2014

Level-Set Shape Reconstruction of Binary Permittivity Distributions from Near-Field Focusing Capacitance Measurements

S. H. Taylor  
*Purdue University*

S V. Garimella  
*Purdue University*, sureshg@purdue.edu

Follow this and additional works at: [http://docs.lib.purdue.edu/coolingpubs](http://docs.lib.purdue.edu/coolingpubs)

[http://dx.doi.org/doi:10.1088/0957-0233/25/10/105602](http://dx.doi.org/doi:10.1088/0957-0233/25/10/105602)

This document has been made available through Purdue e-Pubs, a service of the Purdue University Libraries. Please contact epubs@purdue.edu for additional information.
Level-set shape reconstruction of binary permittivity distributions using near-field focusing capacitance measurements

Stephen H. Taylor and Suresh V. Garimella
Cooling Technologies Research Center, an NSF I/UCRC
School of Mechanical Engineering
Purdue University
West Lafayette, IN 47907 USA

Abstract

A near-field focusing capacitance sensor consists of an array of long, coplanar electrodes offset by a small interface gap from an identical orthogonal array of electrodes. The sensor may be used to characterize permittivity inhomogeneities in thin dielectric layers. The sensor capacitance measurements represent a tessellated matrix of integral-averaged values describing void content in a series of zones corresponding to the electrode crossing points (junctions) of the sensor. The sensor does not lend itself to computed tomography because the individual capacitance measurements do not represent overlapping regions of sensitivity. An evolving level-set algorithm is proposed to reconstruct a binary permittivity distribution. A mathematical construct, based on the physics of inverse-square fields, is used to approximately reconstruct shape features too small to be captured by the raw measurements. The method accommodates the non-uniform area-sensitivity of the junction capacitance measurement. Effective use of the algorithm requires active management of the convergence criterion and evolution rate. The algorithm is demonstrated on a series of phantoms as well as measurements of a voided dielectric thermal interface material using a near-field focusing sensor.

Keywords: shape reconstruction, level-set, tomography, capacitance, void fraction, thermal interface

1. For Measurement Science and Technology
2. Corresponding author, sureshg@purdue.edu
1. Introduction

In capacitive sensing, a set of electrodes is used to characterize permittivity in a domain of interest, providing knowledge about the material distribution by inference. Capacitive sensing is a low-cost, adaptable technique that has been demonstrated in a variety of applications. General design principles for capacitive sensors have been concisely reviewed by Hu and Yang [1].

In tomography applications, multiple electrodes are located around the periphery of the domain of interest. Up to $N(N-1)/2$ independent capacitance measurements may be taken through combinatorial actuation of different electrode pairs, where $N$ is the total number of electrodes used. Individual measurements exhibit sensitivity to overlapping regions within the domain of interest, and a tomographic reconstruction algorithm is required to resolve the physical permittivity distribution. Standard tomographic reconstruction algorithms, including linear back projection (LBP), Tikhonov methods, Landweber methods, and the algebraic reconstruction technique (ART) have been discussed and compared by Yang and Peng [2]. Novel improvements to tomographic reconstruction have been recently demonstrated, including image reconstruction based on electrical field lines [3], gradient-based local mesh refinement [4], and computational efficiency gains through Jacobian reduction [5].

Standard reconstruction algorithms produce reconstructions of continuous permittivity throughout the domain of interest; however, in many applications, the desired outcome of tomographic imaging is binary in nature, such as when used to distinguish fluid phases. Efforts have been made to develop capacitance tomography reconstruction algorithms for constructing discrete phase distributions. Censor [6] introduced a method of ‘binary steering’ that can be applied to most standard algorithms to drive them toward binary solutions. Ren et al. [7] developed a specialized reconstruction method to resolve a permittivity interface contained within a cube containing four electrodes on each side. For a mathematical framework on capacitance tomography of piecewise-constant permittivity distributions, the reader is referred to Tanushev and Vese [8]. One regularization strategy which guarantees piecewise-constant reconstructions is to define the boundary between contrasting permittivity regions in two dimensions as a level contour of a three-dimensional function. For examples of level-set tomographic
Level-set shape reconstruction using near-field focusing measurements

reconstruction, the reader is referred to Kortschak et al. [9] and Fang [10]. Impedance tomography with level-set reconstruction has been experimentally demonstrated to identifiably reconstruct objects within a pipe [11] and map human lung cavities [12]. Conditioning of level-set formulations may be tailored to the application. For example, Alvarez et al. [13] used multiple level-set functions to recreate long, narrow regions of permittivity contrast in a domain, characteristic of cracks.

In contrast to tomography, some capacitive sensor applications provide single-point or area-localized measurement information, such as for monitoring liquid level in a tank [14, 15] or nonintrusive phase fraction measurement in a finite segment of a microchannel [16], which may also be performed with resistive measurements [17, 18] for conductive fluids.

