Exploring the Effects of Conversational Repair as a Scaffolding Strategy to Promote Mathematics Explanations of Students with Learning Disabilities

Jia Liu
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By Jia Liu

Entitled Exploring the Effects of Conversational Repair as a Scaffolding Strategy to Promote Mathematics Explanations of Students with Learning Disabilities

For the degree of Doctor of Philosophy

Is approved by the final examining committee:
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Approved by Major Professor(s): Yan Ping Xin

Approved by: Ala Samarapungavan 09/30/2013
Head of the Graduate Program Date
EXPLORING THE EFFECTS OF CONVERSATIONAL REPAIR AS A SCAFFOLDING STRATEGY TO PROMOTE MATHEMATICS EXPLANATIONS OF STUDENTS WITH LEARNING DISABILITIES

A Dissertation
Submitted to the Faculty
of
Purdue University
by
Jia Liu

In Partial Fulfillment of the
Requirements for the Degree
of
Doctor of Philosophy

December 2013
Purdue University
West Lafayette, Indiana
To All the Days as a Student
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ABSTRACT

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Conversational repair often occurs in conversations when people attempt to address communicative breakdowns or inaccuracy by way of repeating what have been said or putting them in another way. The review of literature on conversational repair revealed that as an important concept in pragmatic aspect of language, it is an effective strategy to improve communication of different populations with disabilities. However, it is rarely studied in the domain of mathematics and with the population with learning disabilities/difficulties (LD). In current reform-based, discourse-oriented mathematics classrooms, students with LD encounter difficulties articulating or explaining well their reasoning processes due to the mathematical and communicative difficulties they may have. As such, the ability to repair communicative breakdowns or inaccuracy – making conversational repair – is important for them to make progress in classroom discourse and team work.

This study designed the intervention based on the different repair request techniques in the implicit-explicit continuum to elicit repair of self-explanation from students with LD. Using a multiple-baseline-across-participants design, the study found that the intervention
was effective for improving the students’ problem solving and reasoning ability measured by their word-problem-solving performance and self-explanation performance, respectively. It provided implications for future studies concerning the use of conversational repair in subject domain classroom discourse, especially for individuals with LD.
CHAPTER 1. INTRODUCTION

Mathematics plays an important role in life and human society. Current reform in mathematics education calls for the social and cultural aspects of mathematics learning to develop students’ competencies, which means not only being able to “do” mathematics, but also to “speak” mathematics. Thus, communication in mathematical classroom has become an essential part in mathematics education. As stated by The Principles and Standards for School Mathematics (National Council of Teachers of Mathematics, 2000) (hereafter called the Standards), to learn mathematics, students were not merely required to conduct the mathematical procedures, but also to make ideas public for reflection, discussion and refinement. All students were expected to learn to be clear and convincing in expressing their own ideas and to listen to, understand, and make connections with others’ ideas and hence sharpen their thoughts.

The communication between teachers and students in classrooms is the classroom discourse. As a fully studied area, classroom discourse within specific domain areas, like mathematics, was not given much attention until recently. There have been increasing interests in teaching and learning within academic content areas where discourse has become a major mechanism for students’ knowledge production.
In traditional classrooms, teachers are regarded as “controlling” or “dominating” classrooms. They take the initiation of the conversations, generally by asking a closed, known-answer question; students reply to the question, and teachers provide evaluation or feedback of students’ reply. This pattern, known as initiation-response-evaluation (IRE), or initiation-response-feedback (IRF) (or IRE/F) (Mehan, 1979; Sinclair & Coulthard, 1975), almost covers all the teacher-student talk as the criterial discourse structure.

In contrast, the reform movement can be interpreted as supporting discourse-oriented teaching (Williams & Baxter, 1996). Teachers play a still important but different role in reformed classrooms from traditional ones. Instead of being the “dominator” or “controller”, teachers are encouraged to be the “orchestrator” of the classroom discourse. Their instructional strategies should be able to enhance students’ communication skills for organizing and expressing their ideas, sharing their ideas with others and evaluating other’s thinking.

Given the current spotlight cast on mathematics classroom discourse, it is worth the endeavor to explore how to achieve a quality classroom discourse to engage all students. Problems exist that hinder the achievement of this goal.

First, quality mathematics pedagogy for enhancing effective classroom discourse is still in its formative stage and teachers encounter difficulties in establishing the classroom community that can enact mathematics reform, such as sense of less efficacy, students’ disengagement in more challenging tasks, difficulties in managing mathematical directions set by instruction, in anticipation for where a lesson goes, and in preparation for their own roles in instruction.
Second, on the students’ side, discourse-oriented instruction has not been
generally applied to students with disabilities and there was little research concerning the
impact of the *Standards* on special education. Baxter, Woodward, and Olson (2001)
observed 16 low-achieving students, either identified as LD or not, of 5 elementary
mathematics classrooms which implemented reform-based instruction. They found that
these students were only minimally involved in whole-class discussions. They rarely
spoke and were easily distracted. Their passivity might be attributed to cognitive
overload from the reform-based curriculum, teachers’ subtle passive behaviors, and
inadequate opportunity to speak. However, we know for now that though it is hard for
students with LD to involve in classroom discourse to articulate or explain their thinking
or problem-solving processes due to the mathematical and communicative difficulties
they may have, they still can increase their participation if teachers use effective
instructional strategies (Baxter, Woodward, & Olson, 2001). They can also benefit from
the discourse-oriented classroom as their normally-achieving peers do (Berry & Kim,
2008; Kroesbergen & Van Luit, 2002, 2003; Woodward & Baxter, 1997), and learn the
thinking behaviors such as asking questions, disagreeing, explaining and suggesting
solutions. To achieve this goal, teachers’ help in classroom discourse is not merely to set
up classroom norms, and to give encouragements and expectations. What is more
important is to provide necessary help to affect students’ mathematical ideas so that they
could efficiently and reasonably construct knowledge.

One major task in classroom discourse, according to the *Standards*, is to have
students provide clear explanations of their thinking and problem solving. The ability to
explain plays an important role in learning. It is a metacognitive process which helps
learners to reflect on their own thinking process and establish schema. Studies have demonstrated that the strategy of asking learners to give explanations were effective on understanding (Aleven & Koedinger, 2002). Specifically, this metacognitive strategy is especially effective when combined with experimenter-provided worked-out examples. A number of studies have shown that across various subject domains, students improved their learning outcomes if they were asked to self-explain worked-out examples (Ainsworth & Loizou, 2003; Aleven & Koedinger, 2002; Chi, Bassok, Lewis, Reimann, & Glaser, 1989; Hausmann & VanLehn, 2007; Sandoval, Trafton, & Reiser, 1995; Tajika, Nakatsu, Nozaki, Neumann, & Maruno, 2007).

Given the significance of self-explanation in classroom discourse and in learning, it necessitates the development of strategies that could cultivate this ability. Studies on such interventions have varied results and limitations. Questions concerned may include: for students of different achievement levels, what are the characteristics of their abilities in explaining their thinking and problem solving? When they can solve the problems, can they explain how they solved it? When they cannot solve the problems, can they explain the part that hindered their success? How and what kind of intervention can help improve their explaining ability? Can the interventions that facilitate explanation be helpful in promoting students’ problem-solving abilities further? As such, there is a call for more and better interventions to improve self-explanation ability (Aleven & Koedinger, 2003; Renkl, 1999).

Scaffolding is an important concept in learning. It is a dynamic system which provides external help to the learner within one’s zone of proximal development (ZPD) and gradually fades away the help as the learner progresses (Stone, 1998). In classroom
discourse where learning mostly depends on communication, it is natural to expect the
dialog between teachers and students can scaffold the explaining process.

In the domain of conversational analysis (CA), conversational repair often occurs in conversations when people attempt to address communicative breakdowns or inaccuracy by way of repeating what has been said or putting them in another way. It has been found that conversational repair is the mechanism people use to address the problems in communication, and it has also been demonstrated to be an effective strategy to improve communication of different populations with disabilities. For example, for students with moderate and severe disabilities who had poor intelligibility and grammatical clarity, peer-mediated conversational repair could help the target students and their normally-achieving peers to communicate longer and better (Weiner, 2005). Also, there is evidence that requesting conversational repair from the speaker repeatedly is better than requesting once in reaching a final mutual understanding. Several studies have been conducted on how individuals with language impairments (LI) (Brinton, Fujiki, & Sonnenberg, 1988; Brinton, Fujiki, Winkler, & Loeb, 1986) or mild to moderate intellectual disabilities (ID) (Brinton & Fujiki, 1996) responded to stacked requests (like “Huh?”, “What?”, and “I didn’t understand that.” [Brinton et al., 1986]). It has been found that the target populations lack persistence in responding to all the requests. Also, they were more likely to respond by merely repeating (instead of doing some revision of or addition to) what they had said and they produced more inappropriate responses compared with normally-achieving peers.

Most of the research on conversational repair is restricted to linguistic analysis or addressing communicative problems for individuals with language deficits. It was rarely
adopted in other various content areas, such as mathematics. As an effective strategy to improve language production, it is reasonable to suppose that it will be helpful to improve the explaining ability of students in mathematics classroom discourse. If so, it will be meaningful to adopt this strategy in mathematics classroom to enhance students’ discourse and even further improve students’ problem solving abilities. Further, since the reform-based mathematics education focusing on classroom discourse is rarely studied concerning students with LD, it is more impending to see how these students explain their mathematical thinking and reasoning and how the scaffolding strategy of conversational repair can help them to better articulate their problem solving and even improve their problem solving abilities.

Given the call for effective strategies to provide analytic scaffolding and encourage verbal reasoning from students, this paper extends the current research in conversational repair to develop it as an intervention for helping students with LD’s verbal reasoning. Based on research in conversational repair and scaffolding instructions, the repair requests used to scaffold students in this paper varied in terms of explicitness in addressing the specific parts of the repairable utterance along the repairing process. The repair process starts with initiations, or repair requests, from others, to ask the target students to repair their original utterance. It is expected that later on as the request process becomes familiar, students with LD would self-generate explanation repair to their own problematic utterances. That is, in addition to the use of repair requests as a scaffolding strategy applied by others, students with LD can gradually self-address explanation deficits without requests from others.

The research questions are:
(a) What are the characteristics of self-explanation utterances of students with LD, pertinent to repair frequencies and types before, during, and after intervention?

(b) Will conversational repair increase the quality of self-explanation of students with LD?

(c) Will the students gradually need fewer and fewer repair requests as they go through the intervention?

(d) How will conversational repair help students’ problem solving performance?
CHAPTER 2. LITERATURE REVIEW

Overview

This chapter will first present the significance of communication in learning and requirement of current reform-based mathematics classroom on discourse. In addition, it will describe how teachers and students can achieve successful discourse-oriented teaching and learning processes, especially when students with LD are involved. Next it will point out the significance of self-explanation in classroom discourse and review relevant literature on self-explanation and interventions used to enhance this strategy. Then it will point out a way to improve current research on ways to cultivate self-explanation and apply it to students with LD.

2.1 Communication for Learning

The importance of communication in learning stems from Vygotsky’s social development theory and social constructivism.

Social development theory deals with the relationship between thought and language. It has three major themes (Vygotsky, 1962; 1978): (1) Social interaction plays a fundamental role in the process of cognitive development; (2) The More Knowledgeable Other (MKO); and (3) The Zone of Proximal Development (ZPD). Vygotsky held that language, learning, and social interaction are mutually dependent.
Social interaction is crucial in shaping cognition in the sense that development of thinking happens from the social to the individual rather than from the individual to the social (Kotsopoulos, 2010). Learning occurs largely through linguistic interaction, where a learner learns with assistance from a more knowledgeable other, who is usually a tutor, a teacher, or an older adult with better understanding or more knowledge than the learner in a certain aspect. From initially learning with guidance and help to at last being able to independently solve a task, learning occurs in one’s ZPD (Hufferd-Ackles, Fuson, & Sherin, 2004).

The concept of scaffolding first proposal by Wood, Bruner, and Ross (1976) originally targeted at the adult assistance to young children during joint problem-solving activities. The beginning researchers were influenced by Vygotsky’s theory. Cazden (1979) first made the connection between ZPD and scaffolding metaphor explicit (Stone, 1998). Since then, the two were closely related. According to Stone (1998), there are four key features of scaffolding metaphor: (1). An adult involved a child in an activity where the child was fully aware of the desirable goal bit was unable to achieve the goal independently; (2). The adult provided titration of assistance during the interaction; (3). The adult’s support could vary in types, such as extensive dialogue, modeling or gestures; (4). The adult’s support faded out gradually so that responsibility could be transferred to the child.

The concept of scaffolding has been widely studied in the contexts of parent-child interactions and teacher-student interactions, both descriptively and experimentally. In the experimentally studies, it has been generally found that students or children in the scaffolded condition outperformed the ones in other
comparison conditions. It is also effective in the field of learning disabilities. For instance, a well-known application of scaffolding is reciprocal teaching (Palincsar & Brown, 1984; Palincsar, 1986; Palincsar & Brown, 1987). In the study involving junior high students with lowest reading scores and poor reading comprehensions (Palincsar & Brown, 1984), students and the adult instructor used reciprocal teaching to take turns to serve as the teacher’s role in leading the group learning of a target text. The person in the teacher’s role was responsible to conduct a routine of activities, such as asking questions, summarizing, and predicting or requesting clarification. The actual teacher provided assistance to the “teacher” when necessary by means of modeling, prompting, and direct explanation. It found that with this intervention, students’ comprehension performance significantly improved. So did their use of target reading strategies.

2.2 Constructivistic Approach for Students with LD

Vygotsky’s theory is one of the foundations of constructivism. Constructivism emphasizes teacher-student interaction. It views students as active participants in learning process with teachers’ assistance (Mercer, Jordan, & Miller, 1994). In terms of the explicitness of instruction, constructivism can be classified into endogenous, exogenous, and dialectical ones (Mercer et al., 1994). The endogenous constructivism holds that instructions should not be explicit. Teacher-student interactions provide challenges and students are supposed to self-explore and self-discover new knowledge; exogenous constructivism believes in more direct instruction “through the use of describing, explaining, modeling, and guiding practice with feedback” (pp.
dialectical constructivism stays in between. It believes in collaborative interactions where teachers provide instructions when needed ("e.g., offering metacognitive explanations, modeling cognitive processes, asking leading questions, and providing encouragement" [pp. 293]).

Constructivism can also be classified as radical and social. Contrast to radical constructivism (von Glasersfeld, 1990) which emphasizes self-organization, social constructivism (Cobb, 1994) takes into account the sociocultural perspective and "asserts that an individual’s learning is affected by participating in a wider culture, the classroom, and the outside world" (Hufferd-Ackles et al., 2004, pp. 83).

This paper takes the perspectives of social development theory, scaffolding metaphor, and dialectical and social constructivism. Mathematics learning is a constructive process where students construct mathematical knowledge by building the new on prior knowledge (Hiebert & Carpenter, 1992). In this process, "not only cognitive development, but also social interactions, affective development, and the context of learning, are regarded as influential factors in mathematical learning" (Montague, 1997, pp. 164).

Documented mathematical deficiencies of students with difficulties in mathematics learning include aspects in cognitive and metacognitive strategies, memory and retrieval processes and transfer (Kroesbergen & Van Luit, 2002). They need much teacher assistance to recognize mathematical relationships (Borkowski, 1992; Cobb, Yackel, & Wood, 1992). Thus researchers state that students with math learning difficulties need direct, or structured, instructions and such instructions have been found to be effective (Gersten, Chard, Jayanthi, Baker, Morphy, & Flojo, 2009;
Van Luit & Naglieri, 1999). If the instruction is too endogenously constructivistic, or too discovery-oriented, it is less effective for these students than for others (Woodward & Baxter, 1997). Based on Mercer et al.’s (1994) review of literature on constructivistic teaching and strategy instruction for students with mild to moderate disabilities (including students with LD), the majority were exogenous and dialectical. Teacher modeling target strategy was the most frequently used instructional component (12 out of 14 articles), followed by engaging in interactive dialogues (9 out of 14 articles), encouraging metacognition and self-regulation, and providing prompts and guidance (8 out of 14 articles respectively).

In Kroesbergen and Van Luit’s (2003) meta-analysis of 58 mathematical interventions, direct instruction was found to be more effective than constructivist instruction in teaching basic skills. Yet, on the other hand, research also found that as long as we adapt the form and content of instruction according to their needs (Baxter et al., 2001), students with LD can also learn from instructions that provide opportunities to develop their own strategies (Kroesbergen & Van Luit, 2002, 2003; Woodward & Baxter, 1997). They can also learn the thinking behaviors such as asking questions, disagreeing, explaining and suggesting solutions (Berry and Kim, 2008), and reasoning with higher order skills, such as critical thinking, to the level and even better than their average-achieving peers (Leshowitz, Jenkens, Heaton, & Bough, 1993).

As such, the teacher-student interaction can be adjusted to be a dialectical one with an “implicit-to-explicit” continuum, which means that students are first given freedom to speak out their own thinking and the teacher provides indirect feedback or
guidance only when needed to lead students’ thinking. Assistance will be more and more explicit and direct in situations where students fail to progress with given guidance. There are limited studies for this combination of direct and indirect instructional interaction. Of relevance are studies that compare guided instruction and direct instruction for students with mild to moderate disabilities.

Kroesbergen and Van Luit (2002) compared guided instruction (GI), structured instruction (SI) and the control condition for teaching multiplication skills to a total of 75 students with low math performance from both regular and special education assigned to one of the three conditions. In SI condition, the teacher had a clear pattern, including repeating the previous lesson, providing an instruction, explaining how to solve the task, practicing and so on. Students were only asked to answer the teacher’s questions and apply taught strategies. They were not allowed to contribute anything that was not introduced by the teacher. While in GI condition, more space was given to individual contributions from students. After reviewing the previous lesson, the teacher presented the topic of the current lesson and students worked together for solutions and demonstrated their own strategies. The teacher only supported their discovery by posing questions and classifying strategies, but never introduced a strategy that was not discussed by students. The control condition was based on regular curriculum. The participants were tested on two tests: automaticity test and ability test. Automaticity test contained 40 multiplicative calculation problems. Ability test contained ten calculation problems and ten short story word problems. Ten items within the ability test contained numbers beyond what the students had been taught (i.e., 10×10), and they served as the transfer test. The study
found that first the two experimental groups (GI and SI) together showed greater improvement on the ability test but not on automaticity or transfer tests, which means that “the two interventions appear to be more effective than regular math instruction” (pp. 374). Then as to the difference between the two intervention conditions, in posttest, the GI group did significantly better than the SI group on both ability and transfer tests. Yet after three months, the best performance of the GI group did not carry over to a delayed follow-up transfer test because students from regular education improved while those in special education declined at follow-up. Automaticity test showed that GI was especially helpful for students in regular education and SI was especially helpful for those in special education. These findings should be interpreted with the limitations of the study that the students in regular education had slightly higher IQ than the ones in special education, and the sample size was relatively small. Yet the study still could conclude that guided instruction was effective for low achieving students.

Another thing to notice about this study is that in order to make a clear distinction between guided and structured instructions, in constructivistic (i.e. guided) instruction condition, the teacher did not provide any direct teaching. If the desired strategy was not brought up by students, the teacher would not mention it. This too discovery-oriented factor could also affect the effectiveness of the instruction. This inspires a consideration that within the context of constructivistic instructions, the degree of freedom to students’ exploration may vary and hence affect the interpretation of effectiveness of constructivistic interventions in general.
2.3 Classroom Discourse

Discourse is a term from linguistics, which basically means the continuous stretch(es) of language (Crystal, 1980) that denotes social relations. As such, the term discourse implies “social communicative practices” (Hicks, 1995-1996, pp. 51). Classroom discourse is the study of language in educational settings, and has been a significant theoretical perspective for studying learning in social settings (Gee & Green, 1998). It refers to the ways to study structural organization of interaction in classrooms. It has significance in the sense that it holds the premise of the close relationship between language and meaning. It gives the theoretical depiction of the relationship between language and meaning in classroom teaching and learning (Hicks, 1995-1996).

