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THE MODELING OF ANISOTROPIC FUSELAGE LINING MATERIAL

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 INTRODUCTION

- Objective:
  - development of a theoretical model that can account for the effect of lining anisotropy on sound transmission through fuselage structures

- Present Work:
  - measurements of physical properties of polyimide foam samples
  - measurements of random incidence transmission loss
  - numerical simulations of random incidence transmission loss with isotropic and anisotropic models
Physical Properties to be Measured

- Bulk Density
- Flow Resistivity*
- Bulk Modulus* & Loss Factor
- Bulk Shear Modulus*
- Porosity
- Tortuosity*

* anisotropic in polyimide foam
HORIZONTALLY CUT vs. VERTICALLY CUT SAMPLES

foam rise direction

150 cm

75 cm

horizontally cut sample

vertically cut sample
Flow Resistivity Measurement

Horizontally cut sample
$5.81 \times 10^4$ mks Rayls/m

Vertically cut sample
$9.64 \times 10^4$ mks Rayls/m
Bulk Modulus Measurement

Horizontally cut sample
$1.5 \times 10^5 (1+j0.33) \text{ Pa}$

Vertically cut sample
$8.7 \times 10^4 (1+j0.28) \text{ Pa}$
Bulk Shear Modulus Measurement

Horizontally cut sample
\(1.1 \times 10^5 (1+j0.06) \text{ Pa}\)

Vertically cut sample
\(6.0 \times 10^4 (1+j0.11) \text{ Pa}\)
\[ \sigma_x = (2N + A)e_x + Fe_x + Me \varepsilon \]
\[ \sigma_z = Fe_z + Ce_z + Qe \varepsilon \]
\[ \tau_{zx} = \tau_{xz} = G\gamma_{zx} \]
\[ s = Me_x + Qe_z + Re \varepsilon \]
EQUATIONS OF MOTION

\[
\begin{align*}
\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xz}}{\partial z} &= \rho_1 \frac{\partial^2 u_x}{\partial t^2} + \rho_2 (q_1^2 - 1) \frac{\partial^2 (u_x - U_x)}{\partial t^2} \\
&\quad + b_3 \frac{\partial}{\partial t} (u_x - U_x) \\
\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \sigma_z}{\partial z} &= \rho_1 \frac{\partial^2 u_z}{\partial t^2} + \rho_2 (q_2^2 - 1) \frac{\partial^2 (u_z - U_z)}{\partial t^2} \\
&\quad + b_2 \frac{\partial}{\partial t} (u_z - U_z)
\end{align*}
\]

\[
\begin{align*}
\frac{\partial \sigma_z}{\partial x} + \frac{\partial \tau_{zx}}{\partial z} &= \rho_1 \frac{\partial^2 U_x}{\partial t^2} + \rho_2 (q_1^2 - 1) \frac{\partial^2 (U_x - U_x)}{\partial t^2} \\
&\quad + b_3 \frac{\partial}{\partial t} (u_x - U_x) \\
\frac{\partial \tau_{zx}}{\partial x} + \frac{\partial \sigma_x}{\partial z} &= \rho_2 \frac{\partial^2 U_x^2}{\partial t^2} + \rho_2 (q_2^2 - 1) \frac{\partial^2 (U_z - U_z)}{\partial t^2} \\
&\quad + b_2 \frac{\partial}{\partial t} (U_z - u_z)
\end{align*}
\]
System of Governing Differential Equations in 2-Dim.

From Dynamic Relations and Stress-Strain Relations,

**Solid Phase:**

\[
\left[ N \frac{\partial^2 u_x}{\partial x^2} + G \frac{\partial^2 u_x}{\partial z^2} \right] + \frac{\partial}{\partial x} \left[ N \frac{\partial u_x}{\partial x} + (G + F - A) \frac{\partial u_x}{\partial z} \right] + \frac{\partial}{\partial x} (Ae_s + M\varepsilon) = -\omega^2 (\rho_{111}^* u_x + \rho_{121}^* U_x)
\]

**Fluid Phase:**

\[
\frac{\partial}{\partial x} (Me_s + R\varepsilon) + (Q - M) \frac{\partial^2 u_x}{\partial z^2} = -\omega^2 (\rho_{121}^* u_x + \rho_{221}^* U_x)
\]

where \( u_x, u_z, U_x, U_z \) are solid and fluid displacements

\( e_s = \frac{\partial u_x}{\partial x} + \frac{\partial u_z}{\partial z} \) is volumetric strain of solid phase

\( \varepsilon = \frac{\partial U_x}{\partial x} + \frac{\partial U_z}{\partial z} \) is volumetric strain of fluid phase

\( A, C, F, G, M, N, Q \) are elastic coefficients
STEP 1 Substitute assumed solutions for displacement fields into the system of governing equations.