One type of localized capacitance sensor uses an orthogonal electrode mesh configuration [19]. This sensor uses a fixed array of $n$ parallel, coplanar electrodes, with a second array of $m$ parallel, coplanar electrodes secured such that the electrodes run perpendicular to the first array. The two electrode planes are separated by some small distance, forming the domain of interest between them. The $n \times m$ crossing points (junctions) may be sequentially actuated to gather a map of localized permittivity estimates. Such an orthogonal mesh capacitance sensor was used to intrusively map liquid-gas phase distribution in the cross-section of a pipe using free-wire electrodes placed normal to the flow direction [20]. Orthogonal meshes have also been used to map phase distribution of flow in a narrow channel between the electrode planes [21]. In the latter configuration, termed a field-focusing sensor, the electrodes are not immersed in the flow. This nonintrusive design allows for comparatively large planar electrodes, which exhibit increased sensitivity. Imaging is accomplished through grayscaling the capacitance data, after normalization between the values representing each phase [21]. When significant overlap occurs between sensitivity regions of adjacent electrode junctions, the image may be improved through tomographic reconstruction techniques [22].

A near-field focusing sensor with very small separation between electrode planes (20 $\mu$m – 500 $\mu$m) has been proposed for in situ characterization of thin dielectric layers [23]. For small electrode plane
Level-set shape reconstruction using near-field focusing measurements

separation, moderate electrode pitch is sufficient to guarantee independent detection zones at each electrode junction, even when inactive electrodes are allowed to float during measurement [24]. Virtual grounding of inactive electrodes during sequential actuation allows for detection zones to be approximately independent even with a smaller electrode pitch by damping the electric field outside the vicinity of the actuated junction [23]. When virtual grounding is used, the measurement is area-localized, representing an average value for the detection zone.

Capacitance tomography reconstruction algorithms solve a permittivity distribution by simultaneously satisfying the measured values for all sensitivity distributions. However, when sensitivity distributions do not overlap, as in the near-field focusing sensor, a family of exact solutions exists independently for each region characterized by a sensitivity distribution. This paper presents a new level-set shape reconstruction algorithm that creates a two-dimensional binary distribution with sub-grid-sized features from a map of area-averaged measurements, such as those provided by a near-field focusing sensor. The algorithm differs substantially from extant level-set methods in that it regularizes the reconstruction problem by dramatically reducing the degrees of freedom via an evolving forcing function (velocity function) formed from a small set of pre-defined basis functions. The algorithm delivers information complementary to a reduced-resolution map of void fractions or a grayscale visualization output directly from the sensor. The near-field focusing sensor and mathematical problem statement are first described, followed by development of the shape reconstruction algorithm. The numerical strategies for implementation of the algorithm are discussed, and the algorithm demonstrated on phantoms and on data from a near-field focusing sensor.

Nomenclature

$A$ matrix of areas enclosed by $\omega$
$C$ matrix of capacitance measurements
$E$ error matrix
$E$ error
**Level-set shape reconstruction using near-field focusing measurements**

\[ F \] inverse-square field vector

\[ i \] zone column index

\[ j \] zone row index

\[ L \] velocity function parameter

\[ M \] map of normalized measurements

\[ m \] electrode/zone rows

\[ n \] electrode/zone columns

\[ p \] time step

\[ S \] level-set function

\[ t \] time

\[ u \] unit function

\[ V \] velocity function

**Greek Symbols**

\[ \delta \] pixel dimension

\[ \nu \] zone component of \( V \)

\[ \psi \] weighting function

\[ \omega \] domain of defined region

**Subscripts**

\[ a \] air

\[ \text{base} \] baseline phantom image

\[ \text{conv} \] convergence threshold

\[ d \] dielectric

\[ G \] global

\[ h \] index

\[ I \] image
Level-set shape reconstruction using near-field focusing measurements

k index
L local
pix domain pixels
rec reconstructed image
x x component
y y component
z z component

Superscripts
p time step number
\( \cdot \) variable of integration

Individual matrix and vector entries are written with the matrix or vector name in non-bold type with identifying subscripts, e.g., \( M_{ij} \) indicates the \( i,j \) component of \( M \).

2. Sensor description and problem statement

The near-field focusing sensor considered in this work is composed of two dielectric substrates, each containing a flush-mounted array of five parallel copper electrodes 640 \( \mu \)m wide, with an electrode pitch of 1550 \( \mu \)m, as shown in figure 1(a). The two offset substrates form a 254 \( \mu \)m interfacial gap, with the electrodes exposed to the inside of the gap, and the two arrays positioned orthogonally to each other. The precise gap thickness is maintained with plastic shims. Capacitance measurements are obtained sequentially from each electrode pair on opposite substrates, while holding all other electrodes at virtual ground.

Each \( i,j \) electrode pair has a 1550 \( \mu \)m \( \times \) 1550 \( \mu \)m detection zone centered at their junction, where \( 1 \leq i \leq n \) and \( 1 \leq j \leq m \). The spatial sensitivity distribution corresponds to the strength of the electric field created during measurement, which is most sensitive in the 640 \( \mu \)m \( \times \) 640 \( \mu \)m region where the two active orthogonal electrodes overlap; the detection zone, spatial sensitivity, and material distribution can be
approximated as two-dimensional [23]. Figure 1(b) illustrates the normalized spatial sensitivity, $\psi(x,y)$, applicable to each detection zone within the interior $3 \times 3$ junctions with grounded neighboring electrodes; the TIM layer has a dielectric constant of 4.7. The sensitivity function corresponds to the mid-plane flux model developed in [23], which allocates sensitivity according to the magnitude of the normal projection of the electric field vectors located at the $x$-$y$ plane in the middle of the 254 $\mu$m interfacial gap.

