Localized Heating Near a Rigid Spherical Inclusion in a Viscoelastic Binder Material Under Compressional Plane Wave Excitation

Jesus O. Mares  
*Purdue University*

Daniel C. Woods  
*Purdue University*

Caroline E. Baker  
*Purdue University*

Steven F. Son  
*Purdue University*

Jeffrey F. Rhoads  
*Purdue University*

*See next page for additional authors*

Follow this and additional works at: [http://docs.lib.purdue.edu/herrick](http://docs.lib.purdue.edu/herrick)
Authors
Jesus O. Mares, Daniel C. Woods, Caroline E. Baker, Steven F. Son, Jeffrey F. Rhoads, J Stuart Bolton, and Marcial Gonzalez

This presentation is available at Purdue e-Pubs: http://docs.lib.purdue.edu/herrick/139
Localized Heating due to Stress Concentrations Induced in a Lossy Elastic Medium via the Scattering of Compressional Waves by a Rigid Spherical Inclusion

Jesus Mares, Daniel Woods, Caroline Baker, Dr. Steven Son, Dr. Jeffrey Rhoads, Dr. Stuart Bolton, Dr. Marcial Gonzalez

School of Mechanical Engineering, Purdue University
Energetic materials in a binder

- Common use of energetic crystals involves embedding them in a binder
- Behaviors of interest include mechanical interactions between crystals, mechanical behavior of the binder and crystals separately, and interactions between the binder and crystals.
- Loading conditions include impact and periodic excitation
- Periodic excitation involves high strain-rate behaviors, even if overall strain rate is low
- Complex structure makes coupled mechanisms difficult to isolate
Experimental Motivation

750-800 µm AP crystals to undergo excitation

Surface temperature rise of 750-800 µm AP particles after 2 s excitation


Single-crystal sample under ultrasonic excitation

(image adapted from Miller et al., *J. Appl. Phys.*, 2016)

ΔT ≈ 7°C

Insulated boundary

d = 1.45 mm

q = 0.162 W

ΔT ≈ 74.24°C

Point heat source in semi-infinite medium
Experimental Crystal Heating Rates

- Analytical approximation using semi-infinite medium solution for heat source magnitude \( q \) and depth \( d \) (varies due to morphology)
- Particle surface temperature is found by applying the same solution with given \( q \) at the particle radius
- 37°C/s for 750-950 µm HMX in Sylgard® at 215 kHz
- 125°C/s for 400-500 µm AP in Sylgard® at 215 kHz

Assumptions:
- Rigid spherical particle (no intrinsic heating)
- Linear viscoelastic binder
- Planar incident wave
- Perfect bonding between binder and particle (no particle/binder friction)
- Thermal stresses negligible
- Temperature-independent parameters

Boundary conditions:
- Displacement of binder = displacement of particle at boundary
- Newton’s second law: stresses integrated over particle surface produce acceleration

Numerical Solution parameters:
- 1-µm wave amplitude
- 500-µm HMX particle
- Sylgard binder
- 500-kHz excitation frequency

Simplified diagram of single-particle embedded in a viscoelastic binder

Mares, et.al., IMECE 2016
Stress Solution

- Solved by Pao and Mao in 1963 for linear elastic binder
- Expanded to lossy viscoelastic binder by Gaunard and Uberall in 1978
- Solved for lossy inclusion by Hinders, et. al. in 1994
- Solved with FE simulation by Chervinko in 2007

Real component of radial stress: $\mathcal{R}(\tilde{\sigma}_{rr})$

Real component of shear stress: $\mathcal{R}(\tilde{\sigma}_{r\theta})$

1-μm, 500-kHz harmonic plane wave excitation using Gaunard and Uberall expressions
Periodic Excitation of a Single Spherical Particle

- 1-μm, 500-kHz harmonic plane wave excitation
- 500-μm diameter HMX crystal in Sylgard

Radial Stress, $|\tilde{\sigma}_{rr}|$ (MPa)

