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S. Krishnan

J. Y. Murthy

S V. Garimella

Purdue University, sureshg@purdue.edu

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A Two-Temperature Model for the Analysis of Passive Thermal Control Systems

Shankar Krishnan
Jayathi Y. Murthy
Suresh V. Garimella
e-mail: sureshg@ecn.purdue.edu

Cooling Technologies Research Center,
School of Mechanical Engineering,
Purdue University, West Lafayette, IN 47907-2088

Introduction

Thermal management of electronics is becoming increasingly challenging as chip-level heat fluxes increase at a rapid rate. Cooling can be achieved either by using conventional techniques that utilize active components like fans, or with passive techniques such as phase change thermal storage units and heat pipes. In many emerging cooling designs, thermal conductivity enhancers such as metallic foams, internal fins and metallic particles are used. The interstices contain either fluid or air. In thermal energy storage applications, for example, phase change materials (PCMs) are used. These materials are particularly attractive for transient applications where the heat generation experiences frequent short spikes in its magnitude. In addition to a high latent heat of melting, phase change materials used in electronics cooling must have high thermal diffusivity to quickly conduct away the dissipated heat. Such PCMs contain either fluid or air. In thermal energy storage applications, for example, phase change materials (PCMs) are used. These materials are particularly attractive for transient applications where the heat generation experiences frequent short spikes in its magnitude. In addition to a high latent heat of melting, phase change materials used in electronics cooling must have high thermal diffusivity to quickly conduct away the dissipated heat.

PCMs

~

1

~

order of 1 W/mK or less

2

1

and with a decrease in Biot number based on the interstitial heat transfer coefficient. Lu et al. [8] developed an analytical model for the convection heat transport through metal foams. Simplifying assumptions were made and guidelines for optimum foam structures were proposed for maximum heat transfer for a given power input. Minkowycz et al. [9] performed a theoretical study to analyze the validity of assuming local thermal equilibrium in heat transfer through a porous fluidized bed. It was shown that local thermal equilibrium depends on the size of the porous layer, pore size, interstitial heat transfer coefficient and thermophysical properties. Calmidi and Mahajan [10] performed an experimental investigation of forced convection in metal foams with air and water as fluid phases. An empirical correlation was also reported. Alazmi and Vafai [11] analyzed a variety of porous me-

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dia transport models for forced convection in porous beds. The
effects of variations in existing semi-heuristic models, effects of
porosity, thermal dispersion and local thermal equilibrium were
reported. Calmidi and Mahajan [12] reported an experimental and
numerical study of forced convection in metal foams under local
thermal non-equilibrium conditions. Hwang et al. [13] performed
experiments on metal foams. They measured the interstitial con-
vective heat transfer and frictional drag for forced convective
flows using a transient single-blow technique. Empirical correla-
tions for interstitial heat transfer coefficient were reported. Further
details of the thermal non-equilibrium modeling of forced convec-
tion can be obtained from [14].

Natural convection in porous media is also encountered in
many applications [15,16]. However, while many studies have
investigated forced convective flow through porous media under local non-equilibrium thermal conditions, fewer have considered natural convection. Deiber and Bortolozzi [17] performed a natural convection study in a vertical porous annulus composed of spheres without assuming local thermal equilibrium to exist between the solid matrix and saturating fluid. They reported that a two-energy equation model should be used at high Rayleigh and Darcy numbers. Mohammed [18] performed a parametric study for natural convection in a porous enclosure under steady-state conditions. The effects of varying the Darcy, Rayleigh, and Prandtl numbers and the solid-to-fluid thermal conductivity ratio on the temperature differentials between solid and fluid were reported. Rees and Pop [19] also reported the existence of nonequilibrium conditions for steady-state flows. Beckermann and Viskanta [20] experimentally and numerically analyzed natural convection inside a porous enclosure with a local thermal equilibrium assumption. Recently, Phanikumar and Mahajan [21] reported numerical and experimental results for natural convective flow in a rectangular domain partially filled with a porous medium (metal foam) and heated from below. Heat transfer enhancement due to the presence of the foam against that with no foam was studied. Effects of thermal dispersion and the Darcy number on the heat transfer were explored. They concluded that a local thermal nonequilibrium model better describes the underlying heat transfer phenomena in metal foams.

