Reducing energy consumption for buildings under system uncertainty through robust MPC with adaptive bound estimator

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ABSTRACT

Model Predictive Control (MPC) has emerged as an alternative to traditional control method to reduce building energy consumption. With the presence of model uncertainty, such as mismatch between the plant and control-oriented model, the use of MPC may result in thermal comfort violation or energy waste. The influence of model uncertainty becomes even more significant as the size and complexity of the investigated building increase. Robust MPC (RMPC), which requires knowledge on the system uncertainty, has been investigated for enhancing the stability of MPC. However, the implementation possibility of the RMPC is prevented by increased computational burden and conservativeness of controller performance. This paper deals with the latter issue by presenting a novel adaptive RMPC scheme for temperature regulation in commercial buildings. The novelty comes from the development of a comparison model built based on a nonlinear autoregressive model for worst-case analysis. This comparison model enables us to transform a linear, robust MPC problem into an adaptive one with a time-varying uncertainty bound. The proposed method is tested on a simulation model developed from building data collected from a spacious hall at an airport terminal. By conducting simulation using different MPCs, it is found that the proposed RMPC method is able to behave robustly against uncertainty with the least performance loss. This means the maximum energy saving and the least thermal comfort violation.

1. INTRODUCTION

Recently, the application of model predictive control (MPC) as a supervisory controller for optimising building-energy usage has received an increasing attention. The advantage of MPC over the traditional control strategy is that the former one is able to handle systematically and effectively constraints on control inputs and states, by taking advantage of weather forecast to perform disturbance prediction.

The performance of an MPC is largely dependent on how accurately the specific model describes the real building process. While different model types are available for building modelling, most researchers choose resistance-capacitance (RC) models as the control-oriented model for optimisation (Cai & Braun, 2015a; Razmara et al., 2015; Bengea et al., 2014). This is because the RC models can be accurately identified using experimental data while maintaining physical significance. Even with limited training data, the RC models can be accurately identified by using a component-based approach (Cai & Braun, 2015b).

However, generating highly accurate RC models by using experimental data is theoretically not always possible, because of persistent disturbances and simplification of heat transfer process. Because the MPC saves energy by maintaining the indoor temperature close to the upper (in summer) or lower (in winter) comfort limits, an inaccurate prediction...
of indoor temperature can result in thermal comfort violations or energy waste. Conventionally, the MPC employs linear, time-invariant (LTI) models to predict future dynamics of the system, despite the fact that controlled systems (thermal zones) are non-linear and non-deterministic. The feedback nature of the MPC allows the controller to reject a small degree of uncertainty. Kim et al. (2015) model roof top units in a mid-size commercial using multi-input and multi-output (MIMO) auto-regressive with exogenous input (ARX) model. The simple ARX model could be employed in this case study because the prediction horizon is short and the control input is not strongly correlated to the unmeasured disturbance. However, for systems with large uncertainties and MPC that requires a long prediction horizon, the explicit consideration of the uncertainties becomes especially crucial. This motivates the research on robust MPC (RMPC). The main idea of RMPC is to consider the model–plant mismatch as an uncertain term and address it explicitly in the control algorithm.

The RMPC has been studied to optimise the energy efficiency of the HVAC systems. For example, Kim (2013) designed an RMPC to improve the stability of traditional MPCs under uncertainty conditions. He found that robust MPC outperforms a nominal MPC when uncertainty is dominant and the model mismatch is significant. Maasoumy et al. (2014) compared the performance of a closed-loop robust MPC with rule-based control and nominal MPC under different levels of uncertainties. They suggest that robust MPC should only be chosen within a specific range of uncertainties. Gondhalekar et al. designed a least-restrictive robust MPC law for indoor temperature regulation. The proposed method eliminates the conservativeness of traditional min–max open-loop prediction MPC while guaranteeing reasonable computational complexity. Past studies show that correctly choosing the uncertainty bound is crucial for the design of RMPC: a too narrow bound would cause thermal comfort violation while a too wide uncertainty bound cause performance lose.

