MEASUREMENT OF VISIBLE RADIATION TRANSFER IN WATER UNDER LABORATORY AND FIELD CONDITIONS

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September 1981

PURDUE UNIVERSITY
WATER RESOURCES RESEARCH CENTER
WEST LAFAYETTE, INDIANA
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by

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The work upon which this report is based was supported by funds provided by the United States Department of Interior, Office of Water Research and Technology, as authorized by the Water Resources Research Act of 1964 (PL 83-379 as amended).

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Technical Report No. 140
Purdue University Water Resources Research Center
West Lafayette, Indiana 47907

September 1981
FOREWORD

The study reported in this document was supported by the U.S. Department of Interior, Office of Water Research and Technology as part of the annual allotment program for the Purdue University Water Resources Research Center. Project personnel were Dr. Frank Incropera, principal investigator; T.R. Wagner, W.G. Houf and M.C. Stoddard, graduate research assistants; and R.P. Fish, undergraduate assistant. Acknowledgement of support is given to Dr. Daniel Wiersma, Director of the Indiana Water Resources Research Center.
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ABSTRACT

Because solar radiation transfer in a body of water strongly influences thermal conditions and the growth of photosynthetic organisms, there is considerable interest in the development of methods for accurately predicting this transfer. The problem is complicated by several factors which include the directional distribution of the incident radiation, reflection and refraction at the air-water interface, absorption and multiple scattering within the water, and reflection off the bottom surface. Although theoretical methods have been developed to treat this problem, little has been done to acquire the radiation property data needed for implementation of the methods or to test their accuracy through a comparison of predictions with measurements under controlled laboratory conditions. In addition, little has been done to determine the nature of radiation transfer in natural waters under field conditions. Accordingly, the objectives of this study have been (i) to measure the optical properties of natural waters, (ii) to measure the distribution of radiation of water samples under laboratory conditions and to compare the results with predictions, and (iii) to measure the distribution of radiation under field conditions.
Working with samples of water from the Wabash River, a complete set of optical property measurements has been obtained. For the same samples, measurements of the angular and depth distribution of the radiation have been made under laboratory conditions for which the directional distribution of the incident radiation was approximately isotropic. The measurements have been compared with predictions based on the discrete ordinate and three-flux theoretical methods. In addition, angular and depth distributions of the radiation were obtained under field conditions. The distributions were strongly influenced by radiation scattering from the collimated beam, with the depth distribution being characterized by a maximum below the air-water interface and the directional distribution exhibiting a maximum in the direction of the collimated beam.
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CHAPTER 1
INTRODUCTION

1.1 Rationale

Existing and anticipated applications, as well as concern for the natural environment, have stimulated interest in determining solar radiation transfer in water bodies. For example, ponding systems are extensively used for the dissipation of electric powerplant waste heat and for the treatment of wastewater. In such applications solar irradiation of the pond strongly influences its thermal structure (the variation of temperature with depth), which in turn influences its performance for the intended application. Ponding systems are also being considered for the intensive culture of edible aquatic organisms, as well as for the collection and storage of solar energy. In the former application knowledge of the local radiation field is essential to predicting the growth of photo-synthetic organisms within the water; in the latter application the radiation field and the associated thermal structure strongly influence the system performance.

The problem typically involves one-dimensional transfer in a planar, scattering-absorbing suspension. The incident radiation, which may be composed of collimated and noncollimated
components, experiences reflection and refraction at the air-water interface, absorption and anisotropic scattering within the water, and if the water is sufficiently shallow, reflection at the bottom surface. Calculation of the local radiant flux and the local volumetric heat absorption rate requires knowledge of the spectral radiant intensity, $I_\lambda$, in the $(\theta, \phi)$ direction at each depth, $z$, of the medium. Predictions of this quantity for planar, scattering-absorbing media may be obtained from solutions of the quasi-steady, one-dimensional equation of transfer. Such solutions have been effected on several levels of complexity, with procedures varying according to the manner in which the foregoing effects are treated. Although early efforts [Dake and Harleman, 1969; Foster, 1971] involved the use of a single-flux (Beer's law) solution, for which reflection and scattering effects were neglected, more recent efforts have been based on multi-flux approximations which account for all relevant processes [Viskanta and Toor, 1972, 1973, 1978; Daniel, et al., 1979; Incropera and Houf, 1979]. The relative merits of the various solution methods have been compared by Houf and Incropera [1980].

Properties needed for implementation of the solution methods include the absorption and scattering coefficients and the scattering phase function of the medium. Measurements of those properties have been obtained for a variety of aqueous
suspensions. Hubert [1945], Tyler et al. [1972], and Hale and Querry [1973] have obtained data for both distilled and natural waters, while Privoznik et al. [1978] and Incropera and Privoznik [1978] have obtained data for algal and wastewater suspensions, respectively. Experimental procedures developed for this purpose may be readily extended to any aqueous suspension.

To develop confidence in the use of a solution method with property data to predict radiation transfer in aqueous media, it is necessary to compare predictions with radiation measurements made under controlled laboratory conditions. Although one such comparison has been made [Daniel et al., 1978], it was made under restrictive conditions involving a diffusely irradiated, dense algal suspension having no air interface and negligible bottom reflection. Moreover, extension of laboratory based results to field conditions is complicated by the existence of a collimated, as well as a non-collimated, contribution to the solar irradiation. The radiation field within a liquid depends strongly on the relative magnitudes of these components, which vary with time of day and atmospheric conditions. Although measurements have been made of large scale changes in the radiance distribution which occur in deep bodies of water [Preisendorfer, 1976],
much remains to be learned of solar radiation transfer in liquids under field conditions.

