REAL-TIME RESERVOIR SYSTEM OPERATION

by

Mark H. Houck

1983

PURDUE UNIVERSITY
WATER RESOURCES RESEARCH CENTER
WEST LAFAYETTE, INDIANA
Technical Report 151

REAL-TIME RESERVOIR SYSTEM OPERATION

by

Mark H. Houck

submitted to
Bureau of Reclamation
United States Department of the Interior
Washington, D.C. 20242

The work on which this report is based was supported in part by funds provided by the United States Department of the Interior as authorized under the Water Research and Development Act of 1978. This constitutes the final partial completion report for project B-125-INE.

Water Resources Research Center
Purdue University
West Lafayette, Indiana 47907
September 1983
Project No. B-125-IND
Matching Fund Agreement No. 14-34-0001-1223

Contents of this publication do not necessarily reflect the views and policies of the Office of Water Research and Technology of the Bureau of Reclamation, U.S. Department of the Interior, nor does mention of trade names or commercial products constitute their endorsement or recommendation for use by the U.S. Government.
The work at Purdue University on real-time reservoir operations has been on-going for more than a decade. The most recent work has focused on the use of optimization models to enhance the ability of the reservoir operators to make good decisions. The researchers involved in this work include: Hasan Yazicigil, Bithin Datta, Emre Can, Sharon deMonsabert, and Mohammad Karamouz. The inspiration for the work has been Gerrit H. Toebes who began the research effort many years ago and continued as project director until his death in 1981. To all of these friends, especially Professor Toebes, I offer my heartfelt thanks for an exciting five years of research work.
Several different optimization models developed by researchers at Purdue University for real-time, short term reservoir system operation are described. The models include: an optimization model constructed specifically for a multi-reservoir, multipurpose system in Kentucky; a goal programming model that requires less information than most other models yet yields equivalent or better results; a chance constrained model that explicitly considers errors in streamflow forecasts in the operating environment; and a balancing model that uses cumulative distribution functions to maintain a balance between storage, release, and any other characteristic of the operation. The description of each model and comparisons between models are brief but references to the relevant literature are provided.
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>PREFACE AND ACKNOWLEDGEMENTS</td>
<td>iii</td>
</tr>
<tr>
<td>ABSTRACT</td>
<td>iv</td>
</tr>
<tr>
<td>LIST OF FIGURES AND TABLES</td>
<td>vi</td>
</tr>
<tr>
<td>INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>What Is Reservoir Operation?</td>
<td>2</td>
</tr>
<tr>
<td>What Are The Uses Of A Reservoir?</td>
<td>2</td>
</tr>
<tr>
<td>Is Multi-Reservoir Multipurpose Operation Difficult?</td>
<td>4</td>
</tr>
<tr>
<td>What Is Real-Time Short-Term Reservoir Operation?</td>
<td>5</td>
</tr>
<tr>
<td>What Is The Role Of The Operating Models?</td>
<td>6</td>
</tr>
<tr>
<td>GREEN RIVER BASIN - OPERATIONS OPTIMIZATION</td>
<td>7</td>
</tr>
<tr>
<td>What Are The Current Rules Of Reservoir Operation?</td>
<td>7</td>
</tr>
<tr>
<td>What Is The Objective Of Operation?</td>
<td>13</td>
</tr>
<tr>
<td>What Is The GRBOOM?</td>
<td>14</td>
</tr>
<tr>
<td>How Is The GRBOOM Used In Real-Time Operations?</td>
<td>18</td>
</tr>
<tr>
<td>How Is The GRBOOM Used To Improve Operating Guidelines?</td>
<td>19</td>
</tr>
<tr>
<td>What Is The Worth Of Information In Reservoir Operation?</td>
<td>21</td>
</tr>
<tr>
<td>REAL-TIME OPERATION BY GOAL PROGRAMMING</td>
<td>24</td>
</tr>
<tr>
<td>What Is A Goal Program?</td>
<td>24</td>
</tr>
<tr>
<td>What Are The Goals Of Real-Time Reservoir Operations?</td>
<td>25</td>
</tr>
<tr>
<td>How Does The Goal Program Perform?</td>
<td>28</td>
</tr>
<tr>
<td>REAL-TIME OPERATION BY CHANCE CONSTRAINED PROGRAMMING</td>
<td>32</td>
</tr>
<tr>
<td>What Is Chance Constrained Programming?</td>
<td>32</td>
</tr>
<tr>
<td>How Are Chance Constraints Applied To Real-Time Operations?</td>
<td>33</td>
</tr>
<tr>
<td>How Well Do Chance Constrained Models Work?</td>
<td>35</td>
</tr>
<tr>
<td>BALANCING REAL-TIME RESERVOIR OPERATIONS</td>
<td>36</td>
</tr>
<tr>
<td>What Is The Balancing Model?</td>
<td>36</td>
</tr>
<tr>
<td>How Well Does The Balancing Model Work?</td>
<td>40</td>
</tr>
<tr>
<td>SUMMARY AND RECOMMENDATIONS FOR FUTURE WORK</td>
<td>41</td>
</tr>
<tr>
<td>REFERENCES</td>
<td>42</td>
</tr>
</tbody>
</table>
LIST OF FIGURES AND TABLES