When the set of capacitance measurements, $C$, are normalized between the known values for a gap filled with air, $C_a$, and with dielectric material, $C_d$, the normalized measurement matrix, $M$, is obtained as,

$$
M_{i,j} = \left( \frac{C_d - C}{C_a - C} \right)_{i,j}.
$$

(1)

The matrix, $M$, describes the spatially-weighted void content in each detection zone, with 0 corresponding to a non-voided condition, and 1 corresponding to a completely voided condition. The characterization domain is normalized to a set of $n \times m$ unit squares corresponding to the detection zones. For a two-dimensional binary permittivity distribution throughout the domain, let the estimate of the binary distribution be defined by the unit function $u_\omega$, which takes the value of 1 within a void region $\omega$, and a value of 0 outside the region $\omega$, as shown in figure 2. The normalized measurement matrix, $M_{i,j}$, represents an integral of each zone distribution, weighted by $\psi$, as,

$$
M_{i,j} = \int \int_{\Omega_{i,j}} u_\omega \, dy \, dx.
$$

(2)

For a near-field focusing sensor, the weighting function, $\psi$, is the normalized spatial sensitivity. For a simplified case where $\psi = 1$, the normalized measurements, $M_{i,j}$, are equal to the respective areas $A_{i,j}$ contained by the normalized zones (figure 2).

The objective is to reconstruct the region, $\omega$, from the map of integral measurements, $M$. Two specific properties are desired. The first property is qualitative: reconstructed shapes with rounded contours should be preferred over straight-edged or jagged geometries. This property serves not only to
condition the solution process, but also to create shapes typical of those produced by the underlying surface-energy-based physics of void formation in the target application. The second property is that the reconstructed shapes should not reflect the underlying zone grid, given that the void shape/location is not governed by the placement of electrodes. A simple example illustrating violation of this principle is to define $\omega$ as a set of $n \times m$ circles placed in the center of each zone, sized to satisfy (2).

3. Level-set formulation

A level-set method is proposed to reconstruct a binary distribution satisfying the measurement matrix, $M$, that is reflective of the two specific properties desired as described above. Classical level-set methods, first investigated by Osher and Sethian [25], are governed by the general form,

$$\frac{dS}{dt} + V \cdot \nabla S = 0$$

(3)

where $S$ is the level-set function, and $V$ is a velocity function derived from underlying physical models or otherwise chosen according to the objective of the application. In two dimensions, the output desired is the evolving level-set, $\{x, y\}|_{S(x,y,t)=0}$. A rigorous generalized treatment of level-set theory is offered by Burger [26], including a review of various level-set formulations in the literature.

In the current problem, the level-set function $S$ determines the binary distribution, $u_\omega$, defined as

$$u_\omega(x,y) = \begin{cases} 
1 & S(x,y) > 0 \\
0 & S(x,y) \leq 0
\end{cases}.$$  

(4)

The classical formulation is modified by removing the gradient dependency and defining the velocity as a function of the reconstruction, $\omega$, as,

$$\frac{dS}{dt} = V(\omega) \quad \omega = \{x,y\}|_{S>0}.$$  

(5)

Equation (5) governs the evolution of the level-set function, and is referred to as the level-set equation.

The velocity, $V$, serves to evolve the level-set function, $S$, with increasing time, $t$, until the distribution, $\omega$, satisfies the measurement map, $M$. 

Level-set shape reconstruction using near-field focusing measurements

The velocity function is derived with an inverse-square artifice. Inverse-square principles generally govern physics of diffusive propagation through space, such as concentric wave expansion, acoustic intensity, thermal radiation, electrical charge, and gravitational strength, with intensity decreasing in inverse proportion with squared distance. Furthermore, a distribution of some physical intensity governed by an inverse-square law reflects the shape of the source at close range, but becomes increasingly spherical with distance away from the source. This principle is exploited in the following derivation to produce a velocity function, \( V \), that steers the algorithm toward reconstructions that smoothly span zone/grid boundaries.

Consider a rectangle, used to define an inverse-square field, extending from \( x_{i-1} \) to \( x_i \) and \( y_{j-1} \) to \( y_j \) on the plane \( z = 0 \) as illustrated in figure 3(a). The inverse-square field created by a differential area \( dA \) on the rectangle at \( (x', y', 0) \) and acting on any point with fixed \( z \)-coordinate, \( z = L \), is represented by \( dF \). Let the field magnitude be given by the inverse-square of the distance as

\[
|dF| = \frac{dA'}{(x-x')^2 + (y-y')^2 + L^2}. \tag{6}
\]

The vector components of \( dF \) are obtained through projection onto the unit directions as

\[
dF = \frac{\langle x'-x, y'-y, -L \rangle dA'}{\left((x-x')^2 + (y-y')^2 + L^2\right)^{3/2}}. \tag{7}
\]

