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Thermodynamics of Air/Water-Cycle Air-Conditioning Systems

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INTRODUCTION

Air cycle refrigeration has been used in aircraft air conditioning systems for many years. In this application, an air cycle has the advantage of lower weight relative to vapor-compression units and it can make use of existing high pressure air from the engine compressor as shown in Figure (1).

In ground based applications an air cycle unit requires its own compressor. Here, the compressor and expander efficiencies become important factors governing the competitiveness of air cycle systems relative to vapor-compression systems.

While considerable work has been done on the gas turbine or Brayton cycle, only a few studies of air cycle refrigeration have been published [3], [5], [6] * and even less has been done on ground based applications [1], [2], [7]. The purpose of this paper is to discuss the basic thermodynamics of several air cycle configurations with special emphasis on the role of water.

PERFORMANCE WITH DRY AIR

The basic air cycle unit employing its own compressor is shown in Figure (2). For this unit one may define a coefficient of performance (COP) as

\[ \text{COP} = \frac{\text{cooling effect}}{\text{work input}} = \frac{h_1 - h_4}{w_e - w_e} \]  

(1)

With dry air treated as an ideal gas with constant specific heats and negligible pressure loss in the heat exchanger this becomes

\[ \text{COP} = \frac{1 - \left[ 1 - \frac{2R}{K} \frac{(R-1)}{R} \right] \left[ 1 + \frac{R-1}{2} \right]}{\left( \frac{R}{2} - e \frac{2R}{K} \frac{(R-1)}{R} \right) \left( \frac{R}{2} - 1 - e \frac{2R}{K} \right)} \]  

(2)

where

- \( e \) = heat exchanger effectiveness = \( (T_e - T_{\infty}) / (T_{\infty} - T_{\infty}) \)
- \( K \) = compressor and expander isentropic efficiencies
- \( R \) = pressure ratio \( P_2 / P_1 = P_3 / P_2 \)
- \( \kappa \) = ratio of specific heats \( c_p / c_v \)

For a typical unit using turbomachines, \( \kappa = 0.85 \) is about the maximum possible efficiency and \( \varepsilon = 0.95 \) is the upper limit for a good heat exchanger. Under these conditions the COP is as shown in Figure (3).

Figure (3) indicates that an air cycle unit can be competitive with vapor-compression units if \( T_{\infty} > T_i \). However, this situation cannot be realized in practice when the unit is connected to a region to be cooled. Figure (4) demonstrates that when operating as a system, \( T_{\infty} > T_i \) is a necessary condition. For example, if the region to be cooled is a residence, \( T_i \approx 750°F = 5350R \). When air conditioning is required, \( T_{\infty} \) is generally greater than this particularly if one selects the rating condition of 95°F ambient air applied to vapor-compression systems.
systems. For comparative purposes then, the air cycle should be examined for $T'' = 950^\circ F = 555^\circ R$. In this paper we shall use $75^\circ F = 535^\circ R$ as the temperature of the region to be cooled.

Efficiencies of this magnitude are not currently possible with turbomachines. The alternative is positive displacement machines which must cope with the additional problem of high flow rates.

Suppose, for example, that 95% efficient machines are possible. Under this condition, the volume flow rate of air into the compressor to produce 20,000 Btu/hr of cooling at the rating conditions is shown in Figure (6).

Turbo machines can handle such flow rates in small sizes because they operate at high speeds. Positive displacement machines must operate much slower. At present, no positive displacement machines which can achieve the required efficiency are available although several designs show promise [1], [2].

### AIR/WATER CYCLE SYSTEMS

One way to increase air cycle performance is to change the properties of the working fluid. This can be done by injecting atomized water at selected points in the system. The performance of the resulting air/water cycle system is highly dependent upon where water is injected and the quantity of water used. The behavior of this system will be illustrated with the two
configurations shown in Figure (7).

![Diagram of high pressure system](image)

**FIG. 7: Two Basic Systems Considered**

The high pressure system of Figure (7a) is the same as Figure (4) with the exception that water may be injected or removed at points A, B, C, and D. The heat exchanger operates at pressures above atmospheric. In the low pressure system of Figure (7b) the heat exchanger operates at pressures below atmospheric. Again, water may be injected or removed at points A, B, C, and D. With dry air, the COP of the low pressure system is

\[
\text{COP} = \frac{\varepsilon \left( 1 - \frac{T_i}{T_f} \right) \left[ 1 - \frac{T_f}{T_i} (1 - \iota) \right]}{\left( \frac{\varepsilon}{\varepsilon} \right) \left[ 1 - \frac{T_f}{T_i} (1 - \iota) \right] + \frac{\varepsilon}{\varepsilon} - \frac{\varepsilon}{\varepsilon} \left( \frac{\varepsilon}{\varepsilon} - \frac{\varepsilon}{\varepsilon} \right)}
\]

and it has almost the same behavior as the high pressure system shown in Figure (5). With dry air then, the high and low pressure systems are essentially equivalent.