Incident sound field: $\Phi_i = e^{-j(k_0 x + k_0 z)}$

Assumed solutions:
- $u_x = a_i e^{-j(k_0 x + k_0 z)}$
- $u_z = b_i e^{-j(k_0 x + k_0 z)}$
- $U_x = c_i e^{-j(k_0 x + k_0 z)}$
- $U_z = d_i e^{-j(k_0 x + k_0 z)}$
STEP 2  Solve the characteristic equations for wavenumbers of three types of waves within polyimide foam layer

\[
\begin{bmatrix}
\lambda_{11} & \lambda_{12} & \lambda_{13} & \lambda_{14} \\
\lambda_{21} & \lambda_{22} & \lambda_{23} & \lambda_{24} \\
\lambda_{31} & \lambda_{32} & \lambda_{33} & \lambda_{34} \\
\lambda_{41} & \lambda_{42} & \lambda_{43} & \lambda_{44}
\end{bmatrix} \begin{bmatrix}
a_i \\
b_i \\
c_i \\
d_i
\end{bmatrix} = 0
\]

\[A_1 k_{iz}^6 + A_2 k_{iz}^4 + A_3 k_{iz}^2 + A_4 = 0\]
Rewrite assumed solutions for displacement fields in terms of 6 unknown coefficients using calculated wavenumbers

- Solid Phase Displacements:
  
  \[ u_x = e^{-j k_x x} \left( \sum_{i=1}^{4} \alpha_i C_i e^{-j k_{i_x} z} + \sum_{i=5}^{6} C_i e^{-j k_{i_z} z} \right) \]

  \[ u_z = e^{-j k_x x} \left( \sum_{i=1}^{4} C_i e^{-j k_{i_z} z} + \sum_{i=5}^{6} \alpha_i C_i e^{-j k_{i_z} z} \right) \]

- Fluid Phase Displacements:

  \[ U_x = e^{-j k_x x} \sum_{i=1}^{6} \beta_i C_i e^{-j k_{i_z} z} \]

  \[ U_z = e^{-j k_x x} \sum_{i=1}^{6} \gamma_i C_i e^{-j k_{i_z} z} \]
STEP 4. Express the solid and fluid stresses of the foam in terms of displacement field solutions

- **Solid Phase Stress:**

\[
\sigma_z = - je^{-j k_z x} \left[ \sum_{i=1}^{4} \{ k_x (F \alpha_i + Q \beta_i) + k_{iz} (C + Q \gamma_i) \} C_i e^{-j k_{iz} z} \\
+ \sum_{i=5}^{6} \{ k_x (F + Q \beta_i) + k_{iz} (C \alpha_i + Q \gamma_i) \} C_i e^{-j k_{iz} z} \right]
\]

\[
\tau_{xz} = - je^{-j k_z x} G \left[ \sum_{i=1}^{4} (k_x + k_{iz} \alpha_i) C_i e^{-j k_{iz} z} + \sum_{i=5}^{6} (k_x \alpha_i + k_{iz}) C_i e^{-j k_{iz} z} \right]
\]

- **Fluid Phase Stress:**

\[
s = - je^{-j k_z x} \left[ \sum_{i=1}^{4} \{ k_x (M \alpha_i + R \beta_i) + k_{iz} (Q + R \gamma_i) \} C_i e^{-j k_{iz} z} \\
+ \sum_{i=5}^{6} \{ k_x (M + R \beta_i) + k_{iz} (Q \alpha_i + R \gamma_i) \} C_i e^{-j k_{iz} z} \right]
\]
SOLUTION PROCEDURE

STEP 5  
Apply the appropriate boundary conditions and solve for reflection and/or transmission coefficients along with 6 unknown coefficients.