The \( z \) component of the cumulative field, \( F \), is given by

\[
F_z = \int_{x_{i-1}}^{x_i} \int_{y_{j-1}}^{y_j} \frac{-L \, dx' \, dy'}{\left((x-x')^2 + (y-y')^2 + L^2\right)^{3/2}}, \tag{8}
\]

where \( x' \) and \( y' \) represent variables of integration within the \( i-j \) detection zone. When \( x_{h}, x_{h+1}, y_{k}, \) and \( y_{k+1} \) correspond to the boundaries of the \( i-j \) zone, the magnitude of \( F_z \) is termed \( v_{h,k} \). The analytical solution is a summation of terms given as

\[
v_{h,k} = \sum_{h=i-1}^{j} \sum_{k=j-1}^{l} (-1)^{i+h+j-k} \tan^{-1} \left( \frac{(x-x_h)(y-y_k)}{L\sqrt{(x-x_h)^2 + (y-y_k)^2 + L^2}} \right). \tag{9}
\]
Figure 3(b-d) illustrates the normalized function \( \nu_{2,2} \) for different values of \( L \). In all cases, the function peaks over the zone, and extends smoothly over the domain. If \( L \) is very low, as shown in figure 3(b), the function undergoes a dramatic gradient near zone boundaries. If \( L \) is very high, as shown in figure 3(d), all gradients are shallow. When implemented in the numerical scheme described in the next section, an \( L \) value between 0.01 and 1 is recommended. Outlying values of \( L \) result in greater computation time and poorer shape reconstructions. In this work, a value is heuristically chosen to maximize the curvature of \( \nu \) at the center point \( (L = 0.272) \), illustrated in figure 3(c). This choice compromises between strong localization in the zone and smooth curvature across zone boundaries. The functions \( \nu_{ij} \) form a basis for constructing the velocity function, \( V \).

In order to construct the initial velocity field, the measurement-attenuated functions \( M_{ij} \nu_{ij} \) are summed and normalized as,

\[
V^0 = V|_{t=0} = \frac{\sum_{i=1}^{n} \sum_{j=1}^{m} (M_{ij} \nu_{ij})}{\max \left[ \sum_{i=1}^{n} \sum_{j=1}^{m} \nu_{ij} \right]},
\]

where the superscript ‘0’ represents the initialization value. Figure 4 illustrates an example measurement map \( M \) with resulting initial velocity function \( V^0 \).

In the algorithm, the velocity function drives the surface \( S \), and corresponding unit function \( u_\omega \) with increasing time according to (5). At any time, \( t \), the error matrix, \( E \), is calculated as

\[
E_{ij} = M_{ij} - \int_{\omega} u_\omega \psi \, dy \, dx.
\]

The error matrix represents the difference between \( M \) and a measurement matrix that would be obtained for the region \( \omega \) at time \( t \). An exact (though not unique) solution for the region \( \omega \) is found when the error matrix is driven to zero by marching forward in time. For \( t > 0 \), the velocity map is attenuated by the error matrix as
Level-set shape reconstruction using near-field focusing measurements

\[
V = \max \left[ \sum_{i=1}^{n} \sum_{j=1}^{m} E_{i,j} v_{i,j} \right] \quad (12)
\]

A key attribute of the velocity function is that, as a linear composition of smooth fields, it can be subjected to arbitrary local attenuation through changes in \( E \) and yet remain smooth. The velocity function thus drives the level-set function to produce smooth (i.e. higher resolution) reconstructions. However, due to the coupling of the velocity function to the zone grid, the reconstruction does not have the ability to distinguish two proximate regions with higher accuracy than the zone grid. Thus, for the purposes of identifying multiple voids, the spatial resolution is equal to two zone widths.

An example of the surface evolution is illustrated in figure 5 for the example normalized measurement map in figure 4(a). At the snapshot in time shown, the error matrix \( E \) (figure 5(a)) has been driven close to zero in the upper right zones because the region \( \omega \) has already been defined in those zones. The zones in the lower left, with values of 0.10, still require some region of \( \omega \) to be defined. The velocity function, \( V \), reflects this need through relatively high positive values at these zones (figure 5(b)), where the level-set function is being driven upward (figure 5(c)). Positive values of the level-set function (lighter shade) define the current region, \( \omega \) (figure 5(d)).

4. Numerical implementation

4.1 Initialization

In this work, the level-set equation (5) is discretized with a first-order explicit scheme as

\[
S^{p+1} = S^p + \Delta t^p V^p .
\] (13)

where the superscript ‘\( p \)’ represents the current time step, and \( \Delta t \) is the time step size. The \( x \)-\( y \) domain is discretized into a fixed set of square pixels for each zone of length \( \delta \), where \( \delta \) is much smaller than the normalized zone length of unity. The integration in (11) is performed with midpoint-rule summation over
Level-set shape reconstruction using near-field focusing measurements

the pixelated domain. Using this pixelated approach, the values of \( v_{ij} \) calculated by (9) may be assigned to each pixel \textit{a priori}.