To study the behavior of these systems with air/water mixtures, the working fluid cannot be treated as an ideal gas with constant specific heats. While a mixture of air and water vapor alone behaves as an ideal gas at the low pressures involved, the introduction of liquid or solid water into the mixture complicates the analysis. These complexities as well as the need for rapid computation of psychrometric data for pressures other than atmospheric can only be handled efficiently by computer methods. For this reason a large number of computer subroutines were created such that they could be linked together to model various problems.

**MIXTURE PROPERTIES**

The thermodynamic properties of air/water mixtures were modeled by treating only the air-vapor portion as an ideal gas. Properties of the solid and liquid water were treated as being only temperature dependent. The reference state for air was taken as 0°F, 14.696 psia. The reference state for water is saturated liquid at the triple point. The enthalpy of an air/water mixture per unit mass of air is then

\[
h = \left[ c_{pa} \frac{dT}{T} + \omega_l \left[ h_{lg} + \left( \frac{c_{pl}}{\rho_{lw}} \right) \right] \right] + \omega_v \left[ \frac{c_{pv}}{\rho_{vw}} \frac{dT}{T} \right]
\]

and the mixture entropy per unit mass of air is

\[
S = \left[ c_{pa} \frac{dT}{T} - \rho_{pa} \frac{\partial s}{\partial T} \left( \frac{P_a}{\rho_{pa}} \right) \right] + \omega_l \left[ S_{lg} + \left( \frac{c_{pl}}{\rho_{lw}} \right) \right]
\]

\[\frac{c_{pv}}{\rho_{vw}} \frac{dT}{T} - \rho_{vw} \frac{\partial s}{\partial T} \left( \frac{P_v}{\rho_{vw}} \right)\]

where the partial pressures of the air and vapor follow from the definition of humidity ratio

\[
\omega_v = \frac{0.622 P_v}{P_a - P_v}
\]

The mixture specific heat per unit mass of air is

\[
c_p = c_{pa} + \omega_v c_{pv} + \omega_l c_{pl}
\]

and the specific volume per unit mass of air is

\[
\nu = \frac{\rho_i}{\rho} \left[ 0.622 + \omega_v \right] + \omega_l \nu_l + \omega_v \nu_v
\]

**FLOW MACHINE ANALYSIS**

The analysis of processes occurring in the expander and compressor is difficult since little data is available for the case of air containing water in solid and liquid form. As pressures and temperatures change during these processes the ability of air to retain water vapor changes and evaporation, condensation, freezing, and sublimation will occur.
These are rate processes which depend on local heat and mass transfer coefficients whose values cannot be accurately determined for flow inside these machines. There are, however, two definite limits bounding the real process; frozen flow in which the mixture composition remains unchanged during the process, and equilibrium flow in which thermodynamic equilibrium is continuously maintained.

The technique used to analyse the compression and expansion processes follows standard thermodynamic practice as follows:

1. Given mixture inlet state and process outlet pressure.
2. Find isentropic work/1bm-air, \( W_s \).
3. Find true work from
   \[ W = \frac{W_s}{\eta_c} \text{ (compression) } W = \gamma_e W_s \text{ (expansion)} \]
4. The true outlet enthalpy is then
   \[ h_{out} = h_{in} \pm W \]
   and the other properties at the outlet state may be found by iteration upon equation (4).

For an isentropic process with equilibrium flow the isentropic outlet state is found by setting
   \[ S_{in} = S_{out} \]

With \( S_{out} \), \( P_{out} \), and \( \omega \) known, the outlet state is found by iteration upon equation (5).

For frozen flow with constant mixture composition, the isentropic outlet state is a non-equilibrium state. The isentropic work per unit mass of air is
   \[ W_s = c_p \left( T_{out} - T_{in} \right) \]

and the true work is
   \[ W = \frac{W_s}{\eta_c} \text{ (compression) } W = \gamma_e W_s \text{ (expansion)} \]

The true outlet temperature then follows from
   \[ T_{out} = c_p \left( T_{out} - T_{in} \right) \]

In this state the mixture is still in non-equilibrium and the mixture temperature will adjust itself downstream of the machine. The equilibrium temperature attained will be such that \( h_{out} \) from equation (14) will be satisfied by equation (4).

