac{A_1 l_1}{A_2 l_2}} = 2 \sqrt{\frac{A_2 l_1}{A_1 l_2}} \quad (2.11) \]
The available space for the coil side in the MCR is twice that in SRM, or \( A_2 = 2A_1 \).

Thus from (2.11), each coil of the MCR motor can have as many as \( 2\sqrt{2} \) times the turns of each SRM coil. This is the upper bound on the turns ratio, because the end turns of the MCR motor would be longer than those in the SRM reducing the ratio \( l_1/l_2 \).

### Turns Ratio of CRR Motor to SRM Motor

Following a similar analysis using the ratio of coil resistance between the CRR motor and the SRM motor yields

\[
\frac{\rho_w f A_3 l_3}{a_{w3}} = 2\frac{\rho_w f A_1 l_1}{a_{w1}},
\]

and

\[
\frac{a_{w1}}{a_{w3}} = \sqrt{2} \frac{A_1 l_1}{A_3 l_3}.
\]

The turns ratio of the CRR motor to the SRM is

\[
\frac{N_3}{N_1} = \left( \frac{f A_3}{a_{w3}} \right) = \frac{A_3 a_{w1}}{A_1 a_{w3}} = \frac{A_3}{A_1} \sqrt{2} \frac{A_1 l_1}{A_3 l_2} = \sqrt{2} \frac{A_2 l_1}{A_1 l_3}.
\]

Again, the available slot area for the winding in the CRR motor is twice that of the SRM; thus, the CRR motor can have up to twice the number of turns of the SRM.

### Maximum Inductance Ratio of MCR Motor to SRM Motor

The MMF across one pair of excited poles in the SRM is \( 2N_1 l \) since each phase has two coils of \( N_1 \) turns each. That of the MCR motor is \( 2N_2 l/2 \), which accounts for the two phases carrying \( l/2 \) across a pole pair. Using (2.11), the relationship between the applied MMF of the MCR motor, \( F_2 \), and that of the SRM motor, \( F_1 \), is

\[
F_2 = 2N_2 \frac{l}{2} = \frac{4}{2} \frac{A_2 l_1}{A_1 l_2} N_1 \frac{l}{2} = 2 \frac{A_2 l_1}{A_1 l_2} N_1 l = \sqrt{2} \frac{A_2 l_1}{A_1 l_2} F_1.
\]
Ignoring the reluctive drop in the iron, the total reluctance of the two airgaps along the path of an excited pole pair at full overlap can be expressed as

\[ R = \frac{2g}{\mu_0 Z r g / \theta g} \]  

where \( g \) is the airgap length, \( r_g \) the radius to the airgap, \( Z \) the axial length, \( \theta_g \) the overlap angle of the airgap, and \( \mu_0 \) the permeability of air. Let the gap radii and overlap angle of the SRM and the MCR motor be the same for this comparison. The maximum inductance per pole pair for the SRM is

\[ L_{\text{max}1} = \frac{(2N_1)^2}{R} = \frac{4N_1^2}{R}. \]  

The maximum inductance per pole pair of the MCR motor should include the effects of mutual coupling between the two excited phases. Rather than examining the inductances in terms of self and mutual components, it is simpler to determine the equivalent maximum inductance as seen by the supply source by considering a magnetically equivalent SRM. From Fig 2.1, it is apparent that an MCR motor with \( N_2 \) turns per coil side excited with \( I/2 \) in each coil side is magnetically equivalent to \( I/2 \) flowing in a single phase of an SRM with \( N_2 \) turns per coil. Therefore the maximum inductance per pole pair of the MCR motor is the same as that of an SRM having \( N_2 \) turns per coil. Thus,

\[ L_{\text{max}2} = \frac{(2N_2)^2}{R}. \]  

which yields

\[ L_{\text{max}2} = \frac{4N_2^2}{R} = \frac{16 \left( \frac{A_2l_1}{A_1l_2} \right) N_1^2}{R} = 4 \left( \frac{A_2l_1}{A_1l_2} \right) L_{\text{max}1}. \]  

With \( A_2 = 2A_1 \) the maximum inductance of the MCR motor could be as much as eight times the inductance of the SRM motor.
Maximum Inductance Ratio of CRR Motor to SRM Motor

The MMF of a pole pair in the CRR motor is produced by the excitation of a single phase coil. Using (2.14) and equating the MMFs of the CRR to that of the SRM yields

\[ F_3 = N_3 I = \sqrt{\frac{2 A_{3 I_3}}{A_{1 I_3}}} N_1 I = \sqrt{\frac{2 A_{3 I_1}}{A_{1 I_3}}} F_1. \]  

(2.20)

To compare the inductances of the SRM and CRR, equation (2.17) can be rewritten using (2.16) as

\[ L_{\text{max} 1} = \frac{4 N_1^2}{\left( \frac{2 g}{\mu_0 Z r_g \theta_{g 1}} \right)}. \]

(2.21)

The maximum inductance per pole pair of the CRR motor with only \( N_3 \) turns per coil is given by

\[ L_{\text{max} 3} = \frac{N_3^2}{\left( \frac{2 g}{\mu_0 Z r_g \theta_{g 3}} \right)}. \]

(2.22)

Thus the ratio of the maximum inductances for the CRR and SRM motors is given by

\[ \frac{L_{\text{max} 3}}{L_{\text{max} 1}} = \frac{N_3^2 \theta_{g 3}}{4 N_1^2 \theta_{g 1}} = \left( \frac{A_{3 I_3}}{2 A_{1 I_3}} \right) \frac{\theta_{g 3}}{\theta_{g 1}}. \]

(2.23)

The ratio of the stator pole arc, \( \theta_{s 1} \), to pole period, \( \theta_{p 1} \), illustrated in Fig. 2.5 is typically 0.4 for an SRM [32]. Since the CRR poles are not salient the width of a CRR stator pole is \( 8_s = \theta_{g 1} / 0.4 \). If each coil of a phase winding in a CRR motor is wound around \( P_s \) salient stator poles, the stator pole overlap, \( \theta_{g 3} \), of the CRR motor spans \( P_s \theta_{p 1} \). The ratio of overlap angles is therefore

\[ \frac{\theta_{g 3}}{\theta_{g 1}} = \frac{P_s \left( \frac{\theta_{g 1}}{0.4} \right)}{\theta_{g 1}} = 2.5 P_s. \]

(2.24)
Substituting (2.24) into (2.23) and using $A_3 = 2A_1$, the maximum inductance of the CRR motor could then be as much as $2.5P_S$ times that of the SRM.

![Figure 2.5 Pole Arc Dimensions](image)

Toque Ratio of MCR Motor to SRM Motor

The average torque $T$ per pole pair of an SRM can be written as [68]:

$$T = \frac{1}{2} i^2 \frac{k_L L_{\text{max}}}{\Delta \theta_r} \tag{2.25}$$

where

$$k_L = 1 - \frac{1}{L_{\text{max}} / L_{\text{min}}} \tag{2.26}$$

The angle $\Delta \theta_r$ is the rotor rotation between the minimum and maximum inductance positions. For the comparison between the SRM and the MCR motor, the value for $k_L$ is the same since the rotor and stator construction are the same. For an SRM, $k_L$ is typically 0.9 [68].

Since $\Delta \theta_r$ is the same for the SRM and MCR and assuming that the phase current in the SRM is twice the phase current in the MCR, the ratio of their static torque can be found by using (2.19) and (2.25):

$$\frac{T_2}{T_1} = \frac{L_{\text{max}2}}{4L_{\text{max}1}} = \left( \frac{A_2}{A_1} \right)^2 \tag{2.27}$$

Thus, based on this linear analysis, with $A_2 = 2A_1$, the MCR motor could produce up to twice the static torque of an SRM of similar dimensions.
Torque Ratio of CRR Motor to SRM Motor

The torque ratio of the CRR to SRM is

\[
\frac{T_3}{T_1} = \frac{1}{2} \frac{k_{L3} L_{\text{max}3}}{k_{L1} L_{\text{max}1}} \frac{\Delta \theta_{r3}}{\Delta \theta_{r1}} = \frac{\Delta \theta_{r3} k_{L3} L_{\text{max}3}}{\Delta \theta_{r1} k_{L1} L_{\text{max}1}} \frac{L_{\text{max}3}}{L_{\text{max}1}} = \frac{A_{3} L_{3}}{2 A_{1} L_{1}} \frac{L_{\text{max}3}}{L_{\text{max}1}} \frac{\theta_{3}}{\theta_{1}}.
\]

(2.28)

Using the value for \(k_{L3}\) of 0.8 given in [68] for the CRR motor and a \(k_{L1}\) of 0.9 for the SRM, \(\Delta \theta_{r3} = 2 \Delta \theta_{r1}\), and the values of \(\theta_{3}\)'s and \(A\)'s already mentioned, it can be shown that the static torque produced by the CRR motor can be as much as 1.11\(P_S\) times the torque produced by the SRM. For the motors of Fig. 2.3, with \(P_S=3\), the CRR motor can produce up to 3.3 times the torque of a similarly dimensioned SRM. However, since \(I_3\) is significantly greater than \(I_1\), the actual factor will be less than 3.3. Motors with more than two excited poles per phase will have less difference in the end turn lengths and hence (2.29) will be closer to the ideal limit. The machine studied in [68] has a six pole stator winding and a \(P_S\) of 2. Thus by (2.28) this CRR motor can produce up to 2.2 times the torque of a similar size SRM motor with the same number of excited poles and phases. This factor of 2.2 is only slightly higher than that of 1.997 determined in [68] based on different criteria of comparison between a two-phase six-pole CRR motor and a four-phase two-pole SRM.

**Summary**

A summary of the results from the above linear analysis is shown in Table 2.2. In this analysis, an attempt has been made to make the evaluation on a common basis: equality of outer dimensions, airgap, number of excited poles per phase, number of phases, and also DC copper losses, subject to the same constraints on current draw and operating voltage. It makes no attempt to optimize the power density of the candidate motors, but instead evaluate the excitation requirements and torque capability of these motors with a given dimension. Certain assumptions made here may not be applicable to all situations, as such the results given have to be interpreted with care.

