Point Excitation of a Coupled Structural-Acoustical Tire Model with Experimental Verification

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Point excitation of a coupled structural-acoustical tire model with experimental verification

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I. Introduction

Traffic noise

Vehicle noise

Roadside residences

- Power Unit noise
- Aerodynamic noise
- Tire/pavement noise

Passengers

Transfer paths

In cabin noise
I. Introduction

Objective:
1. Build a model coupling the tire structure and air cavity
2. Identify tire structural vibration and acoustical modes
3. Create forced response model incorporating coupled modes
4. Investigate the dynamics property of the tire structure

Airborne Waves

Structural Waves
II. Literature Review

- Structure-borne sound on a smooth tyre
  Kropp

- A wave model of a circular tyre. Part 1: belt modelling
  Pinnington

- Vibrations of Shells and Plates
  Soedel

- A coupled tire structure/acoustic cavity model
  Molisani, Burdisso & Tsihlas

- The influence of tyre air cavities on vehicle acoustics
  Fernandez

- The wave number decomposition approach to the analysis of tire vibration
  Bolton, Song, Kim & Kang
III. Model description

- The wheel rim is rigid and fixed
- Tire sidewall is represented by springs in radial and tangential directions
- Ring structure allows for flexural and longitudinal waves
- Harmonic point input excitation at arbitrary angle is applied
III. Model description

Solving for natural frequencies

Harmonic displacements are assumed as:

\[ w = \alpha e^{-jk_0 \theta} e^{j\omega t} \]
\[ u = \beta e^{-jk_0 \theta} e^{j\omega t} \]

Substitution into the static coupled ring EOMs and write solutions in matrix form:

\[
\begin{bmatrix}
M_{11} & M_{12} \\
M_{21} & M_{22}
\end{bmatrix}
\begin{bmatrix}
\alpha \\
\beta
\end{bmatrix}
= \begin{bmatrix}
p \\
0
\end{bmatrix}
\]

\( p \) is the acoustic pressure disturbance in the air cavity, which is assumed to be the distributed load in radial direction

\[ p = -\rho_0 \left[ \frac{\partial \psi}{\partial t} + \bar{v}_{flow} \cdot \nabla \psi \right] \]

By applying boundary conditions at two air-structure contact surfaces, we can obtain the pressure \( p \) as a function of radial coefficient \( \alpha \)
III. Model description

Solving for natural frequencies

So the coupled equations can be expressed in homogeneous form

\[
\begin{bmatrix}
M_{11} - FL & M_{12} \\
M_{21} & M_{22}
\end{bmatrix}
\begin{bmatrix}
\alpha \\
\beta
\end{bmatrix}
= 
\begin{bmatrix}
0 \\
0
\end{bmatrix}
\]

Where \( M_{11}, M_{12}, M_{21}, M_{22} \) and \( FL \) are expressions in terms of structure-related constants and the variables \( k_{\theta} \) and \( \omega \).

Setting the determinant to zero gives us a characteristic equation:

\[
f(\omega) = (M_{11} - FL)M_{22} - M_{12}M_{21} = 0
\]

The natural frequencies can be solved for each mode number \( k_{\theta} \).
III. Model description

Obtaining forced response

With the derived natural frequencies, the modal summation method is adopted here to predict the response due to harmonic point excitation.

Generalized loads:

\[ q_r = \frac{F_r}{R} \delta(\theta - \theta^*) e^{j\omega t} \]
\[ q_{\theta} = \frac{F_{\theta}}{R} \delta(\theta - \theta^*) e^{j\omega t} \]

Assumed displacement:

\[ u_r = \sum_{i=1}^{5} \sum_{n=0}^{\infty} \eta_{ni} U_{ri}(\theta) e^{j\omega t} \]
\[ u_{\theta} = \sum_{i=1}^{5} \sum_{n=0}^{\infty} \eta_{ni} U_{\theta ni}(\theta) e^{j\omega t} \]

\( \eta_{ni} \) is the modal participation factor, \( U_{ni}(\theta) \) is the mode shape

Number of modes considered

Number of wave types considered
III. Model description

Assuming displacements

To obtain the complete solution, two sets of orthogonal mode shapes need to be considered:

\[
U_{m(1)}(\theta) = A_n \cos n\theta \quad U_{\theta n(1)}(\theta) = \frac{B_n}{A_n} \sin n\theta
\]  

\[
U_{m(2)}(\theta) = A_n \sin n\theta \quad U_{\theta n(2)}(\theta) = -\frac{B_n}{A_n} \cos n\theta
\]  

The ratio of amplitudes can be found from the characteristic matrix from the previous modal analysis. The derivation process is the same for the two sets and the complete results would be the sum of the two sets of solutions:

\[
\frac{B_n}{A_n} = -\frac{M_{11} - FL}{M_{12}} = -\frac{M_{21}}{M_{22}}
\]
Finding modal participation factors

The next step is to solve for the modal participation factors $\eta_n$. From Love’s equations, for each type of wave, we have

$$\sum_{n=0}^{\infty} \left[ \eta_n L_r \{U_{\partial_n}, U_m\} - \rho h \ddot{\eta}_n U_m \right] = -q_r$$

$$\sum_{n=0}^{\infty} \left[ \eta_n L_\theta \{U_{\theta_n}, U_m\} - \rho h \ddot{\eta}_n U_{\theta_n} \right] = -q_\theta$$

From eigenvalue analysis, we have

$$\rho h \sum_{n=0}^{\infty} \left[ (\omega_n^2 - \omega^2) \eta_n U_m \right] = q_r$$

$$\rho h \sum_{n=0}^{\infty} \left[ (\omega_n^2 - \omega^2) \eta_n U_{r\theta} \right] = q_\theta$$
III. Model description

Finding modal participation factors

By multiplying orthogonal modes $U_{rm}$ and $U_{\theta m}$ respectively and integrating around the ring circumference, we have

$$\rho h \sum_{n=0}^{\infty} \left( \omega_n^2 - \omega^2 \right) \eta_n \int_0^{2\pi} U_{rm} U_{rm} Rd\theta = \int_0^{2\pi} \left( F_r \frac{1}{R} \delta(\theta - \theta^*) \right) U_{rm} Rd\theta$$

$$\rho h \sum_{n=0}^{\infty} \left( \omega_n^2 - \omega^2 \right) \eta_n \int_0^{2\pi} U_{\theta n} U_{\theta m} Rd\theta = \int_0^{2\pi} \left( F_\theta \frac{1}{R} \delta(\theta - \theta^*) \right) U_{\theta m} Rd\theta$$

Perform the integration and add the above two equations, to give

$$\rho h \left( \omega_n^2 - \omega^2 \right) \eta_n = F_r (\theta^*) U_{rn}(\theta^*) + F_\theta (\theta^*) U_{\theta n}(\theta^*)$$

$$N_n = \int_0^{2\pi} (U_{rn} U_{rn} + U_{\theta n} U_{\theta n}) Rd\theta = \begin{cases} \left( \frac{B_n}{A_n} \right)^2 + 1 \pi R & n \neq 0 \\ 2\pi R & n = 0 \end{cases}$$

So the modal participation factors $\eta_n$ are found for each frequency.
III. Model description

Obtaining displacements

The last step is to substitute those factors back into the assumed displacement solutions.

\[
 u_r(\omega) = \sum_{i=1}^{5} \sum_{n=0}^{\infty} \frac{F_r \cos(n\theta - \theta^*) + F_i \sin(n\theta - \theta^*)}{\rho h N_{ni} f(\omega)}
\]

\[
 u_\theta(\omega) = \sum_{i=1}^{5} \sum_{n=0}^{\infty} \left( \frac{B_{ni}}{A_{ni}} \right) \frac{(F_i \sin(n\theta - \theta^*) - F_r \cos(n\theta - \theta^*))}{\rho h N_{ni} f(\omega)}
\]

As for the numerical calculation, we’ll choose a limited numbers of terms for each wave considered.
III. Model description

Calculating mobilities

The frequency-velocity function can be easily calculated as

\[ v_r(\omega) = j\omega u_r(\omega), \quad v_\theta(\omega) = j\omega u_\theta(\omega) \]

If we apply unit excitation forces \( F_r \) and \( F_\theta \), the mobility function will have the same numerical value as the velocity function.

Radial mobility = \( \frac{v_r}{F_r} \)

Circumferential mobility = \( \frac{v_\theta}{F_\theta} \)
IV. Experimental Set up

- Computer
- LDV
- Tire Tread
- Data Acquisition Box
- Force Transducer
- Signal Generator
- Filter
- Shaker
- Amplifier

Arrows indicate:
- radial velocity
- force
V. Results

Dispersion relations

- 3rd acoustical wave
- 2nd acoustical wave
- 1st acoustical wave
- 2nd structural wave (fast extensional wave)
- 1st structural wave (slow flexural wave)
## V. Results

### Tire material properties

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young’s Modulus [Pa]</td>
<td>$4.8 \times 10^8$</td>
<td>Pressure [bar]</td>
<td>3</td>
</tr>
<tr>
<td>Density [kg/m³]</td>
<td>1200</td>
<td>Inner Radius [m]</td>
<td>0.205</td>
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<tr>
<td>Thickness [m]</td>
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<td>Outer Radius [m]</td>
<td>0.338</td>
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<tr>
<td>Radial stiffness [N/m]</td>
<td>$2 \times 10^6$</td>
<td>Tangential stiffness [N/m]</td>
<td>$1 \times 10^6$</td>
</tr>
</tbody>
</table>

### Natural frequencies [Hz]

<table>
<thead>
<tr>
<th>n\name</th>
<th>slow flexural</th>
<th>acoustical (circumferential)</th>
<th>fast extensional</th>
<th>1&lt;sup&gt;st&lt;/sup&gt; acoustical (radial)</th>
<th>2&lt;sup&gt;nd&lt;/sup&gt; acoustical (radial)</th>
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</thead>
<tbody>
<tr>
<td>0</td>
<td>71.47</td>
<td>N/A</td>
<td>318.06</td>
<td>1352.62</td>
<td>2684.33</td>
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<td>1</td>
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<td>447.44</td>
<td>1369.57</td>
<td>2692.58</td>
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<td>2</td>
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<td>411.68</td>
<td>698.78</td>
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<td>3</td>
<td>135.21</td>
<td>612.74</td>
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<td>2757.78</td>
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<tr>
<td>4</td>
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<td>810.88</td>
<td>1278.21</td>
<td>1606.38</td>
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<td>5</td>
<td>243.13</td>
<td>1005.46</td>
<td>1578.68</td>
<td>1735.93</td>
<td>2884.39</td>
</tr>
</tbody>
</table>
V. Results

Mobility functions

Point 1 is input point, Point 2 is $\pi/2$ away from input point

Radial

Tangential

The high amplitude peaks are due to the neglect of damping, which would smooth the peaks otherwise
V. Results

Dispersion relations

Analytical
V. Results

Dispersion relations
V. Results

Dispersion relations

Dispersion

Coherence (velocity & force)

Cavity depth: 5 in

215/60 R16
V. Results

Dispersion relations

225/45 R18

Dispersion

Coherence (velocity & force)

Cavity depth: 4 in
VI. Conclusion

- The depth-direction acoustic modes in a tire’s air cavity were analytically predicted.
- The frequency-mobility functions were derived under point harmonic excitation.
- Experimental data confirmed the analytical results, although disparities exist due to the difference between analytical and actual tire material parameters.
- For precise radial mobility measurement, the laser should be pointed toward the tire in the radial direction.
Previous Papers on this topic


Thank you
Questions?

Thanks to Ford for providing various tires for testing

Go Further