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Abhijit. A. Sathe  
*Purdue University*

Eckhard A. Groll  
*Purdue University*

S V. Garimella  
*Purdue University, sureshg@purdue.edu*

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# Dynamic analysis of an electrostatic compressor

**Abhijit A Sathe, Eckhard A Groll<sup>†</sup> and Suresh V Garimella**

NSF Cooling Technologies Research Center, School of Mechanical Engineering

Purdue University, West Lafayette, Indiana 47907, USA

**Abstract:** This paper presents an analytical approach for modeling the transient dynamic forces in a diaphragm compressor which operates under the action of an electrostatically actuated diaphragm. An experimentally validated, quasi-static model for a diaphragm compressor for electronics cooling was previously developed in which dynamic effects were neglected. In the new model, the dynamic forces induced due to the finite time necessary for deflection of the diaphragm are taken into consideration using the segmentation approach developed earlier. Results from the analytical model compare favorably with those from a detailed numerical simulation as well as with experimental measurements available in the literature. The analytical dynamic model is applied to two different pumping devices to illustrate the effects of the dynamic forces on the overall performance of the device. The effect of pumping frequency of the device on the operating voltage is also explored.

**Keywords:** Diaphragm compressor; Refrigeration; Damper; Simulation.

**Nomenclature:**

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<sup>†</sup> Corresponding author. Email: groll@purdue.edu

$C$	damping coefficient of gas film ( $\text{N s m}^{-1}$ )
$F$	force ( $N$ )
$h$	thickness of squeeze-film (m)
$\dot{h}$	velocity of diaphragm ( $\text{m s}^{-1}$ )
$\ddot{h}$	acceleration of diaphragm ( $\text{m s}^{-2}$ )
$K$	elastic stiffness of diaphragm ( $\text{N m}^{-1}$ )
$k_B$	Boltzmann constant ( $\text{J s}^{-1}$ )
$Kn$	Knudsen number
$m$	mass of diaphragm (kg)
$P$	pressure (Pa)
$R$	radius of diaphragm (m)
$T$	temperature (K)
$t_d$	diaphragm thickness (m)
$V$	volume of chamber ( $\text{m}^3$ )
$y$	depth (m)

#### Greek

$\lambda$	mean free path (m)
$\rho$	density ( $\text{kg m}^{-3}$ )
$\Delta P$	pressure rise (Pa)
$\varphi$	fluid particle diameter (m)
$\mu$	viscosity (Pa s)
$\sigma_d$	squeeze number
$\omega$	frequency of oscillation (Hz)

## 1. INTRODUCTION

Miniature-scale vapor compression refrigeration systems have received increased recent attention for use in electronics-cooling applications (Trutassanawin et al., 2006). An

electrostatically actuated diaphragm compressor offers promise for use in miniature-scale vapor compression systems because of its potential for high efficiency, compactness and scalability. An electrostatically actuated diaphragm compressor (schematically represented in Figure 1) consists of an oval shaped cavity enclosing a thin circular diaphragm clamped at its circumference. The diaphragm is electrically actuated by supplying a DC voltage between the diaphragm and the chamber wall, which generates an electrostatic field inside the chamber. This leads to a deflection of the diaphragm towards the chamber wall, thereby compressing a refrigerant vapor trapped in the chamber, the flow of which is controlled through inlet manifolds and discharge valves. An analytical model for such a diaphragm compressor was previously developed and experimentally validated by the authors (Sathe et al., 2008). The model was used to conduct a detailed optimization study in which the diaphragm compressor array required for removing heat from electronics was identified for two applications: a desktop computer with 200 W cooling capacity and a laptop computer with 80 W cooling capacity (Sathe et al., 2009). In the analytical simulation model, the domain was divided into a number of radial segments and a quasi-static force balance on the diaphragm was carried out for each segment so that the pull-down voltage could be calculated. Details of the principle of operation, analytical modeling and experimental evaluation of the diaphragm compressor are available in Sathe et al. (2008), and are not repeated here. The analysis assumed that the deflection of the diaphragm in each segment was a quasi-static process, *i.e.*, the diaphragm deflects in a series of extremely small steps. The dynamic forces on the diaphragm, developed because of the finite duration of zipping of the diaphragm from the circumference to the center, were therefore neglected. In practice, a diaphragm compressor operates at the finite frequency of diaphragm oscillation. Hence, a dynamic analysis of the diaphragm is necessary to understand the effect of the operating frequency

on the compressor performance. The dynamic forces include the damping force exerted on the diaphragm by the refrigerant being compressed, and the inertia force resulting from the finite mass of the diaphragm.