1.2 Objectives

Since existing measurements of the radiation field in shallow water layers is fragmentary, the primary objective of this study has been to perform such measurements under both laboratory and field conditions. In particular, using water extracted from a local source (the Wabash River), detailed radiation measurements have been made under laboratory conditions for which the water was diffusely irradiated across an air interface. Using measurements of the water optical properties in conjunction with appropriate theoretical methods, predictions of the radiation field have been made and compared with the measurements. In addition, the spatial and directional distribution of the solar radiation has been measured under field conditions. These measurements have provided an indication of the relative contributions of the collimated and noncollimated components of the solar radiation to conditions in the water.
CHAPTER 2
EXPERIMENTAL PROCEDURES

2.1 Optical Property Measurements

The inherent optical properties of an aqueous suspension include the spectral absorption coefficient, $\kappa_\lambda$, scattering coefficient, $\sigma_\lambda$, and the phase function, $\varphi_\lambda$. The absorption and scattering coefficients provide a measure of the extent to which a beam of radiation is attenuated by the suspension, and their sum is defined as the extinction coefficient, $\beta_\lambda$.

$$\beta_\lambda = \kappa_\lambda + \sigma_\lambda$$  \hfill (2-1)

The ratio of scattering to extinction is termed the scattering albedo.

$$\omega_\lambda = \frac{\sigma_\lambda}{\beta_\lambda}$$  \hfill (2-2)

The phase function is a directional distribution which provides the probability of radiation scattering in different directions. The procedure used in this study involved direct measurement of $\beta_\lambda$, $\kappa_\lambda$, and $\varphi_\lambda$, and the inference of $\sigma_\lambda$ and $\omega_\lambda$ from Eqs. (2-1) and (2-2), respectively.
The system used to measure the extinction coefficient is shown in Fig. 2.1. A 200W G.E. quartzline bulb, which was powered by a Sorenson DCB 40-25B power supply, was used as a light source. Light from the source was passed through a monochrometer (Heath, Model EU-700) and focused into a slightly divergent beam. The beam was passed through a test cell which contained the water sample of interest. A fiber optic probe was submerged in the water and used to record the intensity of the beam at two locations, \( z_1 \) and \( z_2 \). The extinction coefficient could then be determined from an expression of the form

\[
\beta = \ln \left( \frac{I_{\lambda}(z_2)}{I_{\lambda}(z_1)} \right) \frac{z_2 - z_1}{L}
\]

Signals from the fiber optic probe are transmitted to the processing system shown in Fig. 2.2. Signals, which are transmitted via a single 40-mil Dupont Cropon optical fiber, are amplified by passage through a photomultiplier tube (EMI 9558Q) and a variable gain current-to-voltage amplifier. A voltage-to-frequency converter (Matrix 1605) is then used to provide a frequency signal, which is counted and averaged over a 10 second interval (Data Precision S740 Multifunction counter). The averaged signal is then recorded on a line printer (Digitec
Fig. 2.2 Signal amplification and recording apparatus
The absorption coefficient was measured by using a Shimadzu multipurpose recording spectrophotometer (Model MPS-50L).

Phase function measurements were made using the system on Fig. 2.3. A helium-neon ($\lambda = 632.8\,\text{nm}$) laser was used as the radiation source, and the fiber optic detector was rotated about the beam, from an angle of 0 degrees (facing the beam) to 180 degrees (along the beam). As the detector is rotated, the amount of scattered radiation is recorded, and the phase function is obtained from the expression

$$
\Phi_\lambda (\xi) = \frac{2I_\lambda (\xi) [U (\xi)]^{-1}}{\int_0^\pi I_\lambda (\xi) [U (\xi)]^{-1} \sin\xi \, d\xi}
$$

(2-4)

where

$$
U (\xi) = \int_0^{\frac{w}{\sin\xi}} \left[ 1 + \beta_\lambda \frac{w}{2} \, \text{ctn} \, \xi - \beta_\lambda \, \xi \, \cos\xi \right] \times
$$

$$
[1 - \beta_\lambda \left( \frac{w}{2 \sin\xi} \right)] \times [1 - \beta_\lambda \left( \frac{w}{\sin\xi} - \lambda \right)] \, dl
$$

(2-5)
2.2 Radiation Measurements under Laboratory Conditions

A schematic of the system used for the laboratory radiation measurements is shown in Fig. 2.4. A 294 liter Plexiglass tank (0.61m deep and 0.56 by 0.86m on the sides) was used as a container for the river water. Neglecting any effect of the azimuthal angle $\phi$, the spectral intensity of the radiation in the suspension, $I_\lambda$, depends upon the depth z and the polar angle $\theta$. Radiation measurements were made by submerging a fiber optic probe in the culture. The probe (Fig. 2.5) was constructed of stainless steel tubing (1.9mm O.D., 1.1mm I.D.) in which a 40 mil monofilament plastic fiber was inserted. The probe tip (Fig. 2.6) was capped with a brass barrel guide having a 0.33mm aperture and a glass cover. The aperture was separated from the tip by a distance of 11 mm, and the half angle of view of the probe was determined to be 0.058 rad (3.3°). The probe was supported by a traversing mechanism which permitted travel in each of the three rectangular coordinates, as well as rotation about a horizontal axis. All probe and traverser components were painted with Nextel velvet black to minimize reflections.

The light source for the experiments consisted of an overhead bank of twelve 150W flood lamps, arranged 4 by 3 on
Fig. 2.4 River water suspension and measurement system.
Fig. 2.5 Geometry of probe used for radiation field measurements
Fig. 2.6 Probe tip geometry
approximately 0.21 m centers. A large, fused, opal (diffusing) glass plate was suspended approximately 1.5 m below the lamps, and white paper sheets were used to form vertical side walls between the lamps and the tank. The sheets minimized light loss and promoted a more uniform distribution of radiation incident on the diffusing glass. A small rectangular section was cut from the glass to permit insertion of the probe traversing mechanism.

The directional distribution of the radiation incident on the suspension was measured by using the fiber optic probe, and the results are shown in Fig. 2.7. The deviation from isotropic conditions is small (less than 10%) for $\theta^* \leq 0.7$ rad ($40^\circ$), but becomes significant for large values of the incident angle, at which the intensity of the incident radiation decreases sharply to zero. The probe was also used to map the variation in the normal intensity ($\theta^* = 0$) over a horizontal plane just above the air interface, and the standard deviation of the measurements was found to be less than 10% over a central region which occupied approximately 60% of the total surface area of the suspension.