The initialization process of the algorithm is illustrated in figure 6. In order for the level-set function, \( S \), to define \( \omega \) as it is driven in the positive \( z \) direction, it must be initialized at a constant negative value. In all examples in this paper, an empirically recommended initialization value is used, as given by

\[
S^0 = -\sqrt{nm}.
\] (14)

The recommended initial time step size, \( \Delta t^0 \), is calculated as

\[
\Delta t^0 = -S^0 / \max(V^0).
\] (15)

When a time step smaller than that indicated by (15) is used to initialize the algorithm, the level-set function may take a large number of time steps to obtain a global maximum above zero. Computational efficiency is gained by using (15), which guarantees that the global maximum of \( S^1 \) is zero. Thus, for positive nonzero values of \( \Delta t^1 \) and \( V^1 \), the global maximum of \( S^2 \) is guaranteed to be greater than zero, defining a nonempty region for \( \omega^2 \) beginning with \( \omega^2 \). Figure 6 illustrates the level-set function, \( S \), for the first three time steps corresponding to the measurement map in figure 4(a).

4.2 Time step management

In any given time step, local pixel changes from the previous reconstruction are highly dependent on local gradients of \( S \). In general, gradients in different regions of \( S \) are largely independent, giving rise to nonlinear relationships between \( V^p \), \( \Delta t^p \), and \( \omega^p \). As the level-set function evolves, these nonlinear relationships prevent a monotonic decrease in the global error, \( E_G \), defined by the error matrix \( E \) as

\[
E_G^p = \sum_{i,j} (E_{ij}^p)^2.
\] (16)

Nonmonotonic behavior of global error may always be attributed to one of two phenomena. Both phenomena are illustrated in the temporal evolution for the example case in figure 4(a). Global error
versus time step for this case is illustrated in figure 7, with the corresponding level-set functions shown in figure 8.

First, regions of $\omega$ that have satisfied their zone requirements may continue to exhibit noisy fluctuations due to propagation effects from distant regions which continue to evolve. This phenomenon occurs during $3 \leq p \leq 12$, when the algorithm has defined only the first region composing $\omega$ (see figure 8(a)). There are small noise fluctuations on top of an otherwise constant global error until the second region of $\omega$ is defined. A particular error increase due to noise is highlighted as ‘A’ in figure 7.

Second, regions that have not satisfied their zone requirements are susceptible to overcorrections, where too many pixels are changed because of a large time step. This phenomenon is illustrated at points ‘B’ and ‘C’ in figure 7. At $p = 15$ (figure 8(b)) the second region of $\omega$ has too many pixels. Local gradients of $S$ are low, resulting in high sensitivity to changes in $S$ and thus, the likelihood of an overcorrection. At $p = 16$ (figure 8(c)) too many pixels have been removed (compare to figure 8(d)). The overcorrection between time steps causes a large upswing in global error.

Although the two phenomena that increase global error during evolution are not mutually exclusive, they represent opposing motivations for the choice of step size. In the first case, convergence is accelerated by maintaining or increasing time step size, so that new isolated regions may be defined more quickly; reduction of step size prolongs plateaus such as the one shown in steps 3-12 of figure 7. In the second case, increasing time step size worsens the magnitude of the overcorrection, preventing convergence; a decrease in step size is required to decrease global error. A constant time step size may trap the algorithm in oscillating overcorrections and prevent convergence; this nonlinear relationship between $S$ and $\omega$ prevents formulation of a generalized stability criterion.

In order to properly manage the evolution of the level-set function, the time step size is actively controlled based on an empirical understanding of the mechanisms affecting the behavior of global error:

1. If the global error decreases from the previous time step, the time step size is maintained for the current time step ($\Delta t^p = \Delta t^{p-1}$).
2. If the global error increases from the previous time step, the previous time step size is attenuated by the current global error ratio, $E_G^{p-1}/E_G^p$, as

$$\Delta t^p = \left(E_G^{p-1}/E_G^p\right) \Delta t^{p-1}.$$ \hfill (17)

This error ratio method accounts for the following:

a. If the increase is due to small fluctuations, attenuation will be negligible, allowing the algorithm to continue forward to define additional isolated regions composing $\omega$.

b. If the increase is due to an actual overcorrection, the step size will be reduced in appropriate proportion.

3. If the global error is exactly the same as that of the previous time step, the previous time step size was too small to effect changes in the solution for $\omega$, and the step size is doubled to speed convergence ($\Delta t^p = 2\Delta t^{p-1}$).

When the time step size is managed as a function of global error in this manner, the algorithm stably drives global error toward zero. To implement actively controlled time-step logic, it is recommended that the step size at $p = 1$ be reinitialized as

$$\Delta t^1 = \sqrt{nm}. \hfill (18)$$

4.3 Convergence criterion

A maximum local error is used to determine convergence, as the algorithm may produce a poor solution where all error stems from a single zone if a global error is used. The maximum local error, $E_L$, is defined as

$$E_L^p = \max \left| E_{ij}^p \right|. \hfill (19)$$

When choosing the local error convergence threshold, $E_{\text{conv}}$, the pixel resolution must be considered. In general, the minimum threshold value that can be satisfied by a pixelated solution is

$$E_{\text{conv,min}} = 0.5 \delta^2 \max(\psi). \hfill (20)$$
When \( \psi = 1 \), this minimum threshold requires that the region \( \omega \) will satisfy the measurement map \( M \) to the nearest pixel in each detection zone.