The factors of torque improvement of the CRR and MCR motors would be reduced when the axial length is short compared to the diameter, because end turn lengths, not considered in this study, will become significant. The MCR motor clearly has the
advantage over the SRM as long as the ratio $\frac{l_2}{l_1}$ is kept small. As the number of poles is increased or the diameter to length ratio is decreased, the torque of the MCR motor approaches the maximum improvement shown in Table 2.2. The inclusion of iron saturation will also reduce the apparent benefits shown for the MCR motor and the CRR rotor. Any comparison of saturated condition should be made for motors of the same number of excited poles per phase.

The MCR motor has a simpler rotor construction than the CRR motor. Although the torque ratio of the CRR motor is larger than that of the MCR motor, the axially laminated CRR rotor will be more expensive. For this reason, the MCR motor is considered as a candidate for replacement of a windshield wiper motor in Chapter 5.

Table 2.2 Comparison of Variable Reluctance Structures

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>SRM Motor</th>
<th>MCR Motor</th>
<th>CRR Motor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Phase Resistance</td>
<td>PR_1</td>
<td>2PR_1</td>
<td>PR_1</td>
</tr>
<tr>
<td>Turns per Coil</td>
<td>N_1</td>
<td>$2\sqrt{\frac{A_2l_1}{A_1l_2}}N_1 \leq 2\sqrt{2}N_1$</td>
<td>$2\sqrt{\frac{A_3l_1}{A_1l_3}}N_1 \leq 2N_1$</td>
</tr>
<tr>
<td>Effective MMF</td>
<td>$F_1$</td>
<td>$\sqrt{\frac{A_2l_1}{A_1l_2}}F_1 \leq \sqrt{2}F_1$</td>
<td>$\sqrt{\frac{A_3l_1}{2A_1l_3}}F_1 \leq F_1$</td>
</tr>
<tr>
<td>Peak Inductance</td>
<td>$L_{\text{max}}$</td>
<td>$4\left(\frac{A_2l_1}{A_1l_2}\right)l_{\text{max}} \leq 8L_{\text{max}}$</td>
<td>$\frac{A_3l_1}{2A_1l_3} \theta_{g1}L_{\text{max}} \leq 2.5P_{s}L_{\text{max}}$</td>
</tr>
<tr>
<td>Static Torque</td>
<td>$T_1$</td>
<td>$\left(\frac{A_2l_1}{A_1l_2}\right)T_1 \leq 2T_1$</td>
<td>$\frac{\Delta\theta_{g1}k_{L3}}{\Delta\theta_{g1}k_{L1}} \left(\frac{A_3l_1}{2A_1l_3}\right)T_1 \leq 1.11P_{s}T_1$</td>
</tr>
</tbody>
</table>
CHAPTER 3. CALCULATION OF PERMEANCE FOR THREE DIMENSIONAL REGIONS CONTAINING NONLINEAR MAGNETIC MATERIAL

The torque for the machines discussed in Chapter 2 can be determined accurately from a solution of the magnetic field distribution of the machines. Since the steel used in their construction has nonlinear properties it is essential that the computational method used to determine the field be able to incorporate nonlinear effects. Chapter 1 detailed the various methods which have been used to study variable reluctance structures. This chapter describes and extends Hammond's dual energy method \cite{72} to the calculation of permeance for three dimensional regions containing magnetically nonlinear material. This method is then applied to the study of variable reluctance structures in Chapters 4 and 5. It contains material published in \cite{90}.

**Background**

Many electric machines and actuators have complex structures and fields that vary significantly in three dimensions, and as such can not be adequately represented in two dimensions. Although three-dimensional finite element and boundary element programs can accurately analyze these problems, such programs are computationally intensive and are often too time consuming for design analysis.

Roters \cite{24} described a method for approximating the permeances in linear three dimensional problems by assuming flux path shapes. Commensurate with the magnetic circuit approach, Roters treated the permeances of the flux paths as separate circuit elements. While this method has the merit of simplicity, it does not account for the effects of the field distribution in saturable materials or for deviations of the actual field from the assumed flux paths. It therefore does not provide the necessary accuracy when faced with complex and irregular geometric details.

More recently Hammond\cite{71-72} proposed a dual energy method based on an approach developed earlier by Lord Rayleigh \cite{73-74} and later used by Maxwell \cite{75} to find the resistance of an irregularly shaped object. This method divides the region under investigation into tubes of constant flux and slices of constant magnetomotive force.
References [83-88] give substantial detail about theoretical and computational aspects of tube and slice calculations. The two representations, tubes and slices, are complementary descriptions which result in upper and lower bounds to the field energy. Besides providing bounds to the solution, the dual energy method has the computational advantage of using simple integrations as opposed to solution of large matrix equations in finite element methods.

Other advantages of using the dual energy concept in finite element formulations are discussed in [76-80]. Dual energy techniques are also suitable for computing time harmonic fields [81-82].

Sykulski [89] successfully developed an interactive program of the dual energy method for studying two dimensional linear problems and demonstrated that this technique can rapidly and accurately determine field parameters of complex structures. The quick solution of the dual energy method makes it suitable for design of switched reluctance machines as demonstrated by one recent paper [16]. Unfortunately, the discretization scheme described in [84,89] can not easily be extended to three dimensional problems. The capability of the dual energy method for handling nonlinear materials has been discussed in the context of calculating the energy bounds but not the permeances [79], and an efficient implementation has been demonstrated only for two dimensional problems using a finite element approach.

This chapter details a technique for applying the dual energy method to the calculation of permeances for three dimensional regions containing nonlinear material. It extends the application of the dual energy concept by using a discretization technique that simplifies the implementation in both two and three dimensional problems. It describes special considerations required for handling nonlinear materials. Also presented is a derivation of the tube and slice permeance integrals from Ampere’s and Gauss’ Laws. Finally, as a validation of the proposed method, it compares the calculated flux in a simple electromagnet to that measured and computed from a linear analysis based on a traditional lumped magnetic circuit technique.

### Analysis Technique

Ampere’s and Gauss’ Laws form the basis of the dual equations for magnetostatic fields. Flux tubes and MMF slices describe the field in complementary fashion as shown in Fig. 3.1. A flux tube contains a constant amount of magnetic flux in a division of the space that surrounds a field source. A MMF slice is a division of the space through which
all flux must flow and across which is a constant amount of MMF.

![Figure 3.1 Concept of Flux Tubes and MMF Slices](image)

**Permeance** Calculated Using Flux Tubes

**Derivation of the Permeance Integral**

A tube of flux is a volume through which a contiguous flux flow occurs. The closed path of each flux tube consumes the total amount of applied MMF along its length. Breaking the field into tubes of constant flux results in a lower bound to the permeance, $P_z$, seen by the source. Ampere's Law gives the integral of the magnetic field intensity, $H$, along a flux tube with differential length, $dL$, that is

$$\oint H \cdot dL = NI, \quad (3.1)$$

where $NI$ is the product of conductor turns $N$ and current $I$.

Tube flux, $\Phi_t$, is the independent variable for the lower bound calculation. The field intensity is related to the tube flux by the reluctivity $\nu$ and tube cross-sectional area $A$,

$$H = \nu B = \nu \frac{\Phi_t}{A}. \quad (3.2)$$

The differential length, $dL$, is assumed to be along the normal to the cross-sectional area at every point along the tube, thus simplifying the dot product in (3.1) to a simple product. The integral relationship between the flux and the dependent variable, magnetomotive force $F$, becomes
where \( R_t \) is the tube reluctance or the inverse of tube permeance, \( P_t \). Along a given flux tube the flux, \( F_t \) is constant. Factoring \( F_t \) out of (3.3) yields

\[
\int \frac{\Phi_t}{A} dL = F_t = \Phi_t R_t .
\]  

(3.3)

The value of \( R_t \) from (3.4) is larger than the actual tube reluctance because of the approximation in discretization: the differential length \( dL \) used in the integration is not always perpendicular to the cross-sectional area \( A \) in an irregular path. If they are not perpendicular, the dot product yields a smaller quantity than that given by (3.4). Additionally, if a path of integration is chosen which does not follow the true field lines, Eq. (3.4) represents a larger quantity than the true dot product in (3.1). Thus the computed tube reluctance is larger than or equal to the actual value. Since the tube permeance \( P_t \) is the inverse of the tube reluctance, it will be smaller than the actual tube permeance. Each tube permeance can be calculated separately. The total region permeance, \( P_z \), as seen by the source is given by the summation of all the tube permeances in parallel, that is

\[
P_z = \sum P_t = \sum \frac{1}{R_t} .
\]

(3.5)

Since each tube permeance is less than or equal to the actual tube permeance, the total permeance \( P_z \) would be less than or equal to the actual region permeance. Thus, the value of \( P_z \) from (3.5) can also be regarded as a lower bound to the permeance seen by the source.

**Nonlinear Materials**

The reluctivity, \( \nu \), describes the nonlinear magnetic behavior of the material in terms of \( \mathbf{B} \) and \( \mathbf{H} \). With nonlinear materials the permeance is a function of the tube flux \( \Phi_t \). In determining the region permeance for a value of total flux \( \Phi \) (the sum of all tube fluxes), it is necessary to properly divide the total flux into each tube flux. Improper disbursement of
the flux could result in inequitable tube saturation. The total flux distributes among the
tubes according to their relative permeances just as current divides among parallel resistors.
Since the tube permeances are functions of the tube flux density an iterative approach is
necessary. Beginning at the zero flux condition where there is no saturation, the
permeances can be calculated. As the total flux is incremented from zero to a desired value,
the distribution of incremental flux in each tube is determined by the relative permeance of
the tubes from the previous flux level. At flux step i each tube flux, $\Phi_i^i$, is computed by
apportioning the change in flux $\Delta \Phi$ using the ratio of the tube permeance, $P_i^{i-1}$, to the total
permeance, $P_z^{i-1}$, of the previous step $i-1$ according to the relationship

$$\Phi_i^i = \Phi_i^{i-1} + \frac{P_i^{i-1}}{P_z^{i-1}} \Delta \Phi,$$

where $\Phi_i^{i-1}$ is the tube flux at the previous flux step.

**Permeance Calculated Using MMF Slices**

**Derivation of the Permeance Integral**

The calculation of an upper bound to the permeance follows much the same
development as the lower bound calculation. Slices of constant MMF connected in series
divide the entire region. The outer boundary of the slices is defined by the outermost
boundary of the tubes, with all the flux $\Phi$ of the system flowing through every slice.

