Correlation of Intensity Variations and False Color Displays of Multispectral Digital Images

Jorge Burkle
Elias Baron

Follow this and additional works at: http://docs.lib.purdue.edu/lars_symp

http://docs.lib.purdue.edu/lars_symp/204

This document has been made available through Purdue e-Pubs, a service of the Purdue University Libraries. Please contact epubs@purdue.edu for additional information.
CORRELATION OF INTENSITY VARIATIONS AND FALSE COLOR DISPLAYS OF MULTISPECTRAL DIGITAL IMAGES

JORGE BURKLE AND ELIAS BARÓN
Centro Científico IBM de América Latina, Mexico, D. F.

I.- INTRODUCTION

Ground resolution elements seen by a multispectral sensor, may in some cases, consist of mixtures of object categories so that many of the pixels of an image are not characteristic of any category or class. It would be desirable to have a model to represent the combination categories in terms of simpler homogeneous ones and then estimate the corresponding proportions. A method of correlation of intensity variations, may reveal if a given image is representable by a linear mixture model. However, the intensity variation may be due to effects as shadows or variations in the incident light conditions. It is shown that these effects would also produce a linear variation and experiments for several LANDSAT images show that this is the case. Consequently, the estimation problem is difficult to solve.

False color displays may filter this kind of linear variations producing uniform combined colors for homogeneous areas. The reason for the above conclusions is that, statistically speaking, the intensity variation from pixel to pixel of an homogeneous image, has the same sign for all spectral channels.

II.- LINEAR MIXTURE MODELS

Let us first review some properties of linear mixture models. If we represent the intensity \( S(i) \) reflected of the \( i \)-th channel by a mixture model of two classes \( x \) and \( y \) we have:

\[
S(i) = K(i)\{aR(i,x)+(1-a)R(i,y)\} \tag{1}
\]

where \( K(i) \) is the incident illumination of the \( i \)-th channel, \( R(i,j) \) the reflectivity of the \( j \)-th type of surface in the \( i \)-th channel and \( a \) the ratio of the area covered by class \( x \) to the total area of the resolution element.

If a given image is representable by a model like (1), then as \( a \) varies from pixel to pixel in the image, \( S(i) \) will have a corresponding variation. Let us see if making an analysis of the variation of \( S(i) \) through the image we can decide if (1) is valid. If that is the case, this means that there are two classes or categories in competition.

Suppose that we have two pixels \( A \) and \( B \) for which (1) is valid and \( K(i) \), \( R(i,x) \) and \( R(i,y) \) have the same value for \( A \) and \( B \). Then we have:

\[
S(i,A) = K(i)\{aR(i,x)+(1-a)R(i,y)\}, \quad S(i,B) = K(i)\{aR(i,x)+(1-a)R(i,y)\} \tag{2}
\]

substracting:

\[
S(i,A)-S(i,B) = (a(A)-a(B))\{K(i)R(i,x)-K(i)R(i,y)\} \tag{3}
\]

making the same for channel \( j \):

\[
S(j,A)-S(j,B) = (a(A)-a(B))\{K(j)R(j,x)-K(j)R(j,y)\} \tag{3}
\]

and dividing:

\[
H(i,j,A,B):S(i,A)-S(i,B) = K(i)\{R(i,x)-R(i,y)\}, \quad S(j,A)-S(j,B) = K(j)\{R(j,x)-R(j,y)\} \tag{4}
\]

As we see from (4), \( H(i,j,A,B) \) does not depend on a so we can expect it to be nearly constant (if the \( K's \) and the \( R's \) have narrow distributions) for all pairs of pixels satisfying (1). For example, if we represent by (1) a lake shore of a LANDSAT image, the two classes in competence would be water and land. In table (1) we have a sequence of such pixels with their intensities in four spectral bands. We can see in table (2) some values of \( H(i,j,A,B) \) for the corresponding pixels. However, these values were selected from a larger sample of shore pixels, most of them did not have such a tendency of \( H(i,j) \) to be constant. The main reason besides random factors affecting (1) is the quantization to integer values of the intensities.
CORRELATION OF INTENSITY VARIATIONS:

I. BASIC CONCEPTS.

When we measured again a sequence of random pairs of pixels from pixel to pixel:

\[ D(i;A,B) = S(i;A) - S(i;B) \]  

It can be verified experimentally that the distribution of \( D(i) \) for random pairs of pixels is normal and for images consisting of two different classes (as a lake and land), \( D(i) \) is multimodal. It is always symmetric about its mean \( E(D(i)) = 0 \). The reason of why \( X_i \) is normal for some cases, may be found by linearity considerations: if \( S(i) \) can be represented by a linear mixture model:

\[ S(i) = \alpha(K(i,x) - K(i,y)) + K(i,y) \]  

or by

\[ S(i) = \beta(K(i)R(i)) \]  

where \( \beta \) is a random modulus for the incident intensity \( K(i) \), we have in fact the same situation. In both cases \( X(i) \) depends on a difference \( \alpha(A) - \beta(B) \) (or \( \beta(A) - \beta(B) \)) whose distribution is significantly normal due to the rapid convergence related to central limit properties.

