6-13-2017

Weight Minimization of Noise Treatments by Balancing Absorption and Transmission Performance

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Automotive Noise

Weight minimization of noise treatments by balancing absorption and transmission performance

Hyunjun Shin and J. Stuart Bolton

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Purdue University
OBJECTIVE

• Minimize the weight of a sound package by balancing between absorption and transmission performance

• Trade-off between transmission and absorption
ENGINE NOISE SPECTRUM

Optimizing the weight of the sound package in the range of 500 Hz to 4000 Hz → allows passengers to communicate without interference

- Engine noise is mainly distributed in the range of 1000-3000 Hz → overlaps with a human speech range

Noise spectrum of automobile engine (A)

Noise spectrum of automobile engine (B)

Engine running at 5000 RPM

Engine running at various RPMs

References:
**HOW TO OBTAIN THE SOLUTIONS**

### Modeling

**Noise Treatment**
- Aluminum Panel (1 mm)
- Limp Porous (30 mm)
- Flexible MPP (0.2 mm)

### Calculation

**Transfer Matrix Method**

\[
\begin{bmatrix} P_1 \\ U_1 \end{bmatrix} = \begin{bmatrix} \text{Aluminum} & \text{Limp} & \text{Flexible} & \text{Air} \end{bmatrix} \begin{bmatrix} P_t \\ U_t \end{bmatrix}
\]

**Space-averaged Pressure Magnitude**

\[
|\text{SAP}| = \sum_{i=1}^{n} \frac{\sqrt{\text{Re}(P_n)^2 + \text{Im}(P_n)^2}}{n}
\]

### Optimization

**Genetic Algorithm function**

Solutions yielding the same SAP magnitude

**Pool of Solutions**

PICK THE LOWEST SURFACE DENSITY COMBINATION
• **Termination Impedance**: 11.25 \( \rho c \) (absorption coefficient of 0.3)

Sound Absorption Average

[ ASTM C423]

\[
\alpha_{\text{average}} = \frac{1}{12} \sum_{i=200 \text{ Hz}}^{i=2500 \text{ Hz}} \alpha_i
\]

Interior Space Averaged Coefficient

\[
\alpha_{\text{Interior}} = \frac{\sum_{i=1}^{i=N} \alpha_i S_i}{S_{\text{total}}}
\]

For Seats, Windows, and Roof
### MODELING OF SOUND PACKAGES

#### Sound Package layout

<table>
<thead>
<tr>
<th><strong>ALUMINUM PANEL</strong></th>
<th><strong>LIMP POROUS LAYER</strong></th>
<th><strong>FLEXIBLE MPP</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>Density: 2700 kg/m³</td>
<td>RIGID POROUS (EQUIVALENT FLUID MODEL) (JCA-model)</td>
<td>RIGID POROUS (EQUIVALENT FLUID MODEL) (JCA-model)</td>
</tr>
<tr>
<td>Thickness: 1 mm</td>
<td>LIMP POROUS</td>
<td>FLEXIBLE MPP (Parallel addition with Mass/Area)</td>
</tr>
<tr>
<td>Young’s Modulus: 69e9</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Poisson’s Ratio: 0.33</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

#### Limp Porous
- Density(solid part): 1 to 15 kg/m³
- Flow resistivity: 3300 to 50000 MKS Rayls/m

#### Flexible MPP
- Surface Density: 0.1 to 3 kg/m²
- Flow resistance: 100 to 1500 MKS Rayls

---

Adjusting the acoustic properties (Density and Flow Resistance) of both Limp Porous and Flexible MPP to minimize the weight while maintaining its acoustic performance.

---


## MATERIAL PROPERTIES

<table>
<thead>
<tr>
<th>Material</th>
<th>Surface Density (kg/m²)</th>
<th>Thickness (mm)</th>
<th>Young’s Modulus (Pa)</th>
<th>Poisson’s Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminum Plate</td>
<td>2.7</td>
<td>1</td>
<td>69x10⁹</td>
<td>0.33</td>
</tr>
<tr>
<td>Limp Porous</td>
<td></td>
<td>30</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Flexible MPP</td>
<td></td>
<td>0.2</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Material</th>
<th>Porosity</th>
<th>Thermal characteristic Length (m)</th>
<th>Viscous characteristic Length (m)</th>
<th>Tortuosity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Limp Porous</td>
<td>0.995</td>
<td>105x10⁻⁶</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Flexible MPP</td>
<td>100x10⁻⁶</td>
<td>100x10⁻⁶</td>
<td>100x10⁻⁶</td>
<td></td>
</tr>
</tbody>
</table>