From Gauss' Law,

$$\int \vec{B} \cdot d\vec{A} = \Phi. \quad (3.7)$$

The flux density $B$ can be expressed in terms of the MMF, that is

$$B = \mu H = \mu \frac{E_x}{L}, \quad (3.8)$$

where $L$ is the slice length and $\mu$ is the permeability. Again, assuming that the
incremental slice area is perpendicular to the slice length, (3.7) simplifies to
\[
\int \mu \frac{F_s}{L} dA = \Phi = F_s P_s.
\]

(3.9)

The MMF \( F_s \) being constant for a given slice can be factored out of (3.9), yielding the following expression for the slice permeance \( P_s \):

\[
\int \mu \frac{dA}{L} = P_s.
\]

(3.10)

By the same reasoning applied to the tube permeance integral, (3.10) will yield a larger permeance than the actual slice permeance whenever the slice length is not orthogonal to the slice cross-sectional area.

The total permeance, \( P_y \), as seen by the source is obtained by combining the slice permeances in series, that is

\[
P_y = \left( \sum \frac{1}{P_s} \right)^{-1}.
\]

(3.11)

The value of \( P_y \) from (3.11) can be regarded as an upper bound to the actual permeance seen by the source.

**Nonlinear Materials**

With nonlinear materials, the permeability \( \mu \) is a function of field intensity \( \overline{H} \) making the permeance a function of the MMF. The total MMF, \( F \), is the independent variable in this case. Analogous to the tube calculations, the correct amount of MMF must be apportioned to the slices. The fraction of the MMF across a given slice is in the ratio of slice reluctance to total reluctance. As the MMF is raised from zero to the desired value in steps, the slice MMFs at the new MMF level are determined using the permeances at the previous MMF value, that is

\[
F_s^i = F^i \left[ \frac{(P_s^{i-1})^{-1}}{(P_y^{i-1})^{-1}} \right].
\]

(3.12)
Implementation
In the implementation of (3.4) and (3.10) the discretization must follow the field distribution in shape. Elements are selected to represent tubes and slices of the field. Hexahedral elements of the Serendipity type [91] are used to discretize the domain. In the finite element method the objective is to compute the field potential at each node. The discretization of the region into flux tubes and MMF slices, however, defines the field geometry and facilitates numerical integration for the permeance.

Shown in Fig. 3.2(a) is a 32 node hexahedral element defining a portion of the field. Since the shape of each element is unique, a parametric representation of the element geometry, described in [91] and shown in Fig. 3.2(b), greatly simplifies the numerical integration. The curvilinear surfaces of the tube and slice elements can be represented by appropriate cubic parametric representations. Shape functions defined in Appendix I transform points within the parametric cube in the \((s,u,t)\) space to corresponding points of the actual element in the \((x,y,z)\) space. The permeance integrals (3.4) and (3.10) are evaluated using Gaussian quadrature integration in the \((s,u,t)\) space. The regular shape and size of the cube in the parametric representation simplify such calculations.

To maintain consistency in the integration, the orientation and the numbering of the nodes of the parametric element should correspond to that of the actual element as shown in Fig. 3.2(b). Flux flows parallel to the \(t\) axis through surfaces mapped by the \((s,u,t = \text{constant})\) plane.

Calculation of the Flux Tube Permeance Integral
The reluctance integral (3.4) is evaluated using four-point Gaussian quadrature integration along the length of each element. The numerical integration requires a function evaluation at each of the quadrature points as described in [91]. The function being iterated is the quotient of the reluctivity and the cross-sectional area. At each integration point the cross-sectional area is computed using

\[
A = \int_1^1 |\mathcal{J}_A| dsdu
\]

(3.13)

where the Jacobian relationship \(|\mathcal{J}_A|\) relates the differential area \(dsdu\) to the differential area \(dA\) in \((x,y,z)\) space as described in Appendix A. The reluctivity \(v\), being a function of the flux density, also varies along the path of integration. The reluctivity is calculated from the
nonlinear magnetization curve that is piece-wise represented by cubic splines described in [94].

![Image](tube_flux.png)

(a) Element of Tube or Slice Discretization  (b) Parametric Representation of Element

Figure 3.2 Discretization of the Flux Tubes and MMF Slices

The reluctance integral (3.4) is evaluated using a similar relationship,

\[
\int_{0}^{L} \frac{v}{A} dL = \int_{-1}^{1} \frac{v}{A} |J_L| dt,
\]

(3.14)

where \(|J_L|\) is a Jacobian relationship between \(dL\) and \(dt\) as described in Appendix A. The length Jacobian, the cross-sectional area, and the reluctivity are computed at each integration point since they vary with position within the element. Since the reluctivity is the only part of the integral which changes with flux level, \(|J_L|\) and \(A\) can be precalculated for each quadrature point in each tube element and used for all subsequent levels.

Calculation of the MMF Slice Permeance Integral

The calculation of MMF slice permeance for each element is similar to the calculation of tube reluctance. The element length at the various integration points is obtained from

\[
L = \int_{-1}^{1} |J_L| dt.
\]

(3.15)
For a given MMF level and value of L from (3.15), the field intensity can be determined. Knowing the field intensity, the corresponding permeability is found from a piece-wise spline fit to magnetization data. The permeance integral (3.10) can then be evaluated using the area Jacobian $|U_A|$ given in Appendix A,

$$\int_{A} \frac{M}{L} dA = \int_{-1}^{1} \int_{-1}^{1} |J_A| ds du.$$  \hspace{1cm} (3.16)

Since the MMF affects only the permeability in (3.16), $|J_A|$ and $L$ need only be calculated once and may be reused at all subsequent MMF levels.

**Example**

To determine the efficacy of this method, a simple electromagnet shown in Fig. 3.3, was modeled. The electromagnet consisted of a U-shaped laminated core with a 150-turn excitation winding and a rectangular laminated armature with a 15-turn search coil. Appendix B lists the magnetization data taken from [93] and the measured dimensions used to model the device.

The two tubes and 10 slices used for half of the electromagnet are shown in Fig. 3.3: each tube consists of 10 elements in series; each slice two elements in parallel. The space modeled by the tubes and slices occupies the regions containing the bulk of the field of interest. However, if necessary, additional tubes and slices can be introduced to improve the approximation of the leakage and fringing fields. With tubes and slices over only a half of the device, calculated permeance is half the actual permeance.

The computed flux vs. MMF curves of the electromagnet were compared with measurements of the actual device and with analytical predictions from a simple linear model. Experimentally, flux in the armature was measured by integrating the induced voltage of the search coil caused by a time-varying current excitation of the core. As the sinusoidal excitation current was increased in steps, the corresponding peak values of flux and excitation current were recorded. The measured results are shown in Figs. 3.4-3.7. In the simple linear magnetic circuit analysis of the device the airgap is significant. With no airgap between the armature and the electromagnet, the measured and calculated results compare favorably as shown in Fig. 3.4. The upper and lower bounds maintain a good approximation to the measured data even as the iron saturates. As an indication of the solution speed, to generate the data shown in Fig. 3.4 required approximately 3.5 minutes.
on a personal computer.

With a small 0.18 mm airgap, the upper and lower bounds consistently contain the measured results as shown in Fig. 3.5. Agreement with the linear model is also very good before saturation occurs.

![Discretization of One-Half of the Electromagnet](image)

Figure 3.3 Discretization of One-Half of the Electromagnet

With a larger airgap there is an underestimation of the flux by the computed bounds as shown in Figs. 3.6-3.7. This discrepancy could be due to an increase in leakage flux through paths not modeled. For example, the end fringing flux between the armature and the core.

**Summary**

This chapter presents a numerical method for computing the upper and lower bounds to the permeance of a three dimensional region containing nonlinear magnetic material. It extends the concept of a complementary bounded field solution by using parametric element representations to compute the permeance integrals and also describes techniques for handling material nonlinearity in the computation.

The accuracy and simplicity of this method make it suitable for analysis and design. Computation time is at least an order of magnitude less than required by finite element methods making it useful for design applications. Further work on this modeling technique could include the incorporation of permanent magnet materials, partial flux
Figure 3.4 Magnetization Curve for an Airgap of 0 mm

Figure 3.5 Magnetization Curve for an Airgap of 0.18 mm
Figure 3.6 Magnetization Curve for an Airgap of 0.36 mm

Figure 3.7 Magnetization Curve for an Airgap of 0.54 mm
linkages and time harmonic fields. Another potential area for research is the synthesis of tubes and slices for complicated devices containing multiple sources.

The representation of magnetostatic fields by tubes of flux and slices of $\text{MMF}$ enables the user to better understand the influence of device geometry on the field distribution. Development of a dual energy model for a structure requires thoughtful selection of tube and slice boundaries. The subsequent calculation of the tube and slice permeances enables the designer to efficiently determine the portions of the machine which are most susceptible to saturation. Lastly, the bounded solution which results can be used as an indicator of the accuracy of the tube and slice selections: by adjusting the tubes and slices to reduce the difference in upper and lower bounds, the field model can be progressively improved.

The representation of the field by tubes of flux and slices of $\text{MMF}$ ties the model directly into the solution. Unlike the finite element technique where the structure is discretized in accordance with the material regions, the dual energy discretization must follow the field distribution for the best accuracy. Thus, by refining a dual energy model to reduce the difference in the bounds, the user becomes aware of the impact of the structure being modeled on the field distribution. The refinement process is illustrated in Chapter 4, where the dual energy method is applied to the study of variable reluctance motors.
CHAPTER 4. MODELING VARIABLE RELUCTANCE MOTORS WITH THE DUAL ENERGY METHOD

The application of the dual energy method to variable reluctance motor (VRM) analysis is demonstrated in this chapter. The first step is the development of a preliminary tube and slice model. The disparity of the upper and lower bounds in the prediction of flux linkage is used as a gauge of the accuracy of the model. From an understanding of the influence of tube and slice shapes on the upper and lower bounds, the tubes and slices are altered to obtain better results. For illustrative purposes, the tubes and slices are altered in succession. The progressive modification of tubes first and slices next, results in added insight into the field distribution and the importance of proper tube and slice selection. These two steps illustrate the iterative process that can be continued for further improvement as needed. For most design purposes, a few iterations of the kind shown in this chapter would usually be adequate.

The static torque calculated by the model is compared to results obtained from a traditional magnetic circuit approach, a finite element method, and experimental measurements. Flux linkages are also used to compare the efficacy of the modifications to the model.