The present work undertakes a parametric analysis of natural convection inside enclosures containing metal foams for the transient thermal management of electronics. Only single-phase flows without phase change are considered to elucidate the basic physics. The paper examines the transient effects of different parameters including the Rayleigh, Darcy and Prandtl numbers, solid-to-fluid thermal conductivities, and the Nusselt number for interphase heat transfer, on the temperature differentials between

Fig. 6 Predicted temperature variation at steady-state for various Rayleigh numbers at the mid-height of the domain (y=0.5): (a) solid-to-fluid temperature difference, and (b) solid (broken line) and fluid (solid lines with symbols) temperature distributions.

Fig. 7 Predicted temporal evolution of thermal field for Ra =10^6, Nu_f=0, Pr=1, and Da=10^{-2} at the mid-height of the domain (y=0.5): (a) solid-to-fluid temperature difference, and (b) solid (broken line) and fluid (solid lines with symbols) temperature distribution. Solid (broken line) reaches a steady state very fast.
the solid and fluid under local thermal non-equilibrium conditions. The physics governing the existence of thermal non-equilibrium during the transient are explored.

Mathematical Formulation and Numerical Modeling

A schematic of the problem considered is shown in Fig. 1. A porous medium saturated with fluid is contained in a square enclosure of side $H$. The left and right walls are held at constant temperatures $T_h$ and $T_c$, respectively. The top and bottom walls are adiabatic. The fluid and solid media are at equilibrium initially, and at temperature $T_c$. At $t=0$, the temperature of the left wall is raised to $T_h$. The objective of the work is to examine the temporal evolution of the solid and fluid temperature fields. The liquid is assumed to be incompressible and Newtonian. The momentum equations include the Brinkman’s term and Forchheimer’s extension to Darcy flow. The solid is assumed to be isotropic and rigid. Thermophysical properties of the solid and fluid phases are assumed to remain constant and isotropic over the range of temperatures considered. The Boussinesq approximation is invoked in the fluid phase. Dispersion effects are neglected. Using the dimensionless variables

\[ \xi = \frac{x}{H}; \quad \eta = \frac{y}{H}; \quad \tau = \frac{t \alpha_f}{H^2}; \quad \frac{\rho u}{\rho_f \nu_f \alpha_f}; \quad P = \frac{p H^2}{\rho_f \nu_f \alpha_f} \]

the dimensionless volume-averaged continuity and momentum equations [20] in the fluid phase are

\[ \nabla \cdot \mathbf{U} = 0 \]
The two energy equations can be closed either using continuum equations for solid and fluid are valid at every point in the domain. The two energy equations can be closed either using an unsteady closure [22] or a steady-state closure [5]. A steady-state closure has been found to be sufficient for most problems [22], and is used in this work. The dimensionless energy equations are

\[
\frac{1}{Pr} \left( \frac{1}{e} \frac{\partial U}{\partial \tau} + \frac{1}{e^2} (U \cdot \nabla) U \right) = -\nabla P + \frac{1}{e} \nabla^2 U - \frac{1}{Da^2} \frac{F}{Pr \cdot Da} \left( U + Ra \cdot T_f^* \right)
\]

The governing intrinsic phase-averaged energy equations are written for the solid and fluid phases separately. These macroscopic energy equations are valid at every point in the domain. The energy equations can be closed either using an unsteady closure [22] or a steady-state closure [5]. A steady-state closure has been found to be sufficient for most problems [22], and is used in this work. The dimensionless energy equations are

\[
\frac{\partial T_f^*}{\partial \tau} = \lambda \nabla^2 T_f^* - \frac{(Nu)}{(1-e)} (T_f^* - T_s^*)
\]

Two different volumetric heat transfer coefficients are used. For forced convection through a packed bed of spheres [16,23] the expression is

\[
(Nu)_p = \frac{h_s d^2}{k_f} = 6 \left( 1 - e \right) \left( 2 + 1.1 Pr^{1/3} (Re)^{0.6} \right)
\]

This correlation is valid up to Re of 8500. For forced flow through metal foams, the following correlation is used

\[
(Nu)_p = \frac{h_s d^2}{k_f} = 0.376 (Re)^{0.644} (Pr)^{0.33}
\]