Three different boundaries are pertinent to radiation transfer in the suspension and include the air interface, the tank sides, and the tank bottom. Although the tank sides represent an unnatural boundary, the tank size and the opacity
Fig. 2.7 Actual and isotropic distributions of the radiation incident at the air interface of the suspension.
of the suspension are large and, since all measurements are made along a vertical line at the center of the tank, it is reasonable to neglect two and three-dimensional effects. The bottom boundary was controlled through the use of two Plexiglass sheets, which were coated with Nextel velvet optical paints. One surface was coated with a diffusely reflecting white paint of reflectance $\rho_d \approx 0.91$, and the other surface was coated with a black paint of reflectance $\rho_d \approx 0$. The air interface is optically smooth and provides for refraction of all incident radiation into a cone of half angle $\theta_c = 0.85$ rad (48.7°). This interface also provides for the reflection of upwelling radiation incident from the suspension.

Radiation measurements were made at 6 different depths in the suspension. The first depth, $z_0 = 6.4$ mm, corresponds to a location just below the air-water interface for which the radiation intensity, $I_\lambda$, is large and is primarily associated with the forward direction ($0 \leq \theta < \pi/2$ rad). For each depth a total of 51 measurements were made in an angular scan of the probe from $\theta = 0$ rad (probe tip facing upward) to $\theta = \pi$ rad (probe tip facing downward) in increments of 0.0628 rad (3.6°). These measurements were, in turn, used to determine radiation fluxes associated with different regions. Assuming the intensity to be independent of the azimuthal angle $\phi$, the radiation flux associated with the region defined by $\theta_1 \leq \theta \leq \theta_2$ is
\[ F_\lambda (z) = 2\pi \int_{\theta_1}^{\theta_2} I_\lambda (z, \theta) \cos \theta \sin \theta d\theta \]  

(2-6)

Specific fluxes of interest include the forward flux \((\theta_1 = 0, \theta_2 = \pi/2)\), the backward flux \((\theta_1 = \pi/2, \theta_2 = \pi)\), and the net flux \((\theta_1 = 0, \theta_2 = \pi)\). Adopting the forward flux at \(z_o\) as a reference condition, a normalized radiation flux may then be expressed as

\[ F_\lambda (z) = \frac{\int_{\theta_1}^{\theta_2} I_\lambda (z, \theta) \cos \theta \sin \theta d\theta}{\int_0^{\pi/2} I_\lambda (z_o, \theta) \cos \theta \sin \theta d\theta} \]  

(2-7)

Equation (2-7) is evaluated from the measurements of this study by assuming \(I_\lambda (z, \theta)\) to be independent of \(\theta\) over the change in \(\theta\) associated with two successive measurements \((\Delta \theta = 0.0628 \text{ rad})\). Replacing the integrations by summations, the normalized flux is then

\[ F_\lambda (z) = \frac{\sum_{i=n_1}^{n_2} [I_\lambda (z, \theta_{i+1}) + I_\lambda (z, \theta_{i})][\sin^2(\theta_{i+1}) - \sin^2(\theta_{i-1})]}{25 \sum_{i=i}^{i} [I_\lambda (z_o, \theta_{i+1}) + I_\lambda (z_o, \theta_{i})][\sin^2(\theta_{i+1}) - \sin^2(\theta_{i-1})]} \]  

(2-8)
where summing from \( n_1 = 1 \) to \( n_2 = 25 \) gives the forward flux, from \( n_1 = 26 \) to \( n_2 = 51 \) gives the backward flux, and from \( n_1 = 1 \) to \( n_2 = 51 \) gives the net flux. When divided by two, the term \([I_{\lambda}(z, \theta_{i-1}) + I_{\lambda}(z, \theta_i)]\) provides the average intensity associated with the angular interval \( \Delta \theta = \theta_i - \theta_{i-1} \).

A measure of the spectral intensity of the radiation was obtained from the output of the fiber optic probe, which was rendered monochromatic by passing it through a 513 nm interference filter (Special Optics 9-2103-5145) having an 8 nm bandwidth. The radiation was transmitted to the data processing system of Fig. 2.2, whose output was in the form of an averaged signal, \( P \). This signal, which was recorded using a line printer, is related to the intensity of the radiation intercepted by the probe by an expression of the form

\[
P = \alpha \int_{\Omega_P} \int_{\Lambda_P} I_{\lambda} g(\Omega) \cos \theta d\Omega d\Lambda
\]

where \( \alpha \) is a proportionality constant determined by the sensitivity of the photomultiplier tube and the characteristics of the signal processing system, \( \Lambda_P \) is the exposed area of the probe, \( \Omega_P \) is the probe solid angle of view, and \( g(\Omega) \) is the
function accounting for the angular sensitivity of the probe. Neglecting the variation of $I_\lambda$ over the probe field of view, it follows that, to a good approximation,

$$P = [\alpha \int_{A_p} \int_{\Omega_{\rho}} g(\Omega) \cos \theta d\Omega d\Omega'] I_\lambda$$  \hspace{1cm} (2-10)

where the term in brackets depends on the characteristics of the probe tip design, the photomultiplier tube and the signal processing system. Accordingly, since all radiation results of this study are normalized, the intensity may be equated to the recorded signal $P$. Experimental errors associated with the radiation measurements are less than 5% for $\theta \leq 0.85$ rad but increase with increasing $\theta$ to as much as 100% for the backward direction ($\theta > \pi/2$ rad).

2.3 Radiation Measurements under Field Conditions

The majority of the measurements were made using the test cell of Fig. 2.4 with the fiber optic probe of Figs. 2.5 and 2.6 and the data processing system of Fig. 2.2. In addition measurements of the total solar irradiation and the total
collimated flux were made by using a pyranometer (Eppley 8-48) and a pyrheliometer (Eppley NIP), respectively.

Since the directional distribution of the solar irradiation strongly influences the radiation field within the water, it is necessary to characterize geometrical conditions and, in particular, to describe the orientation of the test cell and the probe relative to solar coordinates. As shown in Fig. 2.8, the test cell was positioned such that its axis, which is perpendicular to oppositely facing sidewalls and parallel to the probe stem, made an angle of $\phi_A = 37$ deg with respect to due north. The zenith, $\theta^*$, and azimuth, $\phi$, angles of the collimated beam were calculated from standard procedures [Thekaehara, 1977], while the zenith angle, $\theta$, of the refracted beam was determined from Snell’s law. At any depth $z$ below the air–water interface, the probe tip could be rotated in a plane which is perpendicular to the tank axis. To minimize the effects of sidewall reflections, the measurements were confined to the range of depths, $0 \leq z \leq 0.3$ m.