[Diagram showing sound panel and regions with symbols for incident sound, reflection, and transmission coefficients.]
BOUNDARY CONDITIONS

PANEL

(i) \( v_{1z} = j\omega W_t \)
(ii) \( v_{2z} = j\omega W_t \)
(iii) \( R_1 - P_2 = (DK_z^2 - \omega^2 m_z)W_t \)

OPEN SURFACE

(i) \( -hP = s \)
(ii) \( -(1-h) = \sigma_z \)
(iii) \( v_z = j\omega (1-h)u_z + j\omega hU_z \)
(iv) \( \tau_{xz} = 0 \)
BOUNDARY CONDITIONS

SEALED SURFACE

(i) \( v_z = j\omega W_t \)
(ii) \( u_z = W_t \)
(iii) \( U_z = W_t \)
(iv) \( u_x = W_p (-/+ \) \( \frac{h_p}{2} \frac{dW_t}{dx} \)
(v) \( (+/-) \tau_{xx} = (D_p k_x^2 - \omega^2 m_x) W_p \)
(vi) \( (+/-) P (-/+ \) \( q_p - jk_x \frac{h_p}{2} \tau_{xx} \)
\[ = (Dk_x^4 - \omega^2 m_x) W_t \]
Sound Transmission Loss Measurement

Reverberation Room 255 m³

Foam Lined Panel Structure

B & K 4166 Pressure Microphone

Speaker #1

Speaker #2

B & K 2131

Power Amplifier

Bandpass Filters

Source #1

Source #2

B & K 2032

B & K 3520 Probe

Semi-anechoic Enclosure

aluminum panels

foam layer

Bonded-Bonded

aluminum panels

foam layer

Unbonded-Unbonded
Transmission Loss for Horizontally Cut Layer

**Unbonded-Unbonded**

- measured
- isotropic theory
- anisotropic theory

![Graph showing transmission loss vs. frequency](image.png)
### Effect of Parameter Change on the Transmission Loss

### Anisotropy in Flow Resistivity

<table>
<thead>
<tr>
<th></th>
<th>$\text{Res}_x$</th>
<th>$\text{Res}_z$</th>
</tr>
</thead>
<tbody>
<tr>
<td>isotropic</td>
<td>$5.81 \times 10^4$</td>
<td>$5.81 \times 10^4$</td>
</tr>
<tr>
<td>anisotropic (original)</td>
<td>$9.64 \times 10^4$</td>
<td>$5.81 \times 10^4$</td>
</tr>
<tr>
<td>anisotropic (case 1)</td>
<td>$9.64 \times 10^3$</td>
<td>$5.81 \times 10^4$</td>
</tr>
</tbody>
</table>

![Graph showing transmission loss vs. frequency](image)
Effect of Parameter Change on the Transmission Loss

Anisotropy in Flow Resistivity

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<tr>
<th></th>
<th>Res_x</th>
<th>Res_z</th>
</tr>
</thead>
<tbody>
<tr>
<td>isotropic (case 2)</td>
<td>$5.81 \times 10^3$</td>
<td>$5.81 \times 10^3$</td>
</tr>
<tr>
<td>anisotropic (case 3)</td>
<td>$9.64 \times 10^3$</td>
<td>$5.81 \times 10^3$</td>
</tr>
<tr>
<td>anisotropic (case 4)</td>
<td>$9.64 \times 10^4$</td>
<td>$5.81 \times 10^3$</td>
</tr>
</tbody>
</table>

Transmission Loss (dB)

Frequency (Hz)
Effect of Boundary Conditions on Transmission Loss

![Graph showing transmission loss vs. frequency for Unbonded-Unbonded (anisotropic) and Bonded-Bonded (anisotropic) boundary conditions.](image)
CONCLUSIONS

- Development of a theory to model anisotropic fuse lining material.
- Anisotropic theory can give closer agreement to measurement than isotropic theory.
- In the anisotropic case:
  - Layer resonances may exist at higher frequencies.
  - The magnitude of flow resistivity normal to the layer is more important parameter than the anisotropy in flow resistivity.