Acoustic Radiation Modes of a Tire on a Reflecting Surface

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LMS User Conference 2005
Troy, Michigan, USA
March 15-16, 2005
Radiation Mode Approach

Radiation Mode Approach is used in
- Active noise control
- Source reconstruction
- Acoustic design optimization

Objectives
- To implement a radiation mode approach to tire noise radiation by using SYSNOISE ATV function
- To identify tire noise generation mechanism

Problem Definition
- Acoustic radiation modes of a rectangular plate
- 3-dimensional radiation field characteristics resulting from the tire and ground geometry
Acoustic Radiation Modes

- defined on the surface of the structure.
- shows the orthogonal characteristics of the radiation field surrounding the structure.
- acoustic radiation mode identifies the effective radiation region on the structure surface.
- dependent on geometry of the structure, not on surface velocity distribution of the structure.
Acoustic Transfer Vector (ATV)

- provides base for calculation of the radiation field characteristics.
- relationship between surface normal velocities ($\mathbf{v}_b$) and radiated sound pressure ($p_r$) in frequency domain.
- dependent on geometry of vibrating surface, field point location and physical properties of acoustic medium.

\[ p_r = \mathbf{V}^T_{ATV} \mathbf{v}_b \]

- $b$: boundary surface
- $r$: at one field point
Acoustic Transfer Vector (ATV)

- Radiation field representation
  - Helmholtz integral equation
    \[ p(\mathbf{x})\alpha(\mathbf{x}) = \int_S p(\mathbf{y}) \frac{\partial G(\mathbf{x}|\mathbf{y})}{\partial n_y} dS_y + j\rho_0 \omega \int_S \nu(\mathbf{y}) G(\mathbf{x}|\mathbf{y}) dS_y \]
  - Rayleigh integral equation (baffled plate case)
    \[ p(\mathbf{x})\alpha(\mathbf{x}) = j2\rho_0 \omega \int_S \nu(\mathbf{y}) G(\mathbf{x}|\mathbf{y}) dS_y \]

- Discretization
  - On the surface:
    \[ Ap_b = Bv_b \]
  - In far-field:
    \[ p_r = d^T p_b + m^T v_b \]

\[ V^T_{ATV}\ ] = d^T A^{-1} B + m^T

\[ p = V^T_{ATV} v_b \]

Acoustic Transfer Matrix (ATM)
**Sound Radiation Mode**

- Radiated sound power in far-field

\[ W = \sum_{r=1}^{R} \frac{|p_r|^2}{2\rho_0c} S_r = \sum_{r=1}^{R} \frac{p_r^* p_r}{2\rho_0c} S_r \]

\[ W = \sum_{r=1}^{R} \frac{v_b^H V^*_{ATVr} V_{ATVr}^T v_b}{2\rho_0c} S_r = v_b^H R v_b \]

apply ATP relationship

- Sound Radiation Mode resulting from eigenvector decomposition of radiation resistance matrix

\[ R = \sum_{r=1}^{R} \frac{V^*_{ATVr} V_{ATVr}^T}{2\rho_0c} S_r \]

: Radiation Resistance Matrix

\[ R = Q \Lambda Q^H \]

- normalized eigenvector \( Q^H \) : Sound Radiation Mode

- eigenvalue \( \Lambda \) : proportional to radiation efficiency

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*Purdue University*
**Plate Radiation Model**

- **Baffled Plate Radiation Model**
  - hemispherical recovery surface (R7.5)
  - baffled rectangular plate (0.4mX0.24m)

- **ATV Calculation Approach**
  - Theoretical approach using Rayleigh Theory
    \[ p(x) = \frac{jωp_0}{2\pi} e^{jωt} \int S \frac{v_n(y) e^{-jkr(y,x)}}{r(y,x)} dS \]
  - Simulation using SYSNOISE (Ver. 5.6) Rayleigh Option
Sound Radiation Modes (320 Hz)

- Amplitude visualization
- Good match between theory and simulation
- Breathing mode is dominant
Sound Radiation Modes (960 Hz)

- Good match between theory and simulation
- Radiation from central region stronger than that from the edge.
Eigenvalue of Radiation Modes

- Eigenvalue of each radiation mode

- Good match between theory and simulation.
- As frequency increases, eigenvalue of each radiation mode increases. Eigenvalue stops increasing and converges at the coincidence frequency.
- Eigenvalues are proportional to radiation efficiencies of radiation resistance matrix.
Tire Radiation Analysis Procedure

[ Direct BEM ]
radiation field characteristics based on Acoustic Radiation Mode 
(Acoustic Transfer Vector)

[ Structural Harmonic FEM ]
structural wave propagation based on surface normal velocities

SPL & Sound Intensity on a hemisphere surrounding a tire

Sound Power 
Radiation Efficiency
Radiation Mode Contribution
Tire Radiation Model

- **Boundary Element Model**
  - 205/70R14 tire base
  - Two radiation cases: free space radiation / reflecting surface radiation
  - Recovery surface: R7.5 sphere (hemisphere) – related to pass-by noise test
  - For reflecting surface radiation case, the reflecting surface was modeled as rigid.