Referring to Fig. 2.9, the angle between the probe tip and the zenith, $\theta_p$, is 0 deg, +90 deg, -90 deg and 180 deg when the probe is facing upward, westward, eastward, and downward, respectively. The angle $\Delta\theta$ between the probe tip and
Fig. 2.3 Probe and test cell orientation in the field
Fig. 2.9 Probe and solar angles
the refracted beam is of particular interest and may be determined from appropriate unit vectors. Representing the refracted beam by the unit vector

\[ \hat{\mathbf{R}} = \sin \theta \cos \phi \hat{i} + \sin \theta \sin \phi \hat{j} + \cos \theta \hat{k} \]  \hspace{1cm} (2-11)

and the line of sight of the probe by the unit vector

\[ \hat{\mathbf{P}} = \sin \theta_p \cos \phi_p \hat{i} + \sin \theta_p \sin \phi_p \hat{j} + \cos \theta_p \hat{k} \]  \hspace{1cm} (2-12)

where \( \phi_p \) is the angular distance (from north) of the plane of probe rotation \( (\phi_p = \phi_A + 270 = 307 \text{ deg}) \), it follows that

\[ \Delta \theta = \cos^{-1} (\hat{\mathbf{R}} \cdot \hat{\mathbf{P}}) \]  \hspace{1cm} (2-13)

Most of the data were obtained as either depth or angular scans. Depth scans were obtained for a fixed probe angle, \( \theta_p \), and illustrate the variation of the radiance with distance from the air-water interface. Angular scans were obtained at a fixed depth and illustrate the variation of radiance with \( \theta_p \), and hence \( \Delta \theta \). Because the scans could be completed in less than 5 minutes, it is reasonable to assume that the solar angles remain constant during the scan period. Most of the scans
were made during sunny periods, for which the noncollimated contribution to the total solar irradiation did not exceed 40%. Meaningful scans were difficult to obtain under cloudy or hazy sky conditions, since the total solar radiation would typically fluctuate by more than 50% during the scan. In addition, to determine the effect of fluctuating sky conditions, several transient scans were obtained for fixed values of the probe depth and angle. For any given day, all data are typically normalized with respect to the radiance recorded at $z=6.35\,\text{mm}$, $\theta_p = 0\,\text{deg}$, and at an early time of the day.

Although the foregoing experiment provided numerous useful results, it is subject to several limitations. A serious limitation is the fact that no provision was made in the probe traversing mechanism to allow for variation in the solar geometry. With limited azimuthal probe positioning capability, it was necessary to correlate experimental observations in terms of the independent variable $\Delta \theta$, which is a measure of the angle between the refracted collimated beam in the sun's azimuth and the direction of the probe tip. Use of this independent variable obscures important trends associated with $\phi$ and $\theta$. The small size of the tank also made measurements difficult to obtain when the polar angle of incidence of the direct radiation was large and exacerbated the effect of shadows and
sidewall reflections. Moreover, the manner in which each data point was obtained was cumbersome. That is, with 10 seconds required to obtain each data point, the number of useful scans which could be obtained was limited by the changing solar and atmospheric conditions.

In an attempt to eliminate the foregoing problems, a second experimental system was assembled. Measurements were made in a circular tank (galvanized steel) of 4 m diameter and 1 m depth whose interior surface was coated with 3M Nextel black velvet paint of negligible reflectivity. A vertical array of six fiber optic probes were assembled on a traversing mechanism which permitted simultaneous variation of the polar angle of each probe. The probes were positioned on vertical supports which were, in turn, supported by horizontal rails spanning a segment of the tank, fixed in one azimuthal position. The six probes were linked to three photodiodes (VDT PIN-10DFF) by means of an interface shown schematically in Fig. 2.10. This enabled data from three depth positions to be available simultaneously. Upon rotation of the interface dials, the remaining three probe outputs were available. Outputs from the probe system and the radiometers were monitored and recorded with a Datel PDL-10 scanning data logger, scanning and printing at a rate of 1 channel/second. The data acquisition scheme is shown in Fig. 2.11.
Fig. 2.10 Field measurement system
Fig. 2.11 Data acquisition system for field measurements
For meaningful data to be obtained using the six probe system, it was necessary that the probes be calibrated relative to each other. Also, because all measurements were normalized to some reference data point, it was necessary to establish the linearity of each probe response. The in-laboratory calibration which was performed verified the linearity of the photodiode response and established the extent of the linear region. The laboratory calibration scheme is shown in Fig. 2.12.

Following assembly, several difficulties were discovered with the six-probe system. It was found that the probe-detector coupling was sensitive to disassembly, which was necessary for daily movement of the apparatus from storage to the field, and that daily calibrations were required before and after the measurement process. Accordingly, a field calibration scheme was assembled and is shown in Fig. 2.13. Since it was not uncommon for one probe-detector pair to change in relative calibration over the course of daily measurements, the need for continuous calibration seriously detracted from the measurement process. Another problem is related to the fact that, in liquid media characterized by small optical depths and highly forward peaked scattering phase functions, there is a strong dependence of the measured intensity on the angle between the line of the sight of the probe and the refracted
Fig. 2.12 Schematic of laboratory calibration system
Fig. 2.13 Schematic of field calibration system
collimated beam. Therefore, in using clear water, data aberrations among probes were encountered for measurements made within the collimated beam due to very slight differences in the angular position among probes. Increasing the mechanical accuracy of positioning among probes would have involved considerable difficulty. In addition, even if the probes were aligned exactly, the positioning device was unable to resolve differences of less than 3 deg, making the location of the intensity maximum difficult to determine. Collectively, these problems made it impossible to obtain reliable field data using the second measurement scheme, and no such data is presented in this report.
CHAPTER 3

EXPERIMENTAL RESULTS

3.1 Optical Properties

Optical property data were obtained for three samples of Wabash River water, one extracted in early September, when the water was rich in biological organisms, another extracted in late October, when many of the organisms had died, and the third extracted in mid-November, when few organisms remained. As these organisms die and settle out of suspension, the water becomes more optically thin and backscattering becomes more pronounced (due to the higher relative concentration of smaller, nonbiological particles).

