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Model Predictive Control of a Radiant Floor Cooling System in an Office Space

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ABSTRACT

This study presents an optimal control formulation for the operation of a radiant floor system in an open plan office space with an air-cooled chiller as a source. A simulation case study with different control schemes is used to evaluate the potential of the model predictive control for the radiant floor as well as the optimal control coordination of a radiant and air comfort delivery system. The comparison with a reference case of proportional control shows a saving potential for the radiant floor of around 10 to 15.8%. This results from maintaining the operative temperature at the upper bound and precooling or load shifting. Optimal control coordination of radiant floor and air system yields additional saving of around 2%. The proposed intuitive formulation of linear programming can be implemented to other control problems with a linear building model and known COP with respect to weather prediction. The formulation is applicable to other complex systems with two or more control systems such as open-plan spaces with several control units or multiple zones (or buildings) with centralized plant.

1. INTRODUCTION

Radiant floor heating and cooling has been investigated for a long time and its superior performance in terms of energy savings and improved comfort, have been revealed in many studies (Fabrizio, 2012; Kim and Olesen, 2015a; Kim and Olesen, 2015b; Nall, 2013a; Nall, 2013b; Nall, 2013c; Olesen, 2008; Rhee and Kim, 2015). Specific advantages are: (a) The system is operated with moderate temperature so the efficiency of the plant is higher. (b) The room air temperature can be maintained at lower and higher setpoint for the heating and cooling case respectively due to the radiative heat exchange with the large floor surface, thereby less energy is consumed while maintaining equivalent comfort. (c) The large slab surface area yields uniform heat transfer to the room so the thermal comfort is improved.

Many previous studies of the radiant floor system focused on the temperature regulation during the heating season. Conventional feedback control has been implemented to the system (Ahn and Song, 2010; Ahn, 2011; Batista et al, 2013; Cho and Zaheer-uddin, 1999; Rhee et al, 2011) controlling the valve to maintain the room air temperature (Ahn and Song, 2010; Batista et al, 2013; Cho and Zaheer-uddin, 1999), PMV (Rhee et al, 2011) and heat flux directly (Athienitis, 1997). However, for all cases, the room air or operative temperature fluctuated more than 2 °C. Controlling the supply water temperature based on outdoor air temperature to prevent the overheating has been suggested (Ahn, 2011) and it has been applied to a cooling case as well (Lin et al, 2006), followed by cooling and heating simulation studies (Arteconi et al, 2014; Gwerder, 2008; Gwerder, 2009; Lehmann, 2011; Olesen, 2002; Park et al, 2014; Schmelas et al, 2015). In some cases, temperature fluctuation was reduced with supply water temperature control (Park et al, 2014, Song et al, 2008, Seo et al, 2014) and with the help of a Dedicated Outdoor Air System (DOAS) in the cooling case (Song et al, 2008, Seo et al, 2014). However, this improvement was based on a simulation study with a forward modeling approach (Park et al, 2014, Song et al, 2008), while the test-cell experiment in the same study showed large fluctuation (Song et al, 2008). Also, in this case, the cooling load offset by the radiant floor system was reduced due to the cooling rate from the DOAS (Song et al, 2008, Seo et al, 2014).

Using advanced control methods such as MPC has shown good potential for the radiant floor system due to its ability of incorporating exogenous inputs and predicting thermal dynamics. Although this has been the focus in
many studies, its potential has not been fully explored due to the following reasons: (a) The prediction model is not good enough to be used for the MPC and the reason, in some cases, is the use of an unsuitable model structure (Feng et al., 2015). (b) The model is not validated with experiment data. Some studies used a forward approach (Nghiem et al., 2013) and in others a building simulation program such as EnergyPlus and TRNSYS was used to generate the data for the estimation (Bernal et al., 2013; Lehmann et al., 2013; Nghiem et al., 2012; Oldewurtel et al., 2013; Sourbron et al., 2013). (c) Overheating for the room temperature was still shown when ANN-based prediction (Lee et al., 2002), semi-physical modeling (Váňa et al., 2014), and transfer function model (Candanedo et al., 2010; Candanedo et al., 2011a; Candanedo et al., 2011b) were applied.

