1980

Simple Mathematical Models of the Vibration and Force Transmission of Discharge or Suction Tubes as Function of Discharge and Suction Pressures

W. Soedel

Follow this and additional works at: http://docs.lib.purdue.edu/icec


This document has been made available through Purdue e-Pubs, a service of the Purdue University Libraries. Please contact epubs@purdue.edu for additional information.

Complete proceedings may be acquired in print and on CD-ROM directly from the Ray W. Herrick Laboratories at https://engineering.purdue.edu/Herrick/Events/orderlit.html
SIMPLE MATHEMATICAL MODELS OF THE VIBRATION AND FORCE TRANSMISSION OF DISCHARGE OR SUCTION TUBES AS FUNCTION OF DISCHARGE AND SUCTION PRESSURES

Werner Soedel, Professor of Mechanical Engineering, Ray W. Herrick Laboratories, School of Mechanical Engineering, Purdue University, West Lafayette, Indiana 47907

ABSTRACT

On the idealized situation of simply supported discharge tubes it is demonstrated how the discharge and suction pressures influence the resonance frequencies and thus the force transmission from the compressor to the compressor shell. The influence of flow velocity is also briefly discussed. With minor modifications, all results are also applicable to suction tubes in cases of high side compressors.

INTRODUCTION

It occurs frequently that it is observed that the sound pressure radiated from the shell of a refrigeration compressor changes when operating conditions change. One of the possible mechanisms was investigated by Johnson and Hamilton [1]. When the speed of sound of the gas in the shell changes with a condition change, gas resonances change. This effect is typically of importance in the higher frequency range.

This paper points out still another mechanism. As the discharge-suction pressure difference changes, discharge or suction tubes change their resonance frequencies and thus their vibration transmission into the shell. The mathematical models are kept as simple as possible without sacrificing some generality in order to illustrate the most important influences to the designer.

Equation of Motion for a Straight Discharge Tube

Let us examine the idealized case of a straight discharge tube that is free to expand in axial direction. Following a standard approach of derivation, like Hamilton's principle [2], one obtains the following equation of motion:

$$EI \frac{\partial^4 w}{\partial x^4} + \left( \rho v^2 - (P_d - P_s)A \right) \frac{\partial^2 w}{\partial x^2} + 2\rho v \frac{\partial^2 w}{\partial x \partial t} + (m+p) \frac{\partial^2 w}{\partial t^2} = 0 \quad (1)$$

where

- $w =$ transverse deflection [mm]
- $\rho =$ mass of gas per unit length [Ns$^2$/mm$^2$]
- $P_d =$ discharge pressure [N/mm$^2$]
- $P_s =$ suction pressure [N/mm$^2$]
- $A =$ internal area of tube [mm$^2$]
- $v =$ average gas velocity [mm/s]
- $E =$ Young's modulus [N/mm$^2$]
- $I =$ area moment [mm$^4$]
- $m =$ mass of tube per unit length [Ns$^2$/mm$^2$]

From a physical viewpoint, the reader is invited to purchase a tube like a toy balloon and observe how after blowing the initially limp balloon up it becomes a stiff structure capable of beam like vibrations. The balloon beam increases its stiffness with increasing pressure differential, and thus increases its resonance frequency. The effect can also be observed on a Borden tube.
\[ \frac{EI}{\delta x^4} \frac{\partial^4 w}{\delta x^4} = \text{Force per unit length caused by the tube stiffness.} \]

\[ \rho v^2 \frac{\partial^2 w}{\delta x^2} = \text{Force per unit length caused by the fact that the gas is forced to follow a curved path when the tube is deflected. This term is best understood as a centrifugal force effect.} \]

\[ (P_d - P_s) \frac{\partial^2 w}{\delta x^2} = \text{Force per unit length caused by an effect that is similar to that of a stretched string. In the case of a high side compressor where we have a suction tube, this term becomes a buckling term.} \]

\[ \frac{2(P_d - P_s)}{\delta x^2} \frac{\partial^2 w}{\delta x^2} = \text{Force per unit length caused by a Coriolis force that is set up because the traveling fluid mass is forced to rotate by the vibrating tube.} \]

\[ (m + \rho) \frac{\partial^2 w}{\delta t^2} = \text{Force per unit length due to the tube and gas inertia.} \]