It is important to note the limitations of Eq. (6) in modeling interphase heat transfer. The correlation in [13] was developed for metal foams in air, for a Reynolds number Re of \(\frac{uL}{v}\) (\(L\) being the test-section length) in the range 1900<Re<7900. To use it for typical PCMs, a scaling factor for Prandtl number similar to that in Phanikumar and Mahajan [21] was included. Further, for high Pr, the flow in the core of the domain is of relatively low velocity, and the Reynolds number may be far lower than the lower Re limit of Eq. (6); it is also clear that Eq. (6) does not correctly recover the conduction (Re=0) limit. To the authors' knowledge, there are no available correlations in the literature describing the inter-phase heat transfer coefficient in the high-Prandtl number and low-Reynolds number limit. Therefore, Eq. (6) is used despite these limitations.

The volumetric heat transfer coefficient in the above equations can be expressed in terms of the inter-phase heat transfer coefficient as

\[
h_s = h_{sf} a_{sf}
\]

In Eq. (7), \(a_{sf}\) is the specific surface area. These correlations are expected to be approximately valid for natural convection as well. The Reynolds number in Eqs. (5) and (6) was interpreted as the local Reynolds number, defined based on the local mean velocity.
The relations for permeability and inertial coefficient used for spherical particles can be obtained from [5,6]. For metal foams, a porosity of 0.8 is used and the inertial coefficient is 0.068 [13].

The computational domain is discretized into finite volumes using an orthogonal mesh. Pressure, velocity and the two temperatures are stored at the cell centroids. A central differencing scheme with a deferred correction [24,25] is used for convective fluxes. A central differencing scheme is used for discretizing diffusive fluxes. A second order Euler implicit (three time level) scheme is used for the transient terms. The SIMPLE algorithm is employed for obtaining the velocity fields. The linearized systems of equations are solved using the strongly implicit procedure (SIP) [25]. Additional details about the numerical approach are available in [2,26]. The calculations are terminated when the residual has dropped at least below $10^{-6}$ for all governing equations.

**Code Validation**

Because experimental results are not available in the literature for natural convection inside porous metal foams, the code was validated against studies on convection in a packed bed of spheres. Beckermann and Viskanta [20] performed an experimental and numerical study of natural convection in a porous enclosure comprising spherical (glass) particles with gallium as the interstitial fluid. A fixed porosity of 0.385 was considered. Predictions from the present work are compared to those in [20] in Fig. 2 at different vertical (η-direction) locations. In the current predictions, Eq. (5) was used to represent the inter-phase heat transfer coefficient. The predicted thermal fields agree well with the numerical predictions in [20] and also show satisfactory agreement with the experimental measurements [20].

Predictions from the present model are also compared in Figs. 3 and 4 with those of Mohammed [18] who also employed a two-temperature model for the analysis of natural convection in a packed bed of spheres. The comparison is shown for the case of Prandtl number=1.0, and the agreement is seen to be excellent.

Grid and time-step independence for the mesh and time-step sizes used in the present simulations was also established. A Rayleigh number of $10^8$, a Darcy number of $10^{-2}$ and a Prandtl number of 1 were used for this set of calculations. Grid independence was tested using three different non-uniform grids, 46×46 (mesh #1), 92×92 (mesh #2) and 182×182 (mesh #3). For mesh #1, a deviation of 6.48% in the maximum solid-to-fluid temperature difference was observed with respect to mesh #3, with this deviation between mesh #2 and mesh #3 reducing to 0.33%. Therefore, mesh #2 (92×92) was chosen for all the calculations in this study. For establishing time-step independence, three different time step values, 1.25×$10^{-4}$, 2.5×$10^{-4}$ and 5×$10^{-4}$ were evaluated with the 92×92 mesh. The deviation in the results for the maximum solid-to-fluid temperature difference compared to the smallest time step was less than 0.01% for both larger time steps. Since refinement in the time-step did not produce discernible improvements in accuracy, a nondimensional time-step value of 2.5×$10^{-4}$ was used for most of the computations in this work.