In classrooms, teachers’ and students’ deliberate actions can be defined as discourse moves (Krussel, Edwards, & Springer, 2004). Discourse moves can be both verbal and nonverbal. Nonverbal forms are facial expressions, gestures, eye contacts and so on. Verbal forms include: challenge; probe; request for clarification; request for elaboration; request for participation; invitation for attention; piece of information; hint; direction. Yet here in this paper we are consonant with Mercer (2010)’s view that in spite of the existence of several communicative modes, spoken language remains the most significant one and the “prime cultural tool of the classroom” (pp. 10) as it “enables, in unique ways, the development of relationships amongst teachers and learners and the development of children’s reasoning and understanding.” (pp. 10).
In traditional classrooms, teachers are regarded as “controlling” or “dominating” classrooms. The communication pattern between teacher and student in class: *initiation-response-evaluation* (IRE) / or *initiation-response-feedback* (IRF) (or IRE/F) was found by Mehan (1979) and Sinclair and Coulthard (1975) respectively. The teacher takes the initiation of the conversation, generally by asking a “closed, ‘known-answer’ question” (Nathan, Kim, & Grant, 2009); the student replies to the question and the teacher provides evaluation or feedback of the student’s reply. The pattern almost covered all the teacher-student talk in traditional classrooms as the criterial or “unmarked” discourse structure (Cazden, 1986).

2.3.1 Mathematics Classroom Discourse

Research on classroom discourse within specific domain areas, like mathematics, developed recently (Walshaw & Anthony, 2008). There is increasing interest in the role classroom communication plays in teaching and learning within academic “content areas” (Hicks, 1995-1996). Current reform in mathematics education calls for reasoning and communication aspects of mathematics learning to develop student competencies. It necessitated the development of classroom communities where students actively engage in mathematical classroom communication. Discourse has become “a major mechanism by which groups of students working together produce mathematical knowledge.” (Williams & Baxter, 1996, pp. 23). The *Principles and standards for school mathematics* (National Council of Teachers of Mathematics, 2000) (hereafter called the *Standards*) emphasized that “communication is an essential part of mathematics and mathematics
education” (2000, p. 60), because it is a way to make ideas public for reflection, discussion and refinement. Students in this process learn to be clear and convincing in expressing their own ideas and to listen to, understand and make connections with other’s ideas and hence sharpen their thoughts. The Standards includes communication as one of the five process standards by stating that “instructional programs from prekindergarten through grade 12 should enable all (italicized by author) students to organize and consolidate their mathematical thinking through communication; communicate their mathematical thinking coherently and clearly to peers, teachers, and others; analyze and evaluate the mathematical thinking and strategies of others; use the language of mathematics to express mathematical ideas precisely.” (p. 60)

2.3.2 Teacher’s Role in Mathematics Classroom

Given the current spotlight cast on mathematics classroom discourse, it worth the endeavor to explore how to achieve a quality classroom discourse to facilitate students’ learning. On the teacher’s side, the reform movement could be interpreted as supporting discourse-oriented teaching (Williams & Baxter, 1996), which supported mathematical learning through discourse. Teachers play a still important but different role in reformed classrooms from traditional ones. They become the “orchestrator” of the classroom discourse (NCTM, 2000; Williams & Baxter, 1996). Their instructional strategies should be able to enhance students’ communication skills for organizing and expressing their ideas, sharing their ideas with others and evaluating other’s thinking. However, quality mathematics pedagogy for enhancing
effective classroom discourse is still in its formative stage (Walshaw & Anthony, 2008) and several researchers have pointed out the difficulties teachers encounter in establishing the classroom community that could enact mathematics reform (Walshaw & Anthony, 2008; Hufferd-Ackles, Fuson, & Sherin, 2004; Williams & Baxter, 1996; Hicks, 1998; Lampert & Blunk, 1998). According to Hufferd-Ackles et al.’s (2004) review of the studies since 1990 on teachers’ change in their instructions for the goals of reform, some of the dilemmas teachers faced included students disengagement in more challenging tasks, difficulties in managing mathematical directions set by instruction, sense of less efficacy, anticipation for where a lesson went and preparation for their own roles in instruction. Compared with teachers in Japan and Germany, U.S. teachers asked significantly more yes/no questions and less describing/explaining questions (Kawanaka & Stigler, 1999).

Implementing the Standards is especially hard for special education. Based on a survey about secondary general education math and special education teachers’ perceptions of the Standards (Maccini & Gagnon, 2002), compared with general education teachers, special education teachers were less familiar with the Standards, especially for those in middle schools. Also, special education teachers were less confident in teaching students with learning disabilities (LD) and emotional disabilities (ED).

Walshaw and Anthony (2008) did a comprehensive review of recent research on the pedagogical approaches teachers actually used in mathematics classroom discourse to produce expected outcomes for diverse students. The searching in literature found a large amount of evidence that supported the benefits of dialogue
involvement in mathematics classroom. First, classroom discussion was crucial to conceptual understanding, and pedagogical practices that facilitated students’ participation in classroom interactions could have significant impact on students’ mathematical development (e.g. Steinberg, Empson, & Carpenter, 2004).

Second, students had different levels of engagement in classroom discourse and there was need to provide equal opportunities for all students, especially for immigrant students (Planas & Gorgorió, 2004), students with lower academic achievement (Baxter et al., 2001; Ball, 1993) and lower socioeconomic status (SES) (Lubienski, 2002), who were more reluctant to express ideas and contributed less to the discourse. If efforts were not made to get them involved enough, they were equal to being excluded from the mathematics classroom, which would constrain their development in mathematics.

Third, the pedagogy to facilitate classroom discussion and communication was not solely providing social scaffolding, which refers to establishing an encouraging climate, and proving opportunity for students to talk and listen to each other and build on others’ thoughts. Also, teachers should provide analytic scaffolding, which refers to the necessary guidance in forms of explanation, questioning, and clarification to move students’ thinking forward and facilitate discussion toward the goal. Williams & Baxter (1996)’s study echoed the third point. The authors made a clear distinction on the functions of social scaffolding and analytic scaffolding. They observed a seventh-grade mathematics teacher who held the belief that to “orchestrate discourse” was to give social scaffolding only. That is, she merely set up classroom norms and expectations for students’ interactions, and
provided encouragement. The teacher, though successful in establishing proper behavior and encouraging communication, still experienced difficulties in implementing discourse-oriented teaching, as she distanced herself from providing analytic scaffolding. That is, she did not give necessary help to affect students’ mathematical ideas so that they could efficiently and reasonably construct knowledge. The consequence of her orchestration was that some students viewed talk as ritual and an end in itself. They seldom asked questions and the questions they asked were mainly about definitions and procedures. Their communication was irrelevant to the work and could not help their performance in any sense.

Thus it can be seen that within classroom discourse teachers’ analytic scaffolding for students’ mathematical thinking, or cognitive work, is crucial. Contrasting to traditional classrooms where the majority of teachers’ questions were low level analytic scaffolding, or in other terms containing low level cognitive work such as asking for recitation of basic facts, yes/no answers or easy computations, reformed classroom calls for high level analytic scaffolding that provides or asks for explanations and justifications, connection making and generalizing, problem posing and generating conjectures, and multiple views by comparing/contrasting different strategies and claims (reviewed by Pierson, 2008). This scaffolding usually is in the forms of requests for elaborated answers such as description/explanation.

However, it is also warned by Kawanaka and Stigler (1999) that not all moves requesting description/explanation were qualified as higher order questions. Only the ones that elicited high order thinking could be called high order questions. Here comes the question of how to code and measure mathematical ideas involved in
classroom discourse, not only the teacher’s, but also, maybe more importantly, the student’s. The following part introduced several frameworks in existing literature that examined the development of students’ reasoning and thinking ability in mathematics classroom.

2.3.3 Students in Mathematics Classroom

According to Kotsopoulos (2010) students’ mathematical talk could be classified as inter-individual and intra-individual level. Inter-individual talks occur among students. This peer-talk is emphasized in current mathematics education in that it enables students to “both solidify their own understandings and potentially support their peers” (pp. 1052) in mathematical sense-making (Pimm, 1987). Intra-individual talks provide insights into individuals’ cognition usually by looking at think-aloud and talk-aloud protocols. Talk-aloud is usually defined as a self-talk to and for oneself to help organize thinking without communicative intention (Kotsopoulos, 2010; Pimm, 1987).

With video study methodology, Kotsopoulos (2010) examined students’ talk-aloud in peer collaborations in an eighth grade classroom and found that talk-aloud served three purposes: to express clarification, confusion, and the combination of the two. What is especially interesting in the finding was that some students did this for themselves, while others expected the talk-aloud to catch peers’ attention. However, students perceived their peers’ talk-aloud behavior as intra-individual instead of a communicative gesture. As such, the talk-aloud process does not help much with mathematical communication among peers. However, as a way of verbalizing one’s
thinking, from Erricsen and Simon’s (1980) information-processing view of cognition, it helped expose the “internal stages of the cognitive processes employed during problem solving” (Montague & Applegate, 1993, pp. 20) and the verbalization would not impose big additional cognitive load.

Students with LD have limited cognitive and metacognitive knowledge. Think-aloud can be a means to explore their cognitive and metacognitive characteristics. For example, Montague & Applegate (1993) asked 30 sixth-, seventh-, and eighth-graders to think aloud while solving one-step, two-step, and three-step mathematical word problems to see their problem-solving processes and strategies. The students were with LD, average-achieving, or gifted. The study found that when the problem was perceived as easy for participants, such as a one-step division problem, there was no difference in verbalizing cognitive and metacognitive strategies between the three groups. When the problems became more and more cognitively challenging, such as two- and three-step word problems, the average-achieving and gifted students brought their cognitive strategies and processes under conscious control. Thus they increased their cognitive verbalization while solving the problem. Yet students with LD reached cognitive overload with these problems, which resulted in them “shutting down” their processing (pp. 29). For the three-step problem, average-achieving students could still control their cognitive strategies, yet their metacognitive strategies were limited by cognitive demand of the problem. As to the total number of cognitive and metacognitive verbalizations, it found that though gifted students made more verbalizations than the other two groups, no significant difference was found between average-achieving students and students with LD, as
opposed to people’s expectations. Yet a closer look at the types of strategies would reveal that average-achieving students used more problem representation strategies such as paraphrasing, visualizing, and hypothesizing, whereas students with LD relied more on reading and computing strategies.

2.3.4 Classroom Discourse Involving Students with Disabilities

Discourse-oriented instruction has not been generally applied to students with disabilities, and there was little research concerning the impact of the Standards on special education (Berry, & Kim, 2008). Baxter et al. (2001) observed 16 low-achieving students, either identified as LD or not, of 5 elementary mathematics classrooms which implemented reform-based instruction. They found that these students were only minimally involved in whole-class discussions. They rarely spoke and were easily distracted. Their passivity might be attributed to cognitive overload from the reform-based curriculum, teachers’ subtle passive behaviors, and inadequate opportunity to speak.

Baxter et al.’s (2002) study reported mathematics discourse changes over time from a teacher-directed one towards a student-centered one in a fourth-grade classroom where teacher intentionally included a wide range of students during problem-solving instructions. Three of the twenty-eight students were at risk for or receiving special education services. The goal was to identify systematic patterns in teacher-student interactions. For that, the authors developed a coding scheme to analyze the video transcripts (though the coding scheme was not clearly provided in the article). The results showed that the classroom discourse was more and more
student-centered, but low-achieving students still had difficulties engaging in the complex discussions and they contributed marginally to the classroom discourse. They needed a great amount of assistance to be able to participate.

Still in the same series of studies on reformed mathematics classrooms involving low-achieving students, Baxter, Woodward, and Olson (2005) explored the use of math journal as an alternative form of communication that could reflect the conceptual understanding, representation, and adaptive reasoning of these students. Their study focused on four 7th graders whose mathematics achievements were in the lowest third of the class. The participants were taught to write down their mathematical thinking and their journals were coded into different levels that represented their levels of conceptual learning. For instance, if students were able to state “memory of concrete experience in her/his own words with no inferences” (pp. 123), it revealed that students had conceptual understanding and strategic competence (problem representation). If they were able to generalize (applying mathematical concepts in similar contexts) and even note the relationships between generalizations, it revealed that they had developed adaptive reasoning. The study found that though the classroom observation suggested the target students marginally involved in classroom discussion, they were willing to explain their reasoning and feeling in journals. Over half of each student’s journals were describing their problem solving steps. The authors concluded that writing could be an alternative strategy for low-achieving students to communicate mathematically.

Berry and Kim (2008) studied 4 teachers’ instructions in a first-grade mathematics inclusion classroom involving students with special education services.
Teacher utterances were coded into six categories: (a) questioning/eliciting, (b) responding to students’ contributions, (c) organizing/giving instructions, (d) presenting/explaining, (e) evaluating, and (f) sociating. The study showed that the instructions of the teachers were mainly recitational and missing the characteristics of interactions required by the Standards. That is, teachers seldom asked questions related to explanations, idea sharing or peer assisting. This study reflected the necessity to implement discourse-oriented instruction for students with disabilities. It suggested that students with disabilities might benefit from scaffolding techniques that facilitated thinking and communication. Teachers could start with new questioning patterns, which echoed the call for more teachers’ elicitation in reformed classrooms by Fraivillig, Murphy, & Fuson (1999). The use of more questions like “How do you know?” and “What if we try…?” could assign a different role to students and make them share authority and accountability for idea contributing.

According to existing literature, though it is difficult for students with disabilities to participate in discourse-oriented learning (Baxter, Woodward, Voorhies, & Wong, 2002; Maccini & Gagnon, 2002), they could still learn the thinking behaviors such as asking questions, disagreeing, explaining, and suggesting solutions (Berry & Kim, 2008). They can also benefit from the discourse-oriented classroom as their normally-achieving peers do (Berry & Kim, 2008; Kroesbergen & Van Luit, 2002, 2003; Woodward & Baxter, 1997) if teachers use effective instructional strategies (Baxter, Woodward, & Olson, 2001).

**Coaching Students with LD’s Reasoning.** Some studies examined the effect of coaching on students with LD’s explanation of reasoning. For instance, Scruggs and
his colleagues found that these students’ explanation of reasoning could be promoted by coaching with the topics about animals (Scruggs, Mastropieri, & Sullivan, 1994; Scruggs, Mastropieri, Sullivan, & Hesser, 1993; Sullivan, Mastropieri, & Scruggs, 1995). In Sullivan et al. (1995)’s study, 63 fourth- and fifth-grade students with LD were assigned to three conditions and provided with information about animals. In the coaching condition, students were provided with different levels of coaching after they heard the information like “The honey bear has a double layer of fur.” In the first part of the procedure, Coaching-One, the question was “Why would that make sense?” If a student failed to produce an appropriate explanation, Coaching-Two was implemented where the student was asked another question with relation information that served to giving some hint, such as “What would honey bears eat and where would they get their food?” If the student failed again, Coaching-Three gave a statement which gave clearer hint and asked the reason again, such as “remember, honey bears steal honey from bee hives, so why would it make sense that the honey bear has a double layer of fur?” If the student still failed, in Coaching-Four the students were given the reasoning directly in the form of a questions and they only needed to give a yes or no response, such as “since honey bears need protection from the bees when they steal their honey, would it make sense that the honey bear has a double layer of fur?”

In provided-explanation condition, the experimenter told the reasoning after the basic information and asked students to repeat what they heard. In no-explanation control condition, the experimenter provided only the basic information without explanation and asked students to listen carefully and repeat what they heard because
repeating helped remembering. After the practice, there were immediate and one-week-delayed tests on recalling the information and explanations about the animals. Results showed that for immediate recall and explanation tests, the coached condition outperformed the explanation condition which outperformed the control condition. The same result also occurred for delayed recall test, but for delayed explanation test, the coached and the explanation condition both outperformed control condition with no significant difference between the coached and the explanation condition. It indicated that when students with LD were coached to generate their own explanations, they remembered substantially and significantly more. Also, students with LD did not voluntarily use strategies and they needed guided assistance. In this case, asking “Why” enabled them to remember more than those who were not encouraged to do so. Nevertheless, a survey at the end of the study showed that students enjoyed the process of actively engaging in thinking during learning. A supplemental analysis of the coaching condition revealed that various levels of coaching helped all students construct explanation and they could accomplish 80% of the items without the fourth level coaching.

The above study brings indications. First, students with LD’s thinking and reasoning level can be coached to improve. Second, asking “why” questions to students with LD with several coaching layers containing more and more scaffolding information can help their thinking and reasoning.
2.3.5 Summary

In mathematics classrooms emphasizing discourse, as stated earlier in this paper, students learn to use the language of mathematics to express mathematical ideas precisely, so that they could make themselves understood by others and convince others, and connect to others’ ideas to sharpen their own thoughts. As such, it seems that the ability to explain is more relevant and crucial than think- or talk-aloud about all the things going on in one’s mind during problem-solving, which mainly serves an intra-individual function. In order to communicate ideas with the teacher and other peers, a student should in the first place be able to explain his/her ideas clearly, either to oneself or to others. In this sense, we say that explanation is an important component in mathematics classroom discourse. Self-explanation is to explain to oneself problem-solving process and reasoning behind the process. Explaining to others is in some sense the same as explaining to oneself (Renkl, 1999). Thus if a student is able to make good self-explanation, s/he can also make him/herself clear when explaining to others.

In a constructivist learning process, to help a student produce a satisfactory explanation of thinking or reasoning, a teacher should collaborate with the student through dialogues. Specifically, scaffolding techniques can be implemented as analytic assistance whenever the student needs. Teacher-student interaction was also called socratic dialogue, interactive discourse, reciprocal teaching, collaborative discussion, and scaffold (Mercer et al., 1994).

In general education, studies on the effects of students’ self-explanation as a strategy for learning outcomes is relevant and reviewed below.
2.4 **Self-Explanation**

We borrow the operational definition of self-explanation by Neuman, Leibowitz, and Schwarz (2000) here: self-explanations are “utterances that involve not only inference of new knowledge but also clarification of the problem and justification of activities that occur during the problem-solving process” (pp. 199). Researchers have shown that it is an effective strategy for learning in the sense that learners who do self-explanation perform better in various cognitive tasks (Aleven & Koedinger, 2002; Neuman & Schwarz, 2000; Tajika et al., 2007). This is known as the *self-explanation effect*.

For example, for university students who had no prior knowledge in programming, those who were trained to use self-explanation strategies had greater performance gains and increased more on self-explanation strategy applications than those without training (Bielaczyc, Pirolli, & Brown, 1995). Rittle-Johnson (2006) examined the effect of self-explanation when combined with direct instruction or discovery learning, and whether the effect could promote transfer. Eighty-five third-through fifth-graders were assigned to one of four conditions across two factors: direct instruction versus discovery learning, and self-explanation versus no self-explanation, to work on mathematical equivalence problems. In the self-explanation conditions, after students solved each problem and were showed the correct answer, they were presented with a computer screen with one correct and one incorrect solution by students from another school. Then they were prompted to explain verbally why each was correct or incorrect. It found that self-explanation led to greater learning and transfer (2-week delay), regardless of instructional condition.
The reasons for the effectiveness of self-explanation have been proposed in two aspects. For one, the study materials are often incomplete texts, and the process of self-explaining provides new knowledge usable for later problem solving. When learners make domain-based explanations, they actively construct and draw inferences to fill in the missing information (Calin-Jageman, & Ratner, 2005; VanLehn, Jones, & Chi, 1992). When learners learn from worked-out examples (Chi, Bassok, Lewis, Reimann, & Glaser, 1989), self-explanation is also a way for learners to check their understanding and fill knowledge gap when the inadequacies are in the learner’s mental models instead of the study materials (Chi, 2000). In other words, if they cannot construct an explanation, it means that there is a gap in their knowledge (VanLehn, & Jones, 1993). Yet some (e.g., Neuman et al., [2000]) also pointed out that self-explanation effect could not be oversimplified to refer to a single definition or interpretation, since the phenomenon was context-specific covering various activities.