Model Development

The simultaneous excitation of opposite poles in a switched reluctance motor results in most of the field lines crossing the overlap region of the excited poles as illustrated in Fig. 4.1. As a result there is very little mutual coupling between the phases. In the mutually coupled reluctance machine of Fig. 2.3(b), the excitation is equivalent to that of the VRM as long as the phase currents are identical. For the purposes of calculating the static torque this condition is assumed.

Since the field is almost entirely confined to the overlapping region of the set of excited poles, the complexity of the model can be reduced by ignoring the other rotor and stator poles where almost no flux flows. In machines with more than two poles, only one
excited pole pair need be represented. Additionally, a plane of symmetry can be drawn, as shown in Fig. 4.1, so that only one-half of that region need be modeled. 

![Diagram](image)

**Figure 4.1** Typical SRM Phase Excitation

The model is based on the techniques discussed in Chapter 3, but is limited to a two-dimensional analysis. Since most switched reluctance machines have relatively narrow airgaps and are long when compared to their diameters, the end effects do not play a significant role in their operation. This is especially true for the mutually coupled reluctance machine in which large length to diameter ratios are advantageous as noted in Chapters 2 and 5.

To construct a dual energy model for variable reluctance machines in accordance with the technique described in Chapter 3, the field distribution in the machine must be represented by tubes of flux and slices of magnetomotive force. A preliminary dual energy model for the variable reluctance structure is shown in Fig. 4.2. This preliminary model has six tubes and seven slices. Three tubes to the left of the center of the overlap and three tubes to the right are distinguished by shading in Fig. 4.2(a). Similarly, Fig. 4.2(b) shows the two rotor core slices, one slice of the rotor pole, a single airgap slice, a stator pole slice, and two slices in the stator yoke. The model is parametrized such that it can represent general machine dimensions defined in Appendix C. The parametrization is detailed in Appendix D.
The Preliminary Dual Energy Model

Since there are six tubes and seven slices, the field region of interest is divided into 42 elements. Each element in turn is described by a 12 node parametric element similar to the 32 node elements described in Chapter 3. Shape functions for the element are given in Appendix A. The element nodes are numbered to produce an orientation in which the flux flows along the parametrized t-axis, as discussed in Chapter 3.

Element nodes are not only parametrized to represent machine dimensions but also to reflect changes in geometry resulting from rotor movement. Element nodes are determined using geometric variables which are rotor position dependent. The shapes of these elements are adjusted to represent the approximate field distribution for any rotor angle between the aligned and unaligned positions. This dependence of the tube and slice geometry on the rotor position is captured in expressions given in Appendix D.

Results Obtained With Preliminary Model

An Assessment of the accuracy of the flux linkage calculations using the preliminary dual energy model can be made by comparing the results obtained to those from a finite
element method [47]. To facilitate the comparison, the results for three different excitation currents covering the allowable current range for the subject machine are shown in Fig. 4.3-4.5. Calculations were made for a 4 kW Oulton switched reluctance machine having dimensions given in Appendix E. The phase flux linkage for three different currents are plotted as functions of rotor position $\theta_r$ that is defined in Appendix C.

In Fig. 4.3, at excitation below the nominal current of 9.0 A, the upper and lower bounds are fairly close for all rotor angles. The bounds are especially close for the unaligned rotor angles $\theta_r \geq 21.35^\circ$, where there is no overlap between rotor and stator poles. The slope of the average of upper and lower bounds deviates from that determined using finite elements only as the rotor becomes fully aligned. The slope is closely related to torque, so it provides a good indication of the accuracy of the torque calculation. At 7.5 A, little saturation occurs. The flux linkage predicted by the average of the bounds is less than the flux linkage predicted by the finite element method for $\theta_r \geq 8^\circ$. This can be attributed to the neglecting of leakage flux through the interpolar space and through other rotor poles in this dual energy model.

At higher current levels, shown in Fig. 4.4-4.5, the upper and lower bounds deviate significantly from one another. This is particularly the case when the rotor and stator poles are only partially overlapping. For these current levels the bounds are still closer in the unaligned position. The deviation of the bounds becomes greater as saturation increases since the tubes and slices saturate in a different manner. As saturation increases, the tubes and slices predict different saturation characteristics since they deviate from the true tube and slice field characterization.

**Tube Refinement**

As a first attempt to improve the accuracy of the calculations, the boundaries of the air fringe elements were adjusted to improve the orthogonality of the flux and MMF. This was accomplished by placing an offset in the location of two mid-side nodes on the fringe element sides as shown in Fig. 4.6. If each of the two side nodes is displaced an equal amount $b$ in a direction perpendicular to the line connecting the corner nodes, then the shape functions define a parabola as the new boundary.

The revised dual energy model, shown in Fig. 4.7, has the same number of tubes and slices as the original model but includes the curved fringe boundaries. The displacements
Figure 4.3 Phase Flux Linkage for 7.5 A Current

Figure 4.4 Phase Flux Linkage for 12.5 A Current

Figure 4.5 Phase Flux Linkage for 30.0 A Current
of the side nodes are calculated as fractions of the length of the sides. Details of the side node displacements are given in Appendix D.

Results of Tube Refinement

Calculations of the phase flux linkage made using the improved fringe elements are shown in Fig. 4.8-4.10. Only a 5% bending in the fringe sides was used for this study. As can be seen by comparing Fig. 4.8 with Fig. 4.3, the bending of fringe paths has almost no effect on the flux linkage curves. More severe fringe curvature can produce noticeable changes in the flux linkage curves but tends to produce an undesirable discontinuity in the flux linkage at the aligned position. Tube modifications only produce
minimal improvement in the results because the slice selection is a greater source of error.

With heavy saturation there is significant error in the permeance calculations because of an improper representation of the slice boundaries. As the material in the rotor and stator saturate, the tube and slice boundaries should change shape. The boundaries which are chosen to represent the field in the unsaturated condition can not similarly describe the conditions which exist as the poles saturate.

Slice Refinement

When the iron is unsaturated the iron boundaries are at nearly the same field potential. This observation was utilized in the preliminary choice of slice boundaries shown in Fig. 4.2 and 4.7. This also is the basis for defining slice boundaries in other studies such as [29] which ignore iron saturation. The size of the disparity between the upper and lower bounds serves as an indicator of the inadequacy of the chosen tube and slice discretization. A smaller spread indicates a good fit of the chosen tube and slice model to the field distribution.

Previous studies [41,47-48] have shown that of the two kinds of iron saturation, bulk and local saturation, the latter kind is the cause of most errors in the field computation. High flux density and potential gradient at the pole tips combine to complicate analysis even with most finite element programs. When the pole tips begin to saturate, the equipotential surfaces migrate into the iron by crossing the air-iron interface [31]. The airgap slice must expand to include the pole tips as shown in Fig. 4.11. In the preliminary tube and slice model shown in Fig. 4.2, the stator and rotor pole slices are confined to the iron in the poles, and the airgap slice does not cut through the pole tips as shown in Fig. 4.11. The stator and rotor pole slices can only bulk saturate. In the iterative process of solving for the upper bound permeance, the MMF is divided between the slices based on their reluctance. Since the highest reluctance is always the airgap, it is assigned most of the MMF while the pole slices do not obtain enough MMF to saturate. The slices saturate only when unrealistically high MMF is applied. If the airgap slice were to incorporate the pole tips as shown in Fig. 4.11, the MMF assigned to the airgap slice would tend to produce local saturation of the pole tip material at lower current.

For slices which cross material boundaries, some elements must include both iron and air. Special elements containing a mix of iron and air, like that shown in Fig. 4.12, are defined to accommodate this requirement. When evaluating the permeance integrals, the
Figure 4.8 Phase Flux Linkage for 7.5 A Current

Figure 4.9 Phase Flux Linkage for 12.5 A Current

Figure 4.10 Phase Flux Linkage for 30.0 A Current
position of the quadrature points inside the element are examined to determine whether
the points are in air or iron. The permeability for the integral function evaluation is
determined by the kind of material in which the point is located. Special elements with
other material boundary locations are defined to facilitate a variety of slice
configurations.

The revised dual energy model is shown in Fig. 4.13. Two slices are inserted around
the airgap region to improve the representation of local saturation effects in the pole tips.
Details of the modifications appear in Appendix F. Careful examination of Fig. 4.13
shows that the slice modifications are not precisely the same as that shown in Fig. 4.11.
Rather than including the pole tips in the airgap slice which includes the rotor and stator
overlap, the pole tips are placed in two slices which cut through the fringe field.
Incorporating the pole tips into the airgap slice would introduce too much error in the
airgap permeance calculation. The slice elements at the airgap would have to cross from
air to iron within the airgap thus requiring the pole face boundaries to be represented by element diagonals. Since the diagonals of the hexahedral elements can not be forced to lie exactly on the pole surfaces, error would result in the calculation of the airgap permeance.

Figure 4.13 Discretization With Slice Boundaries Crossing Material Boundaries

Results of Slice Refinement

Flux Linkage Calculations

Results of the slice modifications are shown in Fig. 4.14-16. With the additional slices crossing from the fringe fields through the pole tips, the flux linkage bounds are further apart for unaligned rotor angles, $\theta_r \geq 21.35^\circ$, than in the preliminary model. With no overlap of rotor and stator poles there is little saturation of the pole tips. The slice modifications which are meant to improve accuracy in the saturated state result in a larger spread in the bounds where there is no saturation. However the average of the bounds in the unaligned rotor positions is still accurate.

Even though the bounds are more divergent where there is no overlap, the accuracy is improved where pole tip saturation is a significant factor. For the mid-overlap portion of the flux linkage curves, from $8^\circ \leq \theta_r \leq 17^\circ$, the bounds are reduced and the average of the
flux linkage bounds more closely follow the finite element solution. This is especially the case for mid and high current levels shown in Fig. 4.15 and 4.16. The pole slices in the preliminary model are unable to saturate as much as the refined pole tip slices.

A small discontinuity exists at $2^\circ$ due to a discontinuous representation of the field tubes and slices. Since the rotor poles are slightly larger than the stator poles in this machine, one rotor pole tip passes under a stator pole tip at this point. Different tube and slice shapes are used for rotor angles where the rotor pole tip is located underneath the stator pole face than where the stator pole tip is above the rotor pole face because of the changes in the geometrical relationship between the poles. The discontinuity which exists at this point in the flux linkage is magnified by the slice refinement of the pole tip slices to account for local saturation. This discontinuity does not pose an obstacle to the prediction of either flux linkage or torque because it is small and its occurrence can be anticipated.