A gas film between two closely spaced parallel plates oscillating in normal relative motion (Figure 2) generates a force due to compression and internal friction, which opposes the motion of the plates, so that an additional force ( $F_{damp}$ ) develops in the system as shown in Figure 3 (Li and Hughes, 2000). As the fluid is squeezed out from the edges of the plate, the viscous drag during the flow creates a dissipative mechanical force on the moving plate, the direction of which is opposite to the movement of the plate. Thus, the fluid film acts as a damper that opposes the motion of the plate because of the so-called squeeze-film damping (Griffin et al., 1966; Bao and Yang, 2000; Senturia, 2001). The magnitude of squeeze-film damping is a function of the gap between the two flat plates and the velocity of the moving plate. Hence, the smaller the gap and the higher the velocity, the larger is the damping force. For an electrostatically actuated flat-plate capacitor system, the distance between the capacitor plates is minimized and the area of the electrodes is maximized in order to improve the efficiency of actuation. Under such conditions, squeeze-film damping is accentuated. This effect has also been well studied and mathematically modeled (Nayfeh and Younis, 2004, Younis and Nayfeh, 2007; Veijola et al., 1998).

While these studies in the literature provide useful information on the phenomenon of squeeze-film damping in a flat-plate electrostatic system (such as a MEMS switch), the mathematical damping force calculation models described cannot be directly used for the current analysis, where the domain is non-linear. To the authors' knowledge, a

mathematical model for squeeze-film damping of a curved electrostatic actuator including a combined force balance of static and dynamic forces is not available in the literature.

The present work models the damping force in the non-linear (curved) compressor chamber by extending the segmentation modeling approach developed in Sathe et al. (2008). The results obtained with the dynamic model are compared to those predicted by the quasi-static model for understanding the influence of the dynamic forces. Predictions from the dynamic model are also validated against experimental results from the literature.

## 2. MODELING OF DYNAMIC FORCES

The deflection of the diaphragm under an electrostatic force causes certain static and dynamic effects. Senturia (2001) represented the static and dynamic forces acting on the diaphragm using a spring, mass and damper model for a parallel-plate capacitor (the gas force was not included), as shown in Figure 2. A force balance on the diaphragm can be represented as (Senturia, 2001):

$$F_{electro} = F_{elastic} + F_{damping} + F_{inertia} \quad (1)$$

Adding the gas force and substituting the relationships for the elastic, damping and inertia forces results in:

$$F_{electro} = F_{gas} + K \cdot (h_{max} - h) + C \cdot \dot{h} + m \cdot \ddot{h} \quad (2)$$

Modeling of the static forces in this force balance was described in Sathe et al. (2008). The damping force of the fluid and the inertia force of the diaphragm are analyzed in the following.

## 2a. Squeeze-film damping

Squeeze-film damping is modeled using the Reynolds equation, which is derived by combining the continuity equation, the Navier-Stokes equations, and the ideal gas law (Nayfeh and Younis, 2004). The following assumptions are made:

1. The gas in the gap can be treated as a continuum.
2. The gap  $h$  is very small compared to the radius of the plate  $R$ .
3. The gas flow between the plates is laminar and primarily viscous.
4. The gas between the plates is incompressible.

The validity of the first assumption is dependent on the Knudsen number, which is defined as the ratio of the mean free path of the gas molecules to the gap between the two plates as:

$$Kn = \frac{\lambda}{h} \quad (3)$$

It may also be expressed as:

$$Kn = \frac{k_B \cdot T}{\sqrt{2} \cdot \pi \cdot \varphi^2 \cdot P \cdot d} \quad (4)$$

Characteristic length  $d$  for a dome-shaped compressor chamber is defined as the maximum chamber depth. For a chamber suction pressure of 572 kPa with refrigerant R134a as the working fluid, the Knudsen number is calculated to be  $3.48 \times 10^{-4}$ .

When the Knudsen number is smaller than 0.01, it is reasonable to assume a no-slip boundary condition to describe the continuum flow. A continuum flow regime may therefore be assumed in the present case since the Knudsen number is much smaller than 0.01.