The values are taken out of

FLEXIBLE MPP MODEL

Complex density of the rigid porous using JCA model

\[ \tilde{\rho}_{eq} = \frac{\alpha_\infty \rho_0}{\phi} \left[ 1 + \frac{\sigma \phi}{j \omega \alpha_\infty \rho_0} \sqrt{\left( 1 + \frac{4 j \eta \rho_0 \omega \alpha_\infty^2}{\sigma^2 \Lambda^2 \phi^2} \right)} \right] \]

Complex bulk modulus of the rigid porous

\[ \tilde{K}_{eq}(\omega) = \frac{\gamma P_0 / \phi}{\gamma - (\gamma - 1) \left[ 1 - j \frac{8k}{\Lambda_\infty^2 C_p \rho_0 \omega} \sqrt{\left( 1 + j \frac{\Lambda_\infty^2 C_p \rho_0 \omega}{16k} \right)} \right]^{-1}} \]
FLEXIBLE MPP MODEL

Characteristic impedance and Complex Wavenumber

\[ Z_{eq\_MPP} = \sqrt{\tilde{\rho}_e \tilde{K}_{eq}} \quad k_{eq\_MPP} = \sqrt{\tilde{\rho}_e / \tilde{K}_{eq}} \]

Transfer impedance of the rigid MPP

\[ Z_{t\_Rigid} = \rho_o c \left( \frac{1 + R_a}{1 - R_a} \right) - \rho_o c = \frac{2\rho_o c R_a}{1 - R_a} \]

*R_a*: Reflection coefficient of the rigid porous with an anechoic backing

Transfer impedance and Transfer matrix of a flexible MPP

- Add the impedance of an impermeable barrier in parallel

\[ Z_{t\_Flexible} = \frac{j \omega m_{MPP} * Z_{t\_Rigid}}{Z_{t\_Rigid} + j \omega m_{MPP}} \]

\[ \begin{bmatrix} 1 & Z_{t\_Flexible} \\ 0 & 1 \end{bmatrix} \]
BUILDING TRANSFER MATRICES

Incidence  
1 Pa  
[94dB]

\[
\begin{bmatrix}
P_1 \\
U_1
\end{bmatrix} = \begin{bmatrix}
N_{11} & N_{12} \\
N_{21} & N_{22}
\end{bmatrix} \begin{bmatrix}
L_{11} & L_{12} \\
L_{21} & L_{22}
\end{bmatrix} \begin{bmatrix}
P_3 \\
U_3
\end{bmatrix}
\]

Transfer matrix of Noise Treatment  
Transfer matrix of Air space

\[
\begin{bmatrix}
N_{11} & N_{11} \\
N_{21} & N_{22}
\end{bmatrix} = \begin{bmatrix}
1 & j\omega m_{alum} \\
0 & 1
\end{bmatrix} \begin{bmatrix}
\cos(k_{eq\_limp}h) & jZ_{eq\_limp} \sin(k_{eq\_limp}h)/Z_{eq\_limp} \\
-j\sin(k_{eq\_limp}h)/Z_{eq\_limp} & \cos(k_{eq\_limp}h)
\end{bmatrix} \begin{bmatrix}
1 \\
0
\end{bmatrix}
\]
BUILDING TRANSFER MATRICES

Transfer matrix of an Interior Air Space

\[
\begin{bmatrix}
L_{11} & L_{11} \\
L_{21} & L_{22}
\end{bmatrix} =
\begin{bmatrix}
\cos(k_{air}(x_3 - x_t)) & jZ_{air}\sin(k_{air}(x_3 - x_t)) \\
j\sin(k_{air}(x_3 - x_t))/Z_{air} & \cos(k_{air}(x_3 - x_t))
\end{bmatrix}
\]