Results from the early September measurements are shown in Figs. 3.1 to 3.3. The coefficients and the albedo are of approximately the same magnitude as results obtained for wastewater samples [Privoznik and Incropera, 1978], although the river water sample of this study exhibits a definite absorption peak around 680 nm due to the presence of chlorophyll a. The large value of the albedo (Fig. 3.3) indicates that extinction is dominated by scattering and not by absorption.

Results obtained for the river water sample of late October and mid-November are shown in Figs. 3.4 to 3.7. When compared with each other and with the early September results, it is clear that the coefficients decrease as the months progress
Figure 3.1 Optical coefficients for the river water sample of early September
Figure 3.2 Absorption coefficient for the river water sample of early September
Figure 3.3 Albedo for the river water sample of early September
Figure 3.4 Optical coefficients for the river water sample of late October
Figure 3.5 Albedo for the river water sample of late October
Figure 3.6 Optical coefficients for the river water sample of mid-November
Figure 3.7 Albedo for the river water sample of mid-November.
from September to November. In addition there is a slight, but discernable decline in the albedo. Both trends may be attributed to the settling of biological organisms from the suspension.

Phase function distributions for the river water samples of late October and mid-November are shown in Figs. 3.8 and 3.9, respectively. The data are fitted with a cubic spline approximation (solid line) which is then normalized (dashed line). Both phase functions are highly peaked in the forward direction, although the mid-November results exhibit a higher degree of backscattering. This trend is due to the presence of a higher percentage of smaller particles in suspension during November.

3.2 Radiation Intensity Distributions under Laboratory Conditions

Radiation intensity measurements made under laboratory conditions for the river water sample of late October are plotted in Figs. 3.10 and 3.11 for the black and white bottom conditions, respectively. At \( \lambda = 513 \text{ nm} \), the wavelength at which the radiation intensity measurements were made, the water sample was characterized by a total optical depth of \( \tau_{\lambda,d} = 5.2 \) and a scattering albedo of \( \omega_{\lambda} = 0.72 \). The data have been normalized with respect to the maximum intensity, and a ten-fold expansion has been used for the upwelling (\( \theta > \pi/2 \)) intensities. For both bottom conditions, the directional
Fig. 3.8 The $\log_{10}$ of the phase function for the river water sample of late October.
Fig. 3.11 Directional distribution of the normalized radiation intensity for the river water sample with a white bottom.
distribution of the radiation ($0 \leq \theta < 0.5\pi \text{ rad}$) is concentrated within Region 1 of Fig. 2.4, that is, within a cone of half angle $\theta = 0.27 \pi \text{ rad}$, which corresponds to the critical angle into which all incident radiation is refracted at the air interface. A small amount of radiation appears in Region 2 ($0.27 \pi \text{ rad} \leq \theta \leq 0.5\pi \text{ rad}$) and is due primarily to the scattering of radiation from Region 1. For all values of the angle $\theta$ associated with the forward direction, absorption acts to diminish the radiation intensity with increasing optical depth, while scattering acts to redistribute the radiation from smaller to larger values of $\theta$. The net effect is one of promoting a more uniform (isotropic) directional distribution of the downwelling radiation with increasing $\tau_\lambda$. However, the existence of a phase function which is highly forward peaked precludes this effect from becoming significant until large values of $\tau_\lambda$ are approached.

Although the downwelling radiation is not strongly affected by the bottom condition, the same may not be said for the upwelling radiation of Region 3 ($0.5 \pi \leq \theta \leq \pi \text{ rad}$). For the black bottom, the only contribution to this radiation is made by scattering from Regions 1 and 2, and the intensity of the upwelling radiation decays monotonically to zero with increasing
optical depth. For the white bottom, however, the reflection of downwelling radiation off the bottom provides for upwelling intensities which do not decrease monotonically with increasing $\tau_\lambda$. These intensities are approximately independent of $\tau_\lambda$ near the air interface ($\tau_\lambda < 1$), but increase with increasing $\tau_\lambda$ as the bottom is approached.

3.3 Radiation Measurements under Field Conditions

Although absent in the laboratory measurements, the contribution of the collimated radiation to the total solar irradiation strongly influences the spatial distribution of the radiance (intensity) under field conditions. Two sets of vertical distributions are presented in Figs. 3.12 and 3.13. The results of Fig. 3.12 were obtained under clear, sunny conditions for which the collimated irradiation, $G_{S,c}$, was approximately 92% of the total irradiation, $G_S$, while the results of Fig. 3.13 were obtained under hazy conditions for which the collimated contribution was approximately 58% of the total. Legends for the figures are provided in Tables 3.1 and 3.2. The most striking feature of Fig. 3.12 is the fact that all of the distributions exhibit a maximum radiance below, and not at, the
Fig. 3.12 Vertical radiance profiles for downwelling radiation when the collimated irradiation was approximately 92% of the total irradiation.
Fig. 3.13 Vertical radiance profiles for downwelling radiation when the collimated irradiation was approximately 58% of the total irradiation.
### Solar Angles

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<th>Solar Time</th>
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<th>$\phi$, deg</th>
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Table 3.1 Conditions associated with the radiance distributions of Fig. 3.12.
### Solar Angles

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Table 3.2 Conditions associated with the radiance distributions of Fig. 3.13.
air-water interface. The trend is most pronounced for the distribution corresponding to $\Delta \theta = 22.1$ deg and diminishes as the angle between the probe and the collimated beam increases. The fact that the trend is absent from Fig. 3.13 suggests that it may be attributed to the existence of a large collimated contribution to the total irradiation. When the collimated flux is dominant, as it is for the conditions of Fig. 3.12, a significant amount of radiation is scattered from the collimated beam into neighboring directions. The process occurs continuously along the path of the beam, causing the radiance associated with neighboring directions to increase with depth. However, as the collimated beam is attenuated, radiation scattering from the beam diminishes, and absorption causes the radiance associated with neighboring directions to decrease with depth. The fact that the maximum depends strongly on the value of $\Delta \theta$ may be attributed to the existence of a phase function which is highly peaked in the forward direction. Hence the probability of scattering from the collimated beam is highest for directions close to this beam and diminishes with increasing $\Delta \theta$.