The damping force consists of two components: viscous damping related to the viscous flow of the fluid, and elastic damping related to the compression of the fluid (Nayfeh and Younis, 2004). The relative importance of the viscous to the elastic damping force is expressed in terms of the squeeze number. The squeeze number of the damping system is given as (Senturia, 2001):

$$\sigma_d = \frac{12 \cdot \mu \cdot R^2}{P \cdot h_0^2} \cdot \omega \quad (5)$$

For a small squeeze number, the viscous damping force dominates and the gas film is not appreciably compressed. The Reynolds equation is applicable for such an incompressible gas.

Assuming  $R = 10$  mm,  $P = 572$  kPa,  $h_0 = 50$   $\mu$ m,  $\omega = 90$  Hz, and  $\mu = 11.88 \times 10^{-6}$  Pa-s, the squeeze number for the given system is calculated using Equation (5) to be  $8.25 \times 10^{-4}$ . Hence, the gas can be considered incompressible in that the elastic damping force generated due to compression of the gas is negligible compared to the viscous damping

force generated due to the viscosity of gas. Therefore, the squeeze-film damping coefficient of the gas can be estimated by using the Reynolds equation in a polar coordinate system as (Bao and Yang, 2000):

$$\frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial}{\partial r} P(r) \right) = \frac{12 \cdot \mu}{h^3} \dot{h} \quad (6)$$

where  $\dot{h}$  is the velocity of the moving plate and  $P(r)$  is the damping pressure on the plate.

It is noted that the Knudsen number and the squeeze number calculated in equations (4) and (5), respectively, are used only for validating the assumptions listed above, and not in the actual force balance equations.

The squeeze-film damping effect discussed thus far is based on a parallel flat-plate system (Figure 2 and Figure 3). The chamber-diaphragm system of interest in a diaphragm compressor, however, does not consist of parallel plates due to the non-linear profile of the compressor chamber (Sathe et al., 2008). For estimating the damping coefficient of this non-linear system, the domain is divided into multiple annular flat plate systems, for which an expression for damping coefficient is available in the literature. For an annular plate (Figure 4), Equation (6) is solved to give the coefficient of damping force as (Bao and Yang, 2000):

$$C = \frac{3 \cdot \pi \cdot \mu \cdot R_o^4}{2 \cdot h^3} \cdot \xi \quad (7)$$

where  $\xi$  is a function of the radii of the annular plate and is given as:

$$\xi = 1 - (R_i / R_o)^4 + \frac{(1 - (R_i / R_o)^2)^2}{\ln(R_i / R_o)} \quad (8)$$

Equation (7) is the equation for the squeeze-film damping coefficient of an annular flat plate. Some additional assumptions are made for the present analysis of the diaphragm compressor:

- In the squeeze-film damping theory for the parallel-plate capacitor, the gas is assumed to escape uniformly along the circumference between the two plates. For the diaphragm compressor, however, as the diaphragm zips to the chamber surface, the movement of the gas in the unzipped portion of the chamber occurs only in one direction: from the circumference to the center. Figure 5 shows the deflection of a moving plate in a flat-plate system vis-à-vis zipping actuation of the diaphragm inside the compressor chamber, where the three different line styles represent the deflection/zipping at different time steps. Since the depth of the dome-shaped chamber is negligibly small compared to its radius, any effects of this one-directional movement of the gas in the chamber on the damping coefficient are considered negligible, and the damping coefficient is still estimated using the same equation used for the parallel-plate capacitor.
- Once the diaphragm starts zipping inside the chamber, its average vertical velocity remains constant. The average diaphragm velocity is computed as the ratio of the maximum diaphragm deflection to the actuation time. The latter is a function of pumping frequency. Of the two governing parameters – operating voltage and pumping frequency – one may be controlled allowing the other to change. In the present analysis, the pumping frequency is fixed and its effect on the operating voltage studied. The average diaphragm velocity is a function of the pumping

frequency and is, therefore, constant for a constant frequency. The variation in the local vertical velocity of the zipping diaphragm front is not considered in the present analysis.