We now deal with joint probabilities for pairs of channels:

\[ P(D(i;A,B) = n, D(j;A,B) = m) \]  

where \( A \) and \( B \) are two pixels selected at random and \( m,n \) are two integer values.

The concept of correlation of intensity variations is related to (8) and has the following meaning: If one of the relations (7) is valid for an image, the pixels will be characterized by \( \alpha \) (or \( \beta \)). In the case of the first relation (7), if \( R(i,y) < R(i,x) \) for a given channel \( i \), then as \( \alpha \) increases, \( S(i) \) tends also to increase but if \( R(i,y) > R(i,x) \) then as \( \alpha \) increases, \( S(i) \) diminishes. Then a correlation between \( D(i) \) and \( D(j) \) would indicate a level of significance for the validity of (1).

The distribution corresponding to (8) must have its principal axis oriented according to the correlation between \( D(i) \) and \( D(j) \). A fast estimate of the slope of the major principal axe in the plane \( D(i)D(j) \) is the mean value \( n(i,j) \). If it is positive, there is positive correlation between \( D(i) \) and \( D(j) \) which would correspond to a situation in which \( R(i,y) < R(i,x) \) and \( R(j,y) < R(j,x) \). Suppose now that in the distribution of (8), we reject those pairs of pixels \( A,B \) for which (5) is true. This means to divide the plane \( D(i)D(j) \) in two regions I and II (figure 1) and I is going to contain the samples not rejected.

The basic concept is the intensity variation from pixel to pixel:

\[ D(i;A,B) = S(i;A) - S(i;B) \]  

It can be verified experimentally that the distribution of \( D(i) \) for random pairs of pixels is normal to a high level of significance for some type of homogeneous images (as mountainous terrains or cities). For other cases \( D(i) \) is not.

\[ \text{Table (1)} \]

<table>
<thead>
<tr>
<th>PIXEL</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>20</td>
<td>13</td>
<td>11</td>
<td>4</td>
</tr>
<tr>
<td>B</td>
<td>23</td>
<td>16</td>
<td>19</td>
<td>9</td>
</tr>
<tr>
<td>C</td>
<td>19</td>
<td>12</td>
<td>11</td>
<td>3</td>
</tr>
<tr>
<td>D</td>
<td>22</td>
<td>15</td>
<td>15</td>
<td>6</td>
</tr>
<tr>
<td>E</td>
<td>22</td>
<td>13</td>
<td>11</td>
<td>3</td>
</tr>
<tr>
<td>F</td>
<td>19</td>
<td>13</td>
<td>12</td>
<td>5</td>
</tr>
</tbody>
</table>

\[ \text{Table (2)} \]

<table>
<thead>
<tr>
<th>PAIR</th>
<th>H(2,1)</th>
<th>H(3,2)</th>
<th>H(4,2)</th>
<th>H(4,1)</th>
<th>H(4,3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>FB</td>
<td>0.75</td>
<td>2.33</td>
<td>1.33</td>
<td>1.00</td>
<td>0.55</td>
</tr>
<tr>
<td>AB</td>
<td>1.00</td>
<td>2.66</td>
<td>1.66</td>
<td>1.66</td>
<td>0.62</td>
</tr>
<tr>
<td>BC</td>
<td>1.00</td>
<td>2.00</td>
<td>1.50</td>
<td>1.50</td>
<td>0.75</td>
</tr>
<tr>
<td>DE</td>
<td>-</td>
<td>2.00</td>
<td>1.50</td>
<td>-</td>
<td>0.75</td>
</tr>
</tbody>
</table>

The concept of correlation of intensity variations is related to (8) and has the following meaning: If one of the relations (7) is valid for an image, the pixels will be characterized by \( \alpha \) (or \( \beta \)). In the case of the first relation (7), if \( R(i,y) < R(i,x) \) for a given channel \( i \), then as \( \alpha \) increases, \( S(i) \) tends also to increase but if \( R(i,y) > R(i,x) \) then as \( \alpha \) increases, \( S(i) \) diminishes. Then a correlation between \( D(i) \) and \( D(j) \) would indicate a level of significance for the validity of (1).