Two issues arise when (20) is used to determine the convergence threshold. First, cases exist for which the algorithm is unable to create a region with this level of fidelity. One such example is illustrated by the ‘Corner Case’ in figure 9, where the exact solution requires sharp-edged reconstructions. Second, though many cases may indeed be reconstructed to this tolerance, such a low error threshold significantly increases the computational expense for little improvement in the reconstruction. Spatial error on the order of several pixels per zone is likely to be insignificant compared to the spatial error associated with the inherent extrapolated nature of the shape construction.

In light of these two issues, convergence of the algorithm is enforced through a progressive relaxation of the convergence criterion. For the reconstructions shown in this paper, the algorithm begins with a convergence criterion of \( E_{\text{conv}} = \delta^2 \max(\psi) \). If the local error remains greater than the threshold, \( E_{\text{conv}} \), after \( nm \) time steps, then the threshold is doubled. Low tolerances are preferred by the algorithm and convergence is guaranteed for all cases. In figure 9, the reconstruction for the corner case is shown at \( p = 100 \), with the local error given as a fraction of pixels per zone. Maximum local error does not reduce after \( p = 60 \). When the convergence criterion is relaxed every \( nm \) time steps, the algorithm converges after 15 time steps. Results are also shown for the example case corresponding to the measurement matrix in figure 4(a).

The entire algorithm is illustrated in the process diagram of figure 10. At initialization, \( p = 0, E_0 = M \), and the other parameters are calculated accordingly. At \( p = 1 \), the region \( \omega \) is an empty set, resulting in no change from the initialized values of \( E, V, E_G \), and \( E_L \); the time step is re-initialized at \( p = 1 \) according to (18).
Level-set shape reconstruction using near-field focusing measurements

5. Results

5.1 Theoretical assessment

Reconstruction of phantoms is used to evaluate the shape reconstruction method developed in this study. In all cases, uniform weighting ($\psi = 1$) is used in (2) to create a measurement matrix equal to the matrix of contained areas. A resolution of $20 \times 20$ pixels per zone is used unless stated otherwise.

Phantom reconstruction is evaluated according to two metrics: the error in total area, $E_A$, and the spatial image error $E_I$, calculated by

$$E_I = \frac{1}{2} \sum_{\text{pix}} \left| u_{\text{co,rec}} - u_{\text{co,base}} \right|,$$

where summation is performed over the pixels of the domain, and the subscripts ‘base’ and ‘rec’ indicate the baseline phantom and the reconstructed image, respectively.

The first desired property of the algorithm is the assured construction of rounded features. This qualitative characteristic is demonstrated in figure 11 by using straight-edged phantoms (figure 11(a)) as inputs to define measurement maps (figure 11(b)). In all cases, the straight edges and corners do not carry over into the reconstruction. In figure 11(d), phantom limbs are significantly narrower than the zone size. These sub-grid features are significantly distorted in the reconstruction. Though the geometries in figure 11 are changed from their source phantoms, they are better representations of geometries that are likely to occur physically for phase incongruities in real materials (voids, bubbles, droplets). Total area is well-preserved for most cases, with $|E_A| < 2\%$ for the cases in figure 11(a-c). In figure 11(d), where the characteristic feature size is smaller than the zone size, area preservation is poorer ($E_A = -6.3\%$).

The second desired property of the algorithm is the decoupling of reconstruction from the underlying zone grid. This is evaluated by comparing a spatially shifted baseline phantom to a reconstruction which is based on a corresponding shifted measurement matrix. Figure 12 illustrates a baseline measurement map chosen to construct a baseline phantom which displays bilateral symmetry and is small enough to be translated to select points on the $3 \times 3$ grid while spanning multiple zones (figure 11...
12(a)). In figure 12(b-d), the baseline phantom is spatially translated, used to calculate the translated measurement matrix, and depicted as a black outline overlaid with the translated reconstruction. In all cases, the total area of the shape is well-preserved ($|E_a| < 1.5\%$). Image error is high ($E_I = 22.5\%$) for figure 12(d), where zones are spanned equally, and the reconstruction exhibits quadrilateral symmetry. In figure 13, a baseline measurement map is chosen to construct a baseline phantom, elongated such that it may be rotated on the $3 \times 3$ grid while spanning multiple zones (figure 13(a)). In figure 13(b-d), the baseline phantom is rotated and re-mapped to the pixel grid, again providing the rotated measurement matrix used for reconstruction. As in the previous case, the correct area of the phantom is well-preserved, but image error runs as high as 20%.