For normal operation of gas, air and refrigeration compressors, we may neglect the Coriolis effect, the centrifugal force effect and gas inertia effect. The simplified equation becomes

\[ EI \frac{\partial^4 w}{\delta x^4} - (P_d - P_s) A \frac{\partial^2 w}{\delta x^2} + \frac{\partial^2 w}{\delta t^2} = 0 \quad (2) \]

**Natural Tube Frequencies Influenced by Pressure Changes**

For the case of a simple supported tube of length \( L \), we obtain as solution the natural frequencies \( f_n \) [Hz] with \( n = 1, 2, \ldots \) (Fig. 1)

\[ f_n^2 = \frac{\pi^2 n^2}{4L^2} \left( 1 + \frac{(P_d - P_s)AL^2}{EI \pi^2 n^2} \right) \quad (3) \]

We observe a stiffening effect of the tube with increased pressure differential. While this stiffening effect shifts the natural frequencies for a typical refrigeration or air conditioning compressor only a few Hertz, it is sufficient to cause a noticeable change in vibration transmission because it may shift the tube in or out of a resonance. To minimize this effect, it is best to introduce damping in form of a friction wire or by other means if the resonance regions cannot be avoided. In case of a suction tube, \( P_d \) and \( P_s \) are reversed and a decrease in natural frequency results from an increase in pressure differential.

**Equation of Motion for a Tube of Constant Curvature, Vibrating in its Plane of Curvature**

To illustrate that the presence of curvature does not lend to any different conclusion, the case of a tube that is a ring segment free to expand circumferentially as shown in Fig. 2 is investigated next. Again, by Hamilton's principle, one obtains, after velocity effects are neglected and the standard simplifications are made,

\[ EI \frac{\partial^4 w}{\delta x^4} + \frac{2(P_d - P_s) Aa^2}{EI \delta x^2} \frac{\partial^2 w}{\delta x^2} + \frac{\partial^2 w}{\delta t^2} = 0 \]

where \( a \) is the radius of the curved tube in [mm]. If \( P_d = P_s \), we recognize that this equation reduces to the ring equation given by Prescott [3]. If we straighten the tube by setting \( a \theta = x \) and \( 1/a = 0 \), we obtain Eq. (2).

**Natural Frequencies of Curved Tubes Influenced by Pressure Changes**

For the case of a simple supported tube of arc length \( L = \alpha \theta \) \((\theta = 0 \text{ to } \theta = a)\) we obtain

\[ f_n^2 = \frac{\pi^2 n^2}{4L^2} \left( 1 - \frac{L^2}{\pi^2 n^2 a^2} \right) \left( \frac{EI}{m} \right) \left( 1 + \frac{(P_d - P_s)AL^2}{EI \pi^2 n^2 a^2} \right) \]

We conclude that the presence of the curvature \( 1/a \) does not change the basic effect observed for the straight tube. Note that the equation reduces to Eq. (3) when the curvature is zero.

**Forces Transmitted into Shell by Tube, Caused by Compressor Vibration**

To illustrate the force transmission mechanism into the shell, let us solve the idealized situation shown in Fig. 3. As equation of motion, Eq. (2) is valid, with the boundary conditions

\[ w(0, t) = W_0 e^{j\omega t} \]

\[ \frac{\partial^2 w}{\delta x^2} (0, t) = 0 \]

\[ w(L, t) = 0 \]
\[ \frac{\partial^2 w}{\partial x^2} (L, t) = 0 \]  

where \( W \) is the amplitude of compressor motion and \( \omega \) is the particular compressor harmonic that is to be investigated.

Making the substitution

\[ w(x,t) = u(x,t) + W_c(1-\frac{X}{L})e^{j\omega t} \]  

(9)

gives

\[ EI \frac{\partial^4 u}{\partial x^4} - (P_d - P_s)A \frac{\partial^2 u}{\partial x^2} + m \frac{\partial^2 u}{\partial t^2} + \lambda \frac{\partial u}{\partial t} = m \frac{\partial^2 W_c}{\partial x^2} (L-\frac{X}{L})e^{j\omega t} \]  

(10)

An equivalent viscous damping coefficient \( \lambda \) in Ns/mm^2 has been introduced. The boundary conditions are now homogeneous

\[ u(o,t) = 0 \]  