The distribution corresponding to (8) must have its principal axis oriented according to the correlation between \( D(i) \) and \( D(j) \). A fast estimate of the slope of the major principal axe in the plane \( D(i)D(j) \) is the mean value \( n(i,j) \). If it is positive, there is positive correlation between \( D(i) \) and \( D(j) \) which would correspond to a situation in which \( R(i,y) < R(i,x) \) and \( R(j,y) < R(j,x) \). Suppose now that in the distribution of (8), we reject those pairs of pixels \( A,B \) for which (5) is true. This means to divide the plane \( D(i)D(j) \) in two regions I and II (figure 1) and I is going to contain the samples not rejected.

\[ \text{Figure (1)} \]
Figure (1) is useful to understand the experiments of reduction of variance explained after (5). For this, it is important to note that the joint distributions of K(i), D(j),..., have elliptic shapes.

From (1) each channel can give a simple estimate of a for each pixel:

$$ a(i) = \frac{S(i)-K(i,y)}{K(i,x)-K(i,y)} \quad (9) $$

An experimental program was written to fit the values of K(i), R(i,x) and R(i,y) to give estimates of the density function f(a(i)) with the same mean and variance for all channels. The correlation of the value of a(i) between several channels, may reach values like 0.98 even if the channel labels are different from 1 and 2 (regarding -5-).

The above correlations and the fact that the distributions of (K(i)) may be significantly normal, lead us to the conclusion that one of the models (7) may be correct for pixels belonging to region I of figure (1). (We say that a pixel A belongs to I if for another random pixel B -5- is not true).

However, we remark that the situation of positive correlation of intensity variations happens in all channels in several cases of homogeneous LANDSAT images. This means according to (1) that R(i,y)<R(i,x) (or vice versa) for all channels. In words: It is possible for several kind of homogeneous subimages to select pixels (those in I) whose intensity variation has the same sign and a high correlation.

The conclusion is that we have two possibilities: Either the second model of (7) is the good one (which would involve variations in the incident light conditions, for example: if the incident angle depends on the inclination of the terrain, that model is correct with b independent of the spectral channel), or one of the two classes corresponds to the shadow of the other (if R(i,y)<R(i,x) then y would be the shadow of x over x, i.e. the shadow of a tree over another tree).

The above implies that incident light conditions or shadow effects are important and if this is the case for an homogeneous image like a mountainous area or a city, some hypothesis for the estimation of a may not be simple in the combination case.

FALSE COLOR DISPLAYS

Some false color techniques may filter these effects because a given combined color is formed by a specific mixture of the basic colors and the mixture is characterized by the ratio of the basic components. This means that homogeneous areas will have uniform colors representative of different classes.

As an application we can make a fast false color classifier taking as classes each of the following sequences: We consider the four spectral intensities of a pixel of a LANDSAT image and put in the place of the band the place it has from maximum to minimum. For example 3124 means that band 2 has the largest value, then follows band 3, then band 1 and finally band 4. The sequence 3124 is considered a false color class.

We then convert each pixel to a sequence which represent the color of that pixel. Experiments show that this technique is useful and efficient to classify a geographic area or an agricultural field. The reason is that, according to the earlier results, statistically speaking, the pixels of an homogeneous area will vary the four spectral bands with the same sign. That is: if band 3 diminishes from pixel A to B, then bands 1,2 and 4 will also diminish. This maintains the same sequence or color for all the pixels of the uniform area.

As example we have the following result from an agricultural field covered mostly by wheat. A sample of 138000 pixels was divided in 'color' sequences in the following way:

<table>
<thead>
<tr>
<th>COLOR</th>
<th>NUMBER OF PIXELS</th>
</tr>
</thead>
<tbody>
<tr>
<td>2314</td>
<td>15405</td>
</tr>
<tr>
<td>2413</td>
<td>9790</td>
</tr>
<tr>
<td>3120</td>
<td>2189</td>
</tr>
<tr>
<td>3214</td>
<td>3924</td>
</tr>
<tr>
<td>3814</td>
<td>3661</td>
</tr>
<tr>
<td>3412</td>
<td>85815</td>
</tr>
<tr>
<td>3413</td>
<td>3522</td>
</tr>
</tbody>
</table>

The sequence 3412 was found characteristic of wheat as the displays indicated. We see then the high percentage of such sequence. This also gives us a physical basis to interpret the information content of an image.

IV.- BIBLIOGRAPHY