Self-explanation also promotes transfer (Chi et al., 1989; Siegler, 2002; Rittle-Johnson, 2006). In learning, often times, learners do not have a deep understanding of the knowledge. Their shallow learning produce inert knowledge that cannot be transferred (Aleven & Koedinger, 2002). Self-explanation deepens understanding and avoids inert knowledge. Rittle-Johnson (2006) pointed out four mechanisms underlying self-explanation effect on transfer: a). aiding invention of new problem-solving approaches; b). broadening the range of problems to which children accurately applied correct procedures; c). supporting the adaptation of procedures to
solve novel problems that did not allow rote application of the procedure; and d).
supporting retention of correct procedures.

To test whether the effect of self-explanation is caused by the extra attention learners pay during this process, or by the explanation-generating activity itself, Hausmann and VanLehn (2007) asked learners to either self-explain or paraphrase (to ensure they paid attention) physics complete and incomplete examples and their result was that the students who were prompted to generate their own explanations, outperformed the paraphrasing group in all tests reflecting both normal and robust learning. Thus though there were previous studies that were in favor of the “attention” hypothesis (e.g., Lovett, 1992), the researchers here supported the hypothesis that the knowledge source mattered. The process of generating explanations from the learners’ own background knowledge is the reason for its effectiveness.

Some explored whether the format of material influences self-explanation effect. In Ainsworth and Loizou’s (2003) study, twenty learners learned about human circulatory system by either text or diagram and generated explanations while learning. The result showed that the diagram condition improved significantly more than the text condition. Students in the diagram condition had a more effective learning in the sense that they produced significantly more self-explanations, while spent significantly less time in learning the material and spoke significantly less. Specifically, most of their explanations were concerning goals and principles.
2.4.1 Self-Explanation with Worked-out Examples

Examples play a critical role in learning. Students cultivate their problem-solving skills by applying the knowledge they have learned in examples. The way examples are processed can affect later problem solving (Sandoval et al., 1995) in the sense that concrete examples offer elaborations on the initial learning and help the appreciation of the relevance between the prior and the new knowledge (Bransford & Schwartz, 1999). Relating a new problem to examples or prior knowledge is easier than constructing a new solution procedure by oneself (Novick, 1990). From the Cognitive Load Theory (CLT) perspective, compared with learning from solving problems, learning from worked-out examples eliminates learners’ impression that the primary goal is to “solve” rather than to “learn”. Learners do not have to perform activities such as subgoal decomposition, and operator selection and execution (Sandoval et al., 1995; Trafton & Reiser, 1993). Thus worked-out examples reduced working memory burden and thus facilitate schema acquisition. Further, by reducing cognitive load, learners can better control their metacognitive processes (Renkl, 1999; Salden, Koedinger, Renkl, Aleven, & McLaren, 2010).

Often times, self-explanation is combined with worked-out problems to lead to deeper understanding, improve problem solving abilities as well as transfer to more complicated tasks. This has been tested over a variety of subject domains such as physics (Chi et al., 1989; Hausmann & Vanlehn, 2007), mathematical word problems (Tajika et al., 2007), geometry (Aleven & Koedinger, 2002), computer programming (Sandoval et al., 1995), and human circulation system (Ainsworth & Loizou, 2003). For example, when undergraduates were tested on computer programming under
different conditions (Sandoval et al., 1995), the result showed that the self-explanation group with solved examples used less time to solve posttest problems than the non-self-explanation group with solved examples. It suggested that self-explanation could improve the efficacy of learning from solved problems.

Tajika et al. (2007) tested six graders’ multi-step ratio word problem solving in three conditions: self-explanation, self-learning, and control. In the self-explanation condition, students were provided with worked-out examples and were asked to indicate whether they understood each description of solution step by saying yes/no judgments first in order to monitor their understanding. Then they wrote down what they understood or did not understand. In the self-learning condition, students were asked to understand each step. In the control group, students were given only numerical expressions and answers without descriptions for each step. The experiment showed that the self-explanation group outperformed the other two groups in the following ratio word problem test and outperformed the control group in a transfer test one month later.

2.4.2 Self-Explainers’ Characteristics

There are different self-explanation patterns between good and poor problem solvers. For example, in a study on algebra problems (Neuman et al., 2000), ninth-graders were divided into three groups of problem solvers: good (upper third), moderate (middle third), and poor (lower third). They were asked to self-explain while they were solving algebra problems. Their think aloud protocols were coded into three types of self-explanations: clarification, inference, and justification, and
two types of control activities: monitoring (i.e., declarative knowledge of what the student knew and did not know) and regulation (i.e., “regulating the execution of procedural knowledge” [pp. 203] such as “I will do it in several steps” and “I am formulating the equation” [pp. 203]). The study found that self-explanation was a significant predictor of problem solving performance, with inference and clarification as the most frequently used categories. The good problem solvers were more likely to produce justifications after regulations (explanation of a control activity). It indicated that for these students, they not only developed planning steps for solving problems, but also used speech to supervise the planning. In this case, speech was used not only to mediate mental functions (e.g., reasoning), but also to supervise their planning, or it was used in a meta-pragmatic way to mediate itself. For poor problem solvers, they were more likely to produce inference followed by clarification (explanation of explanation). Inference was to deduce new knowledge in the form of “if…then” as condition-action rules. The authors held that “the need to explain a deductive inference may reflect students’ difficulty in understanding the inferences they produced” (pp. 208). In addition, the content of the “if…then” conditions was not the solution process itself. Thus good problem solvers’ justification following regulation reflected a more advanced cognitive level than poor problem solvers’ justification following inference.

Chi et al.’s (1989) classic study was on how students learn via self-explaining worked-out examples of Newton’s laws of motion problems. Based on post hoc isomorphic and “far transfer” problems, the top-scoring four students were called Good students and the bottom-scoring four were Poor students. Their think-aloud
protocols showed that the Good students spent longer time on each example. To qualitatively code the ideas expressed by the students, it showed that the Good students had significantly more utterances that belonged to the *explanation* category (compared with *monitoring statements* and *other statements* such as paraphrases), and the Good students gained more understanding of the principles through the learning process.

Different from the above study, Renkl (1997b) controlled for the learning time (quantitative aspect) and only examined the qualitative differences between the Good and the Poor students on studying worked-out probability calculation problems. Renkl found that the quality of self-explanations was significantly related to learning outcomes, and there were distinct characteristics of successful and unsuccessful learners. The successful learners were the ones that either tended to identify the domain principles, or tended to anticipate the next solution step before reading the solution for that part. The less successful learners could also be classified into two types: the passive ones who exhibited very low level of self-explanations, and the superficial self-explainers who touched the explanation activity superficially. It is also worth mentioning that the study found that except for anticipative reasoning, the self-explanation characteristics did not significantly depend on learners’ prior knowledge level.

It should be pointed out that most learners are unsuccessful worked-out-example learners (Renkl, 1999; Aleven, Koedinger, & Popescu, 2003), and according to Renkl (1999) this was related to metacognitive drawbacks of learning with worked-out examples. Because the examples have already been worked out without providing
any feedback, it easily gave learners an illusion of understanding; and many learners are unaware of, or lack the “metaknowledge of how to learn from worked-out examples” (pp. 480). To overcome the drawbacks of learning from worked-out examples, students need to “ascertain the conditions of application of the solution steps beyond what is explicitly stated” (Chi et al., 1989). Generating explanations is a way to help students understand why a solution is taken. Therefore, it is necessary to explore ways to support self-explanation efficiently and practically in educational settings.

2.4.3 Instructional Strategies to Scaffold Self-Explanation

Self-explanation can be facilitated by direct instruction. Direct teaching of self-explanation could be achieved by human one-on-one instruction (Aleven & Koedinger, 2002; Bielaczyc et al., 1995). For instance, Bielaczyc et al. (1995) testified that self-explanation was teachable and it furthered Chi et al.’s (1989) finding that there was not only a positive correlation between particular strategies used in self-explanation and learning outcomes, but also a causal relation between these two. During the intervention, the instructional group received explicit training in three top-level self-explanation strategies that were found to be used by high-performance students, such as identifying and elaborating the relations between the main ideas; determining both the form and meaning of the specific code (for computer programming); and connecting the concepts in the texts and the examples. The training process was structured and systematic one-to-one interaction between the experimenter and each participant in three sessions through a series of
programming lessons. Activities in the sessions included “a). introducing and motivating the self-explanation and self-regulation strategies, b). modeling the strategies using the student model on videotape, and c). verifying a participant’s ability to apply the strategies to instructional materials from a new programming lesson” (pp. 230). The control group interventions paralleled with the instructional group with activities such as discussion, memory recall, and essay writing, but no explicit strategy training. The results showed that the instructional group outperformed in programming performance than the control group. In addition, the instructional group increased the use of the trained self-explanation strategies more than the control group did.

Instead of eliciting or training for self-explanations, indirect interventions use incentives to stimulate motivation (Renkl, 1995, 1997a, 1998). With the name “learning by teaching”, this type of intervention was to assign a teacher’s role to learners. It was assumed that explaining to others was in some sense the same as explaining to oneself, so the explainer’s role would reduce the passive or superficial characteristics of some learners. Studies have tried this “learning by teaching” in three conditions: participants expecting future learners, participants with actual learners, and participants with actual learners and answering “what-if” questions from the co-learners. The studies found that the teaching for future learners condition did not foster self-explanations because it increased stress and reduced intrinsic motivation; the existence of a co-learner increased explanation activities, but not learning outcomes; the co-learners with questions condition increased explanation activities and learning outcomes of those with low intrinsic motivations, but reduced
highly-motivated learners’ learning outcomes because the questions from the co-
learners seemed to affect their spontaneous explanations. In addition, this condition is
most profitable to those with high level of prior knowledge since they were able to
respond to the questions correctly.

Some interventions provided prompts with formats tailored according to
learners’ prior knowledge level. Yeh, Chen, Hung, & Hwang (2010) argued that
reasoning-based prompt environment fit for the lower-knowledge learners and
predicting-based prompt environment fit for the higher-knowledge learners. In their
study, undergraduates were assigned to these two intervention conditions and a
control group while learning a quite complex topic (i.e. AVL tree) in computer
science domain through a computer-based multi-representational learning
environment. The reasoning-based prompts were presented in the format of fill-in-
the-blank statements. The predicting-based prompts involved predicting prompts,
self-checking prompts, as well as fill-in-the-blank statements, which were used when
the learners gave wrong predictions. The result confirmed the hypothesis that when
learning dynamic multimedia materials, self-explaining effects differed for learners
with different expertise levels. In terms of learning outcomes on descriptive
knowledge test, near and far transfer tests, the lower-knowledge learners benefited
most from the reasoning-based prompts and higher-knowledge learners benefited
most from the predicting-based prompts. Yet the most effective prompts did not
reduce cognitive load demand or learning time as expected.

Computer-Based Program as Intervention. Some argued that it is not practical to
give one-on-one help for self-explanation in actual classrooms and thus advocate the
use of computer-based instruction to enhance this ability. For instance, a series of studies have been conducted about intelligent instructional software called the Geometry Cognitive Tutor (Aleven & Koedinger, 2002; Aleven and Koedinger, 2000; Aleven, Koedinger, Sinclair, & Snyder, 1998) in learning geometry. In an early version of it, the software provided guidance only for problem solving but not for self-explanation, and significant learning gains were found (Aleven et al., 1998). Later, with a newer version of the software to support explanation (Aleven & Koedinger, 2002), the students typed in or selected from a reference list the principles that justified problem-solving steps, and the Tutor provided feedback. For example, when the answer was incorrect, it displayed error messages and on-demand hints with multiple levels of hints for each step. The experiment showed that the self-explanation group improved more than the problem-solving group (with older version of the software) in all types of problems in posttest and they are especially better in the terms requiring deeper understanding in transfer the skills learned with the tutor to different test environments. Self-explanation did not change the rate of acquiring knowledge (the self-explanation group did not learn faster), but it changed the nature of acquired knowledge (for instance, the Explanation students had stronger declarative knowledge whereas Problem Solving students had stronger procedural knowledge and shallow knowledge). The researchers thus concluded that prompting and giving feedback to self-explanation can both quantitatively and qualitatively improve learning achievement.

The computer-based tutor providing prompts by reference list (Aleven & Koedinger, 2002; Aleven, Koedinger, & Cross, 1999) had produced positive results.
However, the researchers later realized that “it is important that students explain in their own words, rather than by providing references, or using a structured interface” (Aleven & Koedinger, 2003, pp. 39). The reason is that compared to providing references, a full-blown explanation can show how the principle or rule drawn from the reference can be applied to the current problem. To support self-explanation, natural language dialog (Aleven and Koedinger, 2003) was believed to be better than a menu selection in that (1) natural language is a natural interface already. Learners do not need to learn a new interface; (2) explaining in their own words makes it easier for the learners to build on their prior knowledge; they will spend more time recalling relevant knowledge from memory instead of recognizing what are given in the menu; (3) explaining in learners’ own words reduced problems with unfamiliar terminology. Thus an intervention in the interactive learning environment (ILE) (Aleven et al., 2003) to provide continuous and detailed guidance and feedback would be beneficial to support self-explanation.

To achieve this better natural language self-explanation supported by dialog intervention, the Geometry Cognitive Tutor was further developed into an even newer version called Geometry Explanation Tutor (Aleven, Popscu, & Koedinger, 2001), which evaluated students’ natural explanations (through typing in the computer) and provided feedback through “a restricted form of dialog” (Aleven et al., 2003, pp. 40). The feedback was a sequence of increasingly more directed messages for each explanation category, which ended with a “bottom-out hint” (Salden et al., 2010, pp. 382). Each student utterance should state a geometry role and was categorized to be correct, incorrect, or incomplete. Based on each category, feedback was provided
with “a sequence of increasingly more directed feedback messages” (Aleven et al., 2003, pp. 41). This Explanation Tutor was compared with the older versions that supported explanation by reference list (Aleven et al., 2003). However, this experiment did not find strong evidence for better effect of natural explanation condition and the reasons were discussed: first, giving mathematical explanations was hard itself; second, the computer program was still not a good tutor. It might set too strict criteria for being correct explanations, and its guidance and feedback might not fit in students’ ZPD, or require too much cognitive effort. Therefore, the hypothesis that natural language self-explanation is superior was not supported by the computer-based Cognitive Tutor program, but also was not disconfirmed by it. For now, it might be meaningful to try human-based interaction strategies to support natural language self-explanation first, rather than computer-assisted interaction.

2.4.4 Self-Explanation and Students with LD

Study on self-explanation has not been widely carried out in special education, even though there are intervention studies that contain student verbalization component. According to a meta-analysis of instructional components in mathematics instructions for students with LD (Gersten et al., 2009), “student verbalizations of their mathematical reasoning” (pp. 1210) was one of the major instructional components and eight articles were recognized from existing literature in this category, which resulted in the mean effect size 1.04 ($p < .001$). These eight studies were mostly in the 1980s and a closer look will reveal that though they were all under the umbrella of “verbalization”, the studies covered various topics, such as self-
instruction (e.g., Pavchinski, 1998; Tournaki, 1993), modeling (e.g., Omizo, Cubberly, & Cubberly, 1985) and cognitive strategy instruction (e.g., Hutchinson, 1993), not specifically refers to self-explanation as an effective strategy to improve learning. For instance, in Ross and Braden’s (1991) study, cognitive behavior modification (CBM) was compared with token reinforcement and direct instruction on students with LD’s basic addition and subtraction skills. CBM was to use self-instruction to regulate students’ behaviors while solving problems. They were asked to speak out questions such as “What is my assignment for today?” and “what kind of problem is this, addition or subtraction?” and verbally answer them, as well as verbalize what they did during problem solving process, such as “I add 2 and 1 and I write it down”. The process of self-instruction is a self-monitoring one where verbalization is used to mediate cognition and control behaviors. Verbalization did not focus on students’ explanations of their problem solving process.

Schunk and Cox (1986) conducted a study on 90 middle-school students with LD’s verbalizing out their subtraction problem solving steps and its effect on self-efficacy and subtraction skill. The verbalization here is the same as self-explanation. It found that overt verbalization of problem solving steps facilitated task performance, self-efficacy, and skills, and the verbalization had to be continuous throughout all the problem solving sessions. The group that discontinued verbalization in the middle of the sessions did not improve achievement. The authors pointed out that the effectiveness of verbalization might result from the possibility that as students with LD were often times inattentive to instructions and tasks, the process of verbalizing helped focus their attention on important task features and “as a form of rehearsal,
assists strategy encoding and retention” (pp. 206). It might also give them a sense of self-control over the process and outcome since it made their problem solving strategy salient.

In Schunk and Cox’s study, the middle school students were encouraged to freely verbalize while solving problems. Whereas in Omizo et al.’s (1985) study focusing on the effect of different forms of modeling, it was found that the 6- to 8-year-old students with LD achieve the highest self-efficacy and arithmetic skill scores when they observed a model explaining problem solving and followed the model to verbalize solutions while working on problems. It was better than the condition where students merely observed a model solving problems and the condition where there was no modeling.

To summarize, though the verbalization category in Gersten et al.’s (2009) meta-analysis contained various components, student verbalization was still found to be always effective, and was invariably involved in both explicit instruction and heuristics. The effect of verbalization was interpreted as directly addressing the impulsivity of students with LD, who, when dealing with multistep problems, would tend to randomly combine numbers instead of solving it step by step (Gersten et al., 2009). Yet less has been mentioned about how self-explanation was accounted for improvement in mathematics thinking and reasoning.

2.4.5 Summary

To Date, self-explanation has been implemented in various forms, such as pencil-and-paper-based written form, computer-based written form, oral protocols,
and computer-based multiple-choice form. It should be pointed out that nearly all of the existing studies treat self-explanation as a means to achieve better learning outcomes, instead of exploring it as a part of learning outcomes. There is no doubt that self-explanation is a useful tool for improving learning outcomes, and it is significant to explore it as a tool. However, given that today’s education highlights the constructive process of learning resulting in the importance of social communication in classroom discourse to construct knowledge, it is also significant to explore how to improve the self-explanation activity itself as a way to assess reasoning, which is a part of the outcome performance.

The forms of instructions to scaffold self-explanation were also varied, such as written, oral, and computer-based forms. Based on the reviewed literature above, though existing interventions, either human-based or computer-based, either direct or indirect, were effective to some extent, there are limitations and much to improve in these interventions. According to Renkl (1999), learners have not reached an optimal level of self-explanation in terms of quality and correctness, and the interventions have not solved the problems like the occurrence of incorrect self-explanations and some learners’ substantial comprehension problems. Still, there are many questions that need to be answered. For instance, how to combine the incentive setting with direct training of self-explanation? For the intervention directly aiming at self-explanation behavior, how to provide a specific and clear format so that later researchers can easily replicate and revise it instead of merely stating that a certain kind of prompts were given to the learners? Given all the concerns, it is necessary to explore better scaffolding strategies as interventions to improve self-explanation itself,
as it reflects reasoning. The trend of development for an effective intervention is a natural dialog-based one to scaffold self-explanation with feedbacks being a sequence of increasingly explicit help.