Figure 1. Typical Rule Curve
Figure 2. Reservoir, Tributary, Control Station Example
Figure 3. Effect of Release Schedules On Reservoir Elevation
Figure 4. Typical Penalty Functions
Figure 5. Hypothetical Simulated Operation
Figure 6. Achievable Storage And Flow Penalty Totals In A Season Of Simulated Operation With Different Forecast Horizons (T)
Figure 7. Typical Penalty Function
Table 1. Comparison Of Goal Programming (GP) And GRBOOM Operations
Table 2. Possible One Day Operating Strategies
INTRODUCTION

Over the past three years, considerable headway has been made in the study of real-time reservoir operations. A significant portion of this work has been done at Purdue University by numerous researchers and the results have been or will be published in scientific and engineering journals. But no summary of this work is available for the reader who is currently unacquainted with these latest developments. This report is an attempt to fill this gap.

There are six chapters in this report. The remainder of this chapter is a primer defining real-time, short-term reservoir operation. (Those readers already familiar with real-time reservoir operations should skip these sections and go directly to chapter two.) The second chapter contains a description of one of the first attempts at constructing an optimization model for an existing reservoir system. The model is a linear program and was constructed to enhance the current operation of the four multipurpose reservoirs in the Green River Basin of Kentucky. The third through fifth chapters contain descriptions of different modelling approaches that have been developed. Goal Programming applied to real-time, short-term reservoir operations is described in chapter three. Chance-constrained programming is described in chapter four. And a very effective balancing procedure using cumulative distribution functions of operating performance is described in chapter five. Lastly, in chapter six, a summary of the report and suggestions for future work are given.
What Is Reservoir Operation?

A reservoir is a storage facility for water. The simplest analogy is a bucket into which flows a stream. There are two ways for water to leave the bucket: it may spill over the top if the bucket fills to the brim and it may exit through a faucet located at the bottom of the bucket. Thus, the inflows to the bucket are stochastic and not controllable unless, of course, there is another dam and reservoir upstream. The outflows from the bucket are partially controllable as long as water isn't spilling over the top. The principal decisions in operating the reservoir or bucket are when to open the faucet and how much to open it.

Of course the bucket analogy disregards some important points. For example, reservoirs can lose significant quantities of water through seepage into the groundwater system or under the dam and through evaporation. And operators of a reservoir system usually have several ways to release controlled amounts of water, instead of one faucet. Nevertheless, the reservoir operations problem is how best to use the facilities that are available, given a stochastic inflow and a limited ability to control it.

What Are The Uses Of A Reservoir?

The major uses of reservoirs may be classified into three categories: flow attenuation, reservoir pool stabilization, and hydroelectric energy production. Flow attenuation involves use
of the reservoir to reduce peak flows by only releasing a portion of the inflow and storing the rest. It also involves increasing low flows by supplementing them with water already in storage.

Why is flow attenuation important? Flood damage downstream from the reservoir is reduced if peak flows can be reduced. If low flows can be supplemented on a reliable basis, then the amount of water available for downstream industrial, municipal, commercial and domestic use is increased, thereby facilitating additional development. Navigability of the stream is improved because there will be more days when the flow is sufficient to permit ship or boat traffic. Water quality may be improved due to the dilution of pollutants in more water. And instream recreation below the dam may be enhanced because of the increased low flow. This is not an exhaustive list but does indicate the importance of flow attenuation in the operation of a reservoir.

Pool stabilization is principally for two reasons. Recreation at the reservoir may be adversely affected by significant changes in the pool elevation (or reservoir storage). It is difficult to use a beach when there is fifty yards of muck between the end of the sand and the water; and it is difficult to use a picnic table that is under water. Significant, rapid changes in pool elevation may also result in severe bank sloughing, thereby increasing sediment loads into the reservoir and potentially decreasing the storage capacity. Any land owners abutting the reservoir will also suffer a loss of property.
Hydroelectric energy production is classified separately because the amount of energy produced is a function of storage volume (actually, the depth of water (head) above the turbines) multiplied by the release. Therefore, it does not fit into any category involving storage alone or flow alone.

Is Multi-Reservoir Multipurpose Operation Difficult?

Begin to answer this question by considering a single reservoir with only one purpose and an infinite capacity. If the purpose is flood control, the best operation is to keep the reservoir empty until a flood arrives. Then release only the non-damaging portion and store the rest. If the purpose is water supply, keep the reservoir filled until a drought arrives. Then the maximum amount of water will be available to supplement the very low natural flows. If the purpose is recreation at the reservoir, keep the pool elevation constant and release an amount equal to the inflow. Clearly, this simple example demonstrates the potential conflicts that can exist in a multipurpose reservoir system.