An unattended issue in the previous studies (Kim, 2013; Maasoumy et al., 2014) on RMPC is how to design the uncertain bound properly to reduce conservatism and guarantee robustness. If the uncertainty bound is chosen to be fixed, it will inevitably lead to conservative solutions, because the RMPC has to work conservatively at all times to guarantee that the thermal comfort is satisfied. In this work, we develop a comparison model for worst-case analyse and provide uncertainty bound to the RMPC. The comparison model is built upon a recursive neural-network model (RNN). The idea is to use the recursive nature and nonlinear approximation ability of the RNN model to capture the uncertainty dynamics and nonlinear properties of the buildings. This algorithm allows one to estimate the error bound in an adaptive fashion over the given prediction horizon. Once the uncertainty bounds are obtained, the optimisation problem is solved as a closed-loop min–max RMPC problem, based on nominal model predictions and tightened constraint sets. The proposed method is demonstrated at the terminal building of Adelaide Airport, South Australia. The investigated zone is uncertain due to frequent variation of passenger flow, as well as unknown coupling from both controlled and uncontrolled neighbouring space. The complexity and uncertainty of the investigated building make the case study suitable for testing proposed control methods.

The remainder of this paper will proceed as follows: Section 2 introduces grey-box and RNN modelling methods for building thermal dynamics modelling. The design of RMPC and its adaptive bound estimator are introduced in Section 3. Section 4 discusses the results of using the proposed controllers, including the control performance comparison between different types of MPC. The paper ends with a conclusion and a description of future work.

2. SYSTEM MODELLING

2.1 Case Study Building

The test building is the Terminal-1 (T1) building of Adelaide Airport, South Australia. The perimeter zones of the second floor were selected as the test site for the experiment. The layout of the test area is shown in Figure 1. This area serves as the check-in hall from where most passengers enter or leave the building. A large glass facade is installed to the north to ensure good lighting conditions, while two motorised blinds are installed to reduce the effects of solar gain. The position of the blinds is programmed into BMS and controlled with a fixed schedule. The hall is divided into four adjacent zones. Each zone has an associated CAV (Constant Air Volume) box to condition the space and a sensor to measure the zone temperature. Different from other airport terminals, T1 has no flights during the night, so the HVAC system is scheduled to be switched off during the night. The occupied period for the testing area is 7:00 am to 10:00 pm.
2.2 RC Modelling

Theoretically speaking, buildings’ thermal dynamic can be best described by time-varying nonlinear partial differential equations, which are not suitable for control optimisation. Therefore, simplified RC models are more often used. Past research has compared different structures of RC models in modelling the thermal dynamics of buildings (Sourbron et al., 2013; Fux et al., 2014; Gouda et al., 2002). For control purposes, this study employs a second order RC model to represent the thermal dynamics of the building. In this work, we focus on minimising the supplied thermal energy $Q_u$ at the AHUs level and will not model the efficiency of the AHU system.

The modelling work is a typical multi-input (thermal energy supplied to individual zones) and multi-output (zone temperatures) problem. The parameters that are of concern are the heat transfer coefficients between the air handling units (AHUs), adjacent zones, internal walls and uncontrolled spaces. Zones 1 and 2 located at the east end of the building were selected as the experimental area in this study. Zones 3 and 4 are not considered in this study because they have similar dynamics with zones 1 and 2.

The structure of the RC model is depicted in Figure 2. The corresponding energy and mass balance governing equations for a single zone can be written as:

$$ C_z \frac{dT_z}{dt} = \dot{m}C_a(T_{sa} - T_z) + \frac{T_{out} - T_z}{R_{win}} + \frac{T_w - T_z}{R_w} + \frac{T_n - T_z}{R_c} + Q_{rin} + Q_{p} + Q_{inf}; $$(1)

$$ C_w \frac{dT_w}{dt} = \frac{T_z - T_w}{R_w} + \frac{T_{out} - T_w}{R_w} + Q_{out}; $$

where $C_z$ is the overall thermal capacitance of the air and other fast-response elements, $C_w$ is the thermal capacitance of the interior walls and ceiling, $C_a$ is the specific heat of the air, $T_z$ is the temperature of the investigated zone, $T_n$ is the temperature of the neighbouring zone(s), $T_{out}$ is the outdoor air temperature, $T_w$ is the mean surface temperature of the interior walls and ceiling, $R_{win}$ is the thermal resistance of the windows, $R_w$ represents the convective resistances
between the building envelope and the outdoor and indoor air, \( R_c \) is the convective heat transfer coefficient between adjacent zones, \( Q_{\text{in}} \) and \( Q_{\text{out}} \) denote the inside and outside surface solar radiation heat flux, respectively, \( Q_p \) is the internal heat gain generated by the presence of occupants and their behaviours, and \( Q_{\text{inf}} \) is the internal heat gain from leakage and door openings.