Renkl (1999) called for the combination of self-explanations and instructional explanations from more knowledgeable others such as teachers or tutors to address the shortcomings of sole self-explanations. He proposed that the ways to combine these two should meet five criteria: (1) it should provide as much self-explanations as possible, and as much instructional explanations as necessary; (2) it should provide feedback; (3) it should be appropriately timed and used in learners’ on-going knowledge-construction processes; (4) it should be built on learners’ prior knowledge; and (5) it should emphasize the underlying principles of the content domain.

Corresponding to Renkl (1999) and Aleven and Koedinger (2003)’s call for natural language dialog, it seems that the task is to find how to make the strategy a natural tutorial dialog that builds on the learner’s prior knowledge, gives the learner plenty of space to talk, and helps in when only necessary and highlights the principles of the content domain.

Salden et al. (2010) pointed out that for novice students who have inadequate prior knowledge in longer-term memory, even the Cognitive Tutor mentioned above with immediate feedback and layers of hints may not be enough helpful. They may need a bottom-out hint to move on. It is especially the case for students with LD. Their cognitive and metacognitive deficits have been addressed by modeling cognitive strategies, and findings of their think-aloud studies varied. Few studies addressed self-explanation as a helpful problem-solving strategy for low-achieving
learners or populations with disabilities. To date, the coaching strategy was only studied concerning learning about animals and its effect was only tested concerning recalling ability. It would be meaningful to see whether coaching self-explanation behavior of students with LD through natural tutorial dialog would promote their verbalization of reasoning process during word problem solving in mathematical discourse as well as promote their reasoning ability, which would be indicated in improved ability to solve similar word problems and the ability to transfer to more complicated word problems. Given such a call, the concept of conversational repair will be introduced as a potential strategy, which meets all the requirements as described above.

2.5 Conversational Repair

The concept of repair has been identified by American ethnomethodologists (Schegloff, Jefferson & Sacks, 1977) to address recurrent problems in speaking, hearing, and understanding.

According to Schegloff et al. (1977), the organization of repair is composed of three elements. First of all, there is the “trouble source” or the “repairable” that the repair addresses. Then there is the “initiation” that triggers the repair (Sometimes “repair” happens without initiation.). The result of repair, either a failure or a success, is the “outcome”:

Speaker 1: That is Lucy. [repairable]
Speaker 2: Who? [initiation]
Speaker 1: I said Lucy [repair].
Speaker 2: I see. [outcome: success]
Self-repair and other-repair refer to the success of a repair procedure. If the initiation is done by the speaker of the trouble source, it is called “self-initiation”:

Speaker 1: That is…What is her name? [self-initiation] Oh, right, Lucy.

If it is done by any person other than the trouble source (such as Speaker 2 in the previous example), it is call “other-initiation”.

Schegloff et al. (1977) identified five types of other-initiation: (i) Huh? What?; (ii) the question words who, where, when; (iii) a partial repeat of the trouble-source turn, plus a question word; (iv) a partial repeat of the trouble-source turn (v) Y’mean plus a possible understanding of prior turn.

Similarly, according to who performs the repair, repair can be classified into self-repair and other-repair. Therefore, all in all, repair can be classified into four types: self-initiated self-repair, other-initiated self-repair, self-initiated other-repair and other-initiated other-repair.

Conversational repairs can also be classified in terms of repair requests and responses. Repair requests were essentially the initiators defined by Schegloff et al. (1977). Repair responses were essentially self-repairs following other-initiations. Weiner (2005) classified requests into four types according to their functions: (a) non-specific requests for repair (e.g., “Huh?” and “What?”) (b) request for confirmation (e.g., Speaker: “I saw Sammy”; Listener: “You saw Sammy?”); (c) request for specification or clarification (e.g., Speaker: “Let’s play game”; Listener: “what game?”); (d) request for repetition of specific constituent (e.g., Speaker: “I saw Sammy”; Listener: “you saw who?”). One function could be achieved by different
forms. For instance, to request confirmation from the speaker, the listener can ask non-specifically by saying “Pardon?”, or repeat completely what the speaker has said, or partially repeat the part that needs confirmation, or asks *Y’mean* plus a possible understanding of the speaker’s words.

In normal conversations other-initiations overwhelmingly yielded self-corrections (Schegloff et al., 1977; Schegloff, 1992; 2000). That is, the listener is commonly reluctant to correct the speaker directly in the first place. If communicative problems occur, after the trouble-source turn, the other-initiator located the repairable in the following turn, offering the speaker of trouble source a further opportunity to do self-repair and waiting for him/her to self-repair in the third turn. In this sense, self-repair was preferred in conversation. It could be achieved through both self-initiation and other-initiation, with self-initiation coming first. Other-repair was dispreferred and when it did happen, which was rare, it was either specially modulated or specially positioned. It should be noted that repair is a sequentially organized phenomenon and it goes from trouble source to initiation to outcome, though sometimes a repair can be performed without initiations.

### 2.5.1 Conversational Repairs of/for Students with Disabilities

Other-initiated self-repair is the main type of repair that has been studied for people with disabilities, such as LD, language impairments (LI), and intellectual disabilities (ID). It occurs when the listener specifies a trouble source in the study subject’s conversational turn and usually by means of producing a clarification request to indicate misunderstanding. In response, the speaker repairs what s/he just
said. A successful repair demonstrates the cooperativeness of discourse in nature (Brinton et al., 1988). Some of the studies are reviewed below.

For populations with LD, there is still little research on repair. The limited research showed that students with LD did worse than nondisabled peers in requesting clarification from others -- to initiate questions to elicit other-repairs of communicative breakdowns (like inadequate information) (Donahue, 1984; Donahue, Pearl, & Bryan, 1980).

For populations with mild to moderate ID, stacked requests (“Huh?”, “What?”, “What?”) as a type of other-initiated self-repair were used to examine their conversational abilities (Brinton & Fujiki, 1996). Brinton and Fujiki found that both the young (with a mean chronological age 29) and the older (with a mean chronological age 63) adults with mild to moderate ID were responsive to the first requests, but were less responsive to the following two more requests and no significant difference was found between the young and the old groups. The authors concluded that the individuals with mental retardations were less consistent than expected in providing repairs given their cognitive and linguistic functioning level.

For population with language impairments (LI), a number of researches have demonstrated that children, even with language deficits, are aware of the obligatory nature to respond to clarification requests. For instance, the stacked clarification requests (i.e., two requests [Spilton & Lee, 1977] or three requests [Brinton et al., 1988] in a sequence) were found to be better than a single request in finally reaching a mutual understanding. Brinton et al. (1986) gave three neutral requests (“Huh?”, “What?”, and “I didn’t understand that.”) to 60 children with and without LI in three
different age levels, 5, 7 and 9. In response to the sequenced requests, repetition was the most common type of repair. Children with LI provided more inappropriate responses and frequently ignored the requests. Brinton et al. (1988) did a similar experiment with the same stacked requests to eight 9-year-old students with LI and their language age-matched (LA) and chronological age-matched (CA) peers. As in the previous study, they defined five types of responses, or repair: (a) repetition (of their original utterance); (b) revision (altered form with constant meaning); (c) addition (of detailed or specific information); (d) cue (of background context); (e) inappropriate (unsuccessful outcomes of repair). The LI group produced more inappropriate responses than the other two groups and initial requests often elicited repetitions as responses, but repetitions decreased in subsequent requests and were replaced by other types of responses. LI and LA groups were more likely to repair by revising forms alone than the CA group who tended to repair with supplemental information. LI children were still found to lack persistence in applying the repair strategies to complete the tasks, even though an acknowledgement of success of the repair was provided by the listener at the end of the request sequence in both of the two studies by saying “oh, I see.”

The difficulties children with LI exhibited in these studies may be due to their own language deficits, yet the stacked requests as a strategy and the neutral questions used deserve reconsideration. First, in these studies the three questions given to the subjects (“Huh?”, “What?”, and “I didn’t understand that.”) were all non-specific in requesting clarification. Thus they failed to achieve an approximation of mutual understanding by the listener to let the speaker realize what needed to be said
differently, or to repair. Furthermore, students might misinterpret the requests as disapproval for the content of their utterance (Brinton et al., 1986; Brinton et al., 1988). For children with LD in particular, who were often uncertain about themselves and thought that adults were omniscient, their primary goal during conversations in educational settings was to “conceal comprehension problems and knowledge gaps” (Donahue & Lopez-Reyna, 1998, pp. 400). Thus, repeated questioning as a strategy would make them feel in considerable risks of being “found out” (Donahue & Lopez-Reyna, 1998, pp. 400). Other types of requests, for example, a partial repetition or a partial repetition plus a question word, could be applied in the stacked requests to more explicitly indicate which part of the speaker’s turn needs repair. It is analogous to giving a series of scaffolding instructions to students along their way of solving problem from the least explicit to the most explicit (Day & Cordón, 1993; Day & Hall, 1988). Likewise, stacked questions could vary in terms of explicitness in addressing the specific parts of the repairable along the repairing process.

2.5.2 Scaffolding Questions with Varied Explicitness

Different from past research that has used questions at the same level of explicitness to request repair (e.g., “huh” or “what”), we propose in this paper that repair requests can serve as scaffolding questions which vary in the explicitness of addressing the repairable to achieve successful communication. Some of the examples are listed and discussed below:

A: I have a: - cousin teaches there.
D: Where.
A: Uh:, Columbia.
D: →Columbia?
A: Uh huh.
D: →You mean Manhattan?
A: No. Uh big university. Isn't that in Columbia?
D: Oh in Columbia.
A: Yeah. (Schegloff et al. 1977, pp. 369)

In this example, D first initiated a repair by repeating what A had said, but A did not respond with a clarification because A did not realize the word Columbia could mean both a place and a university and it was confusing to D. As such, D elicited a clarification again – in the form of *You mean* plus a possible understanding of prior turn (in this context, D interpreted Columbia as a place name) – to check whether his/her interpretation was correct. Then A did the repair by clarifying Columbia as a university name. The two conversational partners finally reached mutual understanding.

In another example:

Steven: One, two, three, ((pause)) four five, six, ((pause)) eleven eight nine ten.
Susan: → Eleven? Eight, nine, ten?
Steven: → Eleven, eight, nine, ten.
Nancy: → Eleven?
Steven: Seven, eight, nine, ten.
Susan: That's better.
((Game continues)) (Schegloff et al. 1977, pp. 373)

Susan repeated what Steven had said expecting him to correct his mistake, but Steven did not realize anything wrong there in the mere repetition of his own words. Then Susan specified the place she was questioning (the number eleven) and this explicit request made Steven realize the mistake in counting and through self-correction a repair was fulfilled. Essentially what has been used is a repetition of a prior statement and then highlighting that part of the sentence construction that was
unclear. In conclusion, conversational repair can be the self-righting mechanism for the organization of language (Schegloff *et al.*, 1977) and could be used in social contexts to improve the pragmatic use of language, social acceptance, and the language abilities of students with LD.

2.5.3 Summary

Conversational repair is a mechanism for repairing communicative breakdowns or potential problems in conversations. Predominantly, the conversational partner would leave the speaker the opportunities to correct or readdress his/her own speech, sometimes by giving repair initiations, or, requests. There are different forms and functions of requests and the following repair could be done in different ways. The sequence of conversational repair reflects the cooperative nature of conversation. Studies have been done on the requests with varied explicitness as a scaffolding strategy to improve the speech from students with disabilities. The next chapter, methodology, introduces in detail how different techniques of request were designed to improve the self-explanation of word problem solving by students with LD.
CHAPTER 3. METHODOLOGY

3.1 Pilot Study

To understand the nature of instructional discourse involving students with LD, we have conducted a pilot study (Xin, Liu, Jones, Tzur, & Si, 2012) to explore the reasoning of elementary students with LD in reformed-based mathematical instructional environment. The study was based on the data from a teaching experiment, as part of an NSF-funded project (Xin, Tzur, & Si, 2008). The teaching experiment was designed to nurture multiplicative reasoning of students with LD with constructivist methods.

The participants of the teaching experiment were seven pairs of 4th and 5th graders with LD. The teacher in the teaching experiment was a professor in mathematics education. The teacher closely worked with one or two students at an elementary school in the Midwestern United States for approximately 40 minutes for thirteen sessions. All instructional sessions were videotaped and transcribed.

In this teaching experiment, students were first introduced to a “Please Go and Bring Me …” game (PGBM), which invited the students to play with Unifix Cubes to build towers and based on these experiences to understand the concept of double counting (Steffe & Cobb, 1988). Double counting means in a multiplicative problem
situation, a student coordinates two number sequences. For instance, to solve the problem “If there are 4 towers with 3 cubes in each tower, how many cubes are there in all”, instead of counting the numbers one-by-one from 1 to 12, a student can do double-counting such as: “One tower has 3 cubes, two towers have six cubes, three towers have 9 cubes, and four towers have 12 cubes.” The concept of double counting marks the transition in a child’s counting stage from a unitary one to a binary one (Vergnaud, 1994), and it is critical to the development of multiplicative reasoning (Kouba, 1989).

In the final stage of the teaching experiment, the students were moved from concrete modeling with towers and cubes to conceptual model-based problem solving (COMPS, Xin, 2008; 2012; Xin, Wiles, and Lin, 2008). Specifically, students used diagram equations (e.g., “unit rate” x “# of units” = “product”) (Xin, 2012) to solve equal group problems in various contexts.

3.1.1 The Purpose of the Study

Using the video data from this teaching experiment, the pilot study adapted the framework from Pierson’s (2008) for classroom discourse analysis to quantitatively explore the moment-to-moment teacher-student interactions during the 13-session reform-based small group instruction. The purpose was to examine the nature of the discourse of both the teacher and the student with LD in the constructivism-oriented instructional environment. The discourse was analyzed through the teacher’s and the student’s utterances, in terms of being either a demand move (i.e., asking for information), or a give move (i.e., answering questions or providing information), and the intellectual levels of these moves. For instance, if the utterance is to ask for or provide basic
procedures or to repeat the information given in the problem, such as “Can I use the calculator” or, “How many cubes in each tower”, they are of low level intellectual work. Potential high (PH) level utterances are those that ask for or give a correct answer for a question without any justification or evidence. High intellectual level utterances are those utterances that ask for or give explanation or justifications, making predictions, and/or making comparisons and connections, such as in the context of solving the problem “If you build 20 cubes into towers, with 5 cubes in each tower, how many towers can you build”, the student got the answer “4 towers”. Then the teacher gave a high-level demand by asking “How did you get 4”, and the student gave a high-level response by explaining his solving process: “I guess I counted by 5. Five, ten, fifteen, twenty. That’s 4.”

3.1.2 Participants

The teacher was a professor in mathematics education. The student Tom (a pseudonym) was a male Caucasian fourth-grader, who was classified as having LD. He was from low social economic status and was receiving learning support on mathematics. Fifty percent of his time was in general education classrooms and he had been receiving special education for 3 years. Tom was chosen as the subject of the pilot study because the project team agreed on that compared to other participants of the teaching experiment, Tom was quite willing to communicate with the teacher and articulate his thoughts throughout all the sessions.
3.1.3 Results

The majority of the discourse moves were teacher-demand moves, followed by student-give moves. That is, the interactive pattern was still teacher asking questions and students answering questions. Compared with the demand moves, the teacher provided much fewer give moves, which represented the provision of information and explicitly telling or teaching. There were very few cases when the student initiated a demand.

In terms of the intellectual levels of the moves, it was found that the teacher’s demand moves were more or less evenly distributed across the three levels of intellectual work: high, potential high, and low. So were the teacher’s give moves. The majority of the student’s gives were of low level, and in the few situations students initiated demands, the demands were all of low level intellectual work (See Figure 1).

![Figure 1. Cumulative Frequency of Discourse Moves by Level of Intellectual Work (from Xin et al., 2012)](image-url)
3.1.4 Implication of the Pilot Study

By analyzing the teacher and the student’s conversation and levels of intellectual work involved in their utterances, the pilot study found that even though the teacher believed in constructivist mathematical teaching and there were a lot of communication throughout the sessions, it still followed the traditional classroom pattern where teachers spent most of the time giving demands and students giving short responses. There were more low-level student moves than potential high and high moves. On the other hand, 70% of the time, the student could respond to the teacher’s high-level demands. Apparently, there is a need to study how to scaffold students’ reasoning process and/or better elicit their explanation.

The following excerpts from the teaching experiment provide examples to show the need for improving scaffolding or prompting techniques. Below provides the context for Excerpt 1 during the session on December 2, 2008: the teacher was working with Tom and his partner Ann (a pseudonym). The teacher first made 7 towers with 5 cubes in each tower on the table. Then the teacher covered those towers with a piece of paper. He put 10 single cubes besides the piece of paper, also covered them with another piece of paper, and asked if he wanted to build the 10 single cubes into the same towers with 5 cubes in each, and move the new towers and the 7 towers together, how many towers would there be in all. At the beginning, Ann said it was 21 and Tom could not figure it out. When the teacher removed the paper on the 7 towers, Tom figured out that there were 9 towers, and Ann changed her answer to 54 cubes. The teacher reminded Ann that the question was about how many towers not cubes. Then she changed her answer to 48 towers.
Excerpt 1 *(2008-12-02)*

A: 48.
T: 48 towers?
A: (Smiled) I don’t know.
T: OK. So here’s… Before we go, let me say something that we’ve never done, but we’re probably going to use it once in a while, because I have an advantage in working with both of you. Let’s do what we sometimes do. You know when at the end of the book you have the answer to a question? Say that the answer is 9 [towers]. That’s Tom’s answer. Say that this is at the end of the book. You look at the end of the book, Ann, and Tom told you that the answer is 9 [towers]. We have 7 towers [with 5 cubes in each tower]. We have 10 cubes, and if you put those [10 cubes] into towers of 5, you would have 9 towers here. Can you find a way to explain why Tom might be right? Why 9 should be the answer?
A: I don’t know.
T: Not any idea?
A: No.
T: No. OK, so let’s hear Tom. Why did you think it is going to be 9?

Ann changed her answer from 21 to 54 to 48. None of them were correct, nor was she clear about her answers. It will be interesting and important to examine how she got those three incorrect numbers, since those reflected her incorrect reasoning. However, instead of giving her more opportunities to explain her thinking processes of the answers, the teacher asked Ann to explain Tom’s answer. This request helps in the sense that it asked Ann to make sense of the correct answer, which reflected the correct reasoning. Yet Ann failed to do so. Then the teacher moved on to let Tom explain his thinking of the correct answer. As such, we only know that Ann produced incorrect answers repeatedly, and she could not make sense of the correct answer, but we are not clear how she got those numbers, nor what specifically is hard for her to figure out the correct answer. Still, after Tom explained his correct reasoning, Ann was not provided with an opportunity to repeat it. Therefore, after their working on this problem, it is hard to determine why Ann
failed to figure out the answer by herself, and whether she understood the correct answer after Tom’s explanation.

In the following session on December 9, 2008, the teacher, Tom, and Ann were working on the same type of problem. The three of them first made 7 towers with 6 cubes in each tower on the table. Then the teacher covered those towers with a piece of paper. He put 18 single cubes besides the piece of paper, also covered them with another piece of paper, and asked if he wanted to build the 18 single cubes into the same-sized towers with 5 cubes in each, and move the new towers and the 7 towers together, how many towers would there be in all. After some time working on it, both Tom and Ann got the correct answer: 10 towers. However, Ann found it hard to explain how she got this answer.

Excerpt 2 (2008-12-09)

T: [To Ann] How did you get [10 towers]? You kind of worked it out, then you worked it again. You got 10 towers and you were pretty certain. How did you get it?
A: I don’t know. It’s just… Because you have 7 towers there, and I don’t know how many towers it would make, so I don’t really know. But I know it’s 18…
T: So there are 18 cubes. You don’t know how many towers. There are 7 towers here. How did you figure out it was going to be 10?
A: I don’t know, 7…
T: You kind of guessed? Or…(shrugs shoulders)
A: Hold on, (then mutters under her breath) 7…(moves lips like counting) I don’t know.
T: I think I got it
T: [To Ann] Ok, you got 10 we’ll see if it’s right or wrong because we’re going to open it and look at it, but OK [To Tom] let’s see yours Tom.