**Results and Discussion**

Passive thermal storage units used in electronics cooling would feature solid-to-fluid thermal conductivity ratios ($\lambda$) in the range of $10^{2}$ to $10^{5}$ and Prandtl numbers of 1 to 100. The porosity of the metal foam considered in this study is held constant at 0.8 with a pore size corresponding to $d/H = 0.0135$. The ratio of the average ligament diameter of the foam to the mean cell size of the foam is 0.1875 with average ligament diameter being equal to 0.36 mm [13]. In the range of thermal conductivity ratios of interest ($\lambda\sim10^{-1}$–$10^{5}$), the results did not vary much with $\lambda$, and hence a value of $10^{3}$ was used for all cases.

**Steady State.** The case of zero inter-phase heat transfer ($\text{Nu}_f=0$) is first considered, in which there is no thermal interaction between the solid and fluid phases. Figure 5 shows the variation of solid and fluid temperature profiles for different Rayleigh numbers at mid-height ($\eta=0.5$); the fluid temperature variation at $\eta=0.02$ and $\eta=0.98$ for Ra=$10^8$ is also shown for contrast. The solid profile is expected to be a straight line and is independent of the Prandtl number and $\eta$ location. The fluid profile is also a straight line if the Rayleigh number is low and, as a result, the solid-to-fluid temperature difference $\Delta T^*_f$ is small. As the Rayleigh number increases, the fluid temperature profile deviates from the solid temperature profile as thermal boundary layers develop near the heated and cooled walls. The thickness of the boundary layer scales as $\sim (\text{RaDa}^{2})^{-1/4}$ [27]. It can be seen from Fig. 5 that the numerical predictions agree with the scaling analysis. For example, for Ra=$10^8$ and Da=$10^{-2}$ the numerically predicted boundary layer thickness is approximately 0.09 and that from scaling analysis is 0.1. As the Rayleigh number increases, the thermal boundary layer in the fluid is increasingly confined to a thin layer near the wall, with the core being at a temperature $\theta^*_f\sim0.5$. Thus, the geometric center of the cavity is a point of zero temperature difference between solid and fluid. The
maximum temperature difference is expected to occur at the edge of the thermal boundary layer, with a value $\Delta T^* \sim O(0.5)$ at the mid-height ($\eta=0.5$) of the domain for high Rayleigh numbers. At $\eta=0$ and 1, the maximum temperature difference is $\sim O(1)$ for high Ra as expected.

The next case considered is one in which the solid and fluid exchange energy in a steady-state mode through inter-phase transfer ($\text{Nu} \neq 0$). Figure 6(a) shows the dimensionless solid-to-fluid temperature difference $T_s^* - T_f^*$ at mid-height ($\eta=0.5$) for several Rayleigh numbers, for $\text{Da} = 10^{-2}$, $\lambda = 10^3$, $\Omega = 1$, and $d/H = 0.0135$. Figure 6(b) shows dimensionless solid and fluid temperature variations at mid-height for $\text{Pr}=1$. The effect of the inter-phase heat transfer coefficient is to lower the solid-to-fluid temperature difference by a small extent. The vertical velocity in the rectangular domain scales as $O(0.5)$ at the mid-height of the enclosure ($\eta=0.5$) is not linear but follows the fluid temperature variation due to inter-phase heat exchange between solid and fluid phases.

Figure 6(a) also shows that for a Rayleigh number of $10^6$, increasing the Prandtl number from $\text{Pr}=1$ to $\text{Pr}=100$ increases the solid-to-fluid temperature difference by a small extent. The vertical velocity in the rectangular domain scales as [27]

$$V = \frac{v d H}{a_f d} = \text{Re} \left( \frac{H^2}{d} \right) \sim (\text{RaDa}^2)^{1/2}$$

The inter-phase heat transfer coefficient scales as

$$\text{Nu}_p = \frac{h_s d^2}{k_f} = 0.376 \left( \text{Re} \right)^{0.644} \left( \text{Pr} \right)^{0.37} \sim 0.376 \left( \frac{\text{RaDa}^2}{H^2} \right)^{1/2} \left( \frac{d}{H} \right)^{0.644} \text{Pr}^{-0.274}$$

Thus, the inter-phase Nusselt number depends on the Prandtl number, and tends to decrease as the Prandtl number increases, causing the two media to act in a more uncoupled manner. This dependence assumes that the Prandtl number variation in Eq. (6) is valid at high Pr.

