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Optimum Design of Reciprocating Compressors to Meet Thermodynamic Criteria

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ABSTRACT

A mathematical model of a reciprocating compressor is used in conjunction with a "Complex" method of optimisation to determine the "best" dimensions of a single-stage compressor to meet a prescribed duty. The mathematical procedures provide the basic compressor and valve dimensions by optimising an objective function based, in this instance, on criteria of thermodynamic performance. The consequences of modifying or redefining the objective function are examined. The effect is investigated of varying the constraints which must be imposed to meet practical considerations. The nature of these constraints, and the limits to be imposed on them, have to be declared by the designer.

INTRODUCTION

The many dimensions of a reciprocating compressor and its valves influence parameters which have varying and sometimes opposing effects on performance for a stipulated duty. However an adequate simulation model of a compressor can be employed in conjunction with appropriate mathematical optimisation procedures to determine, in an orderly and economical manner, the dimensions which will give the optimum compromise between the frequently opposing influences.

In this study a relatively simple simulation model, the Kerr model (1) (2), was employed. The optimisation procedure used was a modified form (3) of the Complex method developed by Box (4). This method of solution of constrained optimisation problems stipulates that the constraints imposed must not be exceeded. The method is in contrast with other methods, usually less economical of computer time, which continue to search for the optimum until the limits of the constraints are equalled. With the procedure used the equations which constitute the simulation model of the compressor had to be solved only about 200 times to arrive close to the optimum of the objective function.

The study was concerned primarily with the problem of specifying (a) a suitable criterion of thermodynamic performance of a compressor (the objective function to be optimised), (b) appropriate constraints necessary to take cognisance of various practical considerations and (c) the limits to be imposed on these constraints.

THEORETICAL STUDY

The Objective Function (F(x))

A series of calculations to optimise different forms of thermodynamic criteria (objective function) were performed over a range of compressor speed and cylinder stroke at a fixed compressor pressure ratio, when pumping air. Graphs (Figure 1(a) and 1(b)) were constructed from the results to show lines of constant gas throughput and lines of constant compressor speed on a plot of maximum gas throughput per unit of compressor work input versus compressor cylinder stroke volume (expressed as a ratio of stroke volume to the stroke (swept) volume of the original (reference) compressor, $V_{sw}/V_{swr}$). Hence the combination of compressor dimensions was available which would achieve the minimum power input for a selected gas flow rate at a specified discharge pressure.

The results (set A and set B) in Figure 1(a) and 1(b) differ due to the effect of optimising two differing criteria of thermodynamic performance (objective functions $F(x)a$ and $F(x)b$). Initially the objective function was defined by

$$F(x)a = \frac{\text{volumetric efficiency} \times 100}{1 + \frac{\text{valve losses}}{\text{input work for ideal adiabatic cycle}}}$$ (1)

Superficially this form of $F(x)$ appeared suitable, but experience showed that the optimum occurred at high values of volumetric efficiency but not necessarily at high values of gas throughput per unit of input work.

Equation (1) may be re-written as
The objective function was next modified to be equivalent to gas throughput/input work, again including losses due to gas blowby, loss of volumetric efficiency due to throttling across valves and excess work input due to valve resistance (suction and discharge loops on a pressure-volume diagram). Unfortunately this form of the objective function tended to a maximum as the flow rate tended to zero (when the loss terms became negligible). Hence it was considered that this form of $F(x)$ was also unsuitable: to overcome the difficulty a work term was incorporated into $F(x)$ to account for mechanical losses in the compressor. The work term $\text{ADIAB}(x)$ was arbitrarily chosen such that the mechanical losses were constant at 20% of the total adiabatic work for a compressor operating at a volumetric efficiency of 80%, i.e.

$$\text{ADIAB}(x) = 0.2 \frac{P_i}{P_i} \left(0.8 \frac{V_{sw}}{V_x} \right)^{\frac{n-1}{n}} \left[\frac{P_d}{P_i}^{\frac{n}{n-1}} - 1\right]$$

$\text{ADIAB}(x)$ is independent of volumetric efficiency and ensured that, where a high value of objective function was obtained with a low volumetric efficiency, a penalty resulted due to having a high ratio of friction work to useful work. Thus

$$F(x) = \left(\frac{V_{sw}}{V_x} \right) \frac{100}{\text{volumetric efficiency}} + \text{valve losses} + \frac{\text{ADIAB}(x)}{\text{ADIAB}(a)}$$

This was the form of objective function used to obtain the set B results shown in Figure 1(b).