Observing Equation (1), we found that the only non-linear component in the simplified model is the bilinear term \( mC_0(T_{\text{in}} - T_e) \). Using feedback linearisation, the control input is redefined as \( Q_a = mC_0(T_{\text{in}} - T_e) \). The new control input \( Q_a \) represents the thermal energy supplied to the individual zones. Equations (1) and (2) are discretised using the Euler forward method to obtain:

\[
\begin{bmatrix}
T_z(k+1) \\
T_w(k+1)
\end{bmatrix} = \begin{bmatrix}
1 - \frac{\Delta t}{C_cC_e} - \frac{\Delta t}{e_a} & \frac{\Delta t}{C_cC_e} \\
1 - \frac{\Delta t}{C_cC_e} & 1
\end{bmatrix}
\begin{bmatrix}
T_z(k) \\
T_w(k)
\end{bmatrix} + 
\begin{bmatrix}
\frac{\Delta t}{C_cC_e} \\
0
\end{bmatrix}
\begin{bmatrix}
Q_{\text{inf}}(k) \\
Q_{\text{roof}}(k)
\end{bmatrix},
\]

where \( k \) is the time step, and \( \Delta t \) is the sampling time. There are three disturbance inputs in Equation (3). The first input is the incident solar radiation on the inside and outside of the building envelope. The data used for calculating the solar radiation heat flux is global horizontal irradiation \( I_r (\text{W/m}^2) \). We use \( Q_{\text{in}} = \alpha I_r \) and \( Q_{\text{out}} = \beta I_r \) for calculating the solar radiation heat flux, where \( \alpha \) and \( \beta \) denote the coefficients associated with the area and the absorption rate of the inside and outside building envelop. \( \beta \) also includes the effect of window transmittance. The second input to the system is the internal gain generated by the occupants. The internal gain is proportional to the number of occupants in the hall, so it can be indicated by the carbon dioxide concentration (ppm). Therefore, it is presented by \( Q_p = \gamma CO_2 \). Here \( \gamma \) denotes the coefficient associated with the number of occupants. The two variables are identified together with Equation (3). The third input \( Q_{\text{inf}} \) is not measurable so they are regarded as unmeasured uncertainty. Taking into account the uncertainty and measurement noise, Equation (3) can be re-written as state-space form as:

\[
x(k+1) = Ax(k) + Bu(k) + Ed(k) + Fe(k),
\]

where A, B, E, and F are the corresponding matrices, \( x \) is state vector, \( u \) is input vector, \( d \) denotes measurable disturbance, and \( e \) denotes the uncertainty and measurement noise that is assumed to be Gaussian distributed. The unknown parameters in Equation (4) are identified using measured building data using a Trust-Region reflective algorithm (Moré & Sorensen, 1983). The data used for model identification were collected from between the 1st and 20th of January, 2013. The input is the supplied cooling energy \( Q_c \), the meteorological data of the Bureau of Meteorology of Australia provide disturbance forecast \( (Q_c \text{ and } T_{\text{out}}) \) to the thermal zones, the output is indoor temperature. A source of uncertainty in building control comes from the errors in forecasting. The outside ambient temperature forecast uncertainty increases with longer prediction horizon. However, this study does not specifically investigate the effects of weather forecast error on the modelling results. Instead, we use measured outside air temperature instead of the predicted value to perform analyse. All the other uncertainties are lumped into the error terms.