When more than one reservoir must be considered because they interact in some substantial way the operations problem becomes more complex. Some means of resolving the conflicts must be found.

Cost-benefit analysis offers some help in evaluating alternatives. But so many externalities and intangibles are involved
that economic evaluation is not enough. Multiobjective analysis is often essential.

What Is Real-Time Short-Term Reservoir Operation?

Up to here, two important issues have been ignored. The first is: in what environment are the operating decisions made? Real-time operation implies that the decisions are made in an uncertain and risky environment. Forecasts of future streamflows may be available to the decision makers but the forecasts are not one hundred percent reliable and they are only for a limited time horizon. Therefore, the operator decides how much water to release from the reservoir and then waits. Sometime later, the operator reevaluates the condition of the reservoir system and makes another decision. This process of making a decision with limited uncertain information, waiting, updating the available information, and making another decision is repeated time after time.

The second issue that has been ignored is: how often are decisions made? Short-term implies a time step of hours, days, or at most a week. In the work described in the following chapters, the time step is always taken as one day. However, the time step could be shortened to several hours or increased to several days without any severe consequences.
What Is The Role Of The Operating Models?

Before proceeding to the description of the models in the next five chapters, it is essential that their use be understood. All of the models described in this report are suggested as potential improvements to the existing methods for operating reservoir systems. This in no way implies that these models should or could replace other tools used by the operators in deciding what releases to make.

And the models are definitely not suggested as replacements for the operators. The models are based on certain assumptions. If those assumptions are satisfied in the operating environment, then the operators have the capability to explore a wider array of options by using the models. If the assumptions are not satisfied, then the judgment, intuition and experience of the operators are irreplaceable and invaluable in deciding how to operate the reservoirs.
GREEN RIVER BASIN - OPERATIONS OPTIMIZATION

The Green River Basin is located in Kentucky and contains four multipurpose reservoirs operated by the U.S. Army Corps of Engineers. For more than a decade, this reservoir system has been studied by researchers, led by Professor Gerrit H. Toebes (deceased), at Purdue University in cooperation with the Corps of Engineers. Previous investigations have identified rainfall-runoff properties of the basin and have selected appropriate river routing models. The latest work has focused on constructing an optimization model to assist in real-time, daily operation of the system.

The remainder of this chapter will summarize the work done constructing and testing the Green River Basin Operations Optimization Model (GRBOOM). Because a complete description of this work is contained in Yazicigil (1980) and Yazicigil et al. (1983), some details are omitted. The important features of the operating environment and the model are included.

What Are The Current Rules Of Reservoir Operation?

For the reservoirs in the Green River Basin (GRB) as well as for many other reservoir systems in the U.S., there exists a hierarchy of rules that regulate the operation of the system. In general, these rules can be broken into three categories: the rule curve, release schedules, and operating constraints.
The rule curve is a guide to long-term operation. It defines the desired reservoir storage or elevation level for the entire year. Figure 1 is one example of a rule curve where the reservoir elevation is maintained at one level for about one-half of the year and at another level for the other half of the year.

During actual operation, it will often be impossible or undesirable to maintain the reservoir level at the rule curve. For example, if a large rainfall results in a large inflow to the reservoir, it may be desirable to store some of the inflow to mitigate flood damage downstream. As a result, the reservoir storage will be increased and may no longer equal the rule curve storage level. To accommodate this type of event, release schedules are designed. Each of these comprises a set of rules that defines how the reservoir storage is allowed to deviate from the rule curve.

An example may be helpful. Imagine the single reservoir system shown in Figure 2. Below the dam is a tributary stream and then a control station where a streamflow gauge is located. To reduce flood damage at or near the control station, the release from the dam may be restricted by these rules:

<table>
<thead>
<tr>
<th>Flow At The Control Station [m³/sec]</th>
<th>Maximum Release From The Reservoir [m³/sec]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 - 1000</td>
<td>500</td>
</tr>
<tr>
<td>1001 - 2000</td>
<td>200</td>
</tr>
<tr>
<td>2001 -</td>
<td>50</td>
</tr>
</tbody>
</table>
FIGURE 1. TYPICAL RULE CURVE
FIGURE 2. RESERVOIR, TRIBUTARY, CONTROL STATION EXAMPLE
The results of these rules are that as the uncontrolled tributary flow increases, the controlled release from the reservoir is reduced, and therefore the storage level in the reservoir may be increased.

If the storage level is above the rule curve, another release schedule may define how the storage level is to be reduced to the rule curve. For example, these rules would gradually reduce the storage level to the rule curve when the inflow to the reservoir is below 400 m$^3$/sec.