In order to be able to use Equation (7) for the non-linear compressor geometry, this analysis takes advantage of the segmentation method introduced and developed in Sathe et al. (2008). For the non-linear geometry of the diaphragm compressor chamber, an arrangement of multiple parallel dampers with different initial gaps ( $h$ ) is used as shown in Figure 6. Equation (7) is modified to calculate the damping coefficient of an individual annular-element damper using the following generalized form:

*For*  $i = 1 : N$

$$C_{seg,i} = \frac{3 \cdot \pi \cdot \mu_{seg} \cdot R_i^4}{2 \cdot y_{seg,i}^3} \cdot \xi_{seg,i} \quad (9)$$

where,

$$\xi_{seg,i} = 1 - (R_{i+1}/R_i)^4 + \frac{(1 - (R_{i+1}/R_i)^2)^2}{\ln(R_{i+1}/R_i)}$$

where  $\mu_{seg}$  is the dynamic viscosity of the fluid. An adiabatic compression process is assumed in the chamber and the viscosity of the fluid at each segment is calculated for the temperature and pressure of the fluid in that segment.

The segment damping coefficient is the sum of the damping coefficients of the individual dampers as (Yang and Li, 2006):

*For*  $i = 1 : N$

$$C_i = \sum_{j=i}^N C_{seg,j} \quad (10)$$

Thus, for any segment  $i$ , the coefficient  $C_i$  represents the overall damping coefficient for squeeze-film damping which includes contributions from all other segments constituting the non-linear chamber shape.

As explained in the assumptions, the velocity of the diaphragm is calculated from the net vertical deflection of the diaphragm from the neutral position and the time required for this deflection; the time is, in turn, calculated as the reciprocal of the operating frequency of the diaphragm compressor.

$$t = \frac{1}{Freq} \quad (11)$$

$$Vel = \frac{y_{max}}{t}$$

The damping force on the diaphragm is calculated as the product of the damping coefficient and the velocity of the diaphragm. Hence, the damping force is given as:

$$For\ i = 1:N \quad (12)$$

$$F_{damp,i} = C_i \cdot Vel$$

Equation (12) is used for the estimation of the squeeze-film damping force on the diaphragm at the given operating frequency.

## **2b. Diaphragm inertia force**

The mechanical inertia force on the diaphragm is a function of mass and acceleration of the diaphragm. The mass of the diaphragm is estimated as a product of its volume and density as:

$$\begin{aligned} & \text{For } i = 1: N \\ & m_i = \pi \cdot R_i^2 \cdot t_d \cdot \rho \end{aligned} \quad (13)$$

where  $t_d$  is the thickness of the diaphragm and  $\rho$  is the density of the diaphragm material. The acceleration of the diaphragm starts with a non-zero value and it reduces to zero as the diaphragm velocity remains constant. Hence, the inertia force of the diaphragm is only significant at the start of the zipping actuation.

$$\begin{aligned} & \text{For } i = 1: N \\ & F_{iner,i} = m_i \cdot a \end{aligned} \quad (14)$$

Accounting for the two dynamic forces on the diaphragm discussed above (Equations (12) and (14)), the final force balance on the diaphragm is written as:

$$\begin{aligned} & \text{For } i = 1: N \\ & F_{electro,i} = F_{elastic,i} + F_{gas,i} + F_{damping,i} + F_{iner,i} \end{aligned} \quad (15)$$

These equations are solved using MATLAB<sup>1</sup>.

### 3. VALIDATION OF THE DYNAMIC MODEL

Predictions of the diaphragm actuation time obtained from the analytical dynamic model developed in the present work are compared to those obtained using the computational model and experimental measurements of Athavale et al. (1999). A bidirectional diaphragm pump was investigated in Athavale et al. (1999) based on electrostatic actuation for pumping large flow rates of air. The pump was modeled using coupled

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<sup>1</sup> MATLAB is a registered product of The Mathworks, Natick, MA, USA.

transient computational flow simulations wherein separate solvers were developed for elastic, electrostatic and fluid flow processes and coupled to each other. A contact model was developed to simulate diaphragm contact with the electrodes. A squeeze-film resistance to the diaphragm motion due to the viscous nature of the fluid was also modeled. The diaphragm inertia force was, however, not included. The simulations predicted the actuation time and fluid flow rate for a given applied voltage. The experiments were conducted on a prototype using air as the working fluid. The geometry of the chamber and other operating parameters considered in Athavale et al. (1999) are listed in Table 1. The computed volumetric efficiency of the pump was 87%. The 13% loss in the pumped volume comprised of two loss mechanisms: the reverse flow loss through the suction ports, and the dead volume (clearance volume) at the end of the zipping actuation stroke. However, no estimate of the pump dead volume was provided. Since the maximum diaphragm deflection is a function of the dead volume, the present analytical model considered the following two extreme cases:

1. The dead volume is assumed zero, which indicates that all the volumetric loss is through the suction port. Thus, the volumetric efficiency for the analytical model is assumed to be 100%.
2. The dead volume is assumed 13%, which indicates that all the volumetric loss for the pump is due to the dead volume. The volumetric efficiency for the analytical model in this case is assumed to be 87%.