Included in Figure 1(a) and 1(b) are the values (indicated by black squares) pertaining to the performance of the reference compressor as calculated by the mathematical model. The difference between these reference values in Figure 1(a) and 1(b) is not important to the theme, being due partly to small changes in the manner in which mechanical losses were accounted for and also partly to minor modifications made within the mathematical model, in the interval between calculating set A and set B results. In the computations for Figure 1(a) the mechanical efficiency was assumed to be 80% both for the optimised design and the reference compressor; in Figure 1(b) the mechanical loss term is included in $F(x)b$. In Figure 1(a) the set A results show that the maximum values of gas throughput per unit of work input when employing the objective function $F(x)a$ were less than the value of gas throughput per unit of work input for the reference compressor as originally designed. Although the optimisation procedure had operated correctly, the maximum value of $F(x)a$ was not solely a function of gas throughput but also of the piston area (as equation (2) shows). The procedure using $F(x)a$ produced high values of gas throughput (volumetric efficiency) but these did not occur at the highest values of gas throughput per unit of input work, the ordinate used in Figure 1. This latter quantity is of greater practical interest than gas throughput per unit time and hence it was necessary to define a more appropriate objective function, $F(x)b$. Figure 1(b) shows that the procedure then found optimum values of gas throughput per unit of input work in excess of that of the reference compressor, the improvement becoming less as compressor speed increased.
GAS THROUGHPUT/INPUT WORK VERSUS CYLINDER STROKE VOLUME

Fig. 1 (a) and (b)

COMPUTED OPTIMUM VOLUMETRIC EFFICIENCY Y. CYLINDER STROKE VOLUME

Fig. 2
In Set B the linear clearance could increase above the minimum fixed value prescribed for Set A.

\[ V_c \text{ Limit } = \frac{V_c}{A_p} \]

The constraint limit for suction valve impact velocity at the valve stop was set at 1.82 m/s, an arbitrary value but corresponding approximately to that for the original design when operating at rated conditions. At 300 rev/min the suction valve impact velocity at its stop for the reference compressor, as computed by the mathematical model, was 1.42 m/s. So in this case the optimisation procedure could relax the impact velocity up to the limit of the constraint: this allowed an increase in permitted valve lift with an accompanying reduction in throttling loss across the valve, resulting in an increase in volumetric efficiency and decrease in work input. Consequently, an improvement in the criterion of thermodynamic performance \( F(x)b \) was obtained, amounting to approximately 10%. However, at 500 rev/min the suction valve impact velocity of the reference compressor at the operating condition was 1.89 m/s, about 4% above the specified limit of 1.82 m/s. The optimisation procedure automatically made adjustments to dimensions to reduce the impact velocity to conform to the limit: nevertheless the optimisation achieved an improvement in the criterion of thermodynamic performance \( F(x)b \) of 5%.

It is apparent from Figure 1 that the thermodynamic performance of the compressor as specified by either objective function \( F(x)a \) or \( F(x)b \) would be achieved at compressor speeds below 300 rev/min. However, this would result in unacceptably large compressor dimensions for a given duty and emphasises that criteria other than thermodynamic performance must be considered. Since most refrigerant compressors operate at a fixed speed, usually a function of the frequency of the electrical supply available, the speed would not be a variable in the optimisation of the compressor design.

Figure 2 shows that the optimum volumetric efficiency (maximum gas throughput) relating to
objective function \( F(x) \) (set \( A \) results) was greater than the optimum volumetric efficiency relating to \( F(x) \) (set \( B \) results), yet as seen by comparing Figure 1(a) and 1(b) the optimum compressor efficiency (maximum gas throughput per unit of work input) when employing the objective function \( F(x) \) was less than that when using \( F(x) \). Added to Figure 2 are values of \( f_n \), a function defined by the ratio of piston area to crank radius referred to the ratio for the reference compressor, \( (A_p B)/(C_n/R) \); the results are shown in different form in Figure 3. The values of \( f_n \) (in square brackets in Figure 2) relating to the set \( A \) results using \( F(x) \) indicate that the piston area varied according to the demands of the optimisation procedure. Set \( B \) results using \( F(x) \) tended to have higher values of optimum piston area, and at compressor speeds above 300 rev/min reached the limit of the constraint placed on piston size for practical reasons. Larger piston areas infer larger piston clearance volume (for set \( A \) results, the linear clearance between piston and cylinder head had been made constant at the lowest practical value): a larger piston clearance volume results in lower volumetric efficiency (Figure 2). While a larger piston area automatically allowed larger valve port areas to be accommodated and consequently reduced throttling losses and hence improved volumetric efficiency, this improvement was more than offset by the loss of volumetric efficiency as a direct consequence of larger clearance volume.