[ free space radiation ]  [ reflecting surface radiation ]
Radiation BE Analysis

■ D-BEM Analysis
  • Using Direct Collocation Boundary Element Method (D-BEM) in SYSNOISE ver. 5.6
  • Reason to use D-BEM: D-BEM takes less calculation time and allows model simplification for the interior singularity problem
  • Frequency range: 12.5 Hz – 1600 Hz (constant bandwidth 12.5 Hz)

■ CHIEF Method
  • To eliminate the singularity effect in an exterior D-BEM problem, CHIEF points were introduced inside the tire model.
  • The Number and location of CHIEF points were optimized. Finally 18 CHIEF points were applied.
Sound Radiation Modes (350 Hz)

- first and second modes: sidewall and wheel dominant
- grouping characteristics: 3\textsuperscript{rd} and 4\textsuperscript{th}, 5\textsuperscript{th} and 6\textsuperscript{th}, 8\textsuperscript{th} and 9\textsuperscript{th}, 10\textsuperscript{th} and 11\textsuperscript{th}
- 7\textsuperscript{th} mode: ring like mode
Sound Radiation Modes (350 Hz)

- first and second mode: similar to free space radiation case but a peak is added in the contact patch area
- No grouping characteristic.
Sound Radiation Modes (638 Hz)

- first and second modes: sidewall and wheel dominant
- grouping characteristics
- oscillating modes which have higher value on the sidewall appear in the lower modes.
Sound Radiation Modes (638 Hz)

- first and second mode: similar to free space radiation case but a peak is added in the contact patch area
- No grouping characteristic
Sound Radiation Modes (950 Hz)

- first mode: wheel dominant
- oscillating modes which have higher value on the sidewall appear in the lower modes.
- treadband is not efficient radiation region
Sound Radiation Modes (950 Hz)

- similar to free space radiation case but a peak is added in the contact patch area.
- 3<sup>rd</sup> – 5<sup>th</sup> modes: high radiation region on contact patch area
Radiation Efficiency of Radiation Modes

- Radiation efficiency of each radiation mode for a unit surface normal velocity
  - $\sigma_n = \frac{W}{E} = \frac{2\lambda_n}{\rho_0 c S_b}$ proportional to eigenvalue of radiation resistance matrix

- Radiation modes in same group have equal radiation efficiencies.
- Radiation efficiencies of 1$^{\text{st}}$ and 2$^{\text{nd}}$ modes are higher than other modes.
- As frequency increases, radiation efficiency of each radiation mode increases.
  Radiation efficiency stops increasing and converges at the coincidence frequency.
Radiation Efficiency of Radiation Modes

• Grouping characteristic does not appear.
• Radiation efficiencies of 1\textsuperscript{st} and 2\textsuperscript{nd} modes are higher than other modes.
• Big difference from the free space radiation
  - Radiation efficiencies of 3\textsuperscript{rd}, 4\textsuperscript{th} and 5\textsuperscript{th} modes increases above 800Hz.

strong radiation region from the contact patch area: Horn Effect
Structural FE Analysis

■ Tire FE model

- Shell elements were used.
- To consider stiff belt and rubberized carcass, **orthotropic material properties** were applied on treadband and sidewall.
- Wheel and boundary between wheel and tire were modeled as rigid.
- Inflation pressure: 30 psi

■ Structural Harmonic Analysis

- Full matrix method was performed using ANSYS ver. 7.1.
- Harmonic point source was applied at the point in contact with the ground.
- Frequency range: 12.5 Hz – 1600 Hz (constant bandwidth 12.5 Hz)
Orthotropic Material Properties