With the stated assumptions and the operating parameters specified in Table 1, the time required for the diaphragm to deflect to the chamber surface from the equilibrium position as a function of applied voltage was computed from the dynamic model developed in the present work (Figure 7). The results from the computational model and the experimental

measurements of Athavale et al. (1999) are also plotted. In general, the actuation time decreases with an increase in the applied voltage as expected. At higher applied voltages, the stronger electrostatic field in the chamber helps the diaphragm to overcome the viscous forces more readily, thereby increasing the velocity of the diaphragm and reducing the actuation time. The two limiting curves are obtained for the cases with volumetric efficiencies of 100% (upper limit) and 87% (lower limit). The actuation time predicted and measured by Athavale et al. (1999) falls generally within these limits. While the computational model of Athavale et al. (1999) predicts a near-linear relation between the actuation time and the applied voltage, the analytical model developed in the present work predicts that the actuation time is inversely proportional to the square of the applied voltage. This non-linear relation is a more faithful representation of the system behavior and would be expected from physical arguments.

#### **4. APPLICATION OF THE DYNAMIC MODEL**

The dynamic model of an electrostatically actuated diaphragm-based pumping device which has been developed and validated above is applied in the following to a refrigerant compressor and a water pump to understand the effects of dynamic forces on the overall performance of the devices.

##### **4a. Electrostatic actuation of a refrigerant compressor**

The diaphragm compressor for use in a miniature refrigeration system for electronics cooling that was proposed in Sathe et al. (2008) is considered. The important parameters of the diaphragm refrigerant compressor are given in Table 2. The quasi-static forces on

the diaphragm are compared to the dynamic forces estimated using the analysis described in Section 2. A non-dimensional radius defined in Sathe et al. (2008) is used for plotting the variations of the squeeze-film damping force and the diaphragm inertia as shown in Figure 8. The damping force asymptotically increases as the diaphragm zips from the circumference to the center. As the gas is compressed, its viscosity increases; also, as the diaphragm moves closer to the chamber surface, the overall gap between the diaphragm and the chamber decreases which results in a higher damping coefficient (Equation (9)). Both these effects contribute equally to the increase in damping force with progressive zipping. It is noted that the magnitude of the dynamic forces is negligible compared to the static forces calculated in Sathe et al. (2008) for the same compressor. This is mainly because of the very low viscosity of refrigerant R134a vapor. The diaphragm inertia force is also very small and does not affect the diaphragm force balance.

The variation of the pull-down voltage with the non-dimensional radius from the static and dynamic analyses is shown in Figure 9. The static pressure rise in the chamber and the shape of the pull-down voltage curve are discussed in detail in a previous publication by the authors (Sathe et. al., 2008). Since the dynamic forces are significantly lower than the static forces, the pull-down voltage is not affected by the inclusion of dynamic effects in the model. Hence, for all practical purposes, the dynamic effects in the chamber of the given diaphragm compressor can be neglected.

#### **4b. Electrostatic actuation of a liquid pump**

As a second application of the dynamic model, an electrostatically actuated liquid pump is analyzed for use, for example, in liquid cooling of electronics. Based on the pressure drop

expected in a typical microchannel heat sink for electronics cooling (Liu and Garimella, 2004), a pressure rise of 30 kPa is selected for a water pump. The operating principle of the pump is essentially the same as that of the electrostatic compressor described in Sathe et al. (2008). Since liquid water is an incompressible fluid, the fluid pressure force on the diaphragm is estimated as (Moran and Shapiro, 2008):

$$\begin{aligned} & \text{For } i=1 \text{ to } N: \\ & F_{water,i} = \frac{V \cdot \Delta P}{y_{max} - y_{seg,i}} \end{aligned} \quad (16)$$