This excerpt demonstrates that after working out the correct answer, Ann still could not explain how she got it. The teacher tried several times to ask her to explain by providing general prompts such as “How did you get it”, “How did you figure out it was
going to be 10”, and “you kind of guessed? Or…” But these prompts were not successful in eliciting a solid articulation from Ann that could reflect her thinking. Then the teacher moved on to ask Tom’s thinking. This excerpt indicates that merely providing general prompts are not enough for eliciting students’ explanation sometimes. Prompts may be modified in terms of explicitness to better target at the problematic part in a student’s thinking. In the cases where prompting does not work, the teacher may need to model a complete and correct explanation to the student so that the student will know how to articulate the problem-solving process.

Based on these findings, there is a need to explore scaffolding strategies that teachers can use to elicit better explanations from students with LD. The following part of the chapter describes the experiment of the current study with a purpose to explore an intervention strategy that is based on the concept of conversational repair for helping students with LD’s verbal reasoning.

3.2 Participants and Setting

Participants were three students with LD from 4th grade in two Midwest urban public elementary schools in the United States. They met school district criteria for labeling as having a specific learning disability (SLD). Their pre-intervention performance on the criterion test was ≤ 70% correct in problem solving and in self-explanation, as 70% correct could be viewed as an average grade or “C”, according to Montague and Bos (1986).
Table 1. Students’ Demographic Characteristics

<table>
<thead>
<tr>
<th>Variable</th>
<th>Amy</th>
<th>Bill</th>
<th>Carl</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gender</td>
<td>Female</td>
<td>Male</td>
<td>Male</td>
</tr>
<tr>
<td>Ethnicity</td>
<td>Caucasian</td>
<td>Caucasian</td>
<td>African American</td>
</tr>
<tr>
<td>Age</td>
<td>9 years 1 month</td>
<td>10 years 6 months</td>
<td>10 years 11 months</td>
</tr>
<tr>
<td>Grade</td>
<td>4</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>Classification</td>
<td>SLD</td>
<td>SLD</td>
<td>SLD</td>
</tr>
<tr>
<td>SES</td>
<td>medium</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Reduced/free lunch</td>
<td>paid</td>
<td>reduced</td>
<td>reduced</td>
</tr>
<tr>
<td>Years in special education</td>
<td>1 year</td>
<td>2 years</td>
<td>3 years</td>
</tr>
<tr>
<td>Learning support classroom</td>
<td>Reading Spelling Writing</td>
<td>Reading Math</td>
<td>Reading Math</td>
</tr>
<tr>
<td>Percentage of time in general education class</td>
<td>65%</td>
<td>40%-79%</td>
<td>80%</td>
</tr>
<tr>
<td>IQ</td>
<td>WISC-IV</td>
<td>OLSAT</td>
<td>WISC-IV</td>
</tr>
<tr>
<td>Full scale</td>
<td>94</td>
<td>98</td>
<td>73</td>
</tr>
<tr>
<td>Verbal</td>
<td>100</td>
<td>97</td>
<td>83</td>
</tr>
<tr>
<td>Performance (Non-verbal)</td>
<td>94</td>
<td>100</td>
<td></td>
</tr>
</tbody>
</table>

Note. WISC-IV = Wechsler Intelligence Scale for Children – Fourth Edition (Wechsler, 2003); OLSAT = Otis-Lennon School Ability Test (Otis & Lennon, 1995); RTI = Response to Intervention.

The participants were pulled from their classrooms to a quiet room of their respective schools five times a week to participate in the experiment. The teacher’s lounge was used in one school for conducting the experiment. It was equipped with a table and several chairs in the center, printers, and storage cabinets. In the other school a teacher’s office was used for conducting the experiment. It was equipped with a table and several chairs in the middle of the room, surrounded by a computer desk, and several book and toy storage boxes. The researcher brought pencils, scratch papers, all necessary
booklets, calculators, and a camera with a tri-pod each time. All sessions of the experiment were video-taped.

3.3 Dependent Measures

Dependent measures included students’ word problem solving performance on a criterion test and a transfer test, and the quality of self-explanation in solving the problems in these two types of tests. There was also a survey to measure students’ perception and satisfaction with the scaffolding strategy.

3.3.1 Criterion Test

The criterion test and its alternate forms were used in the pre-test, the intervention, the post-test, and the maintenance test phases. The word problems in the criterion test and its alternate forms were all one-step *equal group* (EG) problems extracted from the problem database in Xin, Wiles, and Lin (2008). Specifically, the criterion test was comprised of 6 EG problems with the conceptual model “‘unit rate’ × ‘# of units’ = ‘product’” (Xin, 2012) (“unit rate” is the number of item[s] in each group, the “# of units” is the number of group[s], and the “product” is the number of total item[s]). In each test worksheet of the 6 problems, two were “unit rate” unknown problems, two were “# of units” unknown problems, and two were “product” unknown problems. The order of the problems was randomized across the alternate test forms. Table 2 illustrates the variations mentioned above with a same story context.
Table 2. Sample Problems of Three Variations of EG Problems

<table>
<thead>
<tr>
<th>EG Problem Variations</th>
<th>Sample Problem Situation</th>
</tr>
</thead>
<tbody>
<tr>
<td>The “unit rate” is unknown.</td>
<td>A sewing machine can sew a type of dress. The sewing machine worked for 96 hours without stopping and finished sewing 8 dresses. How many hours does the sewing machine use to finish one dress?</td>
</tr>
<tr>
<td>The “# of units” is unknown.</td>
<td>It takes a sewing machine 8 hours to finish sewing a dress. If the sewing machine worked for 96 hours without stopping, how many dresses can it finish sewing?</td>
</tr>
<tr>
<td>The “product” is unknown.</td>
<td>It takes a sewing machine 8 hours to finish sewing a dress. If there are 96 dresses to sew, how long will it take the sewing machine to finish all the dresses?</td>
</tr>
</tbody>
</table>

Each problem was printed on top of a sheet of 8 ½-inch by 11-inch unlined paper, leaving the space below for participants to work on the solutions. The 6 sheets of paper for a test were stapled together. Appendix A presents three sample EG word problems in the criterion test. Per Xin and Zhang (2009), Cronbach’s alpha of the criterion test was .86, and the parallel-form reliability of the alternate forms of the criterion test was .85.

3.3.2 Transfer Test

The transfer test had the same format as the criterion test. It has 4 two-step EG word problems extracted from the textbooks adopted by the schools, first three from enVision MATH Common Core Edition (enVision MATH, Charles et al., 2012, pp. xviii; 156; 179), and the last one from Harcourt Math Indiana Edition (Harcourt Math, Maletsky et al., 2004, pp. 224). The problems involved the four possible combinations of
four basic operations (i.e., ××; ÷÷; ×÷; ÷×). Appendix D presents the transfer test. The transfer test was used in the baseline, the post-test, and the maintenance test phases.

3.4 Data Scoring

All the tests taken by the participants were analyzed for two dependent variables: accuracy of problem solving and quality of self-explanation.

3.4.1 Accuracy of Problem Solving

Accuracy of problem solving referred to the percentage of problems solved correctly in each test. It was calculated as the total points earned divided by the total possible points. For the criterion test and its alternate forms, each problem was assigned 2 points, so the total possible score for each test was 12 points. Two points were assigned in one of the following scenarios: (1) the participant only wrote the correct number as the result; (2) the participant provided correct problem solving process (no matter what strategy was used), and it led to the correct number as the result. One point was assigned in one of the following scenarios: (1) the participant provided a problem-solving process which reflected the correct understanding and reasoning, but ended up without a number as the result or with an incorrect number as the result; (2) the participant had the correct number as the result, but the problem solving process provided was incorrect (reflecting incorrect understanding or reasoning of the problem context). A solution was scored 0 when (1) both the problem solving process and the answer were incorrect, or (2) only an incorrect number was written as the answer.
Each problem in the transfer test had two steps. Each step was assigned 2 points, and the scoring criteria were the same as for the criterion test and its alternate forms mentioned above. Thus the total possible score for each problem is 4 points, and the total possible score for the transfer test was 16 points.

Table 3. Sample Problem Solving Performances for 2 Points, 1 Point, and 0 Points

<table>
<thead>
<tr>
<th>Scoring</th>
<th>Sample Explanations (in the context of solving the word problem: It takes a sewing machine 2 hours to finish sewing a dress. If the sewing machine worked for 6 hours without stopping, how many dresses can it finish sewing?)</th>
</tr>
</thead>
</table>
| 2 points | (1) The participant only wrote the correct number as the result. e.g., “3”.
(2) The participant provided correct problem solving process (no matter what strategy was used), and it led to the correct number as the result. such as:
(a) “6÷2=3”
(b) “6-2-2-2=0, so 3.” |
| 1 point | (1) The participant provided a problem-solving process that reflected the correct understanding and reasoning, but ended up without a number as the result or with an incorrect number as the result. e.g., “6÷2=2”.
(2) The participant had the correct number as the result, but the problem solving process was incorrect (reflecting incorrect understanding or reasoning of the problem context). e.g., “6-2-2, so 3”.
| 0 points | (1) Both the problem solving process and the answer were incorrect. e.g., “6-2=4”.
(2) Only an incorrect number (e.g., “4”) or nothing was written as the answer. |

3.4.2 Quality of Self-Explanation

The participants’ original explanations were scored to measure the quality of self-explanation. For the criterion test and its alternative forms, each problem was assigned 2 points for self-explanation, so the total possible score for each test was 12 points. Table 4 illustrated the scoring rubric and sample explanations for 2-point, 1-point, and 0-point
explanations (the scoring rubric was developed based on the reasoning scoring rubric of
the Test of Problem Solving Elementary [3rd Edition] (Bowers, Huisingh, & LoGiudice,
2005), which assesses a school-aged child’s thinking, reasoning, and problem-solving
abilities through linguistic expressions).

Table 4. Sample Explanations for 2 Points, 1 Point, and 0 Points

| Scoring | Sample Explanations (in the context of solving the word problem: It takes a
| sewing machine 2 hours to finish sewing a dress. If the sewing machine
| worked for 6 hours without stopping, how many dresses can it finish
| sewing?) |
|---|---|
| 2 points | The statement expresses clearly and correctly the meaning of the numbers
| mentioned in the statement, and it shows a correct understanding of the
| multiplicative relationships between quantities. In other words, it correctly
| expresses the meaning of “multiple groups of something” such as:
| (a). “It takes 2 hours to sew one, another 2 to sew another one, another 2…so
| it is like a pattern, totaling 6 hours. So I need to find out how many 2s there
| are in 6, so I divide and got it can sew 3 dresses.”
| (b). “It is an equal group problem because it takes equal number of hours to
| finish every dress. We know the total number of hours for sewing dresses is
| 6. We also know sewing one dress takes 2 hours. To answer how many
| dresses we do 6÷2.”
| (c). “It is to find out if you put 6 in groups of 2, how many groups can there
| be.” |
| 1 point | (1) The statement contains less advanced reasoning or strategies (such as
| repeated addition or repeated subtraction) that could lead to the correct
| answer, but does not indicate an awareness of the “multiple groups of
| something” situation.
| e.g., “I minus 2 from 6 and keep going down and down.”
| (2) The statement contains important information, but is limited, reflecting
| vagueness, confusion, incompleteness, or immaturity in the understanding of
| multiplicative relationships between quantities (such as repeating the
| information given in the problem followed by a correct algorithm for
| solution; no or inadequate labels for the numbers mentioned in the statement
| of a correct algorithm for solution; and knowing which number is the
| product, but mixing up the “unit rate” and the “number of units”, etc.).
| e.g., (a). “It takes 2 hours to do one, 6 hours in total, so 6÷2.” (It is a mere
| repetition of the given information in the problem without understanding
| shown.)
| (b). “They want to put the 6 hours in 2 groups.” (mixing up the “unit rate”
| and the “number of units”)
| (c). “It is to put the 6 hours into 2.” (inadequate label for the number “2”) |
(d). “Six divided by 2 is 3, so 3 dresses.” (It is a repetition of the algorithm.)

| 0 points | The statement is irrelevant or gives incorrect understanding of the multiplicative relationships between quantities, such as:  
(a) “Two hours to sew a dress and now 6 hours, so I did 6-2.”  
(b) “I do not know how to explain it.”  
(c) “6 groups of 2.” |

3.5 Student Perception and Satisfaction Survey

To examine the social validity of the intervention, the researcher developed a survey to measure the participants’ general perceptions of the experience of explaining problem-solving process and satisfaction of receiving repair requests to scaffold their explaining and learning process (See Appendix E). A five-point scale was adopted to measure the participants’ responses, with “1” assigned to “strongly disagree”, “2” to “disagree”, “3” to “neutral”, “4” to “agree”, and “5” to “strongly agree”. Items in this survey included statements such as “I like explaining my thinking to other people, especially younger students” and “I think that the teacher’s questions helped me to clarify my thinking.”

3.6 Design

A multiple baseline design across participants (Horner, Carr, Halle, McGee, Odom, & Wolery, 2005) was used to evaluate the functional relationship between the intervention and participants’ self-explanation quality and word problem solving performance. A single-subject research design was chosen because this research method is “particularly appropriate for use in special education research” (pp. 174) to test
educational and behavioral interventions and allow detailed analysis of individuals. In particular, with the multiple baseline design across participants, the intervention is introduced to different subjects one at a time after consistent response patterns are observed in each individual baseline. When the subject who is receiving the intervention shows a clear change in the behavior pattern, the second baseline will be introduced to the intervention. If the changes in each baseline occur only when the intervention is introduced, a functional relation is demonstrated (Barlow, Nock, & Hersen, 2009; Kennedy, 2005).

The experimental design of this study included two major conditions: baseline condition (including pre-intervention assessments and post-intervention assessments) and intervention condition. Chronologically, the experiment includes four phases: the pre-test phase, the intervention phase, the post-test phase, and the maintenance phase.

3.7 Procedure

The experiment took place one session per day for each student. The researcher came to the school to work with the participants individually. The participants were told to take as much time as they needed, yet each session was approximately thirty minutes.

3.7.1 Baseline Condition (A)

The participants began the experiment on the same day to complete one criterion test. At the beginning of the experiment, the researcher told a participant that researchers had some mathematical word problems that were interesting to some younger students
who were at early elementary school age. These younger students were eager to learn from their older peers about how to solve these problems, which would help them more than hearing instructions from teachers. The researcher then asked the participant whether s/he was willing to help the younger students learn mathematics by explaining very clearly how they solved the problems. After the participant indicated agreement, the researcher would tell the participant that his/her explanation did not mean to tell or repeat what was already shown on the sheet of paper, but to explain why the problem was solved this way and the reason behind the solutions. The participants were also told that they could always revise what they had already said or started over. If they did not know how to solve a problem, the participants should state clearly what they did not understand. They could ask the investigator to reread the problems, but the investigator did not give any feedback concerning problem solving or explanation of the problem-solving process.

After the preparation, the researcher asked the participants to read through each problem, write down all of the problem-solving processes, including math equation(s) in the space below the problems, and then explain the solving steps to the investigator. After the participants had finished one criterion test the first day and the transfer test the following day, one student (Amy) took four more alternate forms of the criterion test. Once Amy’s scoring of the self-explanation quality showed a stable trend, the intervention was first introduced to Amy. In the intervention phase, when Amy’s scoring of the self-explanation quality showed an accelerating trend, the intervention was introduced to the second participant (Bill) after he took four additional alternate forms of the criterion test (at least three of them were immediately before the intervention began).
The same sequence was followed for the third participant (Carl) until all the participants were introduced to the intervention phase.

3.7.2 Intervention Condition (B)

At the beginning of the intervention condition, the researcher told the participants that they had done an excellent job explaining the problems during the past days, but there were places that they needed to be more explicit so that the younger students could understand better, and the researcher would help them work on the explanation together. Then, same as the Baseline Condition, the researcher gave the participant one alternative form of the criterion test to work on with the same direction. For each problem, after the participant finished the initial explanation, the researcher implemented the intervention according to the quality of the explanation. That is, if the explanation indicated correct and clear reasoning, the participant would not receive any scaffolding for this problem and would move on to the next problem. Otherwise, the explanation needed to be improved and it would fall into one of the three types: (1) emerging reasoning (i.e., the reasoning was relevant but not precise), (2) faulty reasoning (i.e., the reasoning was incorrect), or (3) no reasoning (i.e., offering no response). The treatment had different components that addressed the different situations, which were specified below. The intervention phase for a participant was finished when the participant’s problem solving and self-explanation scores showed a stable trend at a level of no less than 75% correct.

**Treatment Components.** Based on the earlier review and discussion of ways to improve self-explanation, coaching reasoning of students with LD, and conversational repair process, repair request techniques varying in explicitness from the least explicit to
the most explicit were used to scaffold self-explanation for students with LD. The basic principle guiding the process was that the repair requests from the investigator were given from the most non-specific one (i.e. asking to repeat what was said) to the most specific one (i.e. direct modeling explanation or direct teaching of problem solving). This way of scaffolding allows students with LD enough opportunities to construct their understanding, come up and revise explanations by themselves.

Repair requests referred to the prompts from the investigator requesting the participants to self-repair their explanation. The forms of the requests were based on and maximally follow the basic types defined by Schegloff et al. (1977). The functions of repair requests in this study were developed based on Weiner (2005): (1) the general request (according to Schegloff et al., 1977, they usually take the form of “Huh?” or “What?” In this study, for the purpose of being encouraging and clear about the requesting purpose, the request was in the form of “Do you want to try again to make it better”); (2) the request for revision; (3) the request for specification/clarification; (4) direct other-repair (i.e., modeling of self-explanation); and (5) direct teaching of problem solving. Their application contexts were specified below:

(1). The general request “Do you want to try again to make it better” was given after a participant was given enough time to read and think about the problem, and produced a problematic initial explanation (being emerging reasoning, faulty reasoning, or no reasoning) that needed to be improved. After the participant repaired (or refused to repair) the explanation, the following intervention addressed the repaired (or initial, in case the participant did not try again) explanation if it was still problematic.
(2). After the general request, if the explanation reflected faulty reasoning or no reasoning, which reflected incorrect reasoning, the researcher followed up to provide a hint and then requested a revision by saying “This number means… and this number means…” (Tells the participant the meaning of the two known numbers.) “So do you want to revise your explanation?” If the following repaired explanation still reflected faulty reasoning or no reasoning, the researcher implemented direct teaching (see the Direct Teaching section below), and the participant was asked to repeat what the investigator had just said; if the following repaired explanation became relevant but not precise, the investigator implemented the request for specification/clarification (see the next paragraph).

(3). If the repaired explanation was an emerging reasoning, the researcher requested specification/clarification of the unclear parts by repeating the repairable parts in the participant’s response and adding a wh- question word, or, by saying “And could you tell me more about why you did so?” If the following repaired explanation was still an emerging reasoning, the researcher did direct other-repair. That is, the researcher directly modeled a full-scored explanation to the participant (Following the modeling explanations in Appendix C). Then the participant was asked to repeat what the investigator had just said.