For dry air, frozen and equilibrium flow are the same. To illustrate the difference with water present, consider an expander operating at 85% efficiency, a pressure ratio of 2.67, and an inlet state of saturated air at 535°R, 14.7 psia. For this inlet state
   \[ \omega_1 = 0 \]
   \[ \omega_2 = 0 \]
   \[ \omega_y = \omega_{sat} = 0.01882 \text{ 1bm/lbm-air} \]

The frozen flow analysis may be carried out manually. The specific heat is essentially
   \[ c_p = 0.24 + c_y 0.01882 = 0.2484 \text{ 8tu/lbm-air} \]

and the isentropic outlet temperature from equation (11) is
   \[ T_{out} = 535 \left( \frac{1}{2.67} \right) \frac{0.2484}{0.01882} = 405°R \]

The isentropic work is
   \[ W_s = 0.2484 \left[ 535 - 405 \right] = 32.3 \text{ Btu/lbm-air} \]

The inlet enthalpy is 38.6 Btu/lbm-air so the outlet enthalpy is 38.6 - 27.5 = 11.1 Btu/lbm-air. To find the equilibrium outlet state, one may plot h from equation (4) at \( P = 14.7/2.67 = 5.5 \text{ psia, } \omega = 0.01882 \) as shown in Figure (8). The equilibrium outlet state is thus 483°R.

\[ \omega \]

**FIG. 8: Equilibrium Mixture Enthalpy at 5.5 psia, \( \omega = 0.01882 \text{ 1bm/lbm-air} \)**

At 483°R and 5.5 psia, the saturated humidity ratio is 0.00667 1bm/lbm-air which is the outlet \( \omega_y \). The remaining water is solid.
The expander performance for frozen and equilibrium flow is shown in Figure (9). The computer was used for the equilibrium case. The difference in work is 27.5 Btu/lbm-air for frozen flow and 29.4 Btu/lbm-air for equilibrium flow.

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The number of parameters influencing air/water cycle system behavior is quite large so that in order to study the system it is advantageous to examine individual components first.

If the water content of the mixture entering the compressor increases while the inlet temperature remains constant, compressor work increases unless the inlet degree of saturation

\[ D_s = \frac{\text{total water content}}{\text{water content of saturated air}} = \frac{w}{w_{sat}} \]

and/or the pressure ratio is quite high. This is illustrated in Figure (10) for the compressor of Figure (7a) which compresses the mixture up from 14.7 psia to a higher pressure and the compressor of Figure (7b) which compresses the mixture up to 14.7 psia from a lower pressure.

One can decrease the compressor work by cooling the entering mixture through water injection upstream of the entrance. This cooling effect is illustrated in Figure (11) for the case of dry air entering the injector and 100% saturating effectiveness. The magnitude of this cooling increases as the temperature of the injected water and the relative humidity of the entering air decrease.

The resulting decrease in compressor work thus depends on several parameters. In order to decide whether it is advantageous to inject water one must consider the system design conditions. As one example, consider the compressor of Figure (7a) under typical conditions of 75°F, 50% relative humidity air entering at 14.7 psia and assume that water is available at 60°F. The effect of upstream injection of this water on compressor performance is shown in Figure (12). Injection is thus helpful in this case. In fact, upstream injection is always advantageous when the water supply is colder than the incoming air.

Figure (13) illustrates the effect of water content on expander performance for a constant inlet temperature for the expander of Figure (7b) which expands the mixture down from 14.7 psia to a lower pressure and the expander of Figure (7a) which expands the mixture down to 14.7 psia from a higher pressure. Water is helpful in this case provided the inlet temperature remains unchanged. Injection of water is not advantageous because the cooling effect results in a net decrease in expander work.
Predicting the performance of the heat exchanger with water present becomes a rather involved task. For a proper analysis one requires considerable information about the mixture flows and heat exchanger construction. In order to keep the heat exchanger analysis on the same level as that for the flow machines, we shall define an effectiveness as

$$\varepsilon = \frac{h_{\text{out}} - h_{\text{in}}}{h_{\text{out, max}} - h_{\text{in}}}$$

where $h$ refers to the enthalpy of the mixture inside the heat exchanger. The air flow over the heat exchanger is ambient air at $T_0$ for the high pressure system and air from the region to be cooled at $T_x$ for the low pressure system.