**Unsteady Flow.** As in the steady-state cases, in order to understand the effect of the inter-phase heat transfer on the solid-to-fluid temperature difference we first consider the $\text{Nu}=0$ case. In the absence of inter-phase heat exchange, the solid-to-fluid temperature difference depends on the relative response time of the two phases. The solid reaches steady state in time scales of order $(H^2/a_f)$ or a dimensionless time scale of $\tau_f \sim 1/\Omega$. The fluid response time depends on Ra. For low Rayleigh numbers the time to reach steady state scales as $H^2/a_f$, corresponding to a dimensionless scale $\tau_f \sim O(1)$. For high Rayleigh numbers, the fluid time-to-steady-state scales as the inverse of the vertical velocity, and, hence,

$$V \sim (\text{RaDa}^2)^{1/2}$$

$$\tau_f \sim 1/V \sim (\text{RaDa}^2)^{-1/2}$$

Figure 7(a) shows the variation of dimensionless solid-to-fluid temperature difference along $\eta=0.5$ at different times for $\text{Ra} = 10^6$, $\text{Da} = 10^{-2}$, $\text{Pr}=1$, $\Omega = 1000$, and $\lambda = 1$. Figure 7(b) shows the fluid and solid temperature distributions at the mid-height of the domain. The solid response time is much faster than that of the fluid due to its higher thermal diffusivity, and it is seen to reach a steady state much faster. In Fig. 7(b), the solid has reached a steady state even before the fluid field starts to develop. Thus, at

<table>
<thead>
<tr>
<th>Darcy Number</th>
<th>Prandtl Number</th>
<th>Rayleigh Number</th>
<th>Time $^1$ for solid to reach steady state</th>
<th>Time $^1$ for fluid to reach steady state</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^{-2}$</td>
<td>1</td>
<td>$10^6$</td>
<td>0.00075</td>
<td>0.00935</td>
</tr>
<tr>
<td></td>
<td>$10^4$</td>
<td></td>
<td>0.0023</td>
<td>0.00235</td>
</tr>
<tr>
<td>100</td>
<td>$10^6$</td>
<td></td>
<td>0.015</td>
<td>0.0225</td>
</tr>
<tr>
<td></td>
<td>$10^4$</td>
<td></td>
<td>0.0027</td>
<td>0.00275</td>
</tr>
<tr>
<td>10</td>
<td>$10^6$</td>
<td></td>
<td>0.02345</td>
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</tr>
<tr>
<td>100</td>
<td>$10^6$</td>
<td></td>
<td>0.0125</td>
<td>0.15</td>
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<tr>
<td></td>
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<td></td>
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<td>0.2245</td>
</tr>
<tr>
<td>$10^{-3}$</td>
<td>1</td>
<td>$10^6$</td>
<td>0.01475</td>
<td>0.02175</td>
</tr>
</tbody>
</table>

$^1$Nondimensional time.
The ratio of the solid-to-fluid time to reach steady state is given by

\[ \frac{\tau_s}{\tau_f} \sim \frac{\Omega (RaDa)^{1/2}}{\lambda} \]  

(11)

As the Rayleigh number increases, the fluid response time decreases. This is seen in Fig. 8(a) which shows a plot of temperature difference variation along the mid-height of the domain for \( Ra = 10^6, Da = 10^{-2}, Pr = 1 \), and \( Nu_f = 0 \). Figure 8(b) shows the variation of solid and fluid temperature fields along the mid-height of the domain. Due to the high Rayleigh number, the core fluid is at a temperature of \( T_f = 0.5 \) and the boundary layer is confined to a very thin layer near the heated and cooled walls. The time to reach a steady state and the solid-to-fluid temperature difference at steady state are listed in Tables 1(a) and 1(b) for various Darcy, Prandtl and Rayleigh numbers. The process is declared to have reached a steady state if \( T^* \) at \( (\eta, \xi) = (0.78, 0.98) \) is within 1% of its steady-state value. It is clear from Tables 1(a) and 1(b) that for \( Nu_f = 0 \), the time to reach steady state is independent of \( Pr \), as expected. For low Darcy number (\( Da = 10^{-3} \)), the fluid flow is so strongly retarded by the porous medium that the time to reach a steady-state is governed by the fluid diffusion scale, \( \tau_f = O(1) \); correspondingly, the solid-to-fluid temperature difference is relatively low.