(11)

\[ \frac{\partial^2 u}{\partial x^2} (o,t) = 0 \]  

(12)

\[ u(L,t) = 0 \]  

(13)

\[ \frac{\partial^2 u}{\partial x^2} (L,t) = 0 \]  

(14)

The solution is [4]

\[ u(x,t) = \sum_{n=1}^{\infty} \eta_n \sin \frac{n\pi x}{L} \]  

(15)

where

\[ \eta_n = \frac{F_n}{\omega_n^2 \sqrt{1-(\omega_n/\omega)^2} + 4\xi_n^2 (\omega_n/\omega)^2}} \]  

(16)

\[ \phi_n = \tan^{-1} \left[ \frac{2\xi_n (\omega/\omega_n)}{1 - (\omega/\omega_n)^2} \right] \]  

(17)

\[ F_n = \frac{2m^2 W_c}{n\pi} (1-2\cos n\pi) \]  

(18)

and where

\[ \xi_n = \frac{\lambda}{2m\omega_n} \]  

(19)

\[ \omega_n = 2\pi f_n \]  

(20)

and where \( f_n \) is given by Eq. 3.

The force transmitted into the shell is

\[ Q(L,t) = -EI \frac{\partial^3 u}{\partial x^3} (L,t) \]  

(21)

or

\[ Q(L,t) = EI \frac{\omega_n^3}{L} \eta_n \sin n\pi \cos n\pi \]  

(22)

If there is a tube resonance in the vicinity of a compressor harmonic \( \omega \), this reduces approximately to

\[ Q(L,t) = \frac{2EIW_c^2}{L^3} \frac{\omega_n^3}{n^3} \frac{\omega}{\omega_n} \]  

(23)

\[ \sqrt{\left(1 - \frac{\omega^2}{\omega_n^2}\right)^2 + 4\xi_n^2 \left(\frac{\omega}{\omega_n}\right)^2} \]

The unit of the force is [N].

This result illustrates clearly the importance of a small deviation in \( \omega \) that may occur when the suction and discharge pressures change. As the tube damping is increased, this importance diminishes. It also explains some of the other effects that experimenters encounter.

**Effect of Mass Flow Rate on Tube Vibration**

Neglecting all effects but the centrifugal force effect in Eq. (1), we obtain

\[ EI \frac{\partial^4 w}{\partial x^4} + \rho v^2 \frac{\partial^2 w}{\partial x^2} + (m+\rho) \frac{\partial^2 w}{\partial t^2} = 0 \]  

(24)

Following the standard approach, we obtain for a simply supported tube

\[ f_n^2 = \frac{1}{4L^4} \frac{EI}{m+\rho} \left[ 1 - \frac{\rho v^2 L^2}{EI \omega_n^2} \right] \]  

(25)

We see that the natural frequency of the tube decreases as the flow velocity increases. This effect was found to be important for tubes conveying liquids [5], but is less important than the pressure effect in typical refrigeration compressor applications.

However, what is important, but extends beyond the scope of this discussion, is the fact that this result is related to the mechanism with which gas pulsations excite the vibration of tubes.
Summary and Discussion

This paper has given simple models that illustrate one of the mechanisms that causes shell noise to be a function of compressor suction and discharge pressures. The influence of flow velocity was also briefly discussed. All results apply also to the suction tube of a high side compressor, with slight modifications.

It should be noted that tubes that are coiled like a coil spring will theoretically not exhibit this effect for modes where the vibration is controlled by a torsional stiffness (the coils twist rather than bend). Neither will straight tubes where the tubes are clamped between two rigid surfaces exhibit this effect, since in this case no static tension is created in the tube by the pressure differential. The boundary prevents the tube from expanding axially. It is therefore, recommended that theoretical work on non simplified tubes be approached in two parts. The first part should examine the static tensions that are created by the pressure differential. The second part should then solve the vibration equations. It should also be of considerable interest to examine the vibration that results from the pulsating gas velocity in the pipe through the mechanism of a time dependent velocity. This mechanism is different from considering the pulsating internal pressures as excitation loads.

REFERENCES


Fig. 1 Simplified Model of Tube
Fig. 2 Tube of Constant Curvature Vibrating in its Plane of Curvature

Fig. 3 Simplified Model of Dynamic Force Transmission.