To illustrate the use of this effectiveness, consider the heat exchanger for the high pressure system of Figure (7a) under conditions of $T_0 = 95^\circ F$, $T_x = 200^\circ F$, $T_{45} = 660^\circ R$ saturated air at 30 psia and an effectiveness of $\varepsilon = 0.9$.

For the entering mixture, the saturated humidity ratio at 660°F, 30 psia is 0.388 lbm/lbm-air. The mixture thus contains this much water in the vapor phase only and equation (4) gives the inlet mixture enthalpy as $h_{\text{in}} = 496$ Btu/lbm-air.

If the heat exchanger were 100% effective, the mixture would leave in equilibrium at $555^\circ R$, 30 psia (neglecting pressure loss) and $\omega = 0.389$ at 555°F, 30 psia, $\omega_{\text{AT}} = 0.0174$. At the ideal outlet then:

$$\omega_2 = 0$$

$$\omega_2 = 0.0174$$

$$\omega_2 = 0.389 - 0.0174 = 0.3716$$

and equation (4) yields

$$h_{\text{out}} = 65.4 \text{ Btu/lbm-air}$$

The maximum heat transfer from the mixture is thus

$$Q_{\text{max}} = 496 - 65.4 = 430.6 \text{ Btu/lbm-air}$$

At $\varepsilon = 0.9$, the true heat transferred is $\varepsilon Q_{\text{max}}$ or 388 Btu/lbm-air and the true $h_{\text{out}}$ is

$$h_{\text{out}} = 496 - 388 = 108 \text{ Btu/lbm-air}$$

With $h_{\text{out}}$, $\omega$ and $P$ known, the outlet temperature is found by iteration on equation (4) in the same method illustrated graphically in Figure (8). The result is a mixture outlet temperature of 584°F.

This method of treating the heat exchanger allows one to examine heat exchanger performance easily and is adequate for the basic studies involved in this paper.

The effect of water on heat exchanger performance is shown in Figure (14) for conditions typical of the low and high pressure systems of Figure (7). The increase in heat transfer is greatest when a phase change occurs which corresponds to $D_s < 1$ in the low pressure system and $D_s > 1$ in the high pressure system.

The effect of water on heat exchanger performance is shown in Figure (14) for conditions typical of the low and high pressure systems of Figure (7). The increase in heat transfer is greatest when a phase change occurs which corresponds to $D_s < 1$ in the low pressure system and $D_s > 1$ in the high pressure system.
EFFECT OF WATER ON SYSTEM PERFORMANCE

While the effect of water on individual component performance serves as a guide, it does not give the complete picture of what will happen to a system. This is because the components influence each other when joined together. One can, however, use the results of the previous section as a guide in selecting water injection schemes for the high and low pressure systems of Figure (7).

To illustrate, consider first the high pressure system. If the desire is to improve system COP by water injection, then placing an injector upstream of the compressor will help while placing an injector upstream of the expander will not.

From a practical standpoint, it is not possible to transport liquid water from the heat exchanger outlet to the expander inlet in the necessary atomized form without collecting it and pumping it up to a higher pressure for injection. Since this would not appreciably change its temperature and the result of returning it to the already saturated vapor entering the expander offers little or no improvement as shown by Figure (13), this seems hardly worth the added complexity. It would appear then that the best course is to inject water upstream of the compressor and separate and simply discard any liquid in the heat exchanger outlet stream.

Figure (15) shows typical effects of such an injection scheme using 60°F water in the injector. The incoming air from the region to be cooled is at 75°F, 50% relative humidity and the air flow over the heat exchanger is at 95°F. The injection ratio is defined as

\[
R_I = \frac{\text{injected water}}{\text{unit mass of incoming air}}
\]

Figure (16) shows typical effects of such an injection scheme using 60°F water in the injector. The incoming air from the region to be cooled is at 75°F, 50% relative humidity and the air flow over the heat exchanger is at 95°F. The injection ratio is defined as

\[
R_I = \frac{\text{injected water}}{\text{unit mass of incoming air}}
\]

The hoped for improvement in COP does not occur primarily because while water injection does lower compressor work, it also lowers the mixture temperature entering the heat exchanger. This decreases the heat transferred so that the net effect on COP is detrimental.

Computer studies at other efficiencies, temperatures, and using other injection schemes were also run. All showed little if any improvement in system COP over that for dry air. One must conclude then that water injection is not a viable means of improving the performance of the high pressure system. Improvement can only come through higher machine efficiencies.

