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EFFECT OF VALVE PORT GAS INERTIA ON VALVE DYNAMICS - PART I
SIMULATION OF A POPPET VALVE

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ABSTRACT

A theoretical model was developed of the nonlinear dynamic response of a reciprocating air compressor discharge valve system considering unsteady flow. It constitutes an extension of previous modeling procedures where the flow was considered in a quasi steady manner. The most simplified valve system was used which consists of a mechanical spring loaded circular disc valve centered over a single circular orifice. Coupled unsteady nonlinear flow and disc valve motion equations were developed by treating the flow by a sequence of finite control volumes. Unsteady continuity and momentum equations were written for each control volume. The flow in the orifice, valve passage and piston cylinder was modeled as compressible. Contraction coefficients were incorporated to account for streamline effects. The Kirchhoff contraction coefficient was assumed in the orifice area, while the entrance and exit discharge coefficients in the disc valve passage are assumed from experimental flow-force data. Viscosity was included between the disc valve and seat.

INTRODUCTION

The popular assumption in mathematically modeling reciprocating air compressors, is the use of quasi steady flow theory REF. (1-3). In general, this approach assumes that the influence of the motion of the fluid stream is uncoupled from the dynamics of the valve element. The flow through the valve passage is treated in terms of a sequence of steady state processes. Usually the introduction of experimental data, or in the case of simple valve geometry, explicit expressions for the steady state flow and valve lift force, is required. As such, the interaction between the flow and the valve element which occurs due to the feedback motion of the valve element is neglected. In the most recent application of quasi steady flow theory to the modeling of the dynamic behavior of discharge and suction valves REF. (4), a comparison with measured suction valve motion makes one suspect that it is important to include unsteady flow in certain cases.

To demonstrate this, the most simple discharge valve geometry was considered which consists of a spring loaded disc valve element situated over a single square-edge circular orifice FIG.(1). The flow of air through the discharge valve is produced by a reciprocating piston driven by a connecting rod and crank arm mechanism.

In modeling the system, unsteady flow was considered, incorporating the inertia of the gas in the valve passage. The unsteady conservation of mass and momentum laws were developed which relied on a lumped parameterization procedure utilizing finite fluid elements to predict the dynamic response of the flow. This procedure was previously used successfully to predict the sound radiated from a reciprocating compressor orifice without the valve element REF.(5) and the nonlinear response of a pneumatic-mechanical system REF.(6). Four elements were chosen FIG.(2). The first two fluid elements were located in the piston cylinder chamber and circular orifice section and the latter two fluid elements between the spring loaded disc valve and the seat. Compressibility was included by thermodynamically modeling the flow as adiabatic. Viscosity was included in the disc valve passage. This modeling procedure also required a set of contraction coefficients for streamline effects. A contraction coefficient was assumed in the orifice section. Contraction coefficients at the entrance and exit section of the disc valve were determined from measured static discharge and lift force coefficient data. The finite fluid elements were coupled together by the mass and momentum flux and the thermodynamic pressures across adjacent control surfaces. The equation of motion of the mechanical spring loaded disc valve element was coupled to the flow by the stagnation pressure.

In the following, the resulting equations
are presented to predict the coupled flow and valve response from a reciprocating air compressor discharge valve system. Although this development was carried out specifically for a discharge valve system, it is possible to examine the behavior of suction valves with this model. This can be accomplished under certain restrictive conditions which permit the discharge valve response to bear some similarity to that of a suction valve.

THEORETICAL MODEL AND ASSUMPTIONS

The theoretical modeling of the coupled flow valve response in an air compressor requires the application of the laws of fluid dynamics and a description of the flow pattern associated with the motion of the valve element. In most instances, these laws are written in the field form which requires both temporal and spatial evaluation of the system to be modeled. A first modeling procedure would be, of course, a revised quasi steady approach which would include provisions for unsteady flow. Such a technique would require the addition of unsteady flow and force coefficients which may be deduced from experiments or theory. However, a direct evaluation of these coefficients may give rise to many new classes of unsteady flow problems which would perhaps necessitate a special theoretical system coupled with experimental data. Thus, at this time, as a first approximation, a time integration treating the fluid dynamic variables in a spatially lumped manner was considered.