The foregoing results are consistent with trends which have been inferred from the predictions of the radiation field in a waterbody that experiences collimated irradiation [Houf and Incropera, 1980]. Using the method of discrete ordinates to
solve the equation of radiative transfer, it was found that, for directions other than that of the refracted collimated beam, the variance of the radiance with depth is strongly influenced by scattering from the beam. In particular, for a prescribed direction, the radiance increases with optical depth if the effects of scattering from the collimated beam into the direction contribute more radiation than the losses associated with absorption and scattering along the direction. As the collimated beam decays with increasing depth, however, the effect which scattering from the beam has on increasing the radiance becomes less than the effects which absorption and scattering from the direction of interest have on decreasing the radiance. Accordingly, the variation of the radiance with depth is characterized by a maximum.

The directional distribution of the radiance at any position in the water also depends strongly on the composition of the irradiation. For a large collimated contribution to the total irradiation, the directional distribution exhibits a pronounced maximum in the direction of the refracted collimated beam. In addition, due to strong forward scattering from the collimated beam, the radiance associated with directions close to the beam is also large. This behavior is illustrated by the two directional scans of Fig. 3.14, corresponding to separate days for which the collimated irradiation was 92% and 58% of the total irradiation. Both scans were obtained at approximately
Fig. 3.14 Directional radiance distribution of downwelling radiation at a depth of 6.35 mm when the collimated irradiation was approximately 92% and 58% of the total irradiation.
the same time of day and at a probe depth of 6.4 mm. In
the scan obtained for the 58% case, an unavoidable lapse of
15 minutes occurred in the data acquisition process at \( \theta_p = 0 \) deg.
Since the small reduction in the solar zenith angle which occurred
in that time caused a slight increase in the radiance, the scan
is represented by two symbols. With all other parameters being
nearly equal, differences in the two scans may be attributed to
a change in the composition of the irradiation. As the per-
centage of collimated radiation increases, the distribution be-
comes more sharply peaked about directions close to that of
the collimated beam. For completely overcast conditions \( G_{S,C} \leq 0 \),
there is no sharp peak, and the profile is characterized by a
much broader and flatter shape. Such profiles have been ob-
served in limited field measurements of this study, as well as
in the laboratory measurements involving a noncollimated radia-
tion source.

The results of Fig. 3.14 also reveal the effects of re-
fraction at the air-water interface. At this interface, all of
the transmitted solar radiation is concentrated within a cone
of half angle equal to 47.8 deg (the Snell circle). The small
amount of radiation which appears outside this cone is due to
scattering events which occur in the region \( 0 < z < 6.35 \) mm.

With increasing distance from the air-water interface, the
peak in the directional distribution of the radiance becomes less pronounced, as scattering interactions continue to redistribute the radiation. This trend is shown in Fig. 3.15, where directional distributions are plotted for three different depths. With increasing depth, the directional character of the incident radiation becomes less discernable, as scattering redirects radiation from more intense to less intense regions. Whereas absorption acts to decrease the radiance with increasing depth, scattering acts to promote a more uniform distribution. With increasing depth in a deep body of water, these processes cause the magnitude of the radiance to asymptotically approach zero, as the directional distribution approaches an isotropic condition [Preisendorfer, 1976].

All of the foregoing results pertain to downwelling radiation ($|\theta_p| < 90 \text{ deg}$), which results from transmission of solar radiation by the air-water interface. However, some of this radiation may experience multiple scattering interactions which result in the appearance of upwelling radiation ($|\theta_p| > 90 \text{ deg}$). Although the probability of such interactions is small, due to the existence of a phase function which is highly peaked in the forward direction, upwelling radiation was discernable, and representative vertical distributions are shown in Fig. 3.16. Legends for the figure are provided in Table 3.3, which includes
Fig. 3.15 Directional radiance distribution of downwelling radiation at different depths when the collimated irradiation was 56% of the total radiation.
Fig. 3.16  Vertical radiance profiles for upwelling radiation when the
collimated irradiation was approximately 80% of the total
irradiation.
### Solar Angles

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**Table 3.3** Conditions associated with the radiance distributions of Fig. 3.16
the probe angle, the solar time and angles, and the angle between the collimated beam and the detector for each distribution. Considering results obtained for the positive and negative values of $\theta_p$, independently, the trend is consistently one of decreasing radiance with increasing $\Delta\theta$. The trend is expected, since more scattering interactions are required to direct radiation into angles further removed from the collimated beam. When the data are considered collectively, however, the results for positive and negative values of $\theta_p$ are not characterized by a consistent trend of decreasing radiance with increasing $\Delta\theta$. In particular, radiance distributions associated with positive $\theta_p$ generally exceed those associated with negative $\theta_p$. This discrepancy is attributed to limitations in the experimental system, and not to the influence of additional independent variables. Although an antireflective coating was used to minimize sidewall reflections, some reflection did occur and hence could influence the small radiance associated with the upwelling radiation. Since the western sidewall ($+\theta_p$) was exposed to more collimated radiation during the experiment, reflections would influence distributions associated with $+\theta_p$ more than those associated with $-\theta_p$.

From Fig. 3.16 it is evident that, for $|\theta_p| < 140$ deg, the vertical distributions are characterized by a maximum at a finite
depth $z_m > 0$. This behavior may again be attributed to the effect of radiation scattering from the collimated beam. Radiation may be redirected into angles significantly removed from this beam, although the effect becomes less pronounced with increasing $\Delta \theta$.

The directional distribution of the upwelling radiation is plotted in Fig. 3.17 for four different probe depths, and the corresponding conditions are summarized in Table 3.4. Radiance values are due primarily to multiple scattering, and to a lesser extent, to sidewall reflections. For each distribution, the radiance decreases with increasing $|\theta_p|$ from 90 deg to 180 deg, but the distributions are not symmetric about 180 deg. This asymmetry is due to the effect of the angle $\Delta \theta$. For the position of the sun during the measurements, the angle between the collimated beam and the detector was in the range $68.1 \leq \Delta \theta \leq 78.4$ deg for $\theta_p = -90$ deg and $101.6 \leq \Delta \theta \leq 10.8$ deg for $\theta_p = 90$ deg. For all negative values of $\theta_p$, the radiance exceeded that associated with the corresponding positive value of $\theta_p$ due to the smaller values of $\Delta \theta$. It is also evident that, for probe angles in the range $-90 \deg > \theta_p > -125$ deg, the maximum radiance occurs at depths $z_m > 0$, as scattering from the collimated beam dominates the effect of attenuation due to absorption. For negative values
Fig. 3.17 Directional radiance distribution of upwelling radiation at different depths when the collimated irradiation was approximately 80% of the total irradiation.
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Table 3.4 Conditions associated with the radiance distributions of Fig. 3.17
of $\theta_p \leq -125$ deg and for all positive values of $\theta_p$, the collimated radiation must undergo a large number of scattering interactions to be redirected into the corresponding value of $\Delta \theta$, and hence is significantly attenuated by absorption. Note that, due to the orientation of the test cell, reflections would be strongest from the western sidewall and hence would contribute disproportionately to the radiance for $90$ deg $\leq \theta_p \leq 180$ deg. The fact that radiance values are, instead, large for $-180$ deg $\leq \theta_p \leq -90$ deg indicates that sidewall reflection effects are small compared to those of multiple scattering.
CHAPTER 4
COMPARISONS WITH THEORY