In the low pressure system, injection of water upstream of the expander is again not useful but improvement in performance is gained by injection upstream of the heat exchanger as shown in Figure (16). In this case the water temperature is again 60°F, the air from the region to be cooled is 75°F and the inlet air to the expander is at 95°F, 50% relative humidity. A further increase in COP of 5 to 10% is possible if 60°F water is also injected upstream of the compressor.
Figure (16) indicates that water injection can be quite effective in improving low pressure system performance mainly because the low pressure air can hold more water vapor. But certain additional constraints must be considered before a proper judgement can be made.

1. The mixture entering the heat exchanger cannot be too cold or ice buildup will be a problem.
2. The mixture entering the heat exchanger cannot be too warm or the air being returned to the region to be cooled will not be cold enough for comfort and massive amounts of air will have to be circulated to transfer the heat.
3. Massive injection will consume too much water.
4. Heat exchanger effectiveness of 0.95 is unrealistic with moisture on both sides of the exchanger and less than 100% saturating effectiveness in the injector.

A low pressure system with water injection adjusted upstream of the heat exchanger so that the mixture leaving the heat exchanger is just saturated and an expander pressure ratio controlled so that the mixture entering the heat exchanger is at or slightly above 32°F with no ice present is one way of satisfying some of these constraints. Under these conditions there is one pressure ratio and one water injection ratio associated with a given heat exchanger effectiveness and machine efficiency. Figure (17) shows the resulting performance and required water injection ratio for such a system. The required pressure ratio is between 1.9 and 2.4

Figure (18) is a similar plot for the high pressure system with no water injection (since we have shown that water injection is not helpful here). This graph shows the maximum possible COP.

These figures show that the low pressure system with water injection is superior even if we discard the conditions of high heat exchanger effectiveness because of the large injection ratios required and the improbability of achieving such values with "wet" coil conditions.

**CONCLUSIONS**

The basic thermodynamics of ground based air cycle systems shows that a low pressure system with water injection offers the best performance. Even so, machine efficiencies greater than 90% are required if such a system is to be competitive with vapor-compression systems for residential type applications. In other applications where current vapor-compression COPs are lower (i.e. automotive air conditioning where COP = 1.3-2.0), a low pressure system with turbomachines shows greater promise. This is even more promising when one considers the lower complexity, lower cost, and the elimination of the condenser. The low pressure system has the additional advantage that the fluid circulated in the region to be cooled is separate from the fluid passing through the flow machines. The flow machines could thus be operated with some lubrication, something not acceptable in the high pressure system. The disadvantage of the low pressure system is that volume flow rates are higher requiring larger size flow machines.

FIG. 17: COP and Required Injection for Low Pressure System with Controls

FIG. 18: Maximum Possible COP for High Pressure System (No Water Injection)
NOMENCLATURE

COP = cooling effect/work input

\[
\begin{align*}
\eta_{\text{high}} &= \frac{h_a - h_0}{h_a - h_0} \\
\eta_{\text{low}} &= \frac{h_v - h_0}{h_v - h_0}
\end{align*}
\]

\(h\) = Enthalpy/unit mass air
\(s\) = Entropy/unit mass air
\(T\) = Temperature
\(P\) = Pressure
\(W\) = Work/unit mass air
\(\epsilon\) = Heat exchanger effectiveness
\(\gamma\) = Isentropic efficiency
\(\dot{W}_s\) = Isentropic work/unit mass air
\(C_p\) = Specific heat/unit mass air
\(Q\) = Heat transfer/unit mass air
\(\omega\) = Gas constant
\(\vartheta\) = Humidity ratio lbm/lbm-air
\(\nu\) = Specific volume
\(R_\gamma\) = Injection ratio
\(D_s\) = Degree of saturation
\(h_i\) = Enthalpy of fusion of ice
\(s_i\) = Entropy of fusion of ice
\(h_v\) = Enthalpy of vaporization of water
\(s_v\) = Entropy of vaporization of water
\(P\) = Pressure ratio
\(K\) = Ideal gas ratio of specific heats
\(\ln\) = Natural logarithm

SUBSCRIPTS

c = Compressor
\(e\) = Expander
\(a\) = Air
\(s\) = Solid
\(L\) = Liquid
\(v\) = Vapor
\(ra\) = Air reference state
\(rw\) = Water reference state
\(\infty\) = Ambient
\(\text{sat}\) = Saturated air

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