Another feature of the radiant floor system, besides the advantages discussed so far, is the possibility for load shifting. Many studies focused on the energy saving potential from pre-cooling with air system (Park et al., 2014; Turner et al., 2015; Braun et al., 2001; Braun, 2003; Lee and Braun, 2006a; Lee and Braun, 2006b; Gayeski et al., 2011; Favre and Peuportier, 2014), which in some cases included a thermal storage system (Henze et al., 2004). Large capacity favors pre-cooling (Favre and Peuportier, 2014), and in the case of radiant floor the potential of the system can be maximized without thermal storage on the plant side. Also, pre-cooling strategies have clearly shown the energy saving potential even with rule-based controls that have been realized with building energy simulation tools (Park et al., 2014, Turner et al., 2015), well-estimated control-oriented models (Braun et al., 2001; Braun, 2003; Lee and Braun, 2006a; Lee and Braun, 2006b), and field tests (Braun, 2003). Other recent studies demonstrated the potential of the MPC approach using optimization methods such as pattern search (Gayeski et al., 2011) and dynamic programming (Favre and Peuportier, 2014; Henze et al., 2004).

The objective of this study is to develop a new optimal control formulation for a radiant floor system, considering a high performance building at Purdue campus as a case-study. The problem is formulated into a linear programming using the capacity of the radiant floor and the HVAC plant. The performance analysis is based on simulations with different control schemes.

2. METHODOLOGY

2.1 Building model

An open plan office space (9.9m by 10.5m) that can host up to 20 occupants is considered as test-bed for this study. Its main features are a radiant floor slab and a south facing double façade system (Figure 1). The building model is Linear Time Invariant (LTI) with 6 states (Figure 2). Details of the model can be found in Joe and Karava (2016). The number of occupants is assumed to be 10 between 08:00 am and 18:00 pm. Occupant and equipment heat gain is 75 W and 100W per person. The minimum outdoor ventilation rate is the summation of the 5 cfm and 0.06 cfm per person and area (ft²). Initial temperature of all states is assumed to be 24 °C. The operative temperature, which is a linear combination of the air and Mean Radiant Temperature (MRT) is used to control the space.

![Figure 1: Living Laboratory as a simulation test-bed (section view)](image-url)
2.2 HVAC system

An air-cooled chiller is considered as a source for the air and radiant floor system. Performance data are adopted from the EnergyPlus engineering reference and the Energy Input Ratio (EIR) method is applied based on the catalogue data of an actual product, Trane CGAM20, with capacity and Coefficient of Performance (COP) of 68.9 kW and 2.67 COP in nominal condition (EnergyPlus, 2015). For the cases considered in this paper, the chiller capacity is scaled down to 20%. The electricity consumption of the chiller is a multiplication of three curves that represent the capacity, COP, and Part Load Ratio (PLR). The COP according to different outdoor air temperature and PLR is plotted in Figure 3. Lower outdoor air temperature results in a higher COP for a given PLR. In this study, the PLR is neglected in the optimization formulation and the electricity consumption of the chiller is a function of the COP, which can be predicted from the outdoor air temperature, and heat flux from the chiller to the radiant floor. The radiant floor system is assumed to be controlled in different capacity with respect to the concrete temperature, e.g., the capacity is larger for higher concrete temperature. Therefore, the capacity on the radiant floor is a function of the state in the model which is unknown. So the minimum concrete core temperature ($T_{source,LB}$) is implemented in linear programming formulation, which is assumed to be 14 °C in this study. The effectiveness of the radiant floor is found with the NTU method and experimental data from the actual office space. The maximum capacity is calculated as follows:

$$u_{floor,max} = \dot{m}_{water} \times C_{p,water} \times (T_{source,LB} - T_{supply,water}) \times \text{effectiveness}$$ (1)
2.3 Optimal control problem formulation