(4). If in any place in the process the explanation was worth full score, the participant moved to the next problem. The flowchart in Figure 2 showed the moving among the different treatment components as an intervention proceeded. The treatment components, the contexts in which each component was applied, and the sample contexts were demonstrated in Table 5.
Figure 2. Flowchart of the Moving among the Different Treatment Components

Table 5. Treatment Components and Their Application Contexts

<table>
<thead>
<tr>
<th>Treatment components</th>
<th>Application contexts</th>
<th>Examples</th>
</tr>
</thead>
</table>
| Request for general information  
“Do you want to try again to make it better?” | When the participant provides an initial explanation which needs to be improved, being relevant but not precise, or incorrect, or offering no response. | Student: I drew a picture. Teacher: **Do you want to try again to make it better?** |
| Request for revision  
“This number means…and this number means…”  
(*Tells the participant the meaning of the two known numbers.*)  
“So do you want to revise your explanation?” | When the participant provides an explanation which is incorrect, or offers no response. | (In the context of solving the word problem: It takes a sewing machine 8 hours to finish sewing a dress. If the sewing machine worked for 96 hours without stopping, how many dresses can it finish sewing?)  
Student: I got 8 times 96.  
Teacher: **The number 8 means** the number of hours the sewing machine needs to finish one dress. **The number 96 means** the total number of hours the sewing machine worked to sew dresses. **So do you want to revise your explanation?** |
| **Request for specification/clarification** | When the participant provides an explanation which is relevant but not precise. | (In the context of solving the word problem: It takes a sewing machine 8 hours to finish sewing a dress. If the sewing machine worked for 96 hours without stopping, how many dresses can it finish sewing?) (b) Student: I did 96\(\div\)8. Teacher: You did 96\(\div\)8. **And could you tell me more about why you did so?** (a) Student: It takes 8 hours to sew one dress, another 8 to sew a second one…so it is to see how many in 96. Teacher: How many **what**? Student: how many 8s. |
| **Other-repair (Direct modeling of self-explanation)** | When the repaired explanation after the request for specification/clarification is still relevant but not precise. | See the Modeling Booklet for modeling full-scored explanations in Appendix C. |
| **Direct teaching** | When the repaired explanation after the request for revision is still incorrect or offering no response. | See the Teaching Script in Appendix F. |

**Direct Teaching.** The direct teaching method was based on the conceptual model-based approach to teach word problem solving (Xin, 2012; Xin, Wiles, & Lin, 2008; Xin & Zhang, 2009). Specifically, the word problem (WP) story grammar question cards developed by Xin et al. (2008) was adopted to teach students how to identify and represent the three elements in an EG problem, map the information in the equation, and use the equation to solve the problem (see Figure 3). The Teaching Script was in Appendix F.
3.8 Treatment Fidelity

A checklist that contained the treatment components and each component’s corresponding application context was developed to assess the researcher’s adherence to the assigned strategy (see Appendix B for Treatment Implementation Fidelity Checklist). One third of all the sessions were observed by an independent observer to check the treatment delivery of each component listed on the fidelity checklist. The researcher explained the fidelity checklist to the observer. The adherence of the investigator’s scaffolding to the assigned application context was judged according to the presence or absence of the features listed on the fidelity checklist. Treatment fidelity was calculated as the percentage of correctly-implemented treatment components and it was 96%.

Figure 3. EG Problem Prompt Card (adapted from Xin [2012, p. 105])
3.9 Interrater Reliability

The investigator transcribed all the videotaped explanations of the participants and the communication during intervention between the participants and the investigator (totally 159 pages). The investigator scored all the tests using an answer key, the survey, and coded the participants’ original explanations according to the coding scheme in Table 4. A research assistant (RA) who was blind to the purpose of the study independently re-scored 30% of the test items and 30% of the participants’ explanations. To calculate interrater reliability, the number of agreements was divided by the number of agreements and disagreements and multiplying by 100%. The interrater reliability for the test scores was 100%, and for the explanation scores was 100%.

3.10 Data Analysis

Each of the participant’s performance on problem solving and self-explanation of the criterion test (including its alternative forms) and the transfer test across the pre-test, the intervention, the post-test, and the maintenance test phases was plotted in a graphic display. Visual inspection was used to evaluate the quantitative information of the graph (Kennedy, 2005). Specifically, the visual inspection focused on three dimensions of within-phase patterns: (1) the level of the data, which is typically represented by the mean or median, (2) the trend of the data, and (3) variability. It also focused on two dimensions: (1) immediacy of effect, and (2) overlap.

The percentage of non-overlapping data (PND) was used to calculate the effect sizes of the intervention. According to Scruggs, Matropieri, and Casto (1987), PND is the only major evaluative criterion that can measure treatment effectiveness in most cases. It
was calculated as “the number of treatment data points that exceeds the highest baseline data point in an expected direction” divided by “the total number of data points in the treatment phase (Scruggs, Matropieri, & Casto, 1987, p. 27).
CHAPTER 4. RESULTS

4.1 Baseline Analysis

During the baseline condition, the average word problem solving performance on the criterion tests for each of the three participants was 43.3% correct for Amy, 28.2% correct for Bill, and 1.6% correct for Carl. The average self-explanation performance for each of the three participants on the criterion tests was 23.3% correct for Amy, 1.6% correct for Bill, and 0% correct for Carl. For both the WP solving and the self-explanation, the performance across the three participants was low and stable. Though some variations existed, there was no consistent pattern indicating a trend of either increase or decrease.

Amy (1) Problem solving: Amy’s data showed moderate degree of variability, with a mean of 43.3% correct (range, 25% to 66.7%), a median of 41.7% correct, and a moderate downward trend (see Figure 4 in 4.2 Intervention section). Amy's strategy at this phase was to solve the problems by repeated addition/subtraction. She often read a problem out loud, thought about it, and used the calculator to repeatedly add or subtract. In the meantime, she drew short lines on the working space of her worksheets as tallies to mark how many times she had added or subtracted. If she did multiplication, after marking several tallies, she would stop pressing the calculator, count how many tallies she already got, and see if it was enough. For division problems, she would stop when
she got 0 on the calculator. Amy's problem-solving process was tedious and time-consuming, as most of the problems involved two- or three-digit numbers. It took her a long time to solve a problem, and she easily mis-marked or mis-counted the tallies.

After three worksheets, even Amy herself felt tired of the problem solving process, and she suddenly thought of the operation of multiplication. She used multiplication to replace both repeated addition and subtraction for all the rest of the problems (Worksheets 4 and 5) in baseline condition. This decreased her performance from percentages correct of 50%, 41.7%, and 66.7% for Worksheets 1, 2, and 3, respectively, to percentages correct of 25% and 33.3% for Worksheets 4 and 5 respectively (see Table 6).

Table 6: Percentage Correct for Amy’s WP Solving and Self-Explanation Performances during the Baseline Condition

<table>
<thead>
<tr>
<th></th>
<th>Session 1</th>
<th>Session 2</th>
<th>Session 3</th>
<th>Session 4</th>
<th>Session 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>WP solving</td>
<td>50.0%</td>
<td>41.7%</td>
<td>66.7%</td>
<td>25.0%</td>
<td>33.3%</td>
</tr>
<tr>
<td>Self-explanation</td>
<td>33.3%</td>
<td>25.0%</td>
<td>41.7%</td>
<td>16.7%</td>
<td>0.0%</td>
</tr>
</tbody>
</table>

(2) Self-explanation: Amy’s data showed low degree of variability, with a mean of 23.3% correct (range, 0 to 41.7%), a median of 25% correct, and a moderate downward trend (see Figure 4 in 4.2 Intervention section). During the baseline condition, according to the coding scheme in Table 4, most of the time, Amy’s self-explanation would be considered emerging reasoning. This was shown by the following two types of explanations (see Example 1 and Example 2).

Example 1 (from Session 1):
(Uncle Jim is a painter. He says that it takes 8 gallons of paint to paint one entire house. How many houses could he paint with 408 gallons of paint?)
Amy (A): "If you have a calculator, you put 408 in the calculator and minus 8, minus 8 and keep on going. I put tally marks every time I did it. I got 50 8s to get to 0 from 408.
What the calculator does is once you do minus every time it shows what you have and then goes down each time you do it."

Amy’s explanation in Example 1 showed that she used repeated subtraction to solve the problem, and her statement focused on the calculation procedure. She stated in a clear manner what she did with the calculator. However, she did not elaborate on why she chose to do "408 minus 8, minus 8" in the first place. Therefore, Amy’s explanations as shown in Example 1 were scored as 1 point.

**Example 2** (from Session 1):
(Bobby found 7 boxes in the attic of his house. In each box there were an equal number of crystal drinking glasses. If there were 91 total glasses, how many glasses were there in each box?)

A: (She first wrote three 30s on the paper, added them up by mental math, and said “90”. Then she erased the 30s and wrote four 20s on the paper, added them up by mental math, and said “80”. She erased the 20s and wrote seven 10s, added them up and got 70. Then on another part of the paper she tried adding up six 15s. She then erased the 15s and wrote down six 13s. She added two 13s and wrote down 26 on the paper, added the rest four 13s and wrote down 52. She added the 26 and 52 on the paper and got 78. She then added 13 to 78 and got 91)
"You should guess how many. First I did 30, it is too much, then 20, too much. Then 10, it is too low. And I tried 15, no. then I tried 13. And I guess how many it would be. 4 and I guess 2. That would be too short because it made 78. So I added 1 more and it made 91 exactly. So 13 is the answer."

Amy’s explanation in Example 2 showed that she solved the problem by guessing the number. Yet it also showed that she had the awareness of using repeated addition as the strategy, to see adding which number 7 times would get to 91. Such explanations were scored as 1 point.

In the later sessions of the baseline condition (sessions 4 and 5), Amy began to use multiplication overall. Below is an example of her explanations (see Example 3).

**Example 3** (from Session 5):
(Last week, 37 Jeeps carried soldiers to basic training. If each Jeep was full, and a total of 259 soldiers were going to basic training, how many soldiers fit in each Jeep?)
A: “If you have a calculator, you will type 259 times 37. You will get 9583. How I got that was that it is easier to do times instead of doing minus 8 minus 8…or plus 20, plus 80, plus 60…”

Explanation such as the above was coded as 0 points because Amy was not aware of the difference between repeated addition and subtraction, or the difference between multiplication and division.

Bill (1) Problem solving: Bill’s data showed low to moderate degree of variability, with a mean of 28.2% correct (range, 0% to 50%), a median of 33.3% correct, and a moderate upward trend due to the reason that his first session was the only session he got 0 points for problem solving (see Figure 6 in 4.2 Intervention section). Bill used all the four operations (addition, subtraction, multiplication, and division) in the baseline condition. He often worked with the calculator to figure out what he thought was the answer and copied it on the paper. Thus he only had a math equation as the answer, no drawings or tallies as Amy did. (2) Self-explanation: Bill’s data showed low degree of variability, with a mean of 1.6% correct (range, 0% to 8%), a median of 0% correct, and an almost flat trend. The most distinct feature of Bill’s explanations was that he only gave very general statements, from which we could not tell his reasoning or understanding of the problem. For instance, no matter what the problem was, for most of the time he would say “I solved it this way because it would be better for me. It feels better and easier for me to do” or “I did this because it was very hard to add for me or use anything else. That’s why I multiplication because multiplication is easy for me to do”. The following two examples illustrate this nature of his explanations.

Example 4 (in Session 5):
(Tina gets paid an hourly wage for selling cookies at the mall. She worked 16 hours last week and made $96. How much does Tina make each hour?)
B (Bill): If you do minus, you will keep going and going, like if you do 96-16 it will take longer.

Example 5 (in Session 5):
(In Mrs. Wilson's classroom, each row of ceiling tiles has 52 tiles. There are 41 rows of tiles. How many total tiles are there in Mrs. Wilson's classroom ceiling?)
B: I did that because if you do plus it will take longer.

In Example 4, Bill did not express clearly if he understood it as a repeated subtraction or merely 96-16. In Example 5, even though he said plus would be longer to do, from his words it cannot tell if he referred to repeated addition of 52 for 41 times. Explanations such as the above were coded as 0 points since “the statement is irrelevant or gives inappropriate or imprecise information, showing faulty or incomplete understanding of the multiplicative relationships”.

Table 7: Percentage Correct for Bill’s WP Solving and Self-Explanation Performances during the Baseline Condition

<table>
<thead>
<tr>
<th></th>
<th>Session 1</th>
<th>Session 2</th>
<th>Session 3</th>
<th>Session 4</th>
<th>Session 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>WP solving</td>
<td>0</td>
<td>50%</td>
<td>25%</td>
<td>33%</td>
<td>33%</td>
</tr>
<tr>
<td>Self-explanation</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>8%</td>
<td>0</td>
</tr>
</tbody>
</table>

Carl: (1) Problem solving: Carl’s data showed low degree of variability, with a mean of 1.6% correct (range, 0% to 8%), a median of 0% correct, and an almost flat trend (see Figure 7 in 4.2 Intervention section). Throughout baseline condition, Carl used multiplication three times (for three division problems), and he solved all other problems by either addition or subtraction. Sometimes he worked on the calculator first (he might try different operations), then decided on one operation, and reproduced the solution process on the paper in a vertical format. If he solved a problem by addition or subtraction, often times he would check the result obtained from the calculator by repeating the same process on the paper. If he did multiplication, he would copy the
result from the calculator. He needed a long time to finish one problem, as he often changed his solution. Carl’s WP solving performance was fairly low during baseline condition (see Table 8). The example below showed one case where Carl was confused and uncertain in applying strategies in problem solving (see Example 6).

*Example 6* (in Session 3):
Pretend your teacher sent you to buy some yearbooks from the office to distribute to the class. Each yearbook costs $17. How much would you have to pay if you were supposed to purchase 16 yearbooks?
C (Carl): (He repeatedly wrote eight 17s on the paper, counted all the 17s he wrote, and continued to write 17s until there were fifteen 17s on the paper. He counted all the 17s in an unheard manner. However, he wrote $17+16=33$ in the vertical format on the paper).
“I added it, because how much would you have to pay if you were shopping to purchase 16 yearbooks, and it says it cost 17 dollars, so I minus 17, um 16(this part cannot be heard clearly)...I add up to 33 so 33 dollars.”

Example 6 was the only case in the baseline condition where Carl exhibited the awareness of repeated addition, though he only wrote fifteen 17s instead of sixteen 17s due to miscounting. As such, he was given 1 point for the solution process and 0 points for the result.

(2) Self-explanation: Carl’s data showed no variability, since he got 0 points for all sessions. Carl’s most common explanation in the baseline condition focused on calculation process (see Example 7).

*Example 7* (in Session 1):
(Gary made 41 buttons when running for class president. It takes 23 drops of glue to make each button. How many drops of glue did Gary use?)
C: “I add 41 and 23. I put the biggest one on top and I add the 1 and 3 together first, and then add 4 and 2. If I do times, 3 times 1 it will be back to 3. It will get me wrong.”

Such explanations were coded as 0 points as it did not provide an explanation of reasoning on problem solving.

Another strategy Carl often used was the “keyword” strategy (see Example 8).
Example 8 (in Session 5):
(Last week, 37 Jeeps carried soldiers to basic training. If each Jeep was full, and a total of 259 soldiers were going to basic training, how many soldiers fit in each Jeep?)
C: “(he read the problem first) So I added it up because they are getting in the jeep and squeezing it, like adding, more coming and coming and coming. So it is adding up.”

There are some explanations similar to the one shown in Example 8, such like “‘buy’ means take away”, “‘use’ means take away”, and “I times it because it said ‘how many’ so I times it in my head then on the calculator.” In these instances, Carl made the judgment based on one or two words in the problem, but not on the conceptual understanding of the mathematical problem structure. These explanations were scored as 0 points.

Table 8: Percentage Correct for Carl’s WP Solving and Self-Explanation Performances during the Baseline Condition

<table>
<thead>
<tr>
<th></th>
<th>Session 1</th>
<th>Session 2</th>
<th>Session 3</th>
<th>Session 4</th>
<th>Session 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>WP solving</td>
<td>0</td>
<td>0</td>
<td>8%</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Self-explanation</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

4.2 Intervention Analysis

Amy: (1) Problem solving: Amy’s problem solving performance had a big increase at the first session of intervention condition (from 33.3% correct to 100% correct). In the following sessions her WP solving was no lower than 66.7 % correct. The average of Amy’s WP solving increased from 43.3% correct in the baseline condition to 85.7% correct in the intervention condition. The median of Amy’s WP solving in the intervention condition was 83.3%. The PND of Amy’s improvement in problem solving was 85.7%.

Table 9: Percentage Correct for Amy’s WP Solving and Self-Explanation Performances during the Intervention Condition

<table>
<thead>
<tr>
<th></th>
<th>Session 1</th>
<th>Session 2</th>
<th>Session 3</th>
<th>Session 4</th>
<th>Session 5</th>
<th>Session 6</th>
<th>Session 7</th>
<th>Session 8</th>
<th>Session 9</th>
</tr>
</thead>
</table>
(2) Self-Explanation: Amy’s self-explanation performance had a big increase at the first session of intervention condition (from 0% correct to 100% correct). Amy’s self-explanation scores ranged from 41.7% correct to 75% correct, with an average of 60.7% correct, which was 37.4% increased from her average performance in baseline condition. The median of Amy’s self-explanation in the intervention condition was 58.3%. Figure 4 shows the trend lines of Amy’s performance in problem solving and self-explanation in the baseline and the intervention condition respectively. It can be seen that Amy’s performance in both problem solving and self-explanation changed from a decreasing trend to an increasing trend from the baseline to the intervention condition.

The PND of Amy’s improvement in self-explanation was 85.7%.
Note: P-S = Problem Solving; S-E = Self-Explanation

Figure 4. Trend Lines of Amy’s Performance in Problem Solving and Self-Explanation in the Baseline and the Intervention Condition

At the beginning stage of the intervention phase (i.e., during the first three sessions), similar to her explanation in the baseline condition, most of Amy's self-initiated explanation (without any scaffolding from the experimenter) was about calculation procedures (see Example 9) and restating given information (see Example 10).

Example 9 (in Session 1):
(There are 22 ice cream bars in each box. How many ice cream bars would you have if you bought 37 boxes of ice cream bars?)
A: If you have a calculator, you will do either times or divide. I did times because you will have to do 22 times 37 equals 814. How I got it is you would do times or division instead of doing plus plus plus…or minus minus minus…done.

Example 10 (in Session 1):
(It costs a total of $551 to buy 19 super-sized pizzas for a school party. How much did each pizza cost?)
A: If you had 551 to get 19 super-sized pizzas, all you have to do is 551 divided by 19 to see how many pizzas you can get all the way down to it.

The above explanations were scored as 1 point because the statements expressed a correct choice of operation and a correct solution, yet they were limited or incomplete in expressing the reasoning on why such an operation was chosen based on the information given in the question.

Starting from Session 4, Amy started to develop her ways of explanation, the expressions that she felt comfortable with and could work for all three types of problems. For the product unknown problems, she would say “do $m$ $n$ times” ($m$ stands for the “unit rate”, and $n$ stands for the “number of units”) (see Example 11).

Example 11 (in Session 7):
(There are 26 legs on a particular type of centipede. How many total legs would 31 centipedes have?)
A: “So basically what you are doing is um…you have 26 legs on one centipede. One centipede has 26 legs, and they want 31 centipedes, so like centipedes, centipedes (drawing centipedes on paper [see Figure 5]) I am just pretending these are 26 legs, and pretending these are 31. So basically this is 31, so they want 31, they have 31, and 31 is how many times you do 26, so you do 26 31 times.”

Figure 5. One Case of Amy’s Drawing and Writing for a Problem

The explanation above was given 2 points.

For division problems (either the “unit rate” is unknown or the “number of units” is unknown), Amy sometimes still used the “do \( mn \) times” format (see Example 12).

Example 12 (in Session 5):
(Megan made a total of $162 by selling decorative baskets. If each gift basket costs $27, how many decorative baskets did he sell?)
A: “You will have 162, and then 27 is how many, so 162 is how many dollars you have in total and then 27 is how many dollars each basketball cost. You have 6 basketballs, kind of like you would do 27 6 times.”