Shear Stress, $|\tilde{\sigma}_{r\theta}|$ (MPa)

“Polar” Stress, $|\tilde{\sigma}_{\theta\theta}|$ (MPa)

Azimuthal Stress, $|\tilde{\sigma}_{\phi\phi}|$ (MPa)
Viscoelastic Heating Model

- Based on losses in strain energy density per cycle of applied harmonic stress (hysteretic damping)

Time-averaged heat generation ($W/m^3$):

\[
q = \frac{\omega}{2\pi} \int_{t_0}^{t_0+2\pi/\omega} \left( \sigma_{rr} \frac{\partial \varepsilon_{rr}}{\partial t} + \sigma_{\theta\theta} \frac{\partial \varepsilon_{\theta\theta}}{\partial t} + \sigma_{\phi\phi} \frac{\partial \varepsilon_{\phi\phi}}{\partial t} + 2\sigma_{r\theta} \frac{\partial \varepsilon_{r\theta}}{\partial t} + 2\sigma_{r\phi} \frac{\partial \varepsilon_{r\phi}}{\partial t} + 2\sigma_{\theta\phi} \frac{\partial \varepsilon_{\theta\phi}}{\partial t} \right) dt
\]
Heat Generation in HMX-Sylgard System

Fourier’s law of conduction:

\[ m c_p \cdot m = k_m \left[ \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial T}{\partial r} \right) + \frac{1}{r^2 \sin \frac{\partial}{\partial \theta}} \left( \sin \frac{\partial T}{\partial \theta} \right) \right] + q_m \]

Time-averaged Heat Generation (W/mm³)

Temperature Increases (°C) from 0.05 to 0.5 s

For the temperature solution, a free convective surface condition was applied at a large outer radius

\[ 1-\mu m, \; 500\text{-kHz harmonic plane wave excitation} \]

Animation shows 0.05 s increments, with one frame shown per second
Temperature Increase in HMX-Sylgard System

- 1-μm, 500-kHz harmonic plane wave excitation

**Transient Max. Temperature Increase of Crystal and Binder**

**Temperature Distribution (°C) at t = 0.5 s**

Compares well to Mares, et. al. (2014) in which heating rate were estimated to be between 37 to 125°C/s depending on shape, size, and identity of inclusion
Future Modeling Efforts

- Effect of temperature-dependent parameters
- Non-linear viscoelasticity
- Thermal stress effects
- Debonding effects
- Effect of binder and particle properties

*Mares et al., J. Appl. Mech., 2016 (Submitted).*
Future modeling efforts: Particle morphology

- Particle size and shape have a significant effect on stress amplitudes
- Relationship between frequency and particle size affects phase also
- Denser particle has larger vibrational amplitude

Adapted from Oien, 1973
Questions?
System dynamics

Particle motion described by Newton’s second law:

\[
\frac{4\pi a^3}{3} \rho \ddot{U} = \iiint (\sigma_{rr} \cos \theta - \sigma_{r\theta} \sin \theta) a^2 \sin \theta \, d\theta \, d\phi \bigg|_{r=a}
\]

Particle density affects vibration amplitude
Thermal Analysis Boundary Conditions

\[ k_2 \frac{\partial T}{\partial r}(0, \theta, t) = -k_2 \frac{\partial T}{\partial r}(0, \theta + \pi, t), \]

\[ T(a^-, \theta, t) = T(a^+, \theta, t), \]

\[ k_2 \frac{\partial T}{\partial r}(a^-, \theta, t) = k_1 \frac{\partial T}{\partial r}(a^+, \theta, t), \]

\[ k_1 \frac{\partial T}{\partial r}(R, \theta, t) = U_0 [T_0 - T(R, \theta, t)], \]

\[ k_m \frac{\partial T}{\partial \theta}(r, 0, t) = 0, \]

\[ k_m \frac{\partial T}{\partial \theta}(r, \pi, t) = 0, \]