Nonlinear optimization algorithms such as Fmincon, Pattern search, PSO (Particle Swarm Optimization), and Generic Algorithm are easy to be implemented by assigning the control input at each time step as an individual optimization variable but require a lot of computational time and only provide local optimal solutions in the best case scenario. An implementable optimization form in actual controllers is realized with the consecutive state-space equation over the time horizon. When the state-space model is formed, one step ahead temperature is a function of current temperature, and the exogenous (w) and control input (u) as shown in equation 2. Then the temperature trajectory (X) is a linear function of a column vector (Ω) consisting of Ad, matrix, lower triangle matrix (Ωu and Ωw) consisting of Ad, Bdw, and Bdu, matrix, initial state vector (X0), and exogenous (w) and control input vector (u) as shown in equation 3 (capital letter represents the vector). Finally, the input and output trajectories are in an explicit linear relation, which is a suitable form to be implemented in the optimization algorithm to be proposed.

\[
x[k+1] = A_dx[k] + B_{d,w}w[k] + B_{d,u}u[k]
\]

(2)

\[
\begin{bmatrix}
 x[1] \\
 x[2] \\
 \vdots \\
 x[n]
\end{bmatrix} = \begin{bmatrix} A_d & \ldots & A_d \end{bmatrix} \begin{bmatrix} 0 & \ldots & 0 \\
 0 & \ldots & 0 \\
 \vdots & \ddots & \vdots \\
 0 & \ldots & 0 \\
 \end{bmatrix} \begin{bmatrix} w[0] \\
 w[1] \\
 \vdots \\
 w[n-1]
\end{bmatrix} + \begin{bmatrix} B_{d,u} & 0 & \ldots & 0 \\
 A_dB_{d,u} & B_{d,u} & 0 & \ddots \\
 \vdots & \ddots & \ddots & \ddots \\
 A_d^{n-1}B_{d,u} & A_d^{n-2}B_{d,u} & \ldots & B_{d,u}
\end{bmatrix} \begin{bmatrix} u[0] \\
 u[1] \\
 \vdots \\
 u[n-1]
\end{bmatrix}
\]

(3)

The control input, i.e. the heat flux rate from the plant to the building through the HVAC system is typically considered as a decision variable in previous formulations. However, with the formulation proposed in this study, we can utilize the state which is the temperature of each node together with the control input as decision variables. All dynamics are set through the equality constraint, and bounds for the control input and conditioned zone’s temperature are set as bounds on the input and inequality constraint.

2.4 Case study

Four different cases are considered in the simulation study. Case 1 is reference (baseline) case in which the radiant floor system operates with a proportional integral (PI) control. The air system only gives ventilation to the zone by regulating the supply air temperature to be the same with the room air temperature. The coefficients of the PI controller are tuned to maintain the room operative temperature inside the bound.

Case 2 represents a model predictive control strategy of the radiant floor system without considering the COP and with the air system providing only ventilation. In this way, the operative temperature is maintained at the upper bound and the starting time of the radiant floor system is precisely controlled. The objective function is the summation of heat flux from the chiller to the radiant floor system over the prediction horizon. The equality constraint is for the dynamics and the inequality constraints are for the operative and concrete core temperature. Bounds are given to the decision variables considering the capacity of the chiller and radiant floor. The optimization formulation is shown in equation 4. u, X, and T represent the input trajectory, state trajectory, and temperature trajectory vectors. \( \Omega \) represents the lower triangle matrix from equation 3. \( \text{C}_{\text{op}} \) represents the vector consisting of C matrix calculating the operative temperature in state space equation.