Although the asymmetry of the intensity distribution precludes the use of theoretical methods to predict the field measurements, such predictions are possible for the laboratory measurements. The radiation field within a planar, scattering-absorbing medium is determined by the equation of transfer, which may be expressed as

\[ \frac{dI_{\lambda}}{\mu d\tau_{\lambda}} = -I_{\lambda}(\tau_{\lambda}, \mu, \phi) + \frac{\omega_{\lambda}}{4\pi} \int_{0}^{2\pi} \int_{-1}^{+1} p_{\lambda}(\mu', \phi' \rightarrow \mu, \phi) I_{\lambda}(\tau_{\lambda}, \mu', \phi') d\mu' d\phi' \] (4-1)

The dependent variable is the spectral intensity, \( I_{\lambda}(\tau_{\lambda}, \mu, \phi) \), of the radiation at the optical depth, \( \tau_{\lambda} = \beta_{\lambda} z \) and propagating in a direction determined by the polar and azimuthal angles, \( \mu = \cos \theta \) and \( \phi \), respectively. The radiative properties of the medium required to determine the intensity include the extinction coefficient, \( \beta_{\lambda} \), the scattering albedo, \( \omega_{\lambda} \), and the scattering phase function, \( p_{\lambda} \). Each of these properties may be determined from the methods described in Section 2.1. For the conditions of this study, Eq. (4-1) is solved for incident radiation which is assumed to be uniform and diffuse. Fresnel reflection and refraction are assumed to occur at the air interface, and diffuse reflection is assumed to occur at the bottom. Two methods of
solutions have been used.

In the three-flux method [Incropera and Houf, 1979], the radiation field is separated into three isotropic components associated with the following regions (Fig. 2.4): Region 1 \((0 \leq \theta < \theta_c)\), Region 2 \((\theta_c \leq \theta < \pi/2)\), and Region 3 \((\pi/2 \leq \theta \leq \pi)\). The critical angle, \(\theta_c\), was chosen to be the angle into which all incoming radiation is refracted at the air interface and is determined from knowledge of the change in the index of refraction associated with the interface. For an air-water interface \(\theta_c = 0.85\) rad \((48.7^\circ)\). Previous measurements [Daniel, et al., 1978] have indicated that a large fraction of the downward propagating radiation in a diffusely irradiated aqueous suspension is concentrated in Region 1. The three-flux method may be used to determine the variation with optical depth of the radiation fluxes associated with the three regions \((F_1, F_2, F_3)\), from which the total forward flux \((F_f = F_1 + F_2)\), the total backward flux \((F_b = F_3)\), and the net flux \((F_n = F_1 + F_2 - F_3)\) may be determined.

In the more detailed method of discrete ordinates [Viskanta and Toor, 1972; Daniel, et al., 1979] the radiation field is divided into a finite number of directions (ordinates), and a discrete flux is assigned to each direction. Twenty-four ordinates were used in this study, and the method allowed for
for determining the directional distribution of the radiation, as well as the forward, backward and net radiation fluxes, as a function of optical depth.

The three-flux and discrete ordinate methods were used to calculate the radiation field for the experimental system of Fig. 2.4, using two different values of the bottom reflectance, \( \rho_d \approx 0.91 \) (highly reflecting) and \( \rho_d \approx 0 \) (perfectly absorbing). Calculations were performed for the late October sample, and the results are compared with the data of Figs. 3.10 and 3.11. Factors which can contribute to discrepancies between the experimental and theoretical results include: (i) departure of the directional distribution of the laboratory irradiation from the isotropic distribution assumed for the analysis, (ii) experimental uncertainties associated with the radiation intensity measurements, and (iii) experimental uncertainties associated with the radiative property measurements.

The radiation intensity measurements are compared with predictions based on the discrete ordinate method in Figs. 4.1 and 4.2 for the black and white bottom conditions, respectively. Both the experimental and theoretical results are normalized with respect to the intensity corresponding to \( z = z_0 \) and \( \theta = 0 \) rad. Predictions have been made for the actual directional distribution of the incident radiation (dashed curves), as well as for an isotropic distribution.
Fig. 4.1 Comparison of measured radiation intensities with predictions based on the discrete ordinate method (black bottom).
Fig. 4.2 Comparison of measured radiation intensities with predictions based on the discrete ordinate method (white bottom).
(solid curves), which is often assumed in theoretical treatments of the problem (Fig. 2.7).

It is evident that, for both the isotropic and actual directional distributions, the agreement between results is excellent throughout the central portion of Region 1. Predictions based on the isotropic distribution, however, consistently overpredict the data for that portion of the forward direction ranging from $\theta \approx 0.15\pi$ rad to $\theta \approx 0.3 \pi$ rad. This discrepancy is clearly due to the isotropic approximation, since the actual directional distribution of the irradiation is characterized by a significant departure from isotropy at an incident angle of $\theta^* \approx 40^\circ$ (0.22 $\pi$ rad). With an index of refraction of 1.333 for the aqueous suspension, this angle corresponds to a refracted angle of $\theta \approx 29^\circ$ (0.16 $\pi$ rad) in the suspension. Hence, for the assumed isotropic condition, the angle at which the discrete ordinate method begins to overpredict the measured intensities corresponds to the angle at which the incident radiation experiences a significant departure from isotropy. This interpretation is supported by comparing the data with predictions based on the measured directional distribution of the incident radiation (dashed curves). In this case there is far better agreement between the results for that portion of the forward hemisphere corresponding to $\theta \lesssim 0.3 \pi$ rad.
For portions of the forward direction corresponding to $0.3 \, \pi \, \text{rad} \leq \theta \leq 0.5 \, \pi \, \text{rad}$ and for the entire backward direction ($\theta > 0.5 \, \pi \, \text{rad}$), the data are underpredicted for both directional distributions of the irradiation. In this case differences are likely due to uncertainties associated with the radiation intensity and phase function measurements, both of which become large with increasing $\theta$. Since the trend characterizes both black and white bottom conditions, it is unlikely that the discrepancies can be attributed to uncertainties in predicting bottom reflection effects.