Pressure and Velocity Calculation using TMM

\[
\begin{bmatrix}
P_1 \\
U_1
\end{bmatrix} =
\begin{bmatrix}
N_{11} & N_{12} \\
N_{21} & N_{22}
\end{bmatrix}
\begin{bmatrix}
P_1 \\
U_1
\end{bmatrix}
\quad \begin{bmatrix}
P_2 \\
U_2
\end{bmatrix} =
\begin{bmatrix}
L_{11} & L_{12} \\
L_{21} & L_{22}
\end{bmatrix}
\begin{bmatrix}
P_3 \\
U_3
\end{bmatrix}
\]

Therefore,

\[P_2 = (L_{11} + \frac{L_{12}}{Z_t})P_3\]

\[P_3 = \frac{2}{\left(\rho c L_{11}N_{21} + L_{11}N_{11} + L_{12}N_{21} \frac{\rho c}{Z_t} + L_{12}N_{11} \frac{1}{Z_t} + \rho c L_{21}N_{22} + L_{21}N_{12} + L_{22}N_{22} \frac{\rho c}{Z_t} + L_{12}N_{12} \frac{1}{Z_t}\right)}\]

Termination Condition

\[Z_t = \frac{P_3}{U_3}\]
Pressures at location 2 & 3

\[ P_2 = Ae^{i k x_t} + Be^{i k x_t} \]
\[ P_3 = Ae^{i k x_s} + Be^{i k x_s} \]

Since \( P_2 \) and \( P_3 \) were calculated in the previous step,

\[ A = \frac{P_3 - P_2 e^{-i k x_t}}{(e^{-i k x_s} - e^{-2i k x_t} e^{-i k x_s})} \]
\[ B = \frac{P_2(e^{-i k (x_t + x_s)} - e^{-i k (3x_t + x_s)} + e^{-2i k x_s}) - P_3 e^{-i k x_t}}{(e^{-i k x_s} - e^{-2i k x_t} e^{-i k x_s})} \]

Thus, sound pressure at any location between \( x_t \) and \( x_s \) is

\[ P = Ae^{-i k x} + Be^{i k x} \]
\[ x_t \leq x \leq x_s \]
To quantify the interior space acoustic pressure between the noise treatment and the termination, space-averaged pressure magnitude is calculated.

\[
|SAP| = \frac{\sum_{i=1}^{n} \sqrt{Re(P_n)^2 + Im(P_n)^2}}{n}
\]

\(n=1, 2, \ldots, k, \ldots, n\)

Space-averaged pressure magnitudes then averaged over frequency between 500 and 4000 Hz (resolution of 4 Hz).

Target Frequency Range is between 500 Hz – 4000 Hz. (Speech Interference Range)
**OPTIMIZATION PROCESS**

- **Equivalent fluid model**
- **Pressure in the air space calculation**
- **Set the target pressure**

Space-averaged pressure magnitude (frequency range 500 Hz to 4000 Hz)

**Flexible MPP**

\[
\sigma_{\text{low}} \leq \sigma_{\text{MPP}} \leq \sigma_{\text{up}} \\
m_{\text{low}} \leq m_{\text{MPP}} \leq m_{\text{up}}
\]

**Limp porous Layer**

\[
\sigma_{\text{low}} \leq \sigma_{\text{porous}} \leq \sigma_{\text{up}} \\
m_{\text{low}} \leq m_{\text{porous}} \leq m_{\text{up}}
\]

- **Updated** \(\sigma_{\text{MPP}}, m_{\text{MPP}}, \sigma_{\text{porous}}, m_{\text{porous}}\)

**Error Function**

\[
E = \left| 1 - \frac{\text{SAP}_{\text{Current}}}{\text{SAP}_{\text{Target}}} \right|
\]

- **Optimization starts with randomly selected surface densities, and flow resistances for both limp porous and flexible MPP**

**NOISE-CON 2017**
Randomly assigned starting points for optimization

<table>
<thead>
<tr>
<th>Material</th>
<th>Surface Density</th>
<th>Flow Resistance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Limp Porous</td>
<td>0.03 to 0.45 kg/m²</td>
<td>100 to 1500 Mks Rayls</td>
</tr>
<tr>
<td>Flexible MPP</td>
<td>0.1 to 3 kg/m²</td>
<td>100 to 1500 Mks Rayls</td>
</tr>
</tbody>
</table>
Target pressure close to 0.02 Pa [60 dB] case