\[
\begin{aligned}
\min & \quad [I \quad 0] \begin{bmatrix} u \\ X \end{bmatrix} \\
\text{subject to} & \quad [\Omega_u X_0 + \Omega_w w] = \begin{bmatrix} u \end{bmatrix} \\
& \quad \begin{bmatrix} 0 & \text{C}_{\text{op}} \\ 0 & -\text{C}_{\text{op}} \\ 0 & -\text{C}_{\text{op}} \end{bmatrix} \begin{bmatrix} u \\ X \end{bmatrix} \leq \begin{bmatrix} T_{\text{op,UB}} \\ -T_{\text{op,LB}} \\ -T_{\text{op,UB}} \end{bmatrix} \\
& \quad \begin{bmatrix} -\min(u_{\text{chiller, max}}, u_{\text{floor, max}}) \end{bmatrix} \leq \begin{bmatrix} u \end{bmatrix} \leq \begin{bmatrix} 0 \\ \infty \end{bmatrix}
\end{aligned}
\]

(4)
Case 3 represents the optimal control of the radiant floor system with MPC considering the COP. The air system provides ventilation only. The radiant floor system can take advantage of the higher COP with lower outdoor air temperature during the night time, which is typically referred to as pre-cooling or load shifting. The objective function consists of the Energy Efficient Ratio (EER) \((1/ \text{COP})\) and heat flux from the chiller to the radiant floor (equation 5). The same constraints with case 2 are adopted.

\[
\min \left[ -\frac{1}{\text{COP}} \begin{bmatrix} 0 \\ 0 \\ \vdots \end{bmatrix} X \right] u
\]

\[
\begin{bmatrix}
-\Omega_u \\

\Omega \end{bmatrix} u + \Omega w = \Omega \begin{bmatrix} X_0 \end{bmatrix}
\]

where

\[
\begin{bmatrix}
0 \\
0 \\
0 \\
-\Omega_u \\
-\Omega_c \end{bmatrix} u \leq \begin{bmatrix} T_{\text{op,UB}} \\
-T_{\text{op,LB}} \\
-T_{\text{so,LB}} \\
-T_{\text{chiller,UB}} \\
-T_{\text{floor,UB}} \end{bmatrix}
\]

\[
\begin{bmatrix}
-\min(\text{u}_{\text{chiller,UB}}, \text{u}_{\text{floor,UB}}) \\
-\min(\text{u}_{\text{chiller,UB}}, \text{u}_{\text{floor,UB}}) \\
-\min(\text{u}_{\text{chiller,UB}}, \text{u}_{\text{floor,UB}}) \end{bmatrix} \leq \begin{bmatrix} u \\
T_{\text{so}} \end{bmatrix} \leq \begin{bmatrix} 0 \\
\infty \end{bmatrix}
\]

(5)

Case 4 is a coordinated optimal control of the radiant floor \((u_1)\) and air system \((u_2)\). In this case, the air system provides cooling to the space along with the radiant floor while it is only used for ventilation in case 1, 2, and 3. The optimization formulation is similar to that of case 3 but one more control input is added \((u_2)\), and thereby, an additional inequality constraint is incorporated since the two systems share the same source of energy from the air-cooled chiller (equation 6). Two different inputs, with distinct dynamics, are simultaneously controlled to maintain the operative temperature inside the bound while minimizing the energy consumption.

\[
\min \left[ -\frac{1}{\text{COP}} \begin{bmatrix} 0 \\ 0 \end{bmatrix} X \right] \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}
\]

\[
\begin{bmatrix}
-\Omega_u \Omega_u \\
0 \\
0 \\
0 \\
0 \\
-\Omega_u \\
-\Omega_u \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \Omega \begin{bmatrix} X_0 \end{bmatrix}
\]

where

\[
\begin{bmatrix}
-1 \\
0 \\
0 \\
-1 \\
0 \\
0 \\
0 \\
-\Omega_u \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ X \end{bmatrix} \leq \begin{bmatrix} q_{\text{chiller,UB}} \\
-q_{\text{vent}} \\
-T_{\text{op,UB}} \\
-T_{\text{op,UB}} \\
-T_{\text{so,UB}} \\
-T_{\text{chiller,UB}} \\
-T_{\text{floor,UB}} \end{bmatrix}
\]