Further improvement of the flux linkage results can be accomplished by additional refinement of the tubes and slices. As improvements are made, the dual bounds converge. By considering the conditions which produce the largest deviation in the bounds, the proper modifications to the field model can be made. This is in contrast to other techniques, such as the magnetic circuit approach, in which accuracy is always in doubt.

Traditionally the magnetic circuit approach has been used as a basis for first-cut design analysis. The magnetic circuit approach describes the relationship of the applied MMF to the field flux using a circuit containing permeances of the various parts of the machine. This technique is closely related to the tube and slice calculations but does not provide a bounded solution. A magnetic circuit model for variable reluctance motors [41] is shown in Fig. 4.17. Series connected permeances represent the magnetic flux circuit through the poles, airgap, and back-iron portions of the machine. This representation is very similar to the slice representation used in the preliminary dual energy model. Each element of the magnetic circuit is like a slice through which all of the flux passes. Only bulk saturation can be modeled in this simple representation.

Calculations from the magnetic circuit model are compared to the slice calculations from the dual energy model with refined tubes but without refined slices in Fig. 4.18. The magnetic circuit calculations are close to the dual energy slice calculations in both magnitude and shape. This emphasizes that the use of the magnetic circuit approach oftentimes results in either an upper bound or a lower bound to the true field solution. It
Fig. 4.14 Phase Flux Linkage for 7.5 A Current

Fig. 4.15 Phase Flux Linkage for 12.5 A Current

Fig. 4.16 Phase Flux Linkage for 30.0 A Current
also demonstrates the worth of the dual bounds, as they provide a means of determining the validity of the model and suggest ways to improve the calculations.

![Figure 4.17 Magnetic Circuit of Variable Reluctance Motor](image)

**Figure 4.17 Magnetic Circuit of Variable Reluctance Motor**

**Torque Calculations**

The torque, $T_e$, for the variable reluctance motor can be calculated from the partial derivative of the coenergy with respect to the rotor angle:

$$T_e = \frac{\partial W_c}{\partial \theta_r},$$  \hspace{1cm} (4.1)
where $W_C$ is the coenergy and $\theta_r$ is the rotor angle. The coenergy is calculated using [6]

$$W_C = \int \lambda(\theta_r, i) \, d\theta_r, \text{ const.} \quad (4.2)$$

where $\lambda(\theta_r, i)$ is the phase flux linkage. The flux linkage at various currents can be determined from a cubic spline fit [94] of the calculated flux linkage curve for a given rotor angle. Equation (4.2) is integrated in closed form to find $W_C$ at a constant rotor angle. The values for coenergy are computed at the various current levels for each degree of rotor rotation from full alignment to complete unalignment. The derivative of the coenergy with respect to rotor angle is found by fitting a cubic spline to the coenergy at constant excitation.

The torque computed for the various dual energy models is shown in Fig. 4.19. The preliminary model and the model with curved fringe paths predict nearly the same torque. This is not surprising since the flux linkage curves for each of theses were nearly identical. However, the torque predicted by the refined slice model is dramatically different than the other models. For small overlap angles, $15^\circ \leq \theta_r \leq 20^\circ$, and large current the improved representation of local saturation greatly reduces the predicted torque. However, for low current the three models agree closely on the torque.

![Figure 4.19 Torque Calculated With Dual Energy Models](image-url)
The effects of the small discontinuity can also be seen in Fig. 4.19. The discontinuity causes a jump in the torque at 2°. This jump is not serious enough to affect the determination of peak static torque. This can easily be removed from the calculations by using different points to compute the torque or by smoothing.

**Static Torque Comparison**

Computed and measured static torque are compared in Fig. 4.20-22. The static torque computed using the dual energy model with refined slices is shown along with that predicted by finite element and magnetic circuit methods and experimental measurements of a test machine [47]. At the lowest current of 7.5 A, all three computational methods predict the measured torque accurately. The torque predicted by the dual energy model rises to the peak value faster than the measurements but otherwise accurately represents the torque. In the unaligned portion of the curve, 8, \( \geq 21.35^\circ \), the dual energy, finite element, and measurements are very close.

Interestingly, the peak torque predicted by the magnetic circuit approach is higher than that predicted by the other two methods. This is because the magnetic circuit model does not account for pole tip saturation. It is instructive to further examine the effect of local saturation on the calculated torque. There must be sufficient MMF to drive saturation flux density across the airgap before the iron can even begin to saturate. For the machine modeled here, the reluctance at 15° of both airgaps in the magnetic circuit for a single phase is (from 2.16):

\[
\begin{align*}
\mathcal{R} &= \frac{2g}{\mu_0 Z_r \theta_g} \\
&= \frac{2 \times 0.36 \times 10^{-3} \text{m}}{(4\pi \times 10^{-7} \text{H/m})(151 \times 10^{-3} \text{m})(48 \times 10^{-3} \text{m})(6.35^\circ \times \frac{\pi}{180^\circ})} \\
&= 713,269 \text{A/Wb}.
\end{align*}
\]

Assuming a saturation flux density of 1.4 T, the saturation flux level is \( (1.4 \text{T})(\text{overlap area}) = 0.0011 \text{ Wb} \). Ignoring iron reluctances and equating the applied MMF from the two 92 turn coils to the MMF across the airgaps gives
Thus the iron should begin to saturate at a current far below even the 7.5 A in Fig. 4.20. As the current increases to 10.0 A and 12.5 A in Fig. 4.21-22, the differences between the magnetic circuit calculations and the other models increase. The torque predicted by the refined slice dual energy model and the finite element model agree very well with the measured torque. As seen in Fig. 4.19, however, the dual energy models without refined slices begin to predict higher peak torque than the refined slice model as current is increased. The preliminary model, which does not account for local saturation, can not predict the peak torque above the nominal 9.0 A current level as accurately as can the refined slice mode.

At the 30 A current level shown in Fig. 4.23, the magnetic circuit model overestimates the peak torque by a factor of three. While the shape of the torque predicted by the refined slice model is not exactly the same as that of the finite element model, it is accurate enough for most design studies.

For the purposes of design, the peak static torque and the unaligned static torque provide an indication of the capabilities of a specific machine. In Chapter 6 the static torque at mid alignment and with maximum current is used as an indicator of the efficacy of a particular design. As shown in Fig. 4.22 the dual energy model predicts generally constant torque over the range of overlap angles for the maximum current condition. A reliable prediction of the peak torque is had using the torque at mid-overlap, at about 11° for this machine. The minimum torque can be predicted by calculating the torque at the fully unaligned position, which is 22.5° for this machine. In the unaligned position the dual energy model matches the finite element model almost exactly.

**Summary**

The dual energy model is capable of accurately predicting the static torque of variable reluctance motors. The dual energy method provides a means of attaining a reasonably accurate prediction of machine parameters through an iterative refinement process. By altering the tube and slice geometry to reduce the discrepancies in the flux linkage bounds it is possible to develop an accurate model for predicting the static torque. Refinement of the tube and slice model fosters a detailed understanding of the magnetic
Fig. 4.20 Calculated and Measured Torque at 7.5 A

Fig. 4.21 Calculated and Measured Torque at 10.0 A
Fig. 4.22 Calculated and Measured Torque at 12.5 A

Fig. 4.23 Calculated Torque at 30.0 A
field distribution and clarifies the relationship between the geometry of the machine and the magnetic field parameters.

The rapidity with which calculations can be made makes it competitive with magnetic circuit solutions and superior to the finite element method for design analysis. The accuracy of the dual energy modeling makes it superior to magnetic circuit models and competitive with finite element methods.
CHAPTER 5. DESIGN EVALUATION OF SRM AND MCR MOTOR STRUCTURES FOR A LOW SPEED APPLICATION

Windshield wiper systems are representative of systems which require low-speed high-torque actuators. A typical windshield wiper system is comprised of a linkage driven by a DC motor through a gear reduction. Utilizing a variable reluctance motor in such an application would permit the use of a smaller and less expensive gear reduction. In such instances the motor volume is constrained due to packaging, the power source has a fixed voltage and maximum current rating, and the static torque output is specified. Additionally, for automotive applications product cost must be kept to a minimum.

This chapter compares the use of SRM and MCR motor structures discussed in Chapter 2 as a replacement to an existing DC windshield wiper motor. The CRR motor, which is capable of higher torque, is not considered here due to its prohibitively expensive rotor construction costs. Fixing the dimensions to match the basic size of the present motor, the optimum wire gauge and number of poles of each structure is considered. The dual energy model developed in Chapter 4 is used to compute the static torque of the candidate designs. Results are compared to the torque ratios derived in Chapter 2.

**Calculation of Optimum Wire Gauge**

To obtain maximum static torque with a constraint on the current, it is essential to find the optimum wire size which can accomplish this. As suggested in Chapter 2, the winding which fills the allotted space is the optimum choice. Calculation of the best wire gauge begins with computation of the winding resistance.

For optimization it is necessary to relate the winding resistance to the machine geometry. The geometry dictates both the available slot area and the mean turn length which applies to each configuration.
Available Slot Area

The basic dimensions of the stator pole are shown in Fig. 5.1. The slot area available for winding is merely the entire area between the pole face and yoke inside diameter less the area taken up by the stator poles. The area of a stator pole is found approximately from the geometry described in Fig. 5.1:

\[ A_{pole} = \pi \left( r_d^2 - r_c^2 \right) \frac{2\beta_s}{2\pi} - 2\frac{1}{2} (r_d - r_c) \left( r_d - r_c \right) \tan(\beta_s - \phi_s). \]  

(5.1)

The area of a slot, \( A_{slot} \), between stator poles is the total area divided by the number of stator poles, \( N_s \), less the area of a stator pole,

\[ A_{slot} = \frac{\pi \left( r_d^2 - r_c^2 \right)}{N_s} - A_{pole}. \]  

(5.2)

With reference to Chapter 2, the available area, \( A_2 \), for the winding of a coil in a mutually coupled reluctance machine is \( A_{slot} \). For the SRM the available area is \( A_{slot}/2 \).