### 2.3 Recursive Neural Network Modelling

In this section, the thermal dynamics of the same zone will be modelled by an RNN model. Nonlinear autoregressive with exogenous inputs (NARX) model is used to express the RNN structure:

\[
y(k) = f_3(y(k-1), \ldots, y(k-n_y), u(k-1), \ldots, u(k-n_u) + e(k), \]

where \( y(k) = [y_1(k), \ldots, y_p(k)]^T \), \( u(k) = [u_1(k), \ldots, u_m(k)]^T \), and \( e(k) = [e_1(k), \ldots, e_m(k)]^T \) are the system output, input and noise, respectively; \( p \) and \( m \) are the number of outputs and inputs, respectively; \( n_y \) and \( n_u \) are the maximum lags in the outputs and inputs, respectively; \( k \) is the process dead time; and \( f_3 \) is a vector-valued non-linear function. Model order selection is an important step of system identification, but it will not be elaborated this paper. For detailed procedures see (Huang et al., 2015a).
A neural network with three layers of neurons was employed. The network function is expressed with the following equation:

\[ \hat{y}(t) = F \sum_{i=1}^{n_h} W_{f,0}(\sum_{i=1}^{n_u} w_{u,i} \phi_i(k) + b_{u,0}) + B_{j,0}, \]

where \( W_{f,u} \) and \( w_{u,i} \) are weights vector to the hidden layer and output layer respectively. \( b_{j,0} \) and \( B_{u,0} \) are the bias of the hidden units and the output layer, respectively, \( \phi_i(k) \) indicates the vector that contains the regression of the Equation (5) at time step \( k \), \( f_3 \) is sigmoid function expressed by Equation (7), and \( F \) uses a linear function and \( j = 1 \) as only one output is considered. The weight vector \( w \) and bias vector \( b \) at the hidden layer were initialised using the Nguyen-Widrow method to keep the trained model more consistent. Levenberg-Marquardt algorithm was employed to train the neural networks, which minimises mean square error (MSE). For training the RNN, the input-output data cannot be directly fed into the network but should be arranged to follow the structure of the NARX model.

2.4 Model Validation and Comparison

The data used for validation purposes were a completely different set of data, obtained from 21st January 2013 to 24th January 2013. Because the primary purpose of building the models is to achieve predictive control, the multi-step-ahead prediction is needed. However, both of the aforementioned training algorithms only minimise the error over a single step ahead. A more reasonable way to validate our model is to consider multi-step-ahead prediction by taking into consideration the recursive feature of the model (Huang et al., 2015b). The histogram of the residuals generated by the two models is plotted in Figure 3. Overall speaking, compared with the RC model, the residuals generated by the RNN model are smaller and closer to a normal distribution. This is because the recursive nature and nonlinear approximation ability of the RNN model enable it to capture some uncertainty and nonlinear dynamics of the buildings. The complexity and uncertainty of the investigated building make the case study suitable for comparing these two modelling methods.

3. CONTROL DESIGN

3.1 Closed-loop min–max Robust MPC

Conventionally, the MPC employs linear, time-invariant models to predict future dynamics of the system, despite the fact that controlled systems (thermal zones) are non-linear and non-deterministic. This type of MPC is referred to as a deterministic MPC (DMPC). In designing the DMPC, it is assumed that the model can predict the real-world building
plant perfectly. In the context of RMPC, which is designed to address model mismatch explicitly, the effects of model mismatch on state estimations should be addressed. Therefore, Equation (4) is rewritten as:

$$x(k + 1) = Ax(k) + Bu(k) + Ed(k) + Fw(k),$$

where \( w \in \mathbb{W} \) denotes the model uncertainty. \( w \) is bounded but its exact value is not known. The basic idea of min–max RMPC is to determine all possible evolutions of the disturbance sequence over the control horizon, and to minimise worst-case cost. Min–max RMPC can be either in the form of open-loop predictions (Lee, 2011) or closed-loop predictions (Lee, 2011; Löfberg, 2003). The open loop approach fails to take into account that feedback is presented in the receding-horizon implementation of the control, therefore leads to conservative solutions and could even make the optimisation problem infeasible (Löfberg, 2003; Lucia et al., 2014).