The technique of lumped parameterization of the fluid dynamic variables considers control volumes of fluid and assumes that the fluid properties are distributed in some spatial fashion at any instant of time. For the study presented here, four control volumes were considered FIG.(2) which include the piston-cylinder region, orifice region, orifice-disc-valve region, and the disc valve-seat region. Within each control volume, density, pressure and velocity distributions were assigned. Assuming high Reynolds number flow, square velocity and pressure profiles were assigned on each control surface and a static discharge pressure at the exit of the disc valve. The velocity through the disc valve passage was assumed to be at all time subcritical (this was checked during computation) and the flow was assumed to remain parallel to the disc valve element at the exit plane. A reduction in flow area due to separation was also assumed in each control volume by assigning contraction coefficients for various flow conditions in the valve passages. These contraction coefficients were used under the assumption that the flow pattern in the vicinity of the valve and orifice sections does not change appreciably from the static flow patterns during the dynamic response of the disc valve element. For most compressor systems, the mechanical valve velocity can be assumed to be much smaller in magnitude than the velocity of the fluid stream parallel to the valve disc. It was further assumed that the mass of the air behind the disc valve and the turbulent jet mixing region exterior to the disc valve do not significantly influence the dynamic response of the mechanical disc valve system.

The valve dynamic equation is obtained from the free body diagram of FIG.(3). The force component \( F_d \) is a spring preset force, which is typically zero for automatic reed valves but was considered in the model for completeness. The possibility of having an external viscous damper on the valve was also included. The governing equation is

\[
\begin{align*}
\vec{F} &= \vec{F}_d \\
\end{align*}
\]

In the following the forcing function was obtained by applying the continuity equation

\[
\int \frac{\partial p}{\partial t} dV + \int \rho \vec{v} \cdot \vec{n} ds = 0
\]

and the momentum equation

\[
\begin{align*}
\int \partial \rho / \partial t \vec{v} dV + \int \rho \vec{v} \cdot \vec{v} ds &= 0 \\
\end{align*}
\]

It is the unsteady flow force \( F(t) \) which couples the disc-valve to the rest of the system.

CONTROL VOLUMES

Various unsteady flow expressions are presented below where the integral form of the continuity and momentum equations, Eqs 2 and 3, are applied to each of the control volumes, enabling us to calculate the time dependent pressures acting on the disc valve (REF. (7)).

Control Volume \( V_0 \)

Control volume \( V_0 \) was chosen in the interior of the piston cylinder cavity. With the piston instantaneously transmitting a pressure \( P_0 \) across area \( A_0 \) and the wall adjacent to the orifice reacting with a pressure \( P_0 \), the orifice entrance velocity \( V_1 \) becomes, when the continuity equation is applied,

\[
V_1 = \frac{-P_0 V_0 + \rho_0 V_0 A_0 / \rho A_1}{P_0 A_1}
\]

A contraction coefficient was used to account for the reduction in flow area in the orifice. This flow area is defined as

\[
A_1 = \begin{cases} A_p \text{ (discharge flow)} \\ C_c A_p \text{ (suction flow)} \end{cases}
\]

\( C_c \) was assumed to be 0.611.
The velocity an infinitesimal distance from the orifice becomes, from Eq. (2), assuming the flow to be incompressible in this region,

\[ v_{111} = \left( \frac{\rho_1 A_1}{\rho_0 A_0} \right) \]

Next, consider the momentum equation, neglecting viscosity and gravity, and by assuming a linear spatial distribution of velocity at any instant of time in the piston cylinder region and by applying the mean value theorem to the volume integral, the expression for the change of the time rate of change of velocity at the entrance of the orifice becomes

\[ \dot{v}_1 = \left( \frac{\rho_0}{\rho_1} A_1 \right) \frac{\dot{v}_{111}}{\rho_0} \]

where

\[ \dot{v}_{111} = \left\{ \frac{\rho_0 (v_0 + v_{111})}{2 \rho_0} A_0 + (P_0 - P_1) A_1 \right\}
\]
\[ - \frac{\dot{v}_1}{A_0 / \rho_1} \left( \frac{v_0 + v_{111}}{\rho_0 A_0} \right) \]

The absolute magnitude of the velocity notation accounts for the direction of flow.

The volume in the cylinder, \( V_0 \), varies in time as

\[ V_0 = \left\{ \frac{L}{2} \left[ 1 + (2n \pi) \right] \sin^2 \left[ \frac{(2n \pi)}{2} \right] \right\} A_0 + V_0 \]

Here \( L \) and \( L_1 \) are the piston stroke and connecting rod length respectively. The piston velocity, \( v_0 \), is

\[ v_0 = \frac{1}{A_0} \frac{dV_0}{dt} \]

Control Volume \( V_1 \)

A uniform density distribution is assumed in the interior of control volume 1 and a contraction coefficient, \( C_C = 0.611 \), is applied to modify the effective flow area of the orifice. The average velocity in the orifice section is assumed to be

\[ \frac{(v_1 + v_2)}{2} \]

Applying the continuity and momentum equation, the exit orifice velocity and its time rate of change is

\[ v_2 = \frac{(V_2 A_2 - \rho_2 V_1)}{(\rho_2 A_2)} \]

\[ \dot{v}_2 = \left( \frac{\rho_1 V_1 A_1 - \rho_2 A_2 V_2}{2} \right) \frac{V_2}{\rho_1} A_2 + (P_0 - P_2) A_0 \]

\[ - \frac{\rho_2 (v_2 + v_{21})}{2} A_2 - \frac{\dot{v}_2}{\rho_2 A_2} \]

The velocity \( v_2 \), and the rate of change of velocity \( \dot{v}_2 \) at the control volumes interface become, when an infinitesimal control volume is considered,

\[ v_2 = \frac{\rho_2}{\rho_1} V_2 \]

\[ \dot{v}_2 = \frac{\rho_1}{\rho_2} \dot{v}_2 + \frac{\rho_2}{\rho_2} \]

Control Volume \( V_2 \)

Control volume 2 consists of the cylindrical region bounded by the mechanical valve and the exit of the square edge orifice. The diameter of this cylindrical region was chosen to correspond to the location of the minimum value of the radial pressure distribution between the valve disc and the valve seat for small valve positions. The prediction of the minimum pressure location and the role it plays on the discharge coefficient has been studied by many investigators Ref. (8,9,10). A sketch of the separation streamline is shown in FIG.(2). This separation streamline may reattach to the seat of the valve. The reattachment depends on the static displacement of the valve and the pressure difference between the orifice and exit plane of the mechanical disc valve. A contraction coefficient \( C_D \) was specified at the minimum pressure location.

A uniform density, \( \rho_0 \), was assumed in control volume 2, with uniform pressure distributions assumed at the fluid elements control surfaces. See FIG.(3). Square velocity profiles were also assumed with the flow being subcritical at all times. The radial velocity \( v_r \) at the interface of control volumes 2 and 3 was assumed to be parallel to the disc valve and distributed uniformly on an area equal to

\[ A_3 = \pi d_r (h + h_0) / C_D \]

The pressures \( P_2 \) and \( P_3 \) were assumed to act on the disc valve and the exit of the circular orifice, while pressure \( P_3 \) was assumed to act uniformly on the circumferential area \( r_0 (h + h_0) \).

For the pressure and velocity profiles assumed above on each control surface and the uniform density distribution in the interior of the control volume, the continuity equation was used to define this velocity at the minimum pressure location.

\[ v_3 = \left( \frac{\rho_2 V_2 A_2 - \rho_3 A_3 h}{(\rho_2 A_2)} \right) \]

\[ - \frac{\rho_2}{(\rho_2)} \frac{h + h_0}{(\rho_2 A_2)} \]

\[ \frac{v_3}{(h + h_0)} \]

Here \( A_3 = \pi d_r^2 / 4 \) represents the cylindrical cross section of an area of fluid adjacent to the mechanical disc valve element.

To determine the rate of change of velocity at the minimum pressure location and the stagnation pressure acting on the disc valve, consider the momentum equation. For the case when radial symmetric flow is assumed between the disc valve and seat, two components of the momentum equation were evaluated. Consider the \( \chi \)-component. Defining the average velocity components by

\[ v_4 = \frac{(v_4 + h)}{2} \]

in the region between the exit of the orifice and mechanical disc valve bounded by the contracted flow area of the orifice and \( v_4 = h / 2 \) in the remaining annulus region between the separated flow in the orifice region and minimum pressure location, the expression for the stagnation pressure \( P_4 \).
Next, consider the radial component of the momentum equation. Due to the symmetry of flow, a plane was passed through the diameter of the cylindrical control volume and a momentum balance was made. The stagnation pressure \( P_{21} \) was assumed uniformly distributed over the plane splitting the cylinder. An arbitrary velocity distribution was assumed in the radial direction to account for diffusion from which a mean radial velocity was computed and distributed uniformly in the control volume. Since the fluid dynamic forces perpendicular to the plane splitting the cylindrical region are of interest, the radial velocity components were transformed to the original Cartesian coordinate system. The following expression for the time rate of change of velocity is

\[
V_{31} = \left( P_{21} A_{2f} + P_{2f} (A_{1f} - A_{2f}) - P_{3} A_{1f} - P_{3} V_{3} V_{31} A_{2f} - (P_{3} d_{1}^{2} (h + h_{0}) C_{C3} + C_{C3} d_{1}^{2} (h + h_{0}) P_{21}^{2} / 4 + C_{C3} P_{2} d_{1}^{2} (h + h_{0}) / 4) h_{0}^{2} / 2 \right) / \rho_{2}
\]

and the time rate of change of velocity \( V_{3} \) is obtained by differentiating the velocity expression obtained when an infinitesimal control volume is considered.

\[
\dot{V}_{3} = (\rho_{2} \dot{V}_{31} + V_{31} \dot{P}_{2} - \dot{P}_{3} V_{3}) / \rho_{2}
\]

\( \dot{V}_{4} \) is the exit velocity of the disc valve.

**Control Volume \( V_{3} \)**

Control volume \( V_{3} \) consists of an annular region bounded by the disc valve and valve seat. The inside diameter of the annular zone is designated by \( d_{1} \) and the outside diameter by \( d_{p} \). See Figure (2). A contraction coefficient \( C_{C4} \), is introduced at the exit of the disc valve to account for flow separation. Square velocity and pressure profiles were specified on each control surface with the velocity at the entrance and exit of the disc valve passage assumed parallel to the disc valve at all times. It is assumed that \( P_{3} \) acts on the surface of the annulus adjacent to the seat, \( P_{4} \) acts on the exterior surface of the annular control volume with a stagnation pressure \( P_{4} \), acting on the disc valve uniformly distributed over an area \( A_{4} \).

The velocity at an infinitesimal distance from the exit of the disc valve \( v_{4} \) and the time rate of change of velocity \( \dot{v}_{4} \) at the disc valves exit become respectively

\[
v_{41} = v_{4} \rho_{4} / \rho_{3}
\]

\[
\dot{v}_{4} = \left( \dot{V}_{3} \rho_{4} + \rho_{3} \dot{v}_{41} \right) / \rho_{4}
\]

The expression for the stagnation pressure \( P_{31} \), is obtained from the x-component of the momentum equation. When an average velocity \( v_{4} = h/2 \) is assumed in the annular control valve, the stagnation pressure becomes

\[
P_{31} = P_{3} \left\{ \left\{ \rho_{3} \left( \dot{V}_{3} \dot{v}_{41} \right) \right\} \rho_{4} + \rho_{3} \dot{v}_{41} \right\} / 2 A_{4}
\]
The unsteady flow force $F(t)$ is finally deduced by combining Eqs. (15) and (22):

$$F(t) = P_{31}a_4 + P_{21}a_3$$

SUMMARY OF FLOW-VALVE RESPONSE EQUATIONS

The time histories of the system of equations were obtained by using the Runge-Kutta numerical integration technique. The pressures are eliminated when an adiabatic process is assumed, thus leaving nine unknowns.

$$P_n = \text{const} \gamma^n \quad \text{where } n = 1, 2, 3, 4$$

The unknown of the system includes $(\rho_c, \rho_2, \rho_3, h, v_1, v_2, v_3, v_4)$. Since the fluid velocities and their time derivatives, Table I, are functions of density and derivatives, $\mathbf{v} = (\dot{\rho}, \dot{\rho})$ and $\mathbf{v} = (\dot{\rho}, \dot{\rho})$, the equations resulting from the continuity balance can be time differentiated thus eliminating the dependency on the velocity.

<table>
<thead>
<tr>
<th>$v_i$</th>
<th>Equation (4)</th>
<th>$v_2$</th>
<th>Equation (10)</th>
<th>$v_3$</th>
<th>Equation (14)</th>
<th>$v_4$</th>
<th>Equation (18)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>continuity balance</td>
<td></td>
<td>momentum balance</td>
<td></td>
<td>control surface</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The expressions for the twice time differentiated densities can be found in REF. (7), pages 109-110. Final second-order nonlinear ordinary differential equations of the form

$$\ddot{\rho}_n = f(\rho_n, \rho_0, \dot{\rho}_n, v_0, v_0, h, h, \dot{h}) \quad n = 1, 2, 3, 4$$

were numerically integrated to describe the nonlinear dynamic response of a reciprocating and compressor discharge valve system.

NOMENCLATURE

The following nomenclature will apply unless otherwise stated.

- $A_0$ piston surface area
- $A_0$ orifice cross sectional area
- $A_d$ disc valve surface area
- $A_{12}$ orifice entrance flow area
- $A_{22}$ orifice exit flow area
- $A_{32}$ fluid control volume III. Surface area adjacent to disc valve
- $A_{14}$ fluid control volume IV. Surface area adjacent to disc valve.
- $B$ body force vector
- $C_c$ orifice contraction coefficient
- $C_{c1}$ disc valve entrance contraction coefficient
- $C_{c2}$ disc valve exit contraction coefficient
- $C_f$ lift force coefficient
- $d$ orifice diameter
- $d_p$ disc valve diameter
- $D$ piston diameter
- $d_1$ diameter of control volume III
- $F$ external force vector
- $f$ pistons fundamental rotational speed
- $F(t)$ disc valve dynamic force
- $F_d$ pressure force on back side of disc valve
- $A_{3f}$ disc valve entrance flow area
- $A_{4f}$ disc valve exit flow area
- $h$ disc valve displacement
- $h_0$ disc valve, static displacement
- $k$ mechanical disc valve spring constant
- $L$ piston stroke
- $L_1$ connecting rod length
- $\mathbf{n}$ normal vector
- $P$ cylinder pressure
- $P_{1d}$ discharge pressure
- $P_{21}$ pressure in fluid control volume 1
- $P_{22}$ pressure in fluid control volume 2
- $P_{23}$ pressure in fluid control volume 3
- $S$ control surface
- $t$ time
- $V_{01}$ piston clearance volume
- $V_0$ piston cylinder volume
- $V_{10}$ deforming control volume
- $V$ velocity vector
- $v_p$ piston velocity
- $v_0$ velocity input at the bottom end of the piston cavity
- $v_1$ orifice velocity
- $v_2$ orifice exit velocity across control volume 2
- $v_3$ valve entrance velocity across control volume 3
- $v_4$ valve exit velocity across control volume 3
- $v_{11}$ orifice entrance velocity across control volume 2 due to change in density
- $v_{21}$ orifice exit velocity across control volume 2 due to change in density
- $v_{31}$ valve entrance velocity across control volume 3 due to change in density
- $v_{41}$ valve exit velocity across control volume 3 due to change in density
- $v_{111}$ velocity across control volume due to change in density and area
- $\gamma$ isentropic gas constant
- $\mu$ viscosity coefficient
- $\rho$ piston cylinder fluid mass density, control volume
- $\rho_1$ orifice fluid mass density, control volume 2
- $\rho_2$ fluid density in control volume 2
- $\rho_3$ fluid density in control volume 3
- $\theta$ crank angle rotation, degrees
- $\dot{\theta}$ time derivative

REFERENCES

1. Wambganss, M.W., Jr., and Cohen, R., "Simulation of Reciprocating Com-


FIGURE 1
PISTON-ORIFICE-SPRING LOADED DISC-VALVE SYSTEM
FIG. 2a ASSUMED VELOCITY AND PRESSURE PROFILES FOR CONTROL VOLUME $V_0$

A) CONTROL VOLUME $V_1$
B) VELOCITY PROFILE
C) PRESSURE PROFILE

FIG. 2b VELOCITY AND PRESSURE PROFILES ACROSS THE SQUARE-EDGE ORIFICE BOUNDARY

FIG. 2c ASSUMED VELOCITY AND PRESSURE PROFILES, CONTROL VOLUME $V_2$
**FIG. 2d** VELOCITY AND PRESSURE PROFILES, CONTROL VOLUME $V_3$

**FIG. 3** MECHANICAL SPRING DISC VALVE SYSTEM