Figure 5.1 Stator Pole Dimensions for Variable Reluctance Machines

Mean Turn Length

As an approximation, the mean turn of a winding is assumed to be located at a radius \( r_m \) between the stator pole face and the yoke inside radius which encloses \( 1/2 \) the slot area. Thus for both the MCR and the SRM motors the mean turn radius is approximated
by $\sqrt{\left(\frac{r_d^2 + r_c^2}{2}\right)}$. The mean turn is assumed to pass through the center of the area occupied by the winding, placing it at mid slot for the MCR motor or at one-quarter slot away from the edge of the stator pole for the SRM. Since the coils of the MCR motor are wound around multiple stator poles, the angular length of the end-turn for the MCR motor, $\theta_{c2}$, is found to be

$$\theta_{c2} = \frac{2}{P} \pi$$

(5.3)

where $P$ is the number of excited poles per phase. It can be shown for the SRM that the angular length $\theta_{c1}$ is approximately

$$\theta_{c1} = \frac{\pi}{N_s} + \beta_s.$$  

(5.4)

The mean turn length $l_2$ for the MCR motor is then

$$l_2 = 2(Z + r_{c2} \theta_{c2}) = 2\left(Z + \sqrt{\frac{r_d^2 + r_c^2}{2}} \frac{2}{P} \pi\right)$$

(5.5)

and that for the SRM is

$$l_1 = 2(Z + r_{c1} \theta_{c1}) = 2\left[Z + \sqrt{\frac{r_d^2 + r_c^2}{2}} \left(\frac{\pi}{N_s} + \beta_s\right)\right].$$

(5.6)

Copper Wire Resistivity

Wire manufacturers provide information on the resistivity of the standard wire gauges [95]. For the American Wire Gauge (AWG) the resistance per unit length, $r_w$, doubles for every increase of three wire gauges. Thus the resistivity of a certain wire gauge $k$ can be related to the gauge using the expression

$$r_{wk} = 2^{\frac{\Delta}{3}} r_{wo}$$

(5.7)
where \( A = G_k - G_o \), the difference between the gauge under question \( G_k \) and a reference gauge \( G_o \). The doubling of resistance every three wire sizes is due to a commensurate halving of the cross-sectional area. Thus, a relationship between the diameter of \( k \) gauge wire and its gauge number can be found:

\[
D_{wk} = 2^{-6/3} D_{wo}.
\]

Selection of Wire Gauge

The number of turns which can fit in the available slot area can be found from (2.4). To account for random packing of the wires the wire is assumed to occupy a square cross-sectional area having sides equal to the wire diameter [97]. Thus, (2.4) can be expressed in terms of (5.8) to find the number of turns \( N \) of wire gauge \( G_k \) which fit into slot area \( A \):

\[
N = \frac{fA}{a_{wk}} = \frac{fA}{D_{wk}^2} = \frac{fA}{2^{-6/3} D_{wo}^2}.
\]

Manufacturing experience suggests that a typical value for the useful fill factor, \( f \), is about 40%.

The design constraints, as discussed in Chapter 2, determine a value for the coil resistance \( R \). The resistivity of the coil can be found from this using the mean turn length \( l \),

\[
\rho_k = \frac{R}{Nl} = \left( \frac{R}{fA} \right) \left( \frac{2^{-6/3} D_{wo}^2}{l} \right).
\]

The resistivity is also described by (5.7) leading to
Since $A = G_k - G_o$, the wire gauge which fills the allotted slot area and has the desired resistance can be determined, that is

$$G_k = \frac{3\ln \left( \frac{RD_{wo}^2}{fAlr_{wo}} \right)}{\ln 4} + G_o. \quad (5.13)$$

**Calculation of Optimum Number of Poles**

As pointed out in Chapter 2, the MCR motor can produce up to twice the torque of the same size SRM motor. Unfortunately, additional end-turn length and saturation limit this effect. The number of salient stator poles enclosed by an MCR coil is $P_S = \frac{2}{P}N_S$, where $N_S$ is the number of salient stator poles and $P$ is the number of excited poles per phase. For example, the machine shown in Fig. 2.3(b) has $N_S = 6$, $P = 2$, $P_S = 3$. The number of excited poles per phase in the MCR motor is equal to twice the number of coils per phase because each coil acts with a coil in a second excited phase to excite a north and south pole in the machine. For the SRM the number of excited poles per phase is equal to the number of coils per phase because each pair of coils in a given phase act to energize two poles. MCR motors with fewer excited poles per phase have longer end turns because the coils must encircle more salient stator poles.

Utilizing (5.5) and (5.6) it is possible to determine the circumstances for which a mutually coupled winding is superior to the conventional winding for producing static torque at a fixed voltage. To express the results in a more general form, the following ratios are defined:

$$h = \frac{2r_d}{Z}. \quad (5.14)$$
Thus, the mean turn length can be calculated as

\[
1 = 2 \left( Z + \frac{r_d^2 + r_c^2}{2} \theta_c \right) = 2 \left( Z + \frac{hZ}{2} \sqrt{\frac{1 + \gamma^2}{2}} \theta_c \right).
\]  

(5.16)

The ratio of the mean turn length of an SRM to that of an MCR motor with the same dimensions is given by:

\[
\frac{l_1}{l_2} = \frac{1 + \frac{h}{2} \sqrt{\frac{1 + \gamma^2}{2}} \theta_c_1}{1 + \frac{h}{2} \sqrt{\frac{1 + \gamma^2}{2}} \theta_c_2}.
\]  

(5.17)

From 2.26, the ratio of the MCR torque to the SRM torque becomes

\[
\frac{T_2}{T_1} = \frac{A_2}{A_1} l_1 = \frac{2 \left( 1 + \frac{h}{2} \sqrt{\frac{1 + \gamma^2}{2}} \theta_c_1 \right)}{1 + \frac{h}{2} \sqrt{\frac{1 + \gamma^2}{2}} \theta_c_2}.
\]  

(5.18)

The end turn angular span can be expressed in general terms by first defining the ratio between the stator tooth width \(2\beta_s\) and the tooth pitch,

\[
\eta = \frac{2\beta_s}{2\pi} = \frac{\beta_s}{\pi} = \frac{N_s \beta_s}{\pi}.
\]  

(5.19)

This can then be used in (5.4) to compute the angular end-turn width for the SRM motor,
\[ \theta_{ci} = \frac{\pi}{N_s} + \frac{\eta \pi}{N_s} = \frac{\pi}{N_s} (1 + \eta). \] (5.20)

Substituting \( \theta_{ci} \) into (5.18) yields

\[ \frac{T_2}{T_1} = \frac{2 \left( 1 + \frac{h}{2} \sqrt{\frac{1 + \gamma^2}{2}} \left( \frac{\pi}{N_s} (1 + \eta) \right) \right)}{1 + \frac{h}{2} \sqrt{\frac{1 + \gamma^2}{2}} \left( \frac{2\pi}{P} \right)} \] (5.21)

A typical value for \( \eta \) given in [32] is 0.4. With a value for \( \gamma \) of 0.5, the variation of the torque ratio to \( h \) for 2, 4, and 6 pole machines is plotted in Fig. 5.2. The number of salient stator poles, \( N_S \), is equal to the product of the number of phases and the number of poles. It is evident that the MCR winding improves torque over the standard SRM winding when the diameter is small compared to length. This is because the end turns become a smaller part of the total turn length as the ratio \( h \) is decreased. The torque ratio also increases with the number of poles since the pole pitch distance decreases with an increase in the number of poles.

Figure 5.2 Torque Ratio for Various Pole and Phase Combinations
**Design Example**

Automotive actuators are driven from a fixed voltage supply and thus are constrained to operate as suggested by the voltage source analysis presented in Chapter 2. The use of a switched reluctance motor in an automotive actuator application could enable the use of smaller gear reductions. A typical automotive application is the windshield wiper motor. Typically a single motor drives two wiper arms and blades across the windshield through a complex linkage system. The high torque requirements are met by utilizing a permanent magnet DC motor driving through a gear box with a reduction somewhere between 40 and 100. The use of a variable reluctance device could reduce package size by reducing the required gear reduction or could enable direct drive of the wiper arms at each pivot point, thus eliminating the linkages. The objective of this study is to maximize the static torque of a variable reluctance motor obtainable with a given supply voltage.

**Motor Description**

Shown in Fig. 5.3 is a cross-sectional view of a 4 pole 3 phase windshield wiper motor. Dimensions for the device are given in Table 5.1. The rotor has 8 salient poles while the stator has 12 salient poles. For the automotive application, the maximum available voltage is 14 V. The current is limited to 20 A at stall with the full voltage applied.

![Cross-Sectional View of Variable Reluctance Windshield Wiper Motor](image)
The SRM version of the example motor has eight coils per phase. Each of the coils is wound around a single stator salient pole. On the other hand, each phase of the MCR motor has only four coils. Each of the MCR motor coils is wound around three teeth. Using (5.2), (5.5) and (5.6) in (5.13) and (5.9), the wire gauge and number of turns for the SRM and MCR are calculated. Results are shown in Table 5.2. Turns are rounded to the lowest whole number while gauge is rounded to the next higher half gauge. The acceptable fill ratio, f, is taken to be 0.4. The turns ratio N2/N1 for this motor is 2.57 as compared to the maximum possible 2.83. It is interesting to compare the predicted static torque of the two motors. Increasing the static torque at 20 A current yields a higher torque at low steady state speeds for 14 V operation and enables the use of smaller gear reductions.