<table>
<thead>
<tr>
<th></th>
<th>tread band</th>
<th>side wall</th>
</tr>
</thead>
<tbody>
<tr>
<td>circumferential Young’s modulus</td>
<td>750 MPa</td>
<td>7.5 MPa</td>
</tr>
<tr>
<td>cross-sectional Young’s modulus</td>
<td>320 MPa</td>
<td>50 MPa</td>
</tr>
<tr>
<td>shear modulus</td>
<td>50 MPa</td>
<td>1.5 MPa</td>
</tr>
<tr>
<td>Possion’s ratio</td>
<td>0.45</td>
<td>0.45</td>
</tr>
<tr>
<td>density</td>
<td>1200 kg/m³</td>
<td>800 kg/m³</td>
</tr>
<tr>
<td>inflation pressure</td>
<td>30 psi (207 kPa)</td>
<td></td>
</tr>
</tbody>
</table>

- adapted from the work of Kropp [1989] and Pinnington and Briscoe [2002] or based on physical reasoning, or obtained by direct measurement at Continental Tire.
Structural Wave Propagation

Circumferential Wave Number Decomposition

<table>
<thead>
<tr>
<th>Structural velocity distribution in space domain</th>
</tr>
</thead>
<tbody>
<tr>
<td>structural velocity distribution in wave number domain</td>
</tr>
</tbody>
</table>

- Ring mode (ring frequency)
- Flexural wave
- Longitudinal wave
- Cut-on freq. of flexural wave mode

$\begin{align*}
m &= 3 \\
m &= 5 \\
m &= 7 \\
m &= 1
\end{align*}$
Power Calculation

■ **Structural input power**
proportional to space-averaged mean square velocity.

\[ E = \rho_0 c S_b \left\langle \overline{v_b^2} \right\rangle \]

where \( S_b \): tire surface area

■ **Radiated sound power**
calculated at field points on the recovery surface by using D-BEM directly.

\[ W = \sum_{r=1}^{R} \text{Re}[p_r v_r] S_r \]

■ **Sound Radiation Characteristics**
- Strong contribution to sound radiation results from the structural waves with low wave number.
- Sound amplification appear above 800 Hz: **Horn Effect**
Matching of Radiation Modes

- Matching of radiation mode and structural velocity distribution: \(|y_n|\)

\[ W = v_b^H Q \Lambda Q^H v_b = y^H \Lambda y = \sum_{n=1}^{N} W_n = \sum_{n=1}^{N} \lambda_n |y_n|^2 \]

- It shows the relationship between radiation modes and structural velocity on tire surface.
- **All radiation modes do not match with structural velocities.**
- The radiation mode which has a nodal line on the center circumference or on the cross-section including the point in contact with the ground can be neglected.
Sound Power Contribution of Radiation Modes

Sound power contribution of each radiation mode when combined with structural velocities

\[ W = y^H \Lambda y = \sum_{n=1}^{N} \lambda_n |y_n|^2 = \sum_{n=1}^{N} W_n \]

- All radiation modes do not contribute to the sound radiation.
- Reflecting surface radiation: 3rd mode is dominant above 800 Hz (Horn Effect Characteristic)
Cumulative Sound Power Curve

- Free space radiation: 7\textsuperscript{th} radiation mode, a ring-like mode on the treadband, contributes more than 90 percent of the radiated power at the ring frequency.
- Reflecting surface radiation: 3rd mode’s contribution is over 70 % above 800 Hz.
Radiated Sound Power

- Sound Power Calculation
  - direct calculation:
    \[ W = \sum_{r=1}^{R} \text{Re}[p_r v_r] S_r \]
  - radiation mode summation:
    \[ W = v_b^H \Lambda Q^H v_b = y^H \Lambda y = \sum_{n=1}^{N} W_n = \sum_{n=1}^{N} \lambda_n |y_n|^2 \]

- Good match below 1 kHz between direct calculation and radiation mode summation.
- Radiation mode summation method can be used in sound power calculation.
- If the radiation modes are known, it can reduce the time to calculate the sound power.
Conclusions

• Radiation mode approach for the rectangular plate model and the tire model was implemented by using Acoustic Transfer Function (ATV) in SYSNOISE.

• Radiation mode approach can help us to identify the radiated noise generation mechanism.

• Specifically, the radiation characteristics of a three-dimensional tire model in contact with a reflecting surface and enclosed by a hemispherical recovery surface was estimated by using acoustic radiation modes and by comparison with free-space radiation.

• The third radiation mode above 800 Hz is principally responsible for the horn effect in the presence of reflecting surface.

• The significance of the fast, longitudinal wave mode propagating through the treadband was confirmed by the large contribution of the modified ring radiation mode to the radiated sound power at the tire’s ring frequency.
References