It is of particular interest to note from Figure 4 that the optimum performance was not reached with the minimum clearance volume except at the lower compressor speeds. This unexpected result could conceivably have been due to instability and poor convergence in the optimisation procedure. To examine the matter a set of optimisations, set \( C \), was performed using objective function \( F(x) \) but with the clearance volume fixed at a minimum limit, \( V_c \) limit. Figure 5 shows that the optimum performance with set \( C \) tended to be slightly inferior to that with set \( B \) particularly at high compressor speeds and at the highest value of stroke volume permitted by the constraint (Figure 3). Hence the surprising conclusion that the clearance volume should have been larger than the minimum practical value. The increase in piston area above that of the reference compressor permitted a larger valve flow area and reduced throttling loss without reduction in gas flow; the optimisation procedure, when finally faced with the constraint limit on piston area (Figure 3) searched for an alternative strategy and allowed an increase in clearance volume. The consequent reduction in gas flow produced a reduction in valve impact velocity; this allowed an increase in permitted valve lift which in turn reduced losses due to valve throttling and increased the thermodynamic performance, \( F(x) \).

This explanation is further supported by the curves
of suction valve lift ratio, \((H_s/H_{os})\) versus cylinder stroke volume ratio for set B results as presented in Figure 6; the gradients of the curves tend to change from a negative to a positive value as the constraint on piston area becomes operative (Figure 3).

Figure 7 is a plot of optimum volumetric efficiency versus cylinder stroke volume, for both set B and C results. For set C, where the piston area and clearance volume were constrained, the gradients are slightly positive and are accompanied by high values of volumetric efficiency. In particular the graphs pertaining to rotational speeds of 500 and 600 rev/min illustrate this. For set B however, where changes in clearance volume were allowed (with a lower limit on linear clearance) the gradients are predominantly negative and are accompanied by lower values of volumetric efficiency particularly at higher speeds and higher values of cylinder stroke volume.

Figure 8 which is similar to Figure 3, allows comparison to be made of the values of the ratio of piston area to crank radius plotted to a base of cylinder stroke volume for set B and set C. At a rotational speed of 300 rev/min and to a lesser extent at 400 rev/min in set C, the fixed value for the lower limit of clearance volume is beneficial, as this allows the optimum to be achieved with modest values of piston area to crank radius ratio.

In computing the optimum for set B, the optimisation procedure attempts to place the clearance volume on the lower constraint value and at convergence the values of clearance volume lie only a small distance from the constraint limit.

To achieve the optimum at speeds of 500 and 600 rev/min the optimisation procedure attempts to increase the piston area to crank radius ratio and is near to the upper constraint over the range of stroke volumes, particularly at 600 rev/min. Whether the clearance volume is free to adjust (set B) or fixed (set C) the optimisation procedure recommends the highest permitted piston area to crank radius ratio.
CONCLUSIONS

The availability of mathematical models which simulate reciprocating compressors on digital computers in association with optimisation techniques capable of accounting for constraints provides a new design aid. The procedures adjust the dimensions declared to be the relevant variables in order to provide a combination which meets the criterion of optimisation (the objective function). Appropriate changes of the variables, sometimes only small changes of dimensions to which a particular design may be sensitive, can result in significant improvement in the value of the objective function. The magnitude and direction of these individual changes cannot be readily determined by any intuitive process or arrived at by an experimental development program.

Although an indispensable aid to finding the best design, optimisation procedures require even more expertise to be exercised by the designer than is demanded by traditional design methods. The objective function must be defined and this paper shows that a suitable definition of objective function is not obvious even to meet the limited criterion of "best thermodynamic performance". Additionally the designer must exercise acumen in identifying and quantifying the many constraints which have to be imposed in order to arrive at a practical design.

As simulation models become more complete and more generally available, optimisation will inevitably be applied to them, so that a new compressor design is the best compromise of dimensions to meet a specified duty within the constraints imposed to meet practical considerations.

REFERENCES