Comparisons between predictions and measurements may also be made in terms of radiation fluxes. Such fluxes are determined by integrating the radiation intensity over direction and, in normalized form, may be obtained from Eq. (2-7). Using the discrete ordinate method, calculations have been performed to obtain a forward flux, $\bar{F}_f$, which encompasses Regions 1 and 2 of Fig. 2.4, a backward flux, $\bar{F}_b$, which encompasses Region 3, and a net flux, $\bar{F}_n = \bar{F}_f - \bar{F}_b$. Comparisons are made in Figs. 4.3 and 4.4 for the black and white bottom conditions, respectively, and the results are consistent with the trends of Figs. 4.1 and 4.2. The excellent agreement between results for the variation of $\bar{F}_f$ with optical depth is due to the accuracy associated with discrete ordinate predictions of the radiation.
Fig. 4.3 Comparison of measured fluxes with predictions based on the discrete ordinate method (black bottom).
Fig. 4.4 Comparison of measured fluxes with predictions based on the discrete ordinate method (white bottom).
intensity over the central portion of Region 1. Note that, once normalized, there is negligible difference between predictions based on the actual directional distribution of the irradiation and the assumed isotropic distribution. Under-prediction of the backward flux is attributed to uncertainties in the phase function and/or the intensity measurements for this direction.

Comparisons of the experimental fluxes with predictions based on the three-flux method are made in Figs. 4.5 and 4.6 for the black and white bottom conditions, respectively. Five normalized fluxes are considered: the total forward flux, $\bar{F}_f$, the Region 1 and Region 2 components of the forward flux, $\bar{F}_1$ and $\bar{F}_2$, respectively, the backward flux, $\bar{F}_3$, and the net flux, $\bar{F}_n$. In general the agreement with the data is less satisfactory than that obtained from the discrete ordinate method, with $\bar{F}_f$, $\bar{F}_2$, and $\bar{F}_3$ being significantly underpredicted throughout the suspension. This result may be due to uncertainties in the experimental phase function, which is used to generate scattering fractions required for the three-flux method, and/or to the assumption of isotropic radiation over the entire region associated with each of the three fluxes. However, predictions are in reasonable agreement with data for the Region 1 forward flux and for the net flux.
Fig. 4.5 Comparison of measured fluxes with predictions based on the three-flux method (black bottom).
Fig. 4.6 Comparison of measured fluxes with predictions based on the three-flux method (white bottom).
CHAPTER 5

SUMMARY

Radiation measurements have been made in aqueous suspensions under both laboratory and field conditions. In the laboratory, the directional distribution of the incident radiation was nearly isotropic up to an angle of $\theta \approx 40^\circ$ (0.70 rad) from the nadir, but was characterized by a reduction of the intensity with increasing angle beyond $40^\circ$. The laboratory radiation measurements have been compared with predictions based on the discrete ordinate and three-flux methods, and the key conclusions of the study are as follows:

1. The directional distribution of the measured radiation just below the interface is concentrated within a cone of half angle $\theta \approx 48.7^\circ$ (0.27 $\pi$ rad), and much of the radiation which is not absorbed remains within this cone throughout the suspension. Some radiation is scattered out of the cone, but, because the phase function is highly forward peaked, the amount is small. This behavior is likely to characterize radiation transfer in any body of water for which the directional distribution of the irradiation is approximately isotropic.

2. The effects of bottom reflection are discernable for the conditions of this study. In the lower half of the suspension, intensities associated with the backward direction are
significantly larger for a white (highly reflecting) bottom than for a black bottom. Such differences would become more pronounced with decreasing optical depth, \( \tau_d \).

3. The directional distribution of the radiation intensity within the suspension is well predicted by the discrete ordinate method for \( \theta \leq 29^\circ \) (0.16 \( \pi \) rad). In the region \( 29^\circ \) (0.16 \( \pi \) rad \( \leq \theta \leq 54^\circ \) (0.3 \( \pi \) rad), however, the data are overpredicted, and the effect is attributed to differences in the directional distribution of the experimental and theoretical irradiation fields. For the remaining sideward and backward directions, 0.3 \( \pi \) rad \( \leq \theta \leq \pi \) rad, the data are underpredicted by the discrete ordinate method. The effect is attributed to large uncertainties in the phase function measurement for these directions. Similarly, although good agreement is obtained between measured and predicted values of the forward flux, the radiation flux associated with the backward direction is underpredicted by the discrete ordinate method. The ability to accurately predict radiation intensities and fluxes for the backward direction depends strongly on the availability of accurate phase function data for this direction.

4. Radiation flux data associated with both the forward and backward directions are underpredicted by the three-flux method.
Radiation measurements made under field condition were strongly influenced by the existence of a collimated beam, and several unique features may be attributed to scattering from this beam. The effect of such scattering is to redistribute radiation from the direction of the collimated beam into adjoining directions. One consequence of the effect is that, for small to moderate values of the angle between the collimated beam and the line of sight of the probe, $\Delta \theta$, the vertical distribution of the intensity is characterized by a maximum below the air-water interface. The existence of this maximum becomes more pronounced as the ratio of the collimated radiation to the total irradiation increases. This ratio also influences the directional distribution of the intensity, which is characterized by a maximum at the smallest value of $\Delta \theta$ associated with an angular scan. However, as the ratio decreases, the maximum becomes less pronounced and the distribution more closely approximates isotropic conditions. A redistribution of the radiation also occurs with increasing depth, as the effects of scattering and absorption combine to provide a more nearly isotropic field.
REFERENCES