Table 5.1 Dimensions of Example Motor

<table>
<thead>
<tr>
<th>Dimension</th>
<th>Value for Example Motor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Poles</td>
<td>4</td>
</tr>
<tr>
<td>Number of Phases</td>
<td>3</td>
</tr>
<tr>
<td>Number of Stator Teeth</td>
<td>12</td>
</tr>
<tr>
<td>Number of Rotor Teeth</td>
<td>8</td>
</tr>
<tr>
<td>βr = βs</td>
<td>6°</td>
</tr>
<tr>
<td>rsh</td>
<td>6.0 mm</td>
</tr>
<tr>
<td>ra</td>
<td>12.0 mm</td>
</tr>
<tr>
<td>rb</td>
<td>17.0 mm</td>
</tr>
<tr>
<td>rC</td>
<td>17.5 mm</td>
</tr>
<tr>
<td>rd</td>
<td>32.5 mm</td>
</tr>
<tr>
<td>ro</td>
<td>35.0 mm</td>
</tr>
<tr>
<td>Z</td>
<td>75.0 mm</td>
</tr>
<tr>
<td>Vdc</td>
<td>14.0 V</td>
</tr>
<tr>
<td>Maximum Current Draw</td>
<td>20.0 A</td>
</tr>
</tbody>
</table>

Table 5.2 Computed Winding Parameters for Variable Reluctance Wiper

<table>
<thead>
<tr>
<th>Design Parameter</th>
<th>SRM</th>
<th>MCR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Turns Per Coil</td>
<td>35</td>
<td>90</td>
</tr>
<tr>
<td>Wire Gauge</td>
<td>19.5 AWG</td>
<td>20.5 AWG</td>
</tr>
</tbody>
</table>
Static Torque Comparison

Torque computed using the dual energy model discussed in Chapter 4 is shown in Fig. 5.4. At $\theta_f = 12^\circ$ the exact alignment of the pole tips results in a discontinuity in the torque so these points have been shown separate from the torque curves. The torque ratio at the mid-overlap position is 1.57, while at the unaligned position it is found to be 1.63. Using (5.21) the static torque ratio for the MCR to SRM is 1.458. However, the turn count and wire gauge were rounded as mentioned in the previous section, while the torque ratio (5.21) does not incorporate this effect. Rounding of these quantities would result in slightly different peak current draw due to differences in the coil resistance from ideal, but for the sake of comparison the current was set to 20 A for both the SRM and MCR motors. Calculating the torque from the linear model (2.24) with $k_L=0.9$ yields results shown in Table 5.3. Results of (2.24) are multiplied by $(P/2)$ since it is calculation of torque on a per-pole pair basis. The particular dimensions of the machine, chosen to compete with existing DC motor dimensions, allow a 50% increase in torque by the use of the mutually coupled winding.

![Figure 5.4 Torque Computed With the Dual Energy Model](image-url)
Table 5.3 Comparison of Torque Calculations

<table>
<thead>
<tr>
<th>Method of Calculation</th>
<th>$T_1$ (Nm)</th>
<th>$T_2$ (Nm)</th>
<th>$T_1/T_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dual Energy Mid-Overlap</td>
<td>2.61</td>
<td>4.09</td>
<td>1.57</td>
</tr>
<tr>
<td>Dual Energy Unaligned</td>
<td>0.832</td>
<td>1.360</td>
<td>1.63</td>
</tr>
<tr>
<td>Linear (5.20)</td>
<td>---</td>
<td>---</td>
<td>1.46</td>
</tr>
<tr>
<td>Linear (2.24)</td>
<td>2.29</td>
<td>3.79</td>
<td>1.65</td>
</tr>
</tbody>
</table>

Summary

Mutually coupled windings used in place of conventional SRM windings can increase static torque. The static torque of a given machine size can be increased significantly for a fixed supply voltage. Such is the case in automotive applications where the operating constraints suggest voltage source operation. The static torque of an example motor, suitable for use as a windshield wiper motor, can be increased by 50% over a conventional switched reluctance motor by utilizing a mutually coupled winding.
CHAPTER 6. CONCLUSIONS AND FUTURE RESEARCH

Variable reluctance motors are well suited for use as low-speed high-torque actuators. The objective of this thesis is to evaluate the suitability of three variable reluctance motor structures for high-torque low-speed applications and to develop an efficient analysis technique to the design such variable reluctance motors. The relative merits of the SRM, MCR and CRR motors are discussed in Chapter 2. In Chapter 3 a dual energy model of magnetostatic fields applicable to two and three-dimensional structures with nonlinear material is developed from basic principles. Chapter 4 shows how the model can be refined iteratively using knowledge of the modeling assumptions and the physics of the problem at hand. The results obtained by successive improvement of a tube and slice model for variable reluctance motors are compared to those obtained by other methods including measurements. Finally in Chapter 5 expressions for the design of SRM and MCR motors are developed and numerical results of both designs are presented as a substitution for an existing DC windshield wiper motor.

Research Summary

The research contained in this thesis contributes to the study of electromechanical devices in the following ways:

1.) A comparative study is given of static torque obtained from three competing variable reluctance motor structures.

2.) A numerical method is developed for calculation of magnetostatic field parameters suitable for two and three-dimensional nonlinear analysis based on dual energy principles.
3.) Successive refinement of the dual energy model is demonstrated on variable reluctance motors, comparing the results obtained at each stage with similar results from more established methods.

4.) Necessary expressions for SRM and MCR motor &sign are derived and used to evaluate a design candidate for replacement of a DC windshield wiper motor.

**Conclusions**

In Chapter 2, the relative merits of three competing variable reluctance motors are evaluated on the basis of criteria important to low-speed high-torque operation. It is found that the use of phase windings which enclose multiple stator poles in the MCR motor can lead to increased static torque over the same sized conventional SRM. Further improvements are possible by using an axially laminated rotor in conjunction with the mutually coupled phase windings. The results compare favorably with those based on a different set of criteria.

In Chapter 3 and 4 dual energy methods are developed into a numerical method for computation of two and three-dimensional nonlinear magnetostatic field parameters. The successive refinement of the model for accurate prediction of flux linkage and static torque in variable reluctance motors demonstrates the accuracy and utility of the dual energy method. The dual energy model is shown to compete favorably in accuracy with the finite element method.

This work has shown that the dual energy method can be applied to analyze the electromechanical behavior of VRMs. The proposed extension of the modeling technique to handle mixed material tubes or slice elements has been shown to offer improved modeling accuracy in handling the field contours around the pole tips resulting from local saturation.

In Chapter 5 the specific conditions which favor the use of MCR motors over SRM structures are determined. It is shown that MCR motors approach their full potential for increasing static torque when the diameter to length ratio is small or the number of excited poles per phase is large. It is shown by a practical example that the MCR motor can produce higher torque than the SRM when size and operating conditions are predetermined.
**Future Research**

The following areas are of interest for future research:

1.) Extension of the nonlinear dual energy model to include permanent magnet and anisotropic materials.

2.) Development of techniques for automatic generation of tube and slice discretization for general structures.

3.) Further refinement of the dual energy model for variable reluctance motors to improve the accuracy of the computed torque over a wider range of rotor angles.
LIST OF REFERENCES


APPENDICES
Appendix A: Shape Functions for Parametric Representation

Three Dimensional Shape Functions

The transformation from \((s,u,t)\) space to \((x,y,z)\) space is covered in [91]. Discretization utilizes 32 node hexahedral elements as shown in Fig. A.1. The orientation and node numbering enable a consistent representation for the tube and slice integrations.

![Node Numbering and Orientation of Three-Dimensional Element](image)

Figure A.1 Node Numbering and Orientation of Three-Dimensional Element

The point \((x,y,z)\) corresponding to the point \((s,u,t)\) is given by the sums:

\[
x = \sum_{i=1}^{32} N_i x_i,
\]

(A.1)

\[
y = \sum_{i=1}^{32} N_i y_i,
\]

(A.2)

\[
z = \sum_{i=1}^{32} N_i z_i,
\]

(A.3)
where the nodal coordinate points are \((x_i, y_i, z_i)\) and the shape functions are \(N_i\). Definitions of the shape functions depend on the various nodes. For the corner nodes \(\text{where } s = \pm 1, u = \pm 1, t = \pm 1,\)

\[
N_i = \frac{1}{64} (1 + ss_i)(1 + uu_i)(1 + tt_i) \left[ 9(s^2 + u^2 + t^2) - 19 \right],
\]  

(A.4)

for the sides where \(s = \pm \frac{1}{3}, t = \pm 1, u = \pm 1,\)

\[
N_i = \frac{9}{64} (1 - s^2)(1 + 9ss_i)(1 + tt_i)(1 + uu_i).
\]  

(A.5)

Permutation of the \(s, u, t\) variables in (A.5) yields expressions for the other element sides.

Two Dimensional Shape Functions

The transformation from \((s, t)\) space to \((x, y, z)\) space is covered in [91]. The two-dimensional discretization is accomplished using a 12 node hexahedral element with node numbering and orientation shown in Fig. A.2.

![Node Numbering and Orientation of Two-Dimensional Element](image)

\*Figure A.2* Node Numbering and Orientation of Two-Dimensional Element

The point \((x, y, z)\) corresponding to the point \((s, t)\) is given by the sums:

\[
x = \sum_{i=1}^{12} N_i x_i,
\]  

(A.6)
where the nodal coordinate points are \((x_i, y_i)\) and the shape functions are \(N_i\). Definitions of the shape functions depend on the various nodes. For the corner nodes where \(s = \pm 1, t = \pm 1\),

\[
N_i = \frac{1}{32} \left(1 + ss_i\right) \left(1 + tt_i\right) \left[9(s^2 + t^2) - 10\right].
\] (A.8)

for the sides where \(s = \pm \frac{1}{3}, t = \pm 1\),

\[
N_i = \frac{9}{32} \left(1 - s^2\right) \left(1 + 9ss_i\right) \left(1 + tt_i\right).
\] (A.9)

Permutation of the \(s, t\) variables in (A.9) yields expressions for the other element sides.

Jacobian Relationships for Relating Differentials

To evaluate integrals (3.4) and (3.10) using parametric representation of the \((x, y, z)\) variables, it is necessary to relate the differential length \(dL\) and area \(dA\) to corresponding differentials \(dt\) and \(dsdu\) in \((s, u, t)\) variables. Jacobian relationships described in [92] relate the two spaces.