Explanations such as Example 12 were given 2 points, as they showed clearly a correct understanding of the multiplicative relationships between the numbers. The “do \( mn \) times” format is only correct when \( m \) is the “unit rate” and \( n \) is the “number of units”.
Sometimes Amy made the mistake that she confused the product with either the “unit rate” or the “number of units” (e.g., she said “162 groups of 27” or “27 groups of 162” for the problem in Example 12). As such, the explanations were scored as 0 points.

She sometimes used a “go into” format for division problems. The “\( m \) go into \( a \)” format is only correct when \( m \) is the “unit rate” and \( a \) is the product. A common mistake Amy made was confusing the “number of units” with the “unit rate” when she used the “\( m \) go into \( a \)” format (see Example 13).

**Example 13** (in Session 5):
(It costs a total of $576 to buy 8 baseball uniforms. How much does each baseball uniform cost?)

\( A: \) you have 576 dollars in total of all the money you have, and it's...and those...and 8 basketball uniforms of all the basketball uniforms. Those 8...so basically what they are asking is how many times 8 go into 576.

In this case, the explanation was scored as 1 point since it mixed up the “unit rate” and the “number of units”. “8” is the number of uniforms, and as such it cannot “go into” the total amount of dollars.

**Bill** (1) Problem solving: Bill made a quick improvement in problem solving performance at the first session of intervention condition (from 33.3% correct to 83.3% correct) (see Table 10). The average of Bill’s WP solving performance increased from 28.2% correct in the baseline condition to 91.7% correct in the intervention condition, with the median 100% correct. The PND of Bill’s improvement in problem solving was 100%. After reading the problem silently, Bill typically worked with the calculator and then wrote a math equation on the paper as the answer. It was often seen that for a problem he tried both multiplication and division, and if from division he could not get an integer as the result, he then would choose multiplication. (2) Self-explanation: Bill’s
self-explanation performance had an immediate increase at the first session of intervention condition (from 0% correct to 41.7% correct). Bill’s self-explanation performance ranged from 41.7% correct to 75% correct. It increased from an average of 1.6% correct in the baseline condition to an average of 64.2% correct in the intervention condition, with the median 75% correct. The PND of Bill’s improvement in self-explanation was 100%.

Figure 6 shows the trend lines of Bill’s performance in problem solving and self-explanation in the baseline and the intervention condition respectively. Bill got 0 points for problem solving in only the first session of the baseline condition, so his trend line in the baseline condition for problem solving shows an increasing trend. However, his performance level in the intervention phase had a distinct increase when compared with the baseline phase with the PND as 100%.

Table 10: Percentage Correct for Bill’s WP Solving and Self-Explanation Performances during the Intervention Condition

<table>
<thead>
<tr>
<th></th>
<th>Session 1</th>
<th>Session 2</th>
<th>Session 3</th>
<th>Session 4</th>
<th>Session 5</th>
<th>Session 6</th>
<th>Session 7</th>
</tr>
</thead>
<tbody>
<tr>
<td>WP solving</td>
<td>100.0%</td>
<td>66.7%</td>
<td>83.3%</td>
<td>75.0%</td>
<td>75.0%</td>
<td>100.0%</td>
<td>100.0%</td>
</tr>
<tr>
<td>Self-explanation</td>
<td>58.3%</td>
<td>41.7%</td>
<td>50.0%</td>
<td>58.3%</td>
<td>66.7%</td>
<td>75.0%</td>
<td>75.0%</td>
</tr>
</tbody>
</table>
Note. P-S = Problem Solving; S-E = Self-Explanation

Figure 6. Trend Lines of Bill’s Performance in Problem Solving and Self-Explanation in the Baseline and the Intervention Condition

In the early intervention condition, especially the first three sessions, Bill’s most common explanation for a division problem was a mere restatement of the question and what he did, as in Example 14.

*Example 14 (in Session 1):*
(Edgar read 357 pages in his favorite new book. Each day since he bought it, he reads 21 pages. How many days has it been since he bought the book?)

B: I divided to get the answer so you see in each day how many books you would got in each day. It would be 17 pages.

According to Table 4, explanations as the above were scored as 1 point as it provided limited and incomplete reasoning on why he used division.

In the later part of the intervention condition, Bill started to develop his own way of explaining different types of problems, especially for division problems. A commonly-seen problematic explanation was no matter the problem had the “unit rate” unknown or the “number of units” unknown, he would randomly say “divide (whichever was the bigger number given in the problem) equally into (whichever was the smaller number
given in the problem)” (see Example 15) or “divide (whichever was the bigger number
given in the problem) equally into (whichever was the smaller number given in the
problem) groups” for any division problem.

Example 15 (in Session 1):
(Glenn has written in his journal 525 times. If he writes in his journal 25 times each
month, how many months has Glenn been writing in his journal?)
B: You divide 525 equally into 25 to get the answer.

Explanations as in Example 15 showed confusion between the “unit rate” and the
“number of units”. According to Table 4, they were scored as 1 point. At the end of the
intervention condition, Bill developed explanation “formats” for division problems. For
division problems with the “number of units” (n) unknown, he would say “they want to
see how many ms fit into a” (m stands for the “unit rate” and a stands for the product).
For division problems with the “unit rate” (m) unknown, he would say “they want to fit a
equally into n groups” (n stands for the “number of units”).

Compared with division word problems, multiplication word problems (product
unknown) was more difficult for Bill. He made more mistakes when explaining
multiplication problems, and it took him longer time to develop his explanation “format”
for multiplication problems. During the early stage of the intervention condition, his
most common explanation for multiplication problems was to explain them in the
division way (see Example 16).

Example 16 (in Session 3):
(You can pick your own strawberries at the festival. If there are 16 pints of strawberries
in a box, how many pints are in 5 boxes?)
B: you want to multiply 16 to 5 to get the answer because… they want us to see how
many boxes they want…I mean, see how many 16s in 5 boxes.
The “how many 16s in 5 boxes” showed that he was mechanically applying his way for explaining division to multiplication word problems. This type of explanation reflected faulty understanding of the multiplicative relationship between numbers. Thus it was scored as 0 points according to Table 4. Toward the end of the intervention condition, he developed explanation “formats” for multiplication problems: “there are $n ms$” ($m$ stands for the “unit rate” and $n$ stands for the “number of units”).

Carl: (1) Problem solving: Carl made a quick improvement in problem solving performance at the first session of intervention condition (from 0% correct to 33.3% correct). Carl’s problem solving performance increased from an average of 1.6% correct in the baseline condition to an average of 85.7% correct in the intervention condition. During the intervention condition, his problem solving performance ranged from 33.3% correct to 100% correct, with a median of 100% correct. Except for the first session, he got no less than 66.7% correct in other sessions. The PND of Carl’s improvement in problem solving was 100%. After reading the problem, Carl typically worked on the calculator, decided on the solution, and wrote a math equation in the vertical format. (2) Self-explanation: Carl made a quick improvement in self-explanation performance at the first session of intervention condition (from 0% correct to 25% correct). During the intervention condition, Carl’s self-explanation scores ranged from 8.3% correct to 100% correct, with a median of 58.35% correct, and an average of 57.7% correct, which was 57.7% increased from his average performance in baseline condition. The PND of Carl’s improvement in self-explanation was 100%. Yet in the intervention phase the increase seemed to stop at a certain point (he stayed 41.7% correct for Sessions 4, 5, 6, and 7 as shown in Table 11). Then from Session 8 his explanation performance increased again,
and stayed at no less than 75% correct. As such, Carl had more sessions (14 sessions) than both Amy and Bill for the intervention condition. Figure 7 shows the trend lines of Carl’s performance in problem solving and self-explanation in the baseline and the intervention condition. Carl’s performance in both problem solving and self-explanation showed an increasing trend compared with the flat trend in the baseline condition.

Table 11: Percentage Correct for Carl’s WP Solving and Self-Explanation Performances during the Intervention Condition

<table>
<thead>
<tr>
<th>Sessions</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
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<th>11</th>
<th>12</th>
<th>13</th>
<th>14</th>
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</thead>
<tbody>
<tr>
<td>WP solving</td>
<td>33.3%</td>
<td>66.7%</td>
<td>66.7%</td>
<td>83.3%</td>
<td>100.0%</td>
<td>100.0%</td>
<td>66.7%</td>
<td>100.0%</td>
<td>100.0%</td>
<td>100.0%</td>
<td>100.0%</td>
<td>83.3%</td>
<td>100.0%</td>
<td></td>
</tr>
<tr>
<td>Self-explanation</td>
<td>25.0%</td>
<td>33.3%</td>
<td>8.3%</td>
<td>41.7%</td>
<td>41.7%</td>
<td>41.7%</td>
<td>41.7%</td>
<td>75.0%</td>
<td>83.3%</td>
<td>100.0%</td>
<td>75.0%</td>
<td>75.0%</td>
<td>83.3%</td>
<td>83.3%</td>
</tr>
</tbody>
</table>

Note: P-S = Problem Solving; S-E = Self-Explanation

Figure 7. Trend Lines of Carl’s Performance in Problem Solving and Self-Explanation in the Baseline and the Intervention Condition
Same as Amy and Bill, Carl gradually developed his own way of explanation. He
developed the way to explain multiplication problems earlier than division problems,
with the “format” of “$n$ groups of $m$” ($n$ stands for the “number of units”, and $m$ stands
for the “unit rate”) (see Example 17).

Example 17 (in Session 2):
(You can pick your own strawberries at the festival. If there are 16 pints of strawberries
in a box, how many pints are in 5 boxes?)
Carl (C): There are 16 pints. They want to put 5 equal groups of 16.

Before he learned how to explain division problems, his most common mistake
was to explain division problems in a “multiplication way” (see Example 18).

Example 18 (in Session 3):
(Edwin received a total of $374 to buy basketballs for the football team. Each basketball
costs $34. How many basketballs can he buy?)
Carl (C): “They want, there are 374 dollars, and they want 34 basketball, so they want 34
equal groups of 374.”

In Example 18, Carl solved the problem by division correctly, but his explanation
reflected incorrect understanding of the multiplicative relationship among the quantities.
Thus explanations as such were scored as 0 points according to Table 4.

Occasionally he also explained multiplication problems in a “division way” (see
Example 19), and mixed the “unit rate” and the “number of units” (see Example 20).

Example 19 (in Session 3):
(It takes 32 oranges to make one gallon of orange juice. How many oranges would you
need to make 15 gallons of orange juice?)
C: “They want to see 32 equal groups of…how many 15s can come out of 32.”

Example 20 (in Session 4):
(It takes 128 dollars to buy 32 booklets. How much does it take to buy one booklet?)
C: “They want to see how many 32s can come out of 128.”

According to Table 4, explanations as in Example 19 reflected incorrect
understanding of the multiplicative relationship among the quantities. Thus they were
scored as 0 points. Explanations as in Example 20 reflected the confusion on the “unit rate” and the “number of units”. They were scored as 1 point.

In the last sessions of the intervention condition, Carl developed his “format” for explaining division problem. He would say “$n$ groups of $m$” or “$n$ groups of $m$ in $a$” ($n$ stands for the “number of units”, $m$ stands for the “unit rate”, and $a$ stands for the product) for both types of division problems (i.e., the “unit rate” unknown [see Example 21] or the “number of units” unknown [see Example 22]).

*Example 21* (in Session 10):
(It takes 128 dollars to buy 32 booklets. How much does it take to buy one booklet?)
C: “They want 32 groups of 4 in 128.”

*Example 22* (in Session 10):
(Megan made a total of $162 by selling decorative baskets. If each gift basket costs $27, how many decorative baskets did he sell?)
C: “They want 6 groups of 27 out of 162.”

### 4.3 Post-Test

The three participants received post-tests immediately after their intervention conditions.

All of them kept high performance in problem solving (no lower than 83.3% correct) (see Table 12). In terms of self-explanation, compared with their self-explanation performance in the last few sessions of the intervention condition, a little decrease was seen in Amy’s third post-test (50% correct) and Bill’s third post-test (58.3% correct). Yet the median scores of all the three participants’ self-explanation in the post-test (83.3% correct for Amy, 75% correct for Bill, and 83.3% correct for Carl) were higher than or at least equal to their medians in the intervention condition (58.3% correct for Amy, 75% correct for Bill, and 58.35% correct for Carl).
Table 12: *Percentage Correct for the Three Participants’ WP Solving and Self-Explanation(S-E) Performances in the Post-test.*

<table>
<thead>
<tr>
<th></th>
<th>Post-test 1</th>
<th>Post-test 2</th>
<th>Post-test 3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>WP Solving</td>
<td>S-E</td>
<td>WP Solving</td>
</tr>
<tr>
<td>Amy</td>
<td>100%</td>
<td>83.3%</td>
<td>100%</td>
</tr>
<tr>
<td>Bill</td>
<td>100%</td>
<td>75%</td>
<td>100%</td>
</tr>
<tr>
<td>Carl</td>
<td>100%</td>
<td>83.3%</td>
<td>100%</td>
</tr>
</tbody>
</table>

4.4 Maintenance Test

The three participants received maintenance tests one month later than their respective post-tests. All of them solved the problems in the three tests 100% correct. In terms of self-explanation, all participants kept their improvement, and their average scores were all around 69% correct (69.5% correct for Amy, 69.4% correct for Bill, and 69.4% correct for Carl).

Table 13: *Percentage Correct for the Three Participants’ WP Solving and Self-Explanation(S-E) Performances in the Maintenance Test.*

<table>
<thead>
<tr>
<th></th>
<th>Maintenance 1</th>
<th>Maintenance 2</th>
<th>Maintenance 3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>WP Solving</td>
<td>S-E</td>
<td>WP Solving</td>
</tr>
<tr>
<td>Amy</td>
<td>100%</td>
<td>66.7%</td>
<td>100%</td>
</tr>
<tr>
<td>Bill</td>
<td>100%</td>
<td>50%</td>
<td>100%</td>
</tr>
<tr>
<td>Carl</td>
<td>100%</td>
<td>83.3%</td>
<td>100%</td>
</tr>
</tbody>
</table>

4.5 Transfer Test

Recall, the transfer test was administered three times; at pre-test, post-test, and maintenance. On the transfer test which contained two-step multiplication/division word problems, Amy’s overall performance was better than Bill and Carl in terms of both WP
solving and self-explanation. After intervention condition, in the post-test, Amy’s performance in the transfer test had a big increase (from 16.7% correct to 100% correct in WP solving and from 8.3% correct to 83.3% correct in self-explanation), yet it decreased to 50% correct in WP solving and 33.3% correct after one month in the maintenance time. Bill’s WP solving score was lower in the post-test than in baseline. In the maintenance test his WP solving score was the same as his baseline performance. There was some increase in Bill’s self-explanation performance over time (0% correct to 33.3% correct). Carl also increased his WP solving and self-explanation performance in the post-test, yet his performance decreased again in the maintenance test, where his self-explanation score was even lower than that in the baseline condition.

Table 14: Percentage Correct for the Three Participants’ WP Solving and Self-Explanation (S-E) Performances in the Transfer Test.

<table>
<thead>
<tr>
<th></th>
<th>Transfer at Pre-test</th>
<th>Transfer at Post-test</th>
<th>Transfer at Maintenance</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>WP Solving</td>
<td>S-E</td>
<td>WP Solving</td>
</tr>
<tr>
<td>Amy</td>
<td>16.7%</td>
<td>8.3%</td>
<td>100%</td>
</tr>
<tr>
<td>Bill</td>
<td>33.3%</td>
<td>0%</td>
<td>16.7%</td>
</tr>
<tr>
<td>Carl</td>
<td>16.7%</td>
<td>16.7%</td>
<td>33.3%</td>
</tr>
</tbody>
</table>

Figure 8 presents the three participants’ performance for word problem (WP) solving and self-explanation for the criterion tests and the transfer test during the baseline, intervention, and postintervention conditions.
Figure 8. Percentage Correct for Word Problem (WP) Solving and Self-Explanation for the Criterion Tests and the Transfer Test during the Baseline, Intervention, and Postintervention Conditions for the Three Participants

4.6 Social Validity

Amy and Bill took the Student Perception and Satisfaction Survey. Carl moved out of the state before taking the survey. Amy scored 27 out of 30 and Bill scored 28 out of
30. The average of the two participants’ scores was 27.5. The result showed that the participants thought participating in this study was helpful to them. For example, for the statement “This activity is a helpful experience to me”, both of them chose “agree”; the researcher’s input helped them in both word problem solving and explanation. For example, they both “strongly disagree” to the statement that “the teacher’s explanation did not really help my problem solving”, and they both “strongly agree” that “I find it hard to express how I think about the word problems, but the teacher’s prompts are very helpful”. In the early intervention stage, though there were occasions when some participant got impatient with solving and explaining the problems, overall they both enjoyed the learning process and agreed that it was very helpful to them.
CHAPTER 5 DISCUSSION

5.1 Effectiveness of Repair Requests on Self-Explanation and Word Problem Solving

This study aimed to promote the self-explanation and word problem solving ability of students with LD. The intervention was designed to achieve this purpose by the use of conversational repair requests as scaffolding techniques. The repair requests were provided from the most general to the most explicit, with different components in between to be applied depending on the students’ different responses. The data showed that all participants started with low-level performance in both self-explanation and problem solving on the criterion tests. When they entered the intervention condition, there were distinct level changes. All of them have immediate increase in both problem solving and self-explanation performance. Very few overlaps of performance existed between the baseline and the intervention condition (PND as 85.71% for Amy’s problem solving and self-explanation performance, and 100% for both Bill’s and Carl’s problem solving and self-explanation performance). In the intervention phase, Amy and Bill respectively took 7 sessions to reach a stable performance level of no less than 75% correct in both problem solving and self-explanation. In contrast, Carl took 14 sessions to reach a stable performance level of no less than 75% correct. Carl’s low IQ score (73, see Table 1) and low problem solving and self-explanation performances during the pre-test
phase might explain his slower improvement than the other two participants’. Although theoretically, Carl does not satisfy the definition of LD, it was not uncommon for schools to label students such as Carl with lower than normal IQ as SLD. In the post-test, they all kept the increased performance in explaining their reasoning process as well as in solving multiplication/division word problems, and the intervention effect was kept in the maintenance test one-month later. This being said, it may be noticeable that there were fluctuations in both Amy’s and Bill’s performance during the post-test and the maintenance test across both measures on problem solving and explanation. It was observed throughout the experiment that Amy tended to be impulsive. When there was no teacher input, she tended to enjoy working fast, and got stimulated by finishing one problem quickly and hurrying to the next one. It brought mistakes to her explanations. On the day for the last maintenance test, Amy checked with the researcher if this would be the last time to work with the researcher and if the worksheet was her last one. After the researcher confirmed on her questions, she seemed to treasure the last opportunity and slowed down her pace, and her self-explanation performance increased. Bill’s performance was directly related to his state of mind of the day. He sometimes did not get enough sleep at night, which made him very sleepy and could not concentrate for the next day. For instance, on the day for the last post-test and the day for the first maintenance test, he said he was sleepy, and his self-explanation scores went down.

Through communication with the school teachers, it was known that during the time they were in this study, all the participants used enVision MATH (Charles et al., 2012) as the math textbook in the classroom. In particular, Amy’s teacher followed Topic 1 to 9 of the Grade 4 enVision MATH (Charles et al., 2012). They are Numeration;
Adding and Subtracting Whole Numbers; Multiplication Meanings and Facts; Division meanings and Facts; Multiplying by 1-Digit Numbers; patterns and Expressions; Multiplying by 2-Digit Numbers; Dividing by 1-Digit Dividers; and Lines, Angles, and Shapes. Bill’s and Carl’s teachers covered the topics of multiplication/division included the process of multiplication (2 digit × 1 digit & 2 digit × 2 digit), the process of division (2 digit divided by 1 digit & 3 digit divided by 1 digit with no remainders; factors; multiples; the properties of multiplication; rounding (estimating) place value up to the millions; standard form, written form, and expanded form; recognizing multiplication is just repeated addition, and division is just repeated subtraction; and using arrays to write multiplication problems. The researcher found that in the envision MATH (Charles et al., 2012) textbook there are equal group word problems mixed with other types of word problems (such as addition word problems, or word problems involving addition and multiplication at the same time) across Topic 1 to 9. However, word problem was not a focus in any of the classrooms according to the researcher’s communication with the participants’ teachers. Therefore, it is likely that the intervention contributed to the change in participating students’ performance on word problem solving and self-explanation.