In this study, we focus on the application of a closed-loop RMPC to building-energy control. Different from the open-loop approach, the closed-loop RMPC considers the feedback over the prediction horizon and incorporates it into the prediction. In particular, at each step, the closed-loop RMPC considers the future value \( X \) under different disturbance trajectories. The controller generates a family of control sequences, each one corresponds to a different measured state. The maximum costs can be calculated along with some of the worst-case predictions.

Considering the following min–max optimisation problem:

$$\text{min } u \quad \text{max } w \sum_{j=0}^{N-1} Q(u_{k+j}^+) + R(y_{k+j} - r_{k+j}) + S \sum_{k=1}^{N} (|e_{k+j}| + |\pi_{k+j}|),$$

subject to:

$$x_{k+j+1} = Ax_{k+j} + Bu_{k+j} + Ed_{k+j} + Fw_{k+j}, \quad \forall j = 0, \ldots, N-1$$

$$y_{k+j} = Cx_{k+j}, \quad \forall j = 1, \ldots, N$$

$$T_{\min,k+j} \leq y_{k+j} \leq T_{\max,k+j}, \quad \forall j = 1, \ldots, N$$

$$e_{k+j} > 0, \pi_{k+j} > 0, \quad \forall j = 1, \ldots, N$$

$$U_{\max,k+j} \leq u_{k+j} \leq U_{\min,k+j}, \quad \forall j = 0, \ldots, N-1,$$

The constraints that should be met are:

1. \( T_{\text{oc}} \in [21\,\degree\mathrm{C}, 24\,\degree\mathrm{C}] \) Thermal comfort during occupied hours.

2. \( T_{\text{uo}} \in [19.5\,\degree\mathrm{C}, 26\,\degree\mathrm{C}] \) Thermal comfort during unoccupied hours.

3. \( u_q \in [-10\,\text{kW}, 12\,\text{kW}] \) Maximum cooling energy that can be supplied to each zone.

where the double indices \( k+j \) denote the prediction value at time \( k+j \) made at time \( k \), \( U = [u_{1|k}, u_{2|k}, \ldots, u_{N-1|k}] \) is a vector of the control inputs (supplied thermal energy) applied to the model, \( y_{k+j} \) is the predicted output at time \( k \), \( T_{\max} = [T_{\max,k|k}, T_{\max,k+1|k}, \ldots, T_{\max,k+N-1|k}] \) is a vector of the upper comfort temperature band within the horizon, \( T_{\min} \) is a vector of the lower comfort temperature band, variants \( e \) and \( \pi \) denote the temperature violation from the upper and lower bounds, respectively, \( N \) is the prediction horizon, \( \bar{d} \) is the measured disturbance, \( r \) is the set-point temperature, \( U_{\min} \) and \( U_{\max} \) denote the maximum cooling and heating energy that the system can supply, respectively, \( Q \) denotes the penalty on the energy usage, \( R \) denotes the penalty on the comfort constraint violation, and \( S \) denotes the penalty on the set-point temperature deviation. If \( S \) is chosen to be 1, the controller strives to maintain set point temperature during occupied hours; if \( S \) is chosen to be 0, the controller maintains the zone temperature closed to the upper comfort limit. \( \underline{w}_{k+j} \) and \( \overline{w}_{k+j} \) denote the lower and upper uncertainty bounds, respectively. All possible trajectories are included in bands that depend on \( \underline{w}_{k+j} \) and \( \overline{w}_{k+j} \). The values of \( [\bar{w}, \underline{w}] \) depends on the future state of the real system, which is not known. A possible way of estimating the error bounds is to obtain them directly from the historical residuals, e.g. determine the numerical range where a certain probability of the modelling errors happens. However, this method still relies on the choice of the probability density. If the uncertainty set is chosen to be too large, the controller becomes very conservative and control performance will be lost. The conservatism will be demonstrated with an example in a later section.
3.2 RMPC with Adaptive Uncertainty Bound