where  $V$  is the volume of the chamber and  $\Delta P$  is the required pressure rise in the pump chamber. Details of the chamber geometry and the design parameters chosen are included in Table 2. The elastic force and electrostatic force are calculated using the static model (Sathe et al., 2008) while the dynamic forces are calculated using the dynamic model developed in this paper. The variation of these forces with the non-dimensional radius for an operating frequency of 100 Hz is shown in Figure 10. Because of the higher viscosity of the working fluid and the shallower profile of the pump chamber, the effect of the damping force is much more pronounced in this case than for the refrigerant compressor considered above. The diaphragm inertia force still remains negligibly small. The effect of the dynamic forces on the pull-down voltage for the pump is shown in Figure 11, where the pull-down voltage is plotted as a function of the non-dimensional radius for the static and the dynamic analyses. Clearly, a higher pull-down voltage is required to zip the diaphragm when the damping forces are taken into account.

Finally, the effect of frequency on the pull-down voltage is studied by varying the pump operating frequency from 0.1 Hz to 100 Hz. Figure 12 shows the variation of pull-down

voltage for different operating frequencies. At very low operating frequencies, the time-dependent dynamic effects are minimal and the dynamic model predictions reduce to those from the static model. As the frequency is increased, the dynamic effects, mainly due to the viscous damping force, assume greater significance and result in an increase in the pull-down voltage of the pump. Since the static model is independent of the operating frequency, it does not vary with the frequency.

## **5. CONCLUSIONS**

The dynamic effects of diaphragm movement in a pumping device based on electrostatic zipping of the diaphragm to a chamber surface are analyzed. The dynamic forces include the squeeze-film damping of the fluid and the inertia force of the diaphragm. A segmentation-based approach is used for simulating the viscous damping force by dividing the domain into multiple parallel segments, each segment representing a damper with different area and initial separation. The squeeze-film damping force is calculated as a product of the equivalent damping coefficient of the chamber and the velocity of the diaphragm. The squeeze-film damping coefficient is strongly dependent on the viscosity of the fluid. The diaphragm inertia force is also modeled by calculating the mass of the unzipped portion of the diaphragm and its acceleration. The analytical dynamic model predictions are validated based on satisfactory agreement with experimental results and predictions for the diaphragm actuation times as a function of applied voltage from a detailed computational model in the literature.

The analytical model is applied to illustrate the effects of the dynamic forces in an electrostatic refrigerant compressor and an electrostatic liquid pump. Inclusion of dynamic effects had a negligible impact on the operation of the refrigerant compressor, mainly due to the low viscosity of the refrigerant vapor. For the electrostatic liquid pump, on the other hand, the squeeze-film damping force is found to be significant and affects the diaphragm pull-down voltage at higher pumping frequencies. The effect of frequency on the pull-down voltage of the pump is also studied. At very low pumping frequencies, the dynamic model returns the results of the static model since the diaphragm moves slowly enough to simulate quasi-static deflection.

The analytical dynamic model developed in this work may be used for simple yet accurate predictions of dynamic performance of such pumping devices, and is preferable to more complicated numerical models. The analysis relies on physics-based equations and can be modified to accommodate different chamber geometries, different material properties and different working fluids.

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## **Figure Captions**

Figure 1. Schematic layout of an electrostatic compressor.

Figure 2. Senturia's (2001) spring, mass and damper model for parallel-plate capacitor.

Figure 3. (a) Illustration of squeeze-film damping, and (b) profile of damping pressure.

Figure 4. Damping coefficient for an annulus position close to a fixed plate.

Figure 5. Direction of gas escape (represented by block arrows) under the moving plate for (a) a parallel-plate system, and (b) a non-linear diaphragm compressor with zipping actuation.

Figure 6. Damping coefficient for a non-linear chamber geometry.

Figure 7. Variation of diaphragm actuation time with applied voltage from the present analysis and from the computational model and measurements of Athavale et al. (1999).

Figure 8. Variation of dynamic forces with non-dimensional radius for the refrigerant compressor. The inertia force is very small compared to the damping force (inset).

Figure 9. Variation of pull-down voltage with non-dimensional radius for static and dynamic analyses of the refrigerant compressor.

Figure 10. Variation of forces on diaphragm with non-dimensional radius for the liquid pump. The diaphragm inertia force is very small compared to the other forces and essentially lies on the x-axis.

Figure 11. Variation of pull-down voltage with non-dimensional radius for static and dynamic analyses of the liquid pump.

Figure 12. Variation of pull-down voltage with operating frequency for static and dynamic analyses of the liquid pump.