<table>
<thead>
<tr>
<th>Reservoir Elevation Above</th>
<th>Reservoir Release</th>
</tr>
</thead>
<tbody>
<tr>
<td>The Rule Curve</td>
<td></td>
</tr>
<tr>
<td>[m]</td>
<td>[m$^3$/sec]</td>
</tr>
<tr>
<td>10 -</td>
<td>900</td>
</tr>
<tr>
<td>5 - &lt;10</td>
<td>750</td>
</tr>
<tr>
<td>0 - &lt; 5</td>
<td>600</td>
</tr>
</tbody>
</table>

Figure 3 illustrates the results of implementing the release schedules during a period when a heavy rainfall occurs. Because the length of time is so short, the rule curve is essentially flat. The storage begins on the rule curve. Due to the large streamflows in the tributary and into the reservoir, as a result of the rain, the release from the dam is restricted. The reservoir storage increases because water is being stored. At some point, the streamflow tapers off to a point where the mandated release from the reservoir exceeds the inflow, thereby reducing the storage level. Eventually, the storage level returns to the
FIGURE 3. EFFECT OF RELEASE SCHEDULES ON RESERVOIR ELEVATION
rule curve.

Operating constraints are restrictions that take precedence over other operating guidelines and rules. For example, an operating constraint may restrict the storage volume to exceed some minimum storage level. If one of the release schedules would result in the minimum storage constraint being violated, then the release schedule would not be followed. Other operating constraints may restrict releases.

For the Green River Basin, a great deal of work was done to identify all of the rule curves, release schedules, and operating constraints (Toebes and Rukvichai, 1978). Further work was done to compare historical operations to these operating rules (Toebes and Rukvichai, 1978; Yazicigil, 1980). The result was that although the combination of rule curves, release schedules, and operating constraints is fairly complete, not all operations are considered. Furthermore, in discussions with the operators, a need for a model that mimicked the decision-making process that the operators go through was identified. With such a model, the operators could investigate more alternatives during actual operation and could use the model to learn how to improve the operating guidelines.

What Is The Objective Of Operation?

Assume that there exist targets for storage level (perhaps the rule curve), release, and any other measure of reservoir
condition or operation. If the reservoir could be operated so that all of the targets are hit, it would be called ideal operation. Any deviation from ideal operation, therefore, represents non-ideal operation.

Suppose a function exists that relates the deviation of storage level from its target to an amount of penalty, measured in penalty units or dollars. A function could also relate the deviation of the release from its target to an amount of penalty. In general, any non-ideal operation could be valued in terms of the total penalty incurred. Figure 4 illustrates a typical penalty function. The shape of the functions is convex implying that as operations deviate farther from ideal operations (i.e. from their targets), the marginal penalty incurred does not decrease.

The obvious choice of an objective for selecting future operations is to minimize the total penalty incurred by these operations. For the Green River Basin, it was possible to adopt such an objective because the operators were able to identify targets and all of the necessary penalty functions.

What Is The GRBOOM?

The Green River Basin Operation Optimization Model (GRBOOM) is a linear program that minimizes the sum of all penalties incurred for non-ideal operations at all four reservoirs and the nine control stations in the basin over a T day time horizon.
FIGURE 4. TYPICAL PENALTY FUNCTIONS
Typically, the time horizon is between one and five days, which corresponds to the streamflow forecast horizon the the Corps of Engineers uses.

The constraints of the linear program include: mass balance equations (continuity), minimum and maximum storage restrictions, minimum and maximum release restrictions based on the release schedules and operating constraints, minimum and maximum rate of change of release restrictions, and river routing equations. All of these were identified in consultation with the Corps of Engineers.

The CRBOOM's formulation may be illustrated by considering the single reservoir example shown in Figure 2. Equations 1 through 7 provide the simplest version of a CRBOOM type model. Equation 1 is the objective function which is to minimize the total penalty incurred over the T day time horizon. Equation 2 is a mass balance constraint on the contents of the reservoir. Equation 3 is a linear routing model: the tributary flows and reservoir releases are routed to the control station. Equations 4 through 7 limit the range of possible storages and releases.

\[
\text{Minimize } \text{PENALTY} = \sum_{t=1}^{T} PQ(Q_t) + \sum_{t=1}^{T} PS(S_{t+1}) \quad (1)
\]

subject to:

\[
S_{t+1} + R_t - S_t = I_t \quad t = 1, 2, \ldots, T \quad (2)
\]

\[
Q_t - \sum_{j=0}^{J} \alpha_j R_{t-j} - \sum_{j=0}^{J} \beta_j TR_{t-j} = 0 \quad t = 1, 2, \ldots, T \quad (3)
\]