\[
\begin{bmatrix}
-\min(\text{u}_{\text{chiller,UB}}, \text{u}_{\text{floor,UB}}) \\
-\min(\text{u}_{\text{chiller,UB}}, \text{u}_{\text{floor,UB}}) \\
-\min(\text{u}_{\text{chiller,UB}}, \text{u}_{\text{floor,UB}}) \end{bmatrix} \leq \begin{bmatrix} u_1 \\
T_{\text{so}} \end{bmatrix} \leq \begin{bmatrix} 0 \\
\infty \end{bmatrix}
\]

(6)

3 RESULTS ANALYSIS

The prediction horizon for the MPC is 2 days and the optimal control input of the first day is implemented in the simulation. The last state from day one is used as the input to the second day’s optimization as initial state. Actual measurements of outdoor air temperature and solar radiation data from the summer of 2015 were used and the deterministic control formulation assumes perfect weather forecast. After a warm up period of 3 days, MPC simulation runs for 8 days. Lower and upper bounds of the operative temperature are 24 and 26 °C, respectively.

For all four cases, temperature profiles and electricity energy consumption of the air-cooled chiller are compared with the results shown in Figure 4. The first graph of each case in Figure 4 represents the temperatures including air, operative, slab, and source, and the second graph shows the capacity and control input. Transportation energy such
as pump and fan energy is not considered. In case 1, the operative temperature is inside of the bound as the PI controller was tuned, and the maximum control input is available for every iteration whereas it is not possible in other cases of predictive control. The operative temperature for case 2 stays in the upper bound due to predictive control resulting in a 10% energy saving compared to case 1. In case 3, the potential of the system is maximized by considering the COP which provides an additional saving of 5.8%. The pre-cooling and load shifting is clearly seen by comparing case 2 and case 3. The control input is ON when the EER is high so the temperature during the initial occupied period is lower and it is increased with time. Additional 2% energy saving, compared to case 3, can be achieved if the temperature is close to the upper bound. This is the motivation for case 4 which also includes an air system. The daily (top) and total (bottom) energy consumption are shown in Figure 5.

Figure 4: Temperature and energy consumption of all cases
Figure 5: Daily (top) and total (bottom) energy consumption of each case

4. CONCLUSIONS

In this paper, the energy saving potential of a radiant floor cooling system and the coordinated operation of radiant floor and air system in an office building was presented based on a comparative analysis with a simulation study. The optimal control problem was formulated into a linear programming with the decision variables of control input and states considering all constraints of building dynamic and capacity of the plant and radiant floor. The main findings can be summarized as follows:

- 10% of energy saving is achieved with MPC compared to the conventional PI control. Additional saving of 5.8% is achieved from pre-cooling and load shifting.
- Optimal coordination control of radiant floor and air system yields an energy saving of around 2% compared to the optimal control of radiant floor system.

Coordination control is formulated as a centralized optimization problem which requires a central processing unit such as server computer in Building Energy Management System (BEMS). Future work will consider distributed approaches towards plug-and-play building systems with embedded intelligence.

NOMENCLATURE

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>state matrix</td>
<td></td>
</tr>
<tr>
<td>$B$</td>
<td>input matrix</td>
<td></td>
</tr>
<tr>
<td>$C_p$</td>
<td>capacitance</td>
<td>J/kg°C</td>
</tr>
<tr>
<td>$k$</td>
<td>time step</td>
<td></td>
</tr>
<tr>
<td>$m$</td>
<td>mass</td>
<td>kg</td>
</tr>
<tr>
<td>$u$</td>
<td>control input</td>
<td>W</td>
</tr>
</tbody>
</table>
disturbance input (W)
state (temperature) (°C)
effectiveness (—)
stacked matrix in state space formulation (—)

Subscript

d discrete vector
op operative temperature
LB lower bound
UB upper bound
so source

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