The spatial image error of the reconstruction decreases as the zone size shrinks to approach the characteristic feature size of the underlying distribution. In figure 14, a multi-part phantom consisting of rounded shapes is reconstructed on a grid with overall resolution of approximately $80 \times 80$ pixels, with an increasing number of detection zone divisions. When a $2 \times 2$ grid of zones is used (figure 14(a)), the features of the phantom are much smaller than the zones, and a single rounded shape is constructed, with the area spanning each zone. With increasing grid resolution, the zone size approaches the phantom feature size, and the algorithm performs better at recovering the original shape; a $5 \times 5$ grid of zones (figure 14(c)) closely approximates the original phantom. The reconstructions highlight the ability of the algorithm to naturally select either a single, topologically simple shape (figure 14(a)), a single, complex shape (figure 14(b)), or a multi-part geometry (figure 14(c-d)), according to the quantity of information provided. As seen in figure 11(d), the accuracy of the total area reconstructed by the algorithm suffers when the characteristic feature size is smaller than the zone size. Errors in total area, $E_A$, of the images in figure 14(a-d) are 10.9%, 5.6%, -0.5%, and -0.1% respectively. On the right side of figure 14 is a grayscale visualization of the measurement matrix, representing the image offered by unprocessed measurements. This comparison illustrates the improved visualization gained by using the algorithm to reconstruct a smooth, binarized image of the phantom. The reconstructed images on the left may be used to easily obtain an estimate of region location, area, and perimeter.
5.2 Experimental data

The near-field focusing sensor described in Section 2 is used to demonstrate shape reconstruction from experimental data. In the experiment, a 1 cm × 1 cm dielectric thermal interface material, Tflex SF210, is placed between the clear substrates. Spacers are used between the substrates to maintain the interfacial gap precisely at 254 µm, and a fixed weight is used to apply a constant pressure of 12 kPa. The sensor is used to detect through-voids in the interface material. Each sample contains one artificially created void that ranges from 250 µm to 2000 µm in diameter within the 3 × 3 set of internal detections zones, and capacitance is measured with a commercial meter (AD7746) with absolute uncertainty of ±4 fF [27]. The overall uncertainty of the measurement map entries, $M_{ij}$, is ±0.02, governed by repeatability error between cases.

Because of the uncertainty in creating voids of known size and shape, the true x-y boundaries of the void were ascertained optically, by viewing the void through the transparent substrates at 5X magnification. The voids are approximated as two-dimensional. This approximation introduces some error because the TIM is not cleanly excised through the thickness of the layer, and exhibits irregular delamination along the perimeter of each void. The optical evaluation of the true void geometry consists of fitted ellipses representing two bounding cases: a ‘small’ case including the region where the material is completely removed, and a ‘large’ case including material visibly affected by the creation of the void, as shown in figure 15.

The experimental results are shown in figure 16 on grids representing the 3 × 3 set of internal detection zones. The optical estimate for void geometry is shown on the left, with black and gray regions respectively representing the small and large estimates of the true void geometry. Normalized measurement matrices calculated from (1) are shown alongside the void geometries reconstructed using the sensitivity function illustrated in figure 1(b). When only one zone is affected, as in figure 16(a-b), the algorithm constructs a region in the center of the zone. In figure 16(d), a case is shown where a void spans four adjacent zones, but the sensor was not able to detect a change in the center zone, due to the
poor sensitivity in the corner regions of each zone. Tighter electrode spacing would serve to increase uniformity in the junction sensitivity distribution, improving the measurements and the reconstruction.

6. Conclusion

The level-set shape reconstruction algorithm is a suitable method for reconstructing binary shapes from tessellated integral measurements. Proper treatments for active management of evolution time step size and convergence threshold result in stability and consistent performance. The algorithm is capable of reconstructing a solution that accounts for nonuniform spatial sensitivity of each measurement.

The algorithm effectively creates a level-set contour defining a binary reconstruction that may be composed of a simple, complex, or multiple-part shape. The method has been demonstrated on phantoms and experimental data to effectively approximate sub-grid features with smooth contours across the boundaries of the underlying zone grid. The algorithm performs well at reconstructing consistent shapes independent of location on the underlying zone grid when the shape feature size is close to or larger than the zone size, with image error increasing as feature sizes becomes smaller than the zone size. The method may be used for visualization of voids in a dielectric thermal interface layer using capacitance measurements from a near-field focusing sensor.

Acknowledgements

The authors gratefully recognize financial support for this work from Cooling Technologies Research Center, a National Science Foundation Industry/University Cooperative Research Center and Purdue University, as well as technical discussions with Dr. Justin Weibel.
Level-set shape reconstruction using near-field focusing measurements

References


Level-set shape reconstruction using near-field focusing measurements


Level-set shape reconstruction using near-field focusing measurements


List of figures

**Figure 1.** (a) Schematic diagram of near-field focusing sensor (not to scale), and (b) normalized sensitivity function of the capacitance for the detection zone between any internal electrode pair.

**Figure 2.** Diagram of example region, $\omega$, on a $3 \times 3$ system.

**Figure 3.** (a) Diagram illustrating derivation of inverse-square field, $F$. The normalized function $v_{2,2}$, equal to the magnitude of $F$ exerted by $(2,2)$, is shown on a $3 \times 3$ grid for (b) $L = 0.05$ (c) $L = 0.272$, and (d) $L = 1$.

**Figure 4.** (a) Example normalized measurement map, $M$, and (b) corresponding initial velocity function $V_0$.

**Figure 5.** Algorithm process at a snapshot in time showing the (a) error matrix, $E$, (b) velocity function, $V$, (c) level-set function, $S$, with positive region in lighter shade, and (d) defined region, $\omega$.