\[
S_{t+1} \leq \text{SMAX} \quad t = 1, 2, \ldots, T \quad (4)
\]
\[ S_{t+1} \geq S_{MIN} \quad t = 1, 2, \ldots, T \quad (5) \]
\[ R_t \leq R_{MAX} \quad t = 1, 2, \ldots, T \quad (6) \]
\[ R_t \geq R_{MIN} \quad t = 1, 2, \ldots, T \quad (7) \]

\[ I_t \quad = \text{forecasted inflow to the reservoir during day } t \ [m^3] \]
\[ \text{PENALTY} \quad = \text{total penalty incurred over the } T \text{ day time horizon} \]
\[ \text{[penalty points]} \]
\[ PQ (Q_t) \quad = \text{penalty incurred due to a flow of } Q_t \text{ [penalty points]} \]
\[ PS (S_{t+1}) \quad = \text{penalty incurred due to a storage level } S_t \text{ [penalty points]} \]
\[ Q_t \quad = \text{streamflow at the control station during day } t \ [m^3] \]
\[ R_t \quad = \text{release from the reservoir during day } t \ [m^3] \]
\[ R_{MAX} \quad = \text{maximum permissible daily release} \ [m^3] \]
\[ R_{MIN} \quad = \text{minimum permissible daily release} \ [m^3] \]
\[ S_t \quad = \text{storage at the beginning of day } t \ [m^3] \]
\[ S_{MAX} \quad = \text{maximum permissible storage} \ [m^3] \]
\[ S_{MIN} \quad = \text{minimum permissible storage} \ [m^3] \]
\[ T \quad = \text{model and forecast horizon} \ [\text{days}] \]
\[ TR_{t-j} \quad = \text{forecasted } (t-j > 0) \text{ or actual } (t-j < 0) \text{ tributary flow on day } t-j \ [m^3] \]
\[ \alpha_j \quad = \text{fraction of a day's reservoir release that arrives at the control station } j \text{ days later} \]
\[ \beta_j \quad = \text{fraction of a day's tributary flow that arrives at the control station } j \text{ days later} \]
Because the penalty functions in the objective function are all convex, it is possible to approximate them in a linear program. The other typical nonlinearity in such models is the river routing equation. Fortunately, in the Green River Basin, linear routing models (Equation 3) were found to be acceptable. And because the model is relatively small with fewer than 200 constraints and 400 variables for the complete GRBOOM, it is solved as a linear program.

How Is The GRBOOM Used In Real-Time Operations?

Before the model can be used in real-time operations, it must be calibrated. This means that all of the constraints and the objective function must be correct and more importantly that the output of the model produces suggested operations that the operators judge to be feasible, reasonable, and good.

Once the model is calibrated, to use it in the actual decision making process is relatively simple. The GRBOOM is an interactive model so that it is easy to change any portion of the linear program, if the operators decide to do so. Furthermore, it is easy to input the required data for running the model, including current storages, past streamflows that are needed for the river routing equations, and forecasted streamflows. Because the model is small, it takes only seconds to get a solution. If the operators want to explore the effect of changes in the input data (for example, what happens if the forecasted streamflows are
ten percent lower?) it is only a matter of entering the new data and resolving the model.

How Is The GBROOM Used To Improve Operating Guidelines?

It is possible to simulate the operation of the Green River Basin reservoirs over a long period of time by solving the GBROOM time after time while updating the forecast and past input data. If this is done, then changes in the river routing equations or the penalty functions or another part of the model can be evaluated. If changes are made systematically, then it is possible to identify those changes that will result in an improved operation.

One hypothetical example of how better guidelines may be found is illustrated in Figure 5. Under one set of penalty functions, the simulated operation of a single reservoir system like the one shown in Figure 2 may be plotted as dashed lines. Under a different set of penalty functions, the simulated operation is plotted as dotted lines. If the target range for releases and the target storage (rule curve) are correct, then clearly the dotted lines are preferable to the dashed lines. Therefore, the penalty functions used to produce the dotted lines must be preferable. It may seem improbable that the best penalty functions could not be selected apriori. However, experience with this type of model indicates that that is not so (Yazicigil et al., 1983).
FIGURE 5. HYPOTHETICAL SIMULATED OPERATION
What Is The Worth Of Information In Reservoir Operation?

Once the CRBOOM type model is calibrated and then recalibrated as described in the previous section, it is possible to estimate the worth of good forecast data in reservoir operations. One example of this is illustrated in Figure 6. By parametrically adjusting the importance of the storage and flow penalties in the objective function, it is possible to define the relationship between these two types of penalty. Each of the curves in Figure 6 represents this relationship for a particular model or forecast time horizon.

Notice that the curve for a three day forecast horizon is far to the left and down from the curve for the one day forecast horizon. The five day curve is to the left of the three day curve but not as much as the three day curve is away from the one day curve. Because ideal operations would result in a point on Figure 6 at the origin—no flow penalty and no storage penalty—it is possible to evaluate the contribution that additional days of forecast data have on operation.