All combinations which yields averaged pressure of 0.02 Pa

Properties of Lightest and Heaviest materials

<table>
<thead>
<tr>
<th>Material</th>
<th>Flexible MPP</th>
<th>Limp Porous Layer</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lightest Combination</td>
<td>Flow Resistance</td>
<td>Flow Resistance</td>
<td></td>
</tr>
<tr>
<td></td>
<td>[MKS Rayls]</td>
<td>[MKS Rayls]</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1193.7</td>
<td>617.1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Surface Density</td>
<td>Surface Density</td>
<td></td>
</tr>
<tr>
<td></td>
<td>[kg/m²]</td>
<td>[kg/m²]</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.475</td>
<td>0.033</td>
<td>0.508 [kg/m²]</td>
</tr>
<tr>
<td>Heaviest Combination</td>
<td>Flow Resistance</td>
<td>Flow Resistance</td>
<td></td>
</tr>
<tr>
<td></td>
<td>[MKS Rayls]</td>
<td>[MKS Rayls]</td>
<td></td>
</tr>
<tr>
<td></td>
<td>953.2</td>
<td>263.5</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Surface Density</td>
<td>Surface Density</td>
<td></td>
</tr>
<tr>
<td></td>
<td>[kg/m²]</td>
<td>[kg/m²]</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2.796</td>
<td>0.172</td>
<td>2.968 [kg/m²]</td>
</tr>
</tbody>
</table>

Target pressure close to 0.02 Pa
Surface densities under 0.8 kg/m² (Termination Impedance: 11.25ρc)

<table>
<thead>
<tr>
<th>MPP</th>
<th>Flow Resistance (MKS Rayls)</th>
<th>Surface Density (kg/m²)</th>
<th>Limp Porous</th>
<th>Flow Resistance (MKS Rayls)</th>
<th>Surface Density (kg/m²)</th>
<th>Total</th>
<th>Surface Density (kg/m²)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>1193.7</td>
<td>0.475</td>
<td>617.11</td>
<td></td>
<td>0.033</td>
<td>0.508</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1275.6</td>
<td>0.602</td>
<td>135.74</td>
<td></td>
<td>0.068</td>
<td>0.670</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1299.9</td>
<td>0.617</td>
<td>100.30</td>
<td></td>
<td>0.081</td>
<td>0.698</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1445.6</td>
<td>0.435</td>
<td>1464.7</td>
<td></td>
<td>0.277</td>
<td>0.712</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1403.5</td>
<td>0.425</td>
<td>1422.6</td>
<td></td>
<td>0.297</td>
<td>0.722</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1253.5</td>
<td>0.438</td>
<td>1299.5</td>
<td></td>
<td>0.316</td>
<td>0.754</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1421.9</td>
<td>0.558</td>
<td>170.56</td>
<td></td>
<td>0.215</td>
<td>0.773</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1461.3</td>
<td>0.528</td>
<td>254.30</td>
<td></td>
<td>0.246</td>
<td>0.774</td>
<td></td>
</tr>
</tbody>
</table>

All combinations shown above yielded averaged pressure magnitude close to 0.02 Pa
As the space average pressure decreases, “i.e., performance” increases, the range of possible surface densities reduction decreases.
ACOUSTIC PERFORMANCE COMPARISONS

Tradeoff between absorption and transmission performance to reach the certain level of averaged pressure in the interior space.
EFFECTS OF THE TERMINATION IMPEDANCE

Termination impedance is doubled to see its effects

\[ 22.50 \rho c \quad (\alpha = 0.16) \]

Acoustic performances of the noise treatment which yields the average pressure of 0.03 Pa

Absorption Coefficient

Transmission Loss
CONCLUSIONS

- Noise treatment was successfully modeled using JCA equivalent fluid model
- Space-averaged pressure in the interior space was calculated using TMM
- Surface density of a noise treatment was minimized for various averaged pressures in the interior space while maintaining its acoustic performances
- It is verified that there is a tradeoff between absorption and transmission performance of the noise treatment
- In this research, the procedure for the optimization of the weight of the sound package was demonstrated.
- In the future, more complicated acoustic model and the sound package will be studied
REFERENCES