The results for \( Nu_f \neq 0 \) are considered next. Here, the behavior is governed by the relative magnitude of three interacting scales: the response time of the solid, the response time of the fluid, and the response time of the solid-fluid interface. If the interface response time is very long, the solid and fluid evolve separately and large temperature differences exist at steady state. If, on the other hand, the interface response time is much shorter than either the solid or the fluid response time, the two phases will evolve in a coupled manner, with the faster-responding phase imprinting its temperature on the slower-responding phase. A measure of the dimensionless response time of the interface is \( Nu_f^{-1} \) \((=h_fH^2/k_f)^{-1}\). For \( Nu_f \gg 1 \), the two phases will evolve in equilibrium. Figure 9(a) shows the temporal evolution of the solid-to-fluid temperature difference for a Rayleigh number of \( 10^6 \), \( Pr = 1 \) and \( Da = 10^{-2} \). The dimensionless solid and fluid temperature distribution at \( \eta = 0.5 \) is shown in Fig. 9(b). The solid has the fastest response time, and reaches a steady state on a time scale \( \tau_s \sim \Omega/\lambda \). The fluid time scale for this low value of RaDa\(^{1/2}\) is relatively large, leading to a slow approach to steady state. Thus, the fluid is essentially at its initial condition during the time the solid reaches a steady state, and initial temperature differences in Fig. 9(a) are \( O(1) \). However, it is seen that the solid-to-fluid temperature difference is driven to zero over a time scale \( \tau_f = O(0.015) \), far faster than the response time of the fluid. This is a result of the response time of the inter-phase heat transfer, which is intermediate between the solid and fluid scales. This allows the solid to impose its imprint on the fluid temperature profile, which also assumes a straight line shape, leading to nearly zero temperature difference at steady state. For the uncoupled case (Fig. 7(b)), the fluid temperature is not a straight line for the same set of parameters.

In contrast, the interface time-scale is competitive with the solid time scale in Figs. 10(a) and 10(b), which shows the solid-to-fluid...
temperature difference and the solid and fluid temperature variation at mid-height for \( Ra = 10^8 \), \( Pr = 1 \), and \( Da = 10^{-2} \). The ratio of the interface time scale to the solid response time is \( \sim (N_{u} \Omega / \lambda)^{-1} \). Here, the solid is not allowed to reach a steady state before losing heat to the fluid. Instead, the temperature difference begins to fall well before \( \tau \sim \tau_{f} \) and reaches equilibrium with the fluid on a time scale of \( \tau \sim 0.0025 \). Because the fluid time scale is more competitive with the solid for this high value of \( Ra Da^2 (= 10^8) \), we see that the solid temperature profile is not imposed on the fluid and a nonzero temperature difference exists at steady state. The time to reach steady-state and the steady state solid-to-fluid temperature difference for several \( Ra, Pr \) and \( Da \) are listed in Tables 2(a) and 2(b), respectively. Since the two phases are coupled, the time to reach steady state is of the same order for both. This is especially true as \( Ra \) and \( Da \) increase, since \( Nu_{f} \) increases with \( Ra Da^2 \).

The effect of Prandtl number on the solid-to-fluid temperature difference and the temporal evolution of the solid and fluid temperature fields is brought out in Figs. 11(a) and 11(b) for which \( Ra = 10^8 \), \( Pr = 100 \), and \( Da = 10^{-2} \). From Eq. (2), it can be seen that an increase in Prandtl number causes a decrease in the interphase heat transfer coefficient, and, hence, the phases act in a more uncoupled manner. The temporal evolution is faster than the uncoupled case (Figs. 8(a) and 8(b)) and the solid-to-fluid temperature difference is higher than the corresponding \( Pr = 1 \) and \( Nu_{f} \neq 0 \) case. It is important to note, however, that these results are subject to the assumption that Eq. (6) describes the inter-phase heat transfer.

The effect of Darcy number can be seen by comparing Fig. 12 with Fig. 9. Figure 9 has been plotted for \( Ra = 10^8 \), \( Da = 10^{-2} \) while Fig. 12 has been plotted for \( Ra = 10^3 \), \( Da = 10^{-3} \). The fluid velocity scales approximately as \( \sim (Ra Da^2)^{1/2} \). As a result, the temperatures differences in the two figures are almost identical; the response times are also very similar.

Conclusions

A fully transient analysis of natural convection inside a porous enclosure containing a metal foam was carried out using a two-temperature formulation. Based on the results of a parametric study, local thermal equilibrium is not ensured, either during the transient or at steady state in such systems. For transient applications, the temperature difference between the solid and the fluid is governed by the response time of the two phases as well as the time scale of the solid-fluid interface. The results demonstrate that even if the steady-state temperature differences are small, the temperature difference during the unsteady response may be quite large. Single-temperature formulations are expected to yield incorrect predictions when the inter-phase heat transfer time scale is longer than the fastest response time of either the fluid or solid phase. For metal-air or metal/PCM systems, the faster-responding phase for typical ranges of \( Ra \) and \( Da \) is the solid. In this limit, two-temperature formulations are necessary when \( Nu_{f} < 1 \).

The present study did not consider phase change. When phase change is included, an additional time scale associated with the latent heat of melting is introduced in the problem, and depending on the value of the Stefan number, can change the results obtained here. Yet another time scale appears when pulsed heating is used. The interaction of pulse periodicity with these competing scales determines the validity of single-temperature formulations. These effects will be investigated in the future. Another research need is the development of interphase Nusselt number correlations for high Prandtl number and low-Reynolds number flows, as well as correlations for pure conduction heat transfer in PCM-impregnated foams. These correlations are necessary to correctly capture inter-phase heat transfer in the frozen as well as the low-velocity sections of the computational domain.

Nomenclature

- \( a_{ef} \) = Ratio of specific surface area to volume, \( m^{-1} \)
- \( C_{p} \) = Specific heat, \( J kg^{-1} K^{-1} \)
- \( Da \) = Darcy number
- \( d \) = Particle diameter or mean pore diameter, \( m \)
- \( F \) = Inertial coefficient
- \( g \) = Acceleration due to gravity, \( m s^{-2} \)
- \( H \) = Height of enclosure, \( m \)
- \( h_{uf} \) = Inter-phase heat transfer coefficient, \( W m^{-2} K^{-1} \)
- \( h_{v} \) = Volumetric heat transfer coefficient, \( W m^{-3} K^{-1} \)
- \( K \) = Permeability, \( m^2 \)
- \( k \) = Thermal conductivity, \( W m^{-1} K^{-1} \)
- \( Nu \) = Nusselt number based on height of the enclosure
- \( P \) = Pressure, \( N m^{-2} \)
- \( Pr \) = Prandtl number
- \( Ra \) = Rayleigh number
- \( Re \) = Reynolds number
- \( T \) = Temperature, \( K \)
- \( t \) = Time, \( s \)
- \( U \) = Velocity vector, \( m s^{-1} \)
- \( u, v \) = Velocity in x and y directions, \( m s^{-1} \)
- \( x, y \) = Cartesian coordinates

Greek Symbols

- \( \alpha \) = Thermal diffusivity, \( m^2 s^{-1} \)
- \( \beta \) = Thermal expansion coefficient, \( K^{-1} \)
- \( \Delta H \) = Enthalpy of freezing/melting, \( J kg^{-1} \)
- \( \Delta t \) = Time step
- \( \Delta x, \Delta y \) = Spatial mesh sizes, \( m \)
- \( \delta \) = Ratio of mean pore diameter to height of enclosure
- \( \epsilon \) = Porosity
- \( \eta \) = Dimensionless y coordinate
- \( \lambda \) = Ratio of thermal conductivities of solid and fluid
- \( \mu \) = Dynamic viscosity, \( N s m^{-2} \)
- \( \nu \) = Kinematic viscosity, \( m^2 s^{-1} \)
- \( \rho \) = Density, \( kg m^{-3} \)
- \( \tau \) = Dimensionless time
- \( \xi \) = Dimensionless x coordinate
- \( \Omega \) = ratio of volumetric heat capacities of solid and liquid

Subscripts

- \( c \) = cold
- \( ef \) = effective fluid property
- \( es \) = effective solid property
- \( f \) = fluid
- \( h \) = hot
- \( l \) = liquid
- \( p \) = pore or particle
- \( s \) = solid

Superscripts

- * = dimensionless quantity
- ss = steady state

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References


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