In this study, some transfer effect was shown as Amy increased her performances in solving and explaining 2-step multiplication/division word problems (from 16.7% correct in problem solving and 8.3% correct in self-explanation in the pre-test, to 100% correct in problem solving and 83.3% correct in self-explanation in the post-test, and 50% correct in problem solving and 33.3% correct in self-explanation in the maintenance test). However, the intervention effect on the transfer test was not clearly shown for the
other two participants, Bill and Carl. For Bill, in terms of problem solving performance, his score in the post-test phase was even lower than in the pre-test phase, and in the maintenance test phase the score was the same as in the pre-test phase. In terms of Bill’s self-explanation performance, there was some limited increase, from 0 correct in the pre-test phase, to 8.3% correct in the post-test phase, to 33.3% in the maintenance phase. For Carl, his performance in both problem solving and self-explanation in the post-test phase increased compared with the pre-test phase (from 16.7% correct in problem solving and 16.7% correct in self-explanation in the pre-test, to 33.3% correct in problem solving and 33.3% correct in self-explanation in the post-test). In the maintenance test phase, his problem solving performance decreased to the same as in the pre-test phase, and his self-explanation performance dropped to an even lower level (8.3% correct). It is known from their classroom teachers that all the participants learned nothing or very little (one or two problems) about two-step word problems, and during this study they were all on their own to solve the transfer test problems. Compared with the pre-test phase performance, Amy’s performance increase in both problem solving and self-explanation, and Bill’s performance increase in self-explanation suggested some intervention effect in promoting transfer from solving 1-step word problems to solving 2-step word problems for students with LD. Yet overall the intervention did not seem to be able to help all participating students in this study transfer their ability in solving one-step multiplication/division word problems to two-step multiplication/division word problems. It may indicate that this transfer test might be a too far transfer for the participants. As such, the participants may need systematic and explicit instruction on the specific problem type in order for them to show the improvement in performance (Wagner, 2006; Xin & Zhang, 2009).
5.2 Theoretical Implications

Vygotsky’s (1962; 1987) social development theory highlighted the importance of communication in learning. Stemming from that, constructivism emphasizes teacher-student interaction. In response to the call for more reasoning and communication in mathematics classroom discourse, we need to find ways to get students with LD to think and talk in a constructivistic environment. Based on this consideration, and inspired by the literature on coaching students with LD’s reasoning (Scruggs, Mastropieri, & Sullivan, 1994; Scruggs, Mastropieri, Sullivan, & Hesser, 1993; Sullivan, Mastropieri, & Scruggs, 1995) with different layers of coaching questions, this study designed a constructivistic scaffolding strategy consisting of repair requests in an “implicit-to-explicit” continuum. It is a dialectical constructivism in the sense that direct instruction is not provided upfront. Instead, the collaborative interactions between the teacher and the student allow the student the possibility to think and self-construct the articulation of reasoning. Yet they are not left alone in the learning process to figure out everything by themselves. Instructions are provided when needed. The study shows that the students gradually needed less and less scaffolding from the researcher, and they could produce satisfactory self-explanations on their own. When mistake happens, they could realize it and self-initiate the repair. Example 23 is an instance where in the late intervention phase Carl did a self-initiated repair of the initial self-explanation, and made it a satisfactory one.

*Example 23 (in Session 12):*  
(It costs a total of $551 to buy 19 super-sized pizzas for a school party. How much did each pizza cost?)  
C: they want 19 groups of 551. No. 19 groups of 29.)
As such, it cultivates the learning habit of these students that they always need to try by themselves, instead of passively waiting for the teacher to tell them everything. The explanation is also a process to go through their thinking again and reorganize it by way of putting it into words. Thus it helps them self-construct a better understanding of the problem, which leads to better word problem solving ability.

5.2.1 Students’ Self-Explanation Strategy Development

This study supports existing literature (e.g., Baxter et al., 2001; Kroesbergen & Van Luit, 2002, 2003; Woodward & Baxter, 1997) on that if these students were provided with opportunities (e.g., the adapted instruction), they could also develop their own strategies. In this study, all participants developed their own ways of self-explanation for different types of multiplication/division word problems. For instance, at the beginning of the experiment, Amy’s explanation was mainly reiterating what she did on the calculator (e.g., “If you have a calculator, you put 460 minus 20, minus 20…”); Bill started with general statements like “I solved it this way because it would be better for me. It feels better and easier for me to do”, which failed to reflect his mathematical reasoning; and Carl talked about how he added or subtracted two numbers. Then at the late intervention phase, they developed their own ways to explain different types of problems, and they carried on their own “format” of explanation to the post-test and the maintenance test. Table 15 presented the typical expressions the three participants developed as their “formats” for explaining different types of problems.
Table 15: Representative Expressions of Self-Developed Self-Explanations by the Three Participants (in the context of solving equal group problems represented by $m \times n = a$ [product])

<table>
<thead>
<tr>
<th></th>
<th>Multiplication problems (the product $a$ is unknown)</th>
<th>Division problems (the “number of units” $n$ is unknown)</th>
<th>Division problems (the “unit rate” $m$ is unknown)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Amy</td>
<td>“You are doing $m$ $n$ times.” “You are doing $m$ $n$ times.” Or “$m$ goes into $a$ $n$ times”</td>
<td>“They want to see how many $ms$ fit into $a$.” “They want to fit $a$ equally into $n$ groups.”</td>
<td>“$n$ groups of $m$” or “$n$ groups of $m$ in $a$”</td>
</tr>
<tr>
<td>Bill</td>
<td>“There are $n$ $ms$.”</td>
<td>“$n$ groups of $m$”</td>
<td>“$n$ groups of $m$” or “$n$ groups of $m$ in $a$”</td>
</tr>
<tr>
<td>Carl</td>
<td>“$n$ groups of $m$”</td>
<td>“$n$ groups of $m$”</td>
<td>“$n$ groups of $m$” or “$n$ groups of $m$ in $a$”</td>
</tr>
</tbody>
</table>

Carl used the same linguistic expression (“$n$ groups of $m$”) for all three types of problems. Note that Carl had the lowest performance in both problem solving and self-explanation during the pre-test phase; the reason for his uniform self-explanation might be such that he did not need to worry about the linguistic aspects of his explanation. He then could focus on finding out the number of groups and the unit rate, and then generate a satisfactory explanation. For division word problems, Amy preferred the expression of “go into”, and Bill preferred the expression of “fit into.” Amy used the linguistic expression of “You are doing $m$ $n$ times” for multiplication problems, and sometimes she also applied the expression to division problems. That is, Carl, and sometimes Amy, tended to explain division problems multiplicatively.

5.2.2 Students’ Problem-Solving Strategy Development

The participants’ improvement was also shown in their problem solving on the worksheet. Amy’s worksheets at different times were provided here as an example (see Figure 9). At the beginning of the pre-test phase, Amy used repeated addition and
subtraction to solve the problems. She drew short vertical lines on the worksheet as tallies to help her keep track of how many times she had pressed the “+” or “-” button on the calculator. As such, her problem solving process was tiring and she often miscounted the tallies (see Image A in Figure 9). At the end of the pre-test phase and the beginning of the intervention phase when Amy started to use multiplication and division and was not clear about when should the two operations be used, she drew a calculator on the paper instead of the tallies, though the calculator did not contribute to either the problem solving or self-explanation (see Image B in Figure 9). In late intervention phase, she only wrote down the math equation as well as the numbers while she explained her solutions, and her worksheet became neater (see Image C in Figure 9).
Image A (from Baseline Session 1)

Problem 1: There are 22 ice cream bars in each box. How many ice cream bars would you have if you bought 27 boxes of ice cream bars?

[Please write down ALL of your problem-solving process, including each equation(s).]

\[22 \times 27 = 814\]

Image B (from Intervention Session 1)
Therefore, the intervention helped the participants to learn and use the more advanced strategy (i.e., multiplication/division instead of repeated addition/subtraction), and to show their work in mathematics expressions (i.e., math sentences rather than drawing lines or drawing calculators).

5.2.3 Self-Explanation

This study contributes to the literature on self-explanation. Existing research and studies have shown that for normal-achieving students, self-explanation is a good strategy
to improve problem solving ability, which is called the “self-explanation effect”. Yet when it comes to students with LD, little is known if the “self-explanation effect” still exists, or what their self-explanation is like per se. It is only known that in classroom discourse, they rarely participate. For example, Baxter et al.’s (2001) study found that due to limited opportunities offered and their verbal inabilities, it was challenging for these students to actively involve in classroom discussion. Little has been done on how to improve their discourse participation, and how this can promote problem solving ability. This pilot study tried to fill in the gap. It adds to the literature a way to promote students with LD’s self-explanation of reasoning, so that they can better involve in classroom discourse, and better solve mathematics word problems. The result of the study showed that students with LD improved on both self-explanation of their reasoning and word problem solving, which gives evidence that this scaffolding strategy works for these students, and it may be applied in classroom settings and small-group settings. It can help teachers better understand and promote students with LD’s reasoning, and facilitate classroom discourse. Appropriately designed, the scaffolding requests can also be used among peers (in a pair or a group setting) as a collaborative learning strategy. This is a direction for future research and practice.

5.2.4 Conversational Repair

This study contributes to the literature on conversational repair. Conversational repair is a linguistic concept. In pragmatics more specifically, it is typically used as a strategy to improve oral expression for ESL learners and people with disabilities (e.g., Brinton et al., 1988; Weiner, 2005). In this study, the repair requests were designed as
scaffolding techniques to promote reasoning and problem solving in the field of mathematics education, specifically for students with LD. As such, this study is interdisciplinary in nature.

5.3 Practical Implications

This study bears the practical implications on why and how to facilitate self-explanation of students with LD, and eventually promote their problem solving performance. Currently in schools students’ explanation ability is still not given its due attention. Through working with the participants, we find that students easily follow teachers’ modeling. For instance, Amy could imitate very well her teacher’s demonstration (including the teaching words) of how to do the addition calculation, due to the fact that her teacher did that a lot in class. Carl was very used to carrying out the procedures of addition and subtraction, which indicated that in his daily practice, he must have done plenty of exercise on them. None of the students were familiar with how to put their thinking in words to explain how they solved the problem. Yet after the intervention, they all learned how to explain and solve word problems. Given the current reform in mathematics education calls for an active classroom discourse where students know how to talk mathematically, it is suggested that teachers provide more opportunities to engage students with LD in activities such as talking about how they solved the problem and discussing other possible ways to solve the problem to enhance their explanation ability. In addition, when they were confident about articulating their reasoning process, their participation in classroom discourse will be promoted.
5.3.1 Implications on the General Request

At the early stage of the intervention condition, the participants were more likely to respond “No” to the general request asking them if they would like to try again. The reason might be that solving the problem and explaining their thinking process in words had already been a difficult process for them. They were not sure if the problem should be solved and explained like that or how to make it better. As such, they usually refused the opportunity to try again. However, as they went through the intervention, they were more likely to improve their initial self-explanation after the general request. Example 24 and Example 25 contrast two instances of Amy’s responses to the general request at the beginning of the intervention phase (Worksheet 1) and the late intervention phase (Worksheet 7) respectively. In Example 24, she simply refused the opportunity to repair her self-explanation, while in Example 25, the general request suffices to scaffold her to self-repair to a satisfactory explanation.

Example 24 (in Session 1):
(Glenn has written in his journal 525 times. If he writes in his journal 25 times each month, how many months has Glenn been writing in his journal?)
A: You would have 525 and 25 so first I did times, 525 times 25, no that equal to 3884. I knew it could not be that high, so I did 525 divided by 25 equals 21. So that is the answer. Instead of doing plus plus plus you do times. You can do division instead of minus minus minus.
Researcher (R): Do you want to try again to make the explanation better?
A: No.

Example 25 (in Session 7):
(Edgar read 357 pages in his favorite new book. Each day since he bought it, he reads 21 pages. How many days has it been since he bought the book?)
A: You doing 357 into 21 groups total. Equally.
R: Do you want to try again?
A: Yeah. So 357 is how many pages in the whole book, (drawing a book) so you have one book that is 357, and he read 21 pages on one day. Each day he read 21. So they need to find out how many times you would have to do 21, and so it takes 17 days to read this whole book. And so they are doing 21 17 times.
The only concern is that for the beginning of the intervention, if a student had attention problems and easily got impatient or frustrated when they were asked to explain something, teachers could consider skipping this general request depending on the specific case.

5.3.2 Implications on Repeating

If the intervention ends with the teacher direct modeling an explanation or teaching the diagram to solve the problem, an important part of the scaffolding interaction is asking the student to repeat the teacher’s explanation. The reason is that students with LD usually cannot hold much information in their working memory, and after the teacher’s modeling or teaching, even mere repeating is difficult for them (Henry, 2001). Thus it is critical to ask the student to repeat the explanation so that they can internalize the explanation. It usually takes many conversational turns between the teacher and the student until the student can fully repeat the explanation on his/her own, especially at the beginning of the intervention condition. Example 26 is an excerpt of the conversation where the researcher asked Bill to repeat what she had said. Right before the excerpt the researcher had provided a direct modeling of the explanation to the problem ("this is to see how many 4s there are in 240, or say how many groups of 4 in 240").

Example 26 (in Session 2):
(The students in Lee Ann's class collected 240 stamps. They put 4 stamps on each page of an album. How many pages did they need?)
R: Could you repeat?
B: You had to divide 4 into 24? 240.
R: We did 240 divided by 4 because 4 is the number of stamps in each page. So it is like one page 4, another 4...we want to see how many 4s there are. So total is 240, right? So
how many 4s there are in 240, or how many groups of 4 is 240. Could you try one more time?
B: 240 in 4 group each time to get 60.
R: It is 4 each time. Why we do division, because we want to see how many...
B: Groups of 4 there are in each page.

In Example 26, Bill solved the problem correctly, but his original explanation was “I divided 24 into 4 groups because, I mean 240 in 4 groups, because they want 4 stamps on card. We don't know how many cards they needed, so you would divide 240 into 4 groups equally to get 60.” This explanation confused the unit rate (4) and the number of groups (60). The intervention ended with the researcher modeling the explanation to him. This example shows that even in the cases that the student solved the problem correctly and the teacher had modeled how to explain, it may still take many conversational turns between the teacher and the student before the student can independently and clearly express “4” as a “unit rate” in his explanation.

5.3.3 Implications on Students’ Own Ways of Explanation

This study found that during the process of scaffolding, modeling, and helping internalizing an explanation, students developed their own ways of explanation that they are most comfortable with and may not be exactly linguistically the same to the teacher’s. Teachers or instructors should be sensitive to notice the student’s preferred way of expression, value it, help make it correct and stick to it all the time. The reason is that the student’s way of expression may reflect his/her preferred way to mentally organize the information. As mentioned earlier, modeling an explanation and asking students with LD to repeat and internalize the explanation has been a process that takes time and effort, let
alone if the teacher provides multiple ways of explanation in the modeling. Therefore, the suggested way is to find the words the student are comfortable with, and tailor the explanation into one that uses the words. It will be much easier for them to learn if they can use the words they choose and still correctly reflect the reasoning. In the following example, the researcher modeled the correct explanation ("put 195 equally into 15 groups"), and helped Amy to repeat it so that she could say it by herself.

*Example 27 (in Session 7)*:
(1) R: so this is the number of groups (pointing to “15”). Make sense? So we put 195 equally into?
(2) A: 15 groups.
(3) R: Could you say it without help?
(4) A: So it is to see 195 times...
(5) R: We put…
(7) R: We put 195 into…
(8) A: We put 195 into 15 groups.
(9) R: Yes. Equally.
(10) A: Equally.
(11) R: One more time.
(12) A: It is kind of like you are doing 15 13 times
(13) R: Not 15 13 times.
(14) A: I mean 13 15 times.
(15) R: That is great. But if we want to do this directly, explain this (pointing to 195÷15), how do we say? We put…
(16) A: We put 195 into 15 groups equally.
(17) R: Good. One more time.
(18) A: 195 into 15 groups equally.

From line 3 to line 11, the researcher was working on the explanation ("put 195 equally into 15 groups") with Amy, yet in line 12 Amy still did not repeat what she had practiced with the researcher. Instead, she went back to the way of the explanation that she was comfortable with in the “format” of “we do \( m \) \( n \) times”. Though in the following part the researcher still repeated the “put 195 into 15 groups equally” way of explanation, and Amy was able to articulate the explanation by herself successfully, in the post test,
Amy still stick to the “we do \( m \) \( n \) times” way of explanation. Thus as teachers, when we find the way of expression that the student prefers, we better drop the “pre-set” correct explanation and work on enhancing theirs.

As the student learn and work with the teacher, the intervention process will be shorter and shorter. There will be less need to go to the direct teaching/modeling part, and the scaffolding request part will be more and more effective to help student reach a full-score explanation.

5.4 Limitations and Future Directions

This study has a few limitations. First, it involved only three participants. This may affect the generalization of the intervention effect. Future research can extend on this to conduct a group design study with more participants. It will better examine the causal relationship between the intervention and students’ performance. Second, this study focuses on equal group multiplication/division word problems. The participants might gradually realize that the test was all about multiplication/division problems. The current design of the study does not allow an emphasis on the distinction between addition/subtraction and multiplication/division. Yet for students like Carl who only used addition/subtraction in the pre-test phase, it might be an issue that deserves attention. Third, there is little research in existing literature on interventions for students with LD’s self-explanation. As such, there is not much this study could borrow from literature on specific intervention design. The intervention here is exploratory in nature. Future research could try this intervention on normal-achieving students to see how they would respond to it. Or a comparison study could be done which involves both normal-
achieving students and students with LD to see the difference in their performance.

Fourth, this intervention has different components (i.e., the general request; the request for specification/clarification; the request for revision; the direct modeling of self-explanation; and the direct teaching). They were implemented as a package to the participants. A supplementary study could be implemented to record how many steps of the repair requests a student would need, and comparisons between students with different characteristics.

Also, there seems to be a potential correlation between the students’ reasoning (self-explanation) scores and performance scores. Future studies could explore if there should be a correlation for other population (e.g., the gifted students).

5.5 Conclusion

Current reform in mathematics education emphasizes students’ ability to articulate their thinking. They are not only expected to do mathematics, but also to communicate their reasoning with mathematics languages. This is in line with the perspectives of social development theory and social constructivism that communication plays an important role in learning. For normal-achieving students, there are studies on students’ self-explanation characteristics in classroom discourse, and the effectiveness of self-explanation on improving learning outcome. However, when it comes to special education and more specifically students with LD, research shows that they have little quality participation due to their challenges in learning and articulation, and limited discussion opportunities provided in the classroom. More research is needed on strategies to improve their explanation in mathematics classroom discourse.
To fill in this gap, this study borrowed the linguistic concept of conversational repairs, and designed the intervention package with repair requests in an implicit-to-explicit continuum to scaffold students with LD’s self-explanation in the context of solving word problems. This strategy design allowed the students opportunities to self-organize and self-construct their explanation, and still get the scaffolding to lead their thinking when needed. As such, they neither wait passively for the teacher to tell them everything directly, nor feel frustrated about being left alone in the learning process. The study found that this strategy promoted the self-explanation as well as problem-solving ability of students with LD, and they became more confident in solving and explaining the problems as they became more proficient. This result indicated that these students could also benefit from a constructