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Improved Model for Coupled Structural-Acoustic Modes of Tires

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IMPROVED MODEL FOR COUPLED STRUCTURAL-ACOUSTIC MODES OF TIRES

Rui Cao, J. Stuart Bolton, Ray W. Herrick Laboratories, School of Mechanical Engineering, Purdue University
I. Introduction

Traffic noise

Vehicle noise

Roadside residences

Passengers

• Power Unit noise
• Aerodynamic noise
• **Tire/pavement noise**

Transfer paths

In cabin noise

Dominant at high speed
I. Introduction

Objective:
1. Build a model coupling the tire structure and air cavity
2. Identify tire structural vibration
3. Study sound characteristics in interior air cavity
4. Investigate spinning influence

Diagram:
- Fixed axle
- Internal air cavity
- Tire structure
II. Literature Review

Structure-borne sound on a smooth tyre

Kropp

Effects of Coriolis acceleration on the free and forced in-plane vibrations of rotating rings on elastic foundation

Huang & Soedel

A wave model of a circular tyre. Part 1: belt modelling

Pinnington

Effects of rotation on the dynamics of a circular cylindrical shell with application to tire vibration

Kim and Bolton

A coupled tire structure/acoustic cavity model

Molisani, Burdisso & Tsihlas

The Influence of Tyre Air Cavities on Vehicle Acoustics

Fernandez

The wave number decomposition approach to the analysis of tire vibration

Bolton, Song, Kim & Kang
III. Analytical Model

Review of previous models

* Cao & Bolton, NoiseCon 2013

* Cao & Bolton, NoiseCon 2014
III. Analytical Model

Fully coupled circular cavity model

1. The tire rotates about a fixed axle
2. The wheel rim is rigid
3. Tire sidewall is represented by springs in radial and tangential directions
4. Ring structure includes flexural and longitudinal waves
Rotating ring structure

Assume harmonic solutions for displacements:

\[ w = \alpha e^{-jk_\theta \theta} e^{j\omega t} \quad u = \beta e^{-jk_\theta \theta} e^{j\omega t} \]

Substitution into rotating ring EOMs and write solutions in matrix form:

\[
\begin{bmatrix}
M_{11} & M_{12} \\
M_{21} & M_{22}
\end{bmatrix}
\begin{bmatrix}
\alpha \\
\beta
\end{bmatrix} = \begin{bmatrix} 0 \\
0 \end{bmatrix}
\]

Where \( M_{11}, M_{12}, M_{21} \) and \( M_{22} \) are expressions of structure-related constants and variables \( k_\theta \) and \( \omega \). For example:

\[
M_{11} = j \left( \frac{Eh^3}{12R^4} k_\theta^3 + \frac{Eh}{R^2} k_\theta + 2 \frac{\sigma_\theta h}{R^2} k_\theta + 2 \rho h \Omega \omega \right)
\]

\[
M_{12} = \frac{Eh^3}{12R^4} k_\theta^2 + \frac{Eh}{R^2} k_\theta^2 + \frac{\sigma_\theta h}{R^2} k_\theta^2 + \frac{p_0}{R} + k_u + 2 \rho h \Omega k_\theta \omega - \rho h \omega^2
\]
III. Analytical Model

Circular air cavity

Velocity of the flowing air is expressed as

\[ v_\theta = \frac{v_0}{R} r \]

By using velocity potential \( \psi \), the wave equation in the circular air cavity is

\[
\frac{\partial^2 \psi}{\partial r^2} + \frac{1}{r} \frac{\partial \psi}{\partial r} + \frac{1}{r^2} \frac{\partial^2 \psi}{\partial \theta^2} = \frac{1}{c_0^2} \left( \frac{\partial^2 \psi}{\partial t^2} + \frac{2 v_0}{R} \frac{\partial^2 \psi}{\partial t \partial \theta} + \frac{v_0^2}{R^2} \frac{\partial^2 \psi}{\partial \theta^2} \right)
\]

Harmonic solution of pressure is assumed in circumferential direction while Bessel function is assumed in radial direction:

\[ \psi(r, \theta, t) = g_m(\kappa_m r) e^{-jm\theta} e^{j\omega t} \]

\[ g_m(\kappa_m r) = A_m J_m(\kappa_m r) + B_m Y_m(\kappa_m r) \]

\[ p = -\rho_0 \left[ \frac{\partial \psi}{\partial t} + \bar{v}_{flow} \cdot \text{grad} \psi \right] \]
III. Analytical Model

Coupling relations

1. \( f_w = p \)

2. \( v_r = \frac{\partial \psi}{\partial r} \bigg|_{r=R} = v_w = \frac{\partial w}{\partial t} \)

3. \( v_r = \frac{\partial \psi}{\partial r} \bigg|_{r=r_0} = 0 \)
Solving the coupled system

Substituting sound pressure as distributed load in radial direction into the characteristic equations of the ring structure and express $p$ as function of $\alpha$ and $\beta$ by using the boundary conditions:

$$
\begin{bmatrix}
M_{11} & M_{12} \\
M_{21} - FL & M_{22}
\end{bmatrix}
\begin{bmatrix}
\alpha \\
\beta
\end{bmatrix} =
\begin{bmatrix}
0 \\
0
\end{bmatrix}
$$

Where the fluid loading term $FL$ can be expressed as

$$
FL = \frac{p}{e^{-jm\theta}} = (-j\omega \rho_0 + jmv \rho_0)\alpha C_{A/\alpha} (J_m (\kappa_m r) + C_{B/A} Y_m (\kappa_m r)) e^{j\omega t}
$$

By supplying the mode number $m$, which is equivalent to wavenumber, we have

$$
f(\omega) = M_{11}M_{22} - M_{12}(M_{21} - FL) = 0
$$

The values of $\omega$ that satisfy this equations are the natural frequencies of the coupled model.
IV. Testing

Tire mobility measurement set up

- Computer
- LDV
- Data Acquisition Box
- Signal Generator
- Filter
- Force Transducer
- Shaker
- Amplifier
- Tire Tread

Connections:
- Radial velocity from LDV to Tire Tread
- Force from Data Acquisition Box to Force Transducer
IV. Testing

Tire mobility measurement set up

- Computer
- LDV
- Data Acquisition Box
- Signal Generator
- Filter
- Tire Tread
- Force Transducer
- Shaker
- Amplifier

Flow: radial velocity, force
## V. Results

### Table of model parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ring density</td>
<td>$\rho = 1200 \text{ kg/m}^3$</td>
</tr>
<tr>
<td>Air density</td>
<td>$\rho_0 = 1.24 \text{ kg/m}^3$</td>
</tr>
<tr>
<td>Outer radius</td>
<td>$r_1 = 0.3 \text{ m}$</td>
</tr>
<tr>
<td>Inner radius</td>
<td>$r_2 = 0.2 \text{ m}$</td>
</tr>
<tr>
<td>Ring thickness</td>
<td>$h = 0.008 \text{ m}$</td>
</tr>
<tr>
<td>Tire inflation pressure</td>
<td>$p_0 = 20600 \text{ Pa}$</td>
</tr>
<tr>
<td>Radial stiffness</td>
<td>$k_w = 1\times10^5 \text{ N/m}$</td>
</tr>
<tr>
<td>Tangential stiffness</td>
<td>$k_u = 1\times10^5 \text{ N/m}$</td>
</tr>
<tr>
<td>Young’s modulus</td>
<td>$E = 4.8\times10^8 \text{ Pa}$</td>
</tr>
</tbody>
</table>

### Tire measured

*Goodyear 225/55 R17*
V. Results

Dispersion relation (static case)

- 1st structural wave (slow flexural wave)
- 2nd structural wave
  (fast extensional wave)
- 1st acoustical wave
- 2nd acoustical wave
- 3rd acoustical wave

Frequency [Hz] vs Mode number graph.
Dispersion relation (static case)
V. Results

Dispersion relation (static case)

![Dispersion relation graph](image-url)
V. Results

Dispersion relation (static case)
V. Results

**Dispersion relation** (static case)

![Dispersion relation graph](image)

- **Frequency [Hz]** vs. **Mode number**
V. Results

**Dispersion relation** (no fluid loading)

- Acoustical waves disappear
- Fluid loading has minor impact on structural features
V. Results

Dispersion relation (rotating case)

Natural frequencies split into two at each mode of all waves

- Structural Waves
- Airborne Waves

<table>
<thead>
<tr>
<th>Mode number</th>
<th>Frequency [Hz]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1000</td>
</tr>
<tr>
<td>5</td>
<td>2000</td>
</tr>
<tr>
<td>10</td>
<td>3000</td>
</tr>
<tr>
<td>15</td>
<td>4000</td>
</tr>
<tr>
<td>20</td>
<td>5000</td>
</tr>
<tr>
<td>25</td>
<td>6000</td>
</tr>
</tbody>
</table>
V. Results

Dispersion relation (experimental)

- Fast extensional wave
- Slow flexural wave
V. Results

Dispersion relation (experimental)

- Circumferential acoustical modes
- Radial acoustical mode
- 340 m/s line
V. Results

Dispersion relation (experimental)

- 340 m/s line
- Radial acoustical mode
- Circumferential acoustical modes
V. Results

Dispersion relation (experimental)

340 m/s line

radial acoustical mode

circumferential acoustical modes
V. Results

Dispersion relation (experimental)

- 340 m/s line
- Radial acoustical mode
- Circumferential acoustical modes
V. Results

Phase speed (static)

<table>
<thead>
<tr>
<th>Phase Speed [m/s]</th>
<th>Frequency [Hz]</th>
</tr>
</thead>
<tbody>
<tr>
<td>2nd structural wave</td>
<td></td>
</tr>
<tr>
<td>2nd acoustical wave</td>
<td></td>
</tr>
<tr>
<td>3rd acoustical wave</td>
<td></td>
</tr>
</tbody>
</table>

1st acoustical wave
1st structural wave
V. Results

Phase speed (rotating)

![Graph showing phase speed vs. frequency for rotating systems.](image)

Phase Speed [m/s] vs. Frequency [Hz]

![Another graph showing phase speed vs. frequency.](image)

Phase Speed [m/s] vs. Frequency [Hz]
Radial pressure distribution in cavity

Mode number is 2, at the natural frequencies of each wave

1st structural wave

1st acoustical wave
V. Results

Radial pressure distribution in cavity

Mode number is 2, at the natural frequencies of each wave

1\textsuperscript{st} structural wave

1\textsuperscript{st} acoustical wave
V. Results

Pressure distribution in cavity (static)

Mode number is 2, at the natural frequencies of each wave

2\textsuperscript{nd} structural wave

2\textsuperscript{nd} acoustical wave
V. Results

Pressure distribution in cavity (static)

Mode number is 2, at the natural frequencies of each wave

2\textsuperscript{nd} structural wave

2\textsuperscript{nd} acoustical wave
VI. Conclusion

- The ring model allows for motions in radial and circumferential directions, which are associated with flexural waves and longitudinal waves, respectively.
- The air cavity acts as a fluid loading on the ring structure.
- Rotation of tire causes frequency split phenomenon.
- Acoustical wave in tire radial directions exist – “depth modes” detectable in tire surface vibration.
- In circular air cavity, phase speed of circumferential acoustical wave varies with radius due to planar nature of waves.
Thank you

Question?