**Figure 6.** Level-set function, $S$, at (a) initialization and (b-d) for the first three time steps of the example case shown in figure 4(a). Positive region shown in lighter shade.

**Figure 7.** Global error $E_G$ vs time step $p$ for the example case in figure 4(a). Labels correspond to figure 8.

**Figure 8.** Level-set function, $S$, (left) and defined region $\omega$ (right) at select time steps for the example case in figure 4(a).
**Level-set shape reconstruction using near-field focusing measurements**

**Figure 9.** Convergence behavior of an example corner case (left) and the case in figure 4(a) (right) with \( \psi = 1 \) and \( 20 \times 20 \) pixels per zone (\( E_{\text{conv, min}} = 0.5/400 \)).

**Figure 10.** Process diagram of shape reconstruction solution algorithm.

**Figure 11.** Straight-edged phantoms (left), zone maps, \( M \) (center), and reconstructed regions, \( \omega \) (right). Spatial image error, \( E_I \), for each case is (a) 6.8%, (b) 0.8%, (c) 2.6%, and (d) 13.4%.

**Figure 12.** Shape reconstruction on (a) baseline measurement map with (b-d) resulting phantom reconstructed in different locations relative to the zone grid.

**Figure 13.** Shape reconstruction on (a) baseline measurement map with (b-d) resulting phantom reconstructed using different rotations relative to zone grid.

**Figure 14.** Multi-part phantom (top), with shape reconstruction (left) and measurement map grayscale (right) with detection zone divisions and spatial image error, \( E_I \), respectively of: (a) 2 × 2 and 45.6%, (b) 3 × 3 and 25.2%, (c) 5 × 5 and 6.0%, (d) 7 × 7 and 3.7%. Overall resolution is approximately 80 × 80 pixels.

**Figure 15.** Top view of electrode junction (3,3) through the sensor acrylic substrates, which corresponds to figure 16(f).

**Figure 16.** Experimental results for six cases (a-f), with optical measurement of void geometries (left), capacitance measurements normalized as zone map \( M \) (center), and shape reconstruction performed by the current algorithm (right).
Figure 1. (a) Schematic diagram of near-field focusing sensor (not to scale), and (b) normalized sensitivity function of the capacitance for the detection zone between any internal electrode pair.
Figure 2. Diagram of example region, $\omega$, on a $3 \times 3$ system.
Figure 3. (a) Diagram illustrating derivation of inverse-square field, $F$. The normalized function $\nu_{2,2}$, equal to the magnitude of $F_z$ exerted by (2,2), is shown on a $3 \times 3$ grid for (b) $L = 0.05$ (c) $L = 0.272$, and (d) $L = 1$.
Figure 4. (a) Example normalized measurement map, $M$, and (b) corresponding initial velocity function $V^0$. 

*Level-set shape reconstruction using near-field focusing measurements*
Figure 5. Algorithm process at a snapshot in time showing the (a) error matrix, $E$, (b) velocity function, $V$, (c) level-set function, $S$, with positive region in lighter shade, and (d) defined region, $\omega$. 
Figure 6. Level-set function, $S$, at (a) initialization and (b-d) for the first three time steps of the example case shown in figure 4(a). Positive region shown in lighter shade.
Figure 7. Global error $E_G$ vs time step $p$ for the example case in figure 4(a). Labels correspond to figure 8.
Figure 8. Level-set function, $S$, (left) and defined region $\omega$ (right) at select time steps for the example case in figure 4(a).
Figure 9. Convergence behavior of an example corner case (left) and the case in figure 4(a) (right) with \( \psi = 1 \) and 20 \( \times \) 20 pixels per zone \( (E_{\text{conv,min}} = 0.5/400) \).
Figure 10. Process diagram of shape reconstruction solution algorithm.
Level-set shape reconstruction using near-field focusing measurements

**Figure 11.** Straight-edged phantoms (left), zone maps, $M$ (center), and reconstructed regions, $\omega$ (right). Spatial image error, $E_i$, for each case is (a) 6.8%, (b) 0.8%, (c) 2.6%, and (d) 13.4%.
Level-set shape reconstruction using near-field focusing measurements

Figure 12. Shape reconstruction on (a) baseline measurement map with (b-d) resulting phantom reconstructed in different locations relative to the zone grid.
Figure 13. Shape reconstruction on (a) baseline measurement map with (b-d) resulting phantom reconstructed using different rotations relative to zone grid.
Figure 14. Multi-part phantom (top), with shape reconstruction (left) and measurement map grayscale (right) with detection zone divisions and spatial image error, $E_i$, respectively of: (a) $2 \times 2$ and 45.6\%, (b) $3 \times 3$ and 25.2\%, (c) $5 \times 5$ and 6.0\%, (d) $7 \times 7$ and 3.7\%. Overall resolution is approximately $80 \times 80$ pixels.
**Figure 15.** Top view of electrode junction (3,3) through the sensor acrylic substrates, which corresponds to figure 16(f).
**Level-set shape reconstruction using near-field focusing measurements**

Figure 16. Experimental results for six cases (a-f), with optical measurement of void geometries (left), capacitance measurements normalized as zone map $M$ (center), and shape reconstruction performed by the current algorithm (right).