The above-mentioned bounding method can result in conservative solutions, because it is purely based on the historical data but fails to take into account that the errors are also related to the occurrence of future disturbances. The uncertainty can be predicted using a comparison model (Fukushima & Bitmead, 2005). Illustrated by this idea, we propose an uncertainty bound estimator in this study, constructed based on the RNN model presented in the previous sections. The idea is to use the recurrent nature of the RNN model to capture the uncertainties that exist in the building system. The unknown model mismatch between the RC model and the actual building plant can therefore be approximated by the difference between the RC model and the RNN model. The positive uncertainty can be calculated as

\[ \bar{z} = \max_{k \in 1 \ldots n} (\hat{y}_{nn}(k) - \hat{y}_{rc}(k)), \]

where \( \hat{y}_{rc} \) and \( \hat{y}_{nn} \) are open-loop prediction results generated by RC and RNN models, respectively. \( \bar{z} \) denotes the maximum error within \( n \) steps, \( n \leq N \) is the number of steps considered. Figure 4 shows the adaptive RMPC (ARMPC) procedure and the adaptive RMPC algorithm is summarised as follows:

**Algorithm: Adaptive Robust MPC**

1. Choose the initial bounds using sampling method introduced in the previous section.
2. DMPC computes the open loop input trajectory \( U = [u_1, u_2 \ldots u_n] \).
3. The RNN performs \( n \)-steps ahead prediction using input vector \( U \) and disturbance vector \( D \) to obtain a comparison set of output trajectory \( Y = [y_1, y_2 \ldots y_{1+N}] \).
4. Calculate the error bounds based on Equations (11).
5. During occupation period, if \( z \) is positive (underestimation), then update uncertainty bounds to \([-\pi, \pi]\) by setting \( n = 3 \). Else the uncertainty bound is set to the minimum range.
6. During transitional period, if \( z \) is positive (underestimation), then update uncertainty bounds by setting \( n = N \). Else the uncertainty bound is set to the minimum range.
7. Increment \( k \). Go to Step 2.

4. RESULTS AND DISCUSSION

The programs for model training, validation and control optimisation were coded in Matlab, which runs on a PC with Intel Core i7 CPU 2.4 GHz. The optimisation problem are solved using Yalmip (Löfberg, 2004). The optimised control parameter is shown in Table 1.

We present a comparison study on three different MPC approaches, which are DMPC, RMPC and ARMPC, in the presence of model uncertainty. Our goal is to compare the performance of these three controllers in terms of conservatism, stability and computational speed. As illustrated in the previous section, MPC can save energy by keeping the zone temperature close to its upper or lower comfort limit. Because cooling is considered in this study, only underestimation \( T > \bar{T} \) will cause thermal comfort violation on the upper comfort bound.
Table 1: Parameters of the RMPC.

<table>
<thead>
<tr>
<th>RMPC parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sampling interval</td>
<td>10 min</td>
</tr>
<tr>
<td>Prediction horizon, N</td>
<td>30</td>
</tr>
<tr>
<td>Control horizon, P</td>
<td>30</td>
</tr>
<tr>
<td>Weight for uncertainty term</td>
<td>0.1</td>
</tr>
<tr>
<td>Fixed uncertainty bound</td>
<td>[−0.2, 0.25]</td>
</tr>
<tr>
<td>Number of look-ahead steps</td>
<td>10</td>
</tr>
<tr>
<td>Penalty on energy input Q</td>
<td>0.8</td>
</tr>
<tr>
<td>Penalty on soft-constraint violation in R</td>
<td>1</td>
</tr>
<tr>
<td>Penalty on set-point temperature deviation S</td>
<td>1</td>
</tr>
<tr>
<td>Occupancy hour</td>
<td>5:00 am to 9:30 pm</td>
</tr>
</tbody>
</table>

Figure 5: Performance of DMPC, RMPC and ARMPC under model uncertainty.