**Differential Length \(dL\)**

The differential length \(dL\) can be described in terms of the \(x, y\) and \(z\) differentials as

\[
dL = \sqrt{dx^2 + dy^2 + dz^2}.
\] (A.10)

The \(x, y\) and \(z\) differentials relate to the \(s, t\) and \(u\) differentials by the shape functions. For example,

\[
dx = \sum_{i=1}^{12} \left(\frac{\partial N_i}{\partial s} \, ds + \frac{\partial N_i}{\partial u} \, du + \frac{\partial N_i}{\partial t} \, dt\right) x_i.
\] (A.11)
However, $dl$, lies in the direction of flux flow that corresponds to the $t$ axis so that $ds = du = 0$. Combining the equations for $dx, dy,$ and $dz$ from (A.11) with this condition into (A.10) gives

$$
\begin{align*}
\frac{dl}{dt} &= \frac{\sum_{i=1}^{32} \frac{\partial N_i}{\partial x_i}}{\sum_{i=1}^{32} \frac{\partial N_i}{\partial x_i}} \\
&= \left( \sum_{i=1}^{32} \frac{\partial N_i}{\partial x_i} \right)^2 + \left( \sum_{i=1}^{32} \frac{\partial N_i}{\partial y_i} \right)^2 + \left( \sum_{i=1}^{32} \frac{\partial N_i}{\partial z_i} \right)^2 \\
&= \left( \sum_{i=1}^{32} \frac{\partial N_i}{\partial x_i} \right)^2
\end{align*}
$$

which is the Jacobian relationship from (3.10):

$$
\frac{dl}{dt} = \left| J_L \right| dt.
$$

**Differential Area $dA$**

The differential area represents a small portion of the surface through which flux passes. The corresponding surface in the $(s,u,t)$ space is a plane $(s,u,t = \text{constant})$. Thus, $x$ and $y$ are functions of $s$ and $u$ alone:

$$
\begin{align*}
x &= f(s,u) \\
y &= g(s,u) \\
z &= h(s,u).
\end{align*}
$$

The functions $f, g,$ and $h$ are defined by (A.1-3). From [92] the Jacobian relationship between $dA$ and $dsdu$ can be expressed as

$$
\begin{align*}
dA &= \left\{ \left( \frac{\partial(f,h)}{\partial(s,u)} \right)^2 + \left( \frac{\partial(f,g)}{\partial(s,u)} \right)^2 + \left( \frac{\partial(f,g)}{\partial(s,u)} \right)^2 \right\} dsdu, \\
&= \left| J_A \right| dsdu
\end{align*}
$$

where the partial derivatives within the radical of (A.17) are defined to be

$$
\frac{\partial(a,b)}{\partial(v,w)} = \left( \frac{\partial a}{\partial v} \frac{\partial b}{\partial w} - \frac{\partial a}{\partial w} \frac{\partial b}{\partial v} \right).
$$
Appendix B: Characteristics of Electromagnet

Table B.1 Magnetization for Lamination Steel.[18]

<table>
<thead>
<tr>
<th>Flux Density (Wb/m$^2$)</th>
<th>Field Intensity (A/m)</th>
</tr>
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<tbody>
<tr>
<td>0.0</td>
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<tr>
<td>0.106</td>
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</tr>
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<td>2.047</td>
<td>800.0</td>
</tr>
</tbody>
</table>
Figure B.1 Dimensions of Electromagnet (All Dimensions in mm)
Fig. C.1 Dimensioning of Variable Reluctance Geometry
Appendix D: Dual Energy Model for Variable Reluctance Structures

The dual energy model of variable reluctance motors incorporates tube and slice discretizations parametrized to represent a wide range of dimensions and rotor angles. To accomplish this the motor is broken into two halves along the center of overlap. Two general conditions are considered: rotor pole face exposed to stator pole side as shown in Fig. D.1, and stator pole face exposed to rotor pole side as shown in Fig. D.2.

Rotor Pole Face Exposed to Stator Pole Side

Parametrization of the dimensions for the condition where the rotor pole face is exposed to the stator pole side is shown in Fig. D.1. The tube and slices are discretized into elements numbered as shown in Fig. D.1. Element numbering along each half slice of the motor is sequential from the outermost elements to those located closest to the center. Element numbering along the left-half of each slice precedes numbering of the right-half slice. Hence, if Fig. D.1 represents the left half of the machine, as shown, then the numbering is as given. If, however, the right half of the machine is represented by Fig. D.1 the numbering is offset by three to account for the preceding elements in the left half of the machine.

The angle of overlap may be calculated from

\[ \text{overlap} = \beta_s + \beta_r - \theta_r. \]  

**Overlap** overlap, \( \text{maxlap} \), is determined by the maximum of the rotor and stator pole widths. The overlap angle is used to set the width of the tubes by adjusting the parameters \( \alpha_r, \beta_r, F_{s1}, F_{s2} \):

\[ \alpha_r = 0.5 + (\alpha_{r_{max}} - 0.5) \frac{\text{overlap}}{\text{maxlap}} \]  

\[ \beta_r = \beta_{r_{max}} \frac{\text{overlap}}{\text{maxlap}} \]  

\[ F_{s1} = 0.5 - (0.5 - F_{s1_{min}}) \frac{\text{overlap}}{\text{maxlap}} \]
\[ Fs_2 = 1.0 - \left( 1.0 - Fs_{2\min} \right) \frac{\text{overlap}}{\max \text{ lap}} \]  \hspace{1cm} (D.5)

The fringe paths for this condition are **curved** in order to improve orthogonality of the integration paths. Bending of the fringe paths for the condition where the rotor pole face is exposed to the stator pole side is shown in Fig. D.2. The curvature is accomplished by
offsetting the element side nodes an amount proportional to the length of the side. In this way the bending is parametrized to the rotor position.

![Diagram](image)

Figure D.2 Fringe Path Adjustments

Stator Pole Face Exposed to Rotor Pole Side

A similar parametrization is set up for the condition where the stator pole face is exposed to the rotor pole side. Elements are numbered as shown with consideration given for the half of the machine to which they apply as previously discussed. Again, the offset of the mid-side nodes is proportional to the length of the side.

Each half of the machine shown in Fig. D.2-D.3 represents one excited pole of the device. The angular lengths of the yoke elements and rotor core elements are set so that the model meets this requirement.
Figure D.3 Stator Pole Face Exposed to Rotor Pole Side
### Appendix E: Dimensions for Switched Reluctance Machine

Table E.1 Physical Dimensions of Oulton 4 kW SRM [47]

<table>
<thead>
<tr>
<th>Dimension</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_{sh}$</td>
<td>15.0 mm</td>
</tr>
<tr>
<td>$r_a$</td>
<td>30.3 mm</td>
</tr>
<tr>
<td>$r_b$</td>
<td>47.82 mm</td>
</tr>
<tr>
<td>$r_c$</td>
<td>48.18 mm</td>
</tr>
<tr>
<td>$r_d$</td>
<td>78.4 mm</td>
</tr>
<tr>
<td>$r_p$</td>
<td>89.8 mm</td>
</tr>
<tr>
<td>$\beta_r$</td>
<td>11.25°</td>
</tr>
<tr>
<td>$\beta_s$</td>
<td>10.1°</td>
</tr>
<tr>
<td>$\phi_r$</td>
<td>0°</td>
</tr>
<tr>
<td>$\phi_s$</td>
<td>0°</td>
</tr>
<tr>
<td>$Z$</td>
<td>151.0 mm</td>
</tr>
<tr>
<td>Turns Per Pole*</td>
<td>92</td>
</tr>
</tbody>
</table>

*Turns per pole determined using a 15 turn search coil mounted on the rotor to measure induced voltage from each stator phase excited separately. The quantity shown here is one-half the turns per phase.
Table E.2 Steel Magnetization Data for Oulton Machine [47]

<table>
<thead>
<tr>
<th>Flux Density (Tesla)</th>
<th>Field Intensity (A/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>0.5</td>
<td>54.0</td>
</tr>
<tr>
<td>0.55</td>
<td>60.5</td>
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<tr>
<td>0.6</td>
<td>67.6</td>
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<tr>
<td>0.65</td>
<td>74.8</td>
</tr>
<tr>
<td>0.7</td>
<td>83.5</td>
</tr>
<tr>
<td>0.75</td>
<td>93.5</td>
</tr>
<tr>
<td>0.8</td>
<td>104.4</td>
</tr>
<tr>
<td>0.85</td>
<td>116.9</td>
</tr>
<tr>
<td>0.9</td>
<td>131.3</td>
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<tr>
<td>0.95</td>
<td>148.2</td>
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<tr>
<td>1.0</td>
<td>167.1</td>
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<tr>
<td>1.05</td>
<td>187.8</td>
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<tr>
<td>1.1</td>
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<tr>
<td>1.15</td>
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<tr>
<td>1.2</td>
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<td>1.25</td>
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<td>1.3</td>
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<td>79580.0</td>
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<td>2.06</td>
<td>87537.7</td>
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Table E.2 Tube and Slice Parameters for Oulton 4 kW Machine

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
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</thead>
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<td>armax</td>
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<tr>
<td>brmax</td>
<td>0.6</td>
</tr>
<tr>
<td>crmax</td>
<td>0.6</td>
</tr>
<tr>
<td>dr</td>
<td>0.5</td>
</tr>
<tr>
<td>Fs1min</td>
<td>0.1</td>
</tr>
<tr>
<td>Fs2min</td>
<td>0.2</td>
</tr>
<tr>
<td>as</td>
<td>0.3</td>
</tr>
<tr>
<td>bs</td>
<td>0.5</td>
</tr>
<tr>
<td>af</td>
<td>0.05</td>
</tr>
<tr>
<td>bf</td>
<td>0.05</td>
</tr>
<tr>
<td>gf</td>
<td>0.1</td>
</tr>
<tr>
<td>bi</td>
<td>0.0</td>
</tr>
<tr>
<td>bo</td>
<td>0.0</td>
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<tr>
<td>$\gamma$</td>
<td>3°</td>
</tr>
<tr>
<td>dFt</td>
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<tr>
<td>dFs</td>
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<tr>
<td>cfr</td>
<td>3.25</td>
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<tr>
<td>cfs</td>
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</tbody>
</table>
Appendix F: Dual Energy Model of Variable Reluctance Motor with Refined Slices

Tube and slice configurations for the model with refined slices are shown in Fig. F.1 and F.2. The machine is divided in a manner similar to that described in Appendix D, with each half considered separately. The conditions of rotor pole face exposed to stator pole side and stator pole face exposed to rotor pole side are modeled. Much of the machine is modeled as described in Appendix D. Differences in the model and in the element numbering are shown in Fig. F.1 and Fig. F.2.

Elements 26 and 44 in both Fig. F.1 and F.2 are special elements discussed in Chapter 4, made up of both air and iron material. One additional parameter is defined for the construction of the elements:

\[ cr = cr_{\text{max}} \frac{\text{overlap}}{\text{max lap}}. \]  

(F.1)
Figure F.1  Rotor Pole Face Exposed to Stator Pole Side
Figure F.2 Stator Pole Face Exposed to Rotor Pole Side