Is There a Relationship Between the Ideal Carnot Cycle and the Actual Ideal Carnot Cycle and the Actual Vapor Compression Cycle?

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Is There a Relationship Between The Ideal Carnot Cycle and The Actual Vapor Compression Cycle?

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ABSTRACT

Historically the Carnot cycle has been accepted as a reference point for comparison in the design and evaluation of engines and similarly the "reversed" Carnot cycle has been accepted as a reference point for comparison in the design and evaluation of refrigeration systems. It is normally assumed that the Carnot cycle represents the best engine and that the reversed Carnot cycle represents the best refrigeration system of any conceivable engine or refrigeration system operating between an upper and a lower temperature limit. It will be shown herein that in the case of the refrigeration system this assumption may not be accurate; it may be theoretically possible that there could be refrigeration systems that are better than this ideal reversed Carnot Cycle as this reversed Carnot cycle is normally defined. This conclusion is reached as a result of an examination of the Carnot cycle itself and an examination of PH diagrams of hypothetical but theoretically possible refrigerants supports this conclusion. Before attempting to draw similar conclusions in regard to engines it should be borne in mind that in the case of refrigeration it is desirable for the work to be a minimum but in the case of engines it is desirable for the work to be a maximum. The ideal Carnot cycle may represent the best conceivable engine but the ideal "reversed" Carnot cycle, as normally defined, does not always represent the best conceivable refrigeration system.

INTRODUCTION

The term coefficient of performance, COP, is by definition the ratio of the refrigeration received to the work required to obtain this refrigeration. Thus the reversed Carnot cycle COP is the ratio of the reversed Carnot cycle refrigeration received to the reversed Carnot cycle work required to obtain this refrigeration.

THE REVERSED CARNOT CYCLE

Figure 1 illustrates the classical reversed Carnot Cycle for refrigeration. The cycle consist of two isentropic and two isothermal processes as illustrated by points 1,2,3 and 4. In this ideal reversed Carnot cycle gas is compressed isentropically from point 1 to point 2. It is important to note that in the case of the ideal reversed Carnot cycle T_1 is below some available sink temperature, T_0, and that T_2 is above this available sink temperature. The gas rejects heat isothermally along path 2 to 3. It expands isentropically along path 3 to 4 and finally absorbs heat (refrigeration) isothermally along path 4 to 1.
The refrigeration received from this ideal reversed Carnot cycle, as shown in any basic thermodynamics text, is determined by the relationship

\[ \text{Ref} = T_1 \ (S_1 - S_4) \]

The work of this ideal reversed Carnot cycle is determined by the relationship

\[ W = (T_2 - T_1) \ (S_1 - S_4) \]

which is represented by the enclosed area defined by 1,2,3,4,1. The COP is determined by the relationship

\[ \text{COP} = \frac{\text{Ref}}{W} = T_1 / (T_2 - T_1) \]

These relationships are arrived at without any regard to the thermodynamic properties of the refrigeration medium, but as it shall be seen later, all refrigerants are not the same; they do not all result in the same COP even though the compressor isentropic compression efficiency may be the same for all of them.

The work of the isentropic compression process is determined by the relationship

\[ W = \frac{K P_i V_i}{1 - K} \left( \left( \frac{P_2}{P_1} \right)^{(K - 1)/K} - 1 \right) \]
The work of a polytropic compression process would be represented by the expression

\[
W = \frac{N P_1 V_1}{1 - N} \left( \left( \frac{P_2}{P_1} \right)^{\frac{(N-1)}{N}} - 1 \right)
\]

which is similar to the expression for the work of the isentropic compression process except that \( k \) is replaced by \( n \). The value of \( n \) is always less than the value of \( k \) and greater than 1. Thus the polytropic compression work is always less than the isentropic compression work. As \( n \) approaches 1, the expression becomes an indeterminate (infinity x zero) but it can be shown that as \( n \) approaches 1 the expression for work (isothermal or constant temperature) reduces to the expression

\[
W = P_1 V_1 \ln \left( \frac{V_2}{V_1} \right)
\]

Figure 2 and 3 illustrated cycles that do not violate any of the laws of thermodynamics but that have COP’s greater than the ideal reversed Carnot cycle COP. Figure 4 illustrates a cycle that is not possible, it violates the second law of thermodynamics.

The compression process as illustrated in Figure 2 is an isentropic (no heat absorbed or rejected) compression process from point 1 to point 2b which is at a temperature above ambient temperature, \( T_0 \). From point 2b to point 2 the compression process is polytropic (some heat is rejected to the ambient sink). The net effect is that the cycle work is less in this cycle illustrated in Figure 2 than it is in the ideal reversed Carnot cycle as illustrated in Figure 1 but the refrigeration, \( T_1 (S_1-S_4) \), is the same in both cases. Thus the COP of the cycle illustrated in Figure 2 is greater than that of the ideal reversed Carnot cycle COP and there is no violation of any of the laws of thermodynamics.

Figure 3 illustrates another approach to obtaining a COP greater than the ideal Carnot cycle COP and again this cycle does not violate any of the laws of thermodynamics. It is important to observe that the temperature at point 3 is lower than that of point 2 but it is above the available sink temperature, \( T_0 \).

Figure 4 illustrates a cycle that would appear to result in a COP greater than the ideal reversed Carnot cycle COP but there is a fallacy. This cycle illustrates a polytropic compression process from point 2b to point 2 but this is not possible because the temperature at 2b is less than the available sink temperature, \( T_0 \). For this cycle to exist it would require that heat flow from a lower temperature to a higher temperature which is contrary to the second law of thermodynamics.
Figure 5 illustrates a cycle on the TS diagram that is more nearly representative of the actual refrigerant cycle and Figure 6 illustrates this cycle on the PH diagram which is more commonly used to illustrate the vapor compression refrigeration process.

The Carnot cycle COP is defined as \( \frac{T_1}{T_2 - T_1} \) and in the case of the ideal reversed Carnot cycle there is no question as to which temperature is \( T_1 \) and which temperature is \( T_2 \) but this is not the case in the actual cycle. In the actual cycle is \( T_2 \) to be taken as \( T_{2a} \) or \( T_{2b} \) or \( T_{2c} \) or \( T_2 \) or even \( T_0 \). Is \( T_1 \) to be taken as \( T_1 \) or \( T_{2a} \)?

In defining a COP of an actual cycle it is conventional to assume that \( T_1 \) is the evaporator saturation temperature and \( T_2 \) is the condenser saturation temperature but this definitely is not the temperature limits that the cycle operates between as is specified in describing the ideal reversed Carnot cycle. The temperature of the refrigerant entering the condenser is normally much higher than the condenser saturation temperature and the highest temperature of the gas in the compressor is even higher.

There does not appear to be any relationship between the COP of the ideal reversed Carnot cycle and the COP of the actual vapor compression refrigeration cycle. The ideal reversed Carnot cycle does not appear to set any boundary or limit as to theoretically how high the actual COP could be.

An examination of some PH diagrams of the vapor compression cycle using hypothetical refrigerants leads to similar conclusions.
Figure 7 illustrates the conventional vapor compression cycle on a PH diagram except that three alternate compression processes are shown.

An isentropic compression process (as used in the ideal reversed Carnot cycle) is illustrated by line 1-2B and W represents the compression work in this case. In the case of any isentropic compression process $W = \Delta H$. A polytropic compression process is illustrated by line 1-2C and the polytropic compression work is always less than the isentropic compression work. Based on actual measurements in a compressor line 1-2A illustrates approximately the actual compression process and the work of this actual process is normally greater than the isentropic compression work.

Isentropic compression efficiency is defined as the ratio of the isentropic compression work to the actual work. If the actual compression process was polytropic or if the first part of the compression stroke was isentropic and the later part of the compression stroke was polytropic which theoretically it could be, then the isentropic compression efficiency could theoretically be greater than 100%. In actual compressors it is not greater than 100%; it is about 59% in good present day refrigerator freezer compressors, but theoretically it could be greater than 100%. Sometimes the motor losses are separated out to arrive at what could be called a shaft work isentropic compression efficiency which would be about 69.4% (85% efficient motor). Further friction and windage losses could be separated out to arrive at a piston work isentropic compression efficiency which would be about 79.7% (13% friction and windage losses or 87% friction and windage efficiency). The fact that actual present day isentropic compression efficiencies are less than 100% does not preclude the possibility that theoretically they could be greater than 100%. Thus far ideal cycles have been considered without regard to the actual thermodynamic properties of the gas. From Figure 7 it is obvious that the refrigeration obtained is independent of the compression process but it is determined by the thermodynamic properties of the refrigerant. The work and hence the COP does depend on the compression process.
Figure 8 illustrates an extreme case that is not likely to occur in an actual refrigeration machine but under some conditions it possibly could occur. In this figure an isothermal (constant temperature) compression process is illustrated. If the gas was exceptionally hot when it entered the compressor, above the available ambient heat sink temperature, then in theory it could be compressed at a constant temperature. This in effect is saying that \( n \) could approach 1 in the polytropic compression process.

The COP depends on the compression process but it also depends very much on the thermodynamic properties of the refrigerant. This is illustrated in Figure 9A, 9B, 9C, and 9D. For purposes of illustrating the effect that these thermodynamic properties have on the COP an isentropic compression process is illustrated in these four figures (100% isentropic compression efficiency) but bear in mind that theoretically the compression process could be better than isentropic.

The slope of the isentropes (constant entropy lines) and the heat of vaporization have a great effect on determining the work, the refrigeration and the COP of a refrigeration system. The sensible heat of the liquid and especially the sensible heat of the vapor also help to determine the refrigeration and the COP.

Figure 9A can be taken as a base or starting point for comparison.
The length of line 4-1 represents the refrigeration obtained. The length of the horizontal projection of line 1-2 represents the work (isentropic work). Figure 9B represents a different hypothetical but theoretically possible refrigerant. The refrigeration effect is the same as that illustrated in 9A but as a result of the steeper slope of the isentropes which does not violate any of the laws of thermodynamics, the work is less. Figure 9C represents another hypothetical but theoretically possible refrigerant. In this case the work is the same as that of Figure 9A but as a result of the greater heat of vaporization which also does not violate any of the laws of thermodynamics, the refrigeration effect is greater. Figure 9D represents still another hypothetical but theoretically possible refrigerant. In this case the work is the same as that of Figure 9B but the refrigeration effect is the same as that of 9C. This results in a hypothetical super refrigerant that does not violate any of the laws of thermodynamics. There are chemical considerations that have been proposed that suggest that there are limits as to how good a refrigerant may be but these limits are in regard to the chemical make up of the molecule itself and are in no way related to the reversed Carnot cycle or any other cycle. Table 1 lists some calculations of refrigeration, of work and of COP for several different refrigerants.

### Table 1

Theoretical Performance of Refrigerants

<table>
<thead>
<tr>
<th>Refrigerant</th>
<th>Ref (Btu/lb)</th>
<th>Isen. Comp Work (Btu/lb)</th>
<th>COP</th>
</tr>
</thead>
<tbody>
<tr>
<td>R12</td>
<td>61.73</td>
<td>22.70</td>
<td>2.719</td>
</tr>
<tr>
<td>R134a</td>
<td>80.42</td>
<td>29.40</td>
<td>2.735</td>
</tr>
<tr>
<td>R152a</td>
<td>126.46</td>
<td>46.69</td>
<td>2.709</td>
</tr>
<tr>
<td>R22</td>
<td>83.29</td>
<td>32.72</td>
<td>2.545</td>
</tr>
<tr>
<td>R115</td>
<td>43.42</td>
<td>15.79</td>
<td>2.749</td>
</tr>
<tr>
<td>R124</td>
<td>59.40</td>
<td>22.63</td>
<td>2.624</td>
</tr>
<tr>
<td>R134</td>
<td>85.25</td>
<td>30.51</td>
<td>2.794</td>
</tr>
<tr>
<td>R142b</td>
<td>88.75</td>
<td>31.54</td>
<td>2.812</td>
</tr>
<tr>
<td>R143a</td>
<td>78.35</td>
<td>29.57</td>
<td>2.649</td>
</tr>
<tr>
<td>R218</td>
<td>36.17</td>
<td>12.96</td>
<td>2.790</td>
</tr>
<tr>
<td>R290</td>
<td>152.16</td>
<td>56.77</td>
<td>2.704</td>
</tr>
<tr>
<td>RC270</td>
<td>179.36</td>
<td>66.79</td>
<td>2.685</td>
</tr>
<tr>
<td>RC318</td>
<td>44.65</td>
<td>15.33</td>
<td>2.912</td>
</tr>
<tr>
<td>R600a</td>
<td>143.68</td>
<td>49.59</td>
<td>2.897</td>
</tr>
</tbody>
</table>

The calculations assume that the evaporator temperature is -10°F, the condenser temperature is 130°F, the return gas (superheat) temperature is 90°F and the liquid temperature (subcooled) is 90°F. The calculations are based on available NIST data of thermodynamic properties of the refrigerants. As can be seen there is about a 4 to 1 variation in the amount of refrigeration obtained per pound of refrigerant circulated if RC270 is compared to RC318 but the power required to obtain this refrigeration also varies by almost as much. The net effect is that although there is some variation in COP it is only about 12.5%. This table list actual refrigerants that were examined. It is theoretically possible that there could be some better ones.
The likelihood of finding a really super refrigerant may not be great. Some chemical observations have been reported that suggest that there are limits as to how good the refrigerant may be, however, it is theoretically possible that there could be a refrigeration system with a COP greater than the ideal reversed Carnot cycle COP. Present day systems have COP's about 50% of the ideal reversed Carnot cycle COP but in theory they could have COP's greater than 100% of the ideal reversed Carnot cycle COP. It is the chemistry of the refrigerant as well as the quality of the compressor not the ideal reversed Carnot cycle that determines how high or how low the actual COP will be. Present day isentropic compression efficiencies are about 59%. There is no theoretical reason that they could not be greater than 100% and there is no theoretical limit as to how good the refrigerant could be.

In summary neither the isentropic compression process nor the ideal Carnot Cycle COP represent absolute limits as to how efficient a refrigeration system can be.