THANK YOU
APPENDIX A. LIMP POROUS MODEL

Complex density of the limp porous

Complex density of the rigid porous was calculated using JCA model

\[
\tilde{\rho}_{eq}^{\text{limp}} = \frac{A\tilde{\rho}_{eq}^{\text{rigid}} + B}{\tilde{\rho}_{eq}^{\text{rigid}}} + C
\]

\[
A = \rho_{\text{mat}} \ , \ B = -\rho_o^2 \ , \ C = \rho_{\text{mat}} - 2\rho_o
\]

\[
\rho_{\text{mat}} = (1 - \phi)\rho_s + \phi\rho_o.
\]

Complex bulk modulus of the limp porous

\[
\tilde{K}_{eq}(\omega) = \frac{\gamma P_0/\phi}{\gamma - (\gamma - 1) \left[ 1 - j \frac{8k}{\Lambda_l^2 C_p \rho_0 \omega} \sqrt{\left( 1 + j \frac{\Lambda_l^2 C_p \rho_0 \omega}{16k} \right)} \right]^{-1}}
\]
APPENDIX A. LIMP POROUS MODEL

Characteristic impedance of the rigid MPP

\[ Z_{eq \_limp} = \sqrt{\tilde{\rho}_{eq} \tilde{K}_{eq}} \]

Complex wavenumber of the rigid MPP

\[ k_{eq \_limp} = \sqrt{\tilde{\rho}_{eq} / \tilde{K}_{eq}} \]

Transfer matrix for a limp porous

\[
\begin{bmatrix}
\cos(k_{eq \_limp}h) & jZ_{eq \_limp} \sin(k_{eq \_limp}h) \\
\frac{j\sin(k_{eq \_limp}h)}{Z_{eq \_limp}} & \cos(k_{eq \_limp}h)
\end{bmatrix}
\]
APPENDIX C. FLEXIBLE MPP MODEL

\[ \alpha_\infty = 1 + 2 \frac{\varepsilon}{h} \]

\[ \xi = 2 \sqrt{\frac{\phi}{\pi}} \]

\[ \varepsilon = (1 - 1.13\xi - 0.09\xi^2 + 0.27\xi^3) \frac{8r}{3\pi} \]

\[ \sigma = \frac{8\eta}{\phi r^2} \]

\( \alpha_\infty \): Tortuosity

\( \varepsilon \): Corrected length

\( \eta \): Kinetic viscosity

\( \phi \): Open porosity

\( \rho_o \): Ambient air density

\( k \): Thermal conductivity

\( \sigma \): Static air flow resistivity

\( \gamma \): Ratio of specific heats

\( C_p \): Specific heat at constant pressure

\( \Lambda \): Viscous characteristic length

\( \Lambda' \): Thermal characteristic length
APPENDIX B. CAR INTERIOR SPACE ABSORPTION

Ceiling reflects the noise ($\alpha \approx 0.25$)

Dash panel blocks the engine noise and absorb the interior noise

Carpets absorb noise in mid-frequency range

Windows reflect the noise ($\alpha \approx 0.1$)

Leather Seats absorb the noise ($\alpha \approx 0.5$)
APPENDIX C. EFFECTS OF THE TERMINATION IMPEDANCE

Material properties for two different termination impedances both of which yielded the average pressure of 0.03Pa in the interior space.

<table>
<thead>
<tr>
<th>Termination Impedance</th>
<th>Flexible MPP</th>
<th>Limp Porous Layer</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>11.25*ρc</td>
<td>Flow Resistance [Mks Rayls] 935.2</td>
<td>Flow Resistance [Mks Rayls] 1079.9</td>
<td>Surface Density [kg/m²] 0.160</td>
</tr>
<tr>
<td></td>
<td>Surface Density [kg/m²] 0.160</td>
<td>Surface Density [kg/m²] 0.073</td>
<td>Surface Density [kg/m²] 0.233</td>
</tr>
<tr>
<td>22.50*ρc</td>
<td>Flow Resistance [Mks Rayls] 628.6</td>
<td>Flow Resistance [Mks Rayls] 287.4</td>
<td>Surface Density [kg/m²] 0.105</td>
</tr>
<tr>
<td></td>
<td>Surface Density [kg/m²] 0.105</td>
<td>Surface Density [kg/m²] 0.030</td>
<td>Surface Density [kg/m²] 0.135</td>
</tr>
</tbody>
</table>

Surface density for higher termination impedance case came out to be lower than that of lower termination impedance case.