Clearly, in this example, three days of forecast information is much better than only one day of forecast information. Five days however is not that much better than three days. And if seven days of forecast information were available, it may not result in operations any better than the operations based on five days of data. In fact, if the reliability of the forecast information deteriorates with time into the future, it is possible
FIGURE 6. ACHIEVABLE STORAGE AND FLOW PENALTY TOTALS INCURRED IN A SEASON OF SIMULATED OPERATION WITH DIFFERENT FORECAST HORIZONS (T)
that a longer forecast horizon may result in worse operations.
REAL-TIME OPERATION BY GOAL PROGRAMMING

The GRBOOM described in the previous chapter requires a set of penalty functions that relate deviations from ideal operations to an amount of penalty. Obtaining these penalty functions may be very difficult. An alternative approach to modelling the real-time reservoir operations decision making process that does not require the penalty functions is goal programming. Can (1982), Can et al. (1982), and Can and Houck (forthcoming) provide a detailed description of the application of goal programming to real-time reservoir operations. A summary of this work is provided below.

What Is A Goal Program?

There are several types of goal program; only one type is described here. This goal program includes a set of constraints like those in the example of chapter two, Equations 2 through 7. The objective is expressed differently from the form of Equation 1, however. Instead of a single equation that defines the objective, a hierarchy of goals is specified. The goals are ordered by priority with the highest priority goal first and the least important goal last.

The goal program is solved by considering the goals one at a time in order. First, a solution which maximizes attainment of goal 1 is found. A constraint which ensures that goal 1 will always be satisfied to a level equal to the one just found is
added to the constraint set. Goal 2 is now considered. A solution is found that maximizes the attainment of goal 2. Another constraint which ensures that goal 2 will always be satisfied to the level just found is now added to the constraint set. This process continues until all of the goals have been considered.

This procedure is only appropriate if goal 1 is much more important than goal 2 and goal 2 is much more important than goal 3 and so on. It does not permit a reduction in the attainment of a higher priority goal in order to improve the attainment of a lower priority goal. For modelling real-time reservoir operations, goal programming may be appropriate.

What Are The Goals Of Real-Time Reservoir Operations?

Figure 7 is an example of the form of penalty function used in the GRBOOM. Experience with the Corps of Engineers selecting these penalty functions for the Green River Basin indicates that the points a, b, TAR, c, d, and e are relatively easily chosen. They correspond to physical performance levels such as:
FIGURE 7. TYPICAL PENALTY FUNCTION
<table>
<thead>
<tr>
<th>Point</th>
<th>Storage</th>
<th>Flow</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>beaches unusable</td>
<td>fish just surviving</td>
</tr>
<tr>
<td>b</td>
<td>boat ramps unusable</td>
<td>no rafting possible</td>
</tr>
<tr>
<td>TAR</td>
<td>rule curve</td>
<td>flow target</td>
</tr>
<tr>
<td>c</td>
<td>picnic tables underwater</td>
<td>minor flooding</td>
</tr>
<tr>
<td>d</td>
<td>spillway elevation</td>
<td>major flooding</td>
</tr>
<tr>
<td>e</td>
<td>top of dam</td>
<td>disaster</td>
</tr>
</tbody>
</table>

The selection of the penalty values associated with the various storage and flow levels is more difficult. However, it is easier to list the priority of operating in different ranges. For example, operating in the range a - b may be the worst of all ranges. Operating in the d - e range may be next to worst. Operating in the range c - d may be next. One way to summarize these observations is with a hierarchy of goals such as:

1. goal 1. operate in the range a - e
2. goal 2. operate in the range b - e
3. goal 3. operate in the range b - d
4. goal 4. operate in the range b - c
5. goal 5. operate at the target TAR

The first goal may be interpreted as: find a feasible operating policy without regard to the goodness of the policy. The second goal eliminates those operations that are the most detrimental. The remaining goals limit the range of operations step by step,
with the last goal being ideal operation.

How Does The Goal Program Perform?

Specification of the hierarchy of goals requires less information than specification of all the penalty functions. Therefore, it may be expected that operations suggested by the goal programming model could not be better than those suggested by a GRBOOM type model that explicitly uses the penalty functions. This is not true.

If either the GRBOOM type model or the goal programming model is used to simulate operation of a reservoir system over several months, the simulated operation is the result of the solution of a whole sequence of optimization models. Furthermore, only a portion of the optimal solution of each of the models is used in the simulation: only the suggested release for the first day is implemented; the model is updated and resolved for the next day. Therefore, even if the simulated operation is evaluated in terms of the penalty functions used explicitly in the GRBOOM type model, there is no guarantee that the GRBOOM's suggested operations will be better than the goal program's suggested operations.

An example of this initially surprising result is given in Table 1. The GRBOOM was used in a simulation of the operation of the four reservoir system in the Green River Basin. Two simulations were made. Each was for 35 days and the first had a
forecast horizon in the model of one day, the second had a three-day forecast horizon. A goal program based on the penalty functions used in the CRBOOM was also used to simulate the system's operations. The actual cumulative penalties incurred at various times during the 35 days are listed in Table 1.
<table>
<thead>
<tr>
<th>Number of Days of Operation</th>
<th>Cumulative Penalty</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>GRBOOM</td>
<td>GP</td>
<td>GRBOOM</td>
</tr>
<tr>
<td>1</td>
<td>2.13</td>
<td>2.13</td>
<td>2.13</td>
</tr>
<tr>
<td>5</td>
<td>8.18</td>
<td>8.19</td>
<td>8.17</td>
</tr>
<tr>
<td>10</td>
<td>12.26</td>
<td>12.27</td>
<td>12.26</td>
</tr>
<tr>
<td>15</td>
<td>13.34</td>
<td>13.35</td>
<td>13.34</td>
</tr>
<tr>
<td>20</td>
<td>13.35</td>
<td>13.35</td>
<td>13.34</td>
</tr>
<tr>
<td>30</td>
<td>13.81</td>
<td>13.80</td>
<td>13.86</td>
</tr>
<tr>
<td>33</td>
<td>14.85</td>
<td>14.81</td>
<td>14.75</td>
</tr>
<tr>
<td>35</td>
<td>15.28</td>
<td>15.26</td>
<td>15.23</td>
</tr>
</tbody>
</table>

Table 1. Comparison of Coal Programming (GP) and GRBOOM Operations
Although the goal program and GRBOOM penalties for a forecast horizon of one day are not significantly different, over the 35 day simulation, the goal program produces less penalty. For a three day forecast horizon, the GRBOOM operations are less penalized than the goal program operations. However, from days 31 through 33, the goal program outperforms the GRBOOM.

Thus far, no way has been found to determine when the goal program and a GRBOOM type model will yield significantly different operations. The only procedure for discovering which is the preferable approach is to test both.
REAL-TIME OPERATION BY CHANCE CONSTRAINED PROGRAMMING

It is possible to use either the goal programming approach or the linear programming approach (GRBOOM type model) described previously to consider the effects of imperfect streamflow forecasts. One brute force way to measure the effects is to solve the optimization model many times under different streamflow forecast assumptions. A much more elegant procedure that explicitly considers the uncertainties in streamflow forecasts is chance constrained programming. Datta (1981) provides a detailed discussion of this form of optimization modelling applied to real-time, short-term reservoir operation. Only a short description is included in this chapter.

What Is Chance Constrained Programming?

In problems that involve random variables, it is sometimes important to restrict the probability of occurrence of some events. For example, chance constrained programming has been used in long-term planning and operating models for reservoir systems. The random variable has been monthly or seasonal streamflow and the events that have been constrained have been storage levels, release rates, and hydroelectric power production. For example, a chance constraint on release may be

\[ \Pr [\text{release} \leq R_{\text{MAX}}] \geq 0.9 \]  

or
Because release is a function of the reservoir inflow and because inflow is a random variable, the release is a random variable. The selection of an appropriate reliability value such as 0.9 or \( \alpha \), however, is not necessarily easy.

Before a chance constraint can be included in an optimization model, it usually must be converted into a form accepted by the optimization algorithm. Typically, this conversion results in an equation that is called the deterministic equivalent (of the chance constraint) and that is in the proper form for inclusion in a linear program or some other optimization procedure.

---

How Are Chance Constraints Applied To Real-Time Operations?

One constraint and an objective function can be used to illustrate how the real-time reservoir operations problem can use chance constrained programming. The mass balance equation for reservoir storage from the beginning of the first day of operation to the beginning of the second day of operation is:

\[
S_2 = S_1 + I_1 - R_1
\]

In this equation, \( I_1 \) denotes the forecasted inflow to the reservoir during the first day.

The error in the forecast is a random variable whose distribution may be determined. Therefore, the forecast is a random
variable whose distribution may be determined. And through the mass balance equation, the storage at the beginning of the second day \(S_2\) becomes a random variable.

Chance constraints restricting the possible beginning storages for the second day may be formulated as:

\[
\Pr [S_2 > S_{\text{MIN}}] > 0.9 \tag{11}
\]

or

\[
\Pr [S_2 < S_{\text{MAX}}] > \gamma \tag{12}
\]

Appropriate objectives that correspond to these chance constraints (Equations 11 and 12) are: to maximize the value of \(S_{\text{MIN}}\), which is the storage that is exceeded by at least ninety percent of all possible actual storages at the beginning of the second day; or to maximize the value of \(\gamma\), which is the reliability of actual storage at the beginning of the second day not exceeding a specified value \(S_{\text{MAX}}\).

Other performance levels of the operation can be included in either the constraints or objective of the optimization model. For example, hydroelectric power production could be restricted to be within some designated range with high probability. The probability that the storage at the end of five days is within some specified range of the rule curve could be maximized. Or the probability of the flow at a downstream control station falling within a target range could be maximized.
How Well Do Chance Constrained Models Work?

Only limited experience testing chance constrained models for real-time, short-term reservoir operation is available. Datta (1981) describes a single reservoir system whose operation was simulated using a chance constrained model. He also argues very effectively in favor of the chance constrained approach applied to real-time reservoir operations. Only with additional testing, however, will the usefulness of the method be determined.
BALANCING REAL-TIME RESERVOIR OPERATIONS

Another approach to modelling the decision making process of real-time reservoir operation balances different parts of the operation to achieve a good overall operating policy. The balancing is done with cumulative distribution functions of different measures of reservoir operation, such as storage, release, and hydroelectric energy production. Houck (1982), deMonsabert et al. (1983), and deMonsabert and Houck (1982) discuss this approach in detail. A synopsis of the balancing procedure is presented in the remainder of this chapter.

What Is The Balancing Model?

The only difference between the balancing model and a GRBOOM type model or a goal programming model is the form of the objective function; the constraints are identical. The objective of the balancing model is to minimize the maximum probability measure associated with any of the performance criteria over the forecast horizon. An example is the easiest way to explain what this statement of the objective means.

Suppose that it is possible to forecast perfectly, for an infinite time horizon, the necessary streamflows for a GRBOOM type model. Then it would be possible to find the optimal operation for the reservoir system over a long period. It would also be possible to use the historical operations over some long period if it is believed that they represent good operations. In
either case, a long base period of operations is available.

These operations may be used to construct empirical cumulative distribution functions of storages, releases, energy production, and any other physical characteristic of the operation that is deemed important. Although the horizontal axes of the CDFs may have significantly different scales, the vertical axes all have a scale from zero to one. The CDFs can be used to convert any storage, release, etc. to associated values which are called probability measures.

Consider a single day's operation. For a given inflow, the ending storage and the day's release have a one to one relationship. Each unit of water that is released reduces the ending storage by one unit. One possible operation may be to release a large amount, resulting in a small ending storage. Another may be to release a small amount and end with a large storage. Still a third operation may be to release a moderate amount and end with a moderate storage. In all cases, the associated probability measure for ending storage and release could be found. Table 2 summarizes these data.
<table>
<thead>
<tr>
<th>Operations Strategy</th>
<th>Ending Storage</th>
<th>Probability Measure of Ending Storage</th>
<th>Probability Measure of Release</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>small</td>
<td>0.2</td>
<td>large</td>
</tr>
<tr>
<td>2</td>
<td>moderate</td>
<td>0.6</td>
<td>moderate</td>
</tr>
<tr>
<td>3</td>
<td>large</td>
<td>0.9</td>
<td>small</td>
</tr>
</tbody>
</table>

Table 2. Possible One Day Operating Strategies
Operating strategy one results in an ending storage that is so small only twenty percent of the storages in the base operations period were below it; yet eighty percent of the base period releases were below the release made in strategy one. Strategy three results in an ending storage larger than ninety percent of the base period storages and a release larger than only forty percent of the base period releases. Strategy two results in an ending storage and a release that are larger than sixty percent of the storages and releases in the base period.

One conclusion is that strategies one and three are out of balance and strategy two is in balance. Both strategies one and three have one performance level (storage or release) that is very large and another that is very small when compared to the good operations included in the base period. It is possible to reduce the extremes of these two strategies simply by reallocating water among the storage and release. At best, both release and storage will have equal probability measures like strategy two.

The objective of the balancing model is to minimize the maximum probability measure encountered over the operating or forecast horizon. This is equivalent to maximizing the minimum probability measure in this case. And it is equivalent to balancing or equalizing the probability measures encountered.
How Well Does The Balancing Model Work?

The most thorough testing of the balancing model thus far was reported by Houck (1982). In comparisons of a GRBOOM type model, other models, and the balancing model, the balancing model performed extremely well. No model was significantly better than the balancing model, yet most were significantly worse.
SUMMARY AND RECOMMENDATIONS FOR FUTURE WORK

During the past three years, a considerable amount of work has been done investigating how models—especially optimization models—may be used in real-time, short-term reservoir operation. One principal focus of this work has been on the evolving modelling of the Green River Basin multi-reservoir, multipurpose system. In chapter two, a model specifically designed and tested for the Green River Basin is described. In subsequent chapters, other methods of modelling the decision making process are summarized. These methods include goal programming, chance constrained programming, and a balancing model.

The conclusions that can be drawn from these summaries of different modelling approaches are that each appears to have special characteristics that are very useful in real-time operations, and yet not enough testing and comparing have been done to provide guidelines for when to use each of the models. The obvious next step in the investigation of the models, therefore, is to perform further testing so that guidelines for the use of the models can be established.
REFERENCES