Figure 6: Comparison of fixed bound and the adaptive bound.
First, we choose a day on which the LTI control model suffers from a considerable amount of uncertainty to conduct the comparison. The uncertainty is illustrated in Figure 5. The uncertainty bound for RMPC is set to be $[-0.2, 0.25]$, based on the simple sampling methods and the histogram of residuals. The set-point value is chosen to be close to the upper comfort temperature (23 °C). Figure 5 compares the performance of RMPC and DMPC when a uncertainty is presented. The red, dashed line indicates the output of DMPC. It can be seen that, although the uncertainty happened during the steady state, it does not impose an unstable influence on the performance of the DMPC. This is because the closed-loop nature of the DMPC makes it robust to some degree of uncertainty. However, when the temperature is moved towards the upper bound, the comfort constraint is violated by the DMPC, which is due to the large error happens before the end of the occupancy. The blue, solid line shows the performance of the RMPC. The thermal comfort is satisfied by the RMPC all the time, as the modelled uncertainty is located within the designated uncertainty bound for most of the time. However, the RMPC consumes more energy (32% with respect to DMPC), by maintaining the temperature below the set-point value during steady states. This is expected because the RMPC performs conservatively by lowering the indoor temperature to make sure that constraints are safely satisfied at all times. The RMPC works in such a way to prevent a potential comfort violation due to the positive uncertainty. The green, dotted line shows the performance of the ARMPC. It can be seen that the ARMPC tracks the set-point temperature well during the steady states, without wasting too much energy. This is because of the use of a smaller uncertainty bound, as shown in Figure 6. This proves the effectiveness of the proposed method, because a smaller, adaptive uncertainty bound indicates a lower degree of conservatism without violating thermal comfort for RMPC. The ARMPC also satisfies the thermal comfort requirement before the end of occupancy, when the uncertainty bound is expanded to allow a larger uncertainty to happen. Similarly, this is because the ARMPC expand the uncertainty bound during the transitional period to ensure a better robustness.

The simulation period was then extended to two weeks, from 4th to 28th Feb, 2015. Figure 7 shows the simulation results using the three above-mentioned control methods. A summary of the performance of each controller for the investigated days is reported in Table 2. It can be seen that the baseline control consumes the most energy and violated the thermal comfort on 3 days. DMPC consumes the least energy which leads to the greatest energy savings. However, the zone temperature regulated by DMPC violates the comfort constraints in 9 days. This is because the DMPC does not take into account the occurrence of positive uncertainty. The RMPC does not violate any comfort constraints, but consumes 25% more energy than does the DMPC. The ARMP does not violate the thermal constraints on any days and at the same time consumes 12% less energy than the RMPC. This is because the ARMP controls the zone temperature much closer to the set-point value as compared to the RMPC. In terms of computational speed, DMPC is the most efficient one. Both RMPC and ARMPC have slower computational speed, because the exponential increased complexity with the increase of prediction horizon. The ARMPC is slower than RMPC because estimating the uncertainty bounds using RNN estimator requires extra computational efforts.

**Figure 7**: Two weeks’ control performance of DMPC, RMPC and ARMPC.

**Table 2**: Performance comparison between DMPC, RMPC and ARMPC.

<table>
<thead>
<tr>
<th></th>
<th>Baseline</th>
<th>DMPC</th>
<th>RMPC</th>
<th>ARMPC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total energy (kWh)</td>
<td>3580</td>
<td>2440</td>
<td>3052</td>
<td>2656</td>
</tr>
<tr>
<td>Number of infeasible days</td>
<td>3</td>
<td>9</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Simulation time per step</td>
<td>n/a</td>
<td>0.4</td>
<td>4.4</td>
<td>7.1</td>
</tr>
</tbody>
</table>
5. CONCLUSIONS

In this paper, we have presented an adaptive bounds estimator, which allows the uncertainty bound of RMPC to vary according to the dynamic changes of the system. It is shown that the proposed method results in a smaller uncertainty bound when the actual uncertainty is low, which greatly reduces the conservatism of the traditional RMPC. A comparison between two modelling methods shows that when the uncertainty is significant, the RNN model achieves more accurate long-term prediction result as compared with the linear RC model. More interestingly, it is also found the errors generated by the former model are closer to a normal distribution, which makes statistical analysis possible. We believe this is because the RNN model captures the uncertainty and nonlinearity of the system using its recursive property. Additionally, because of the closed-loop nature of the DMPC, the modelling errors do not always lead to poor control performance. The influence of modelling error on control performance also depends on the types of uncertainty and the time when this might happen. It seems that the DMPC usually performs more robustly during the steady states as compared with the transition period. Conclusively, by combining neural network modelling technology with the classical control method, this paper opens up a new path for solving complicated real-world building energy control problem. As a future study, the proposed control method will be tested experimentally at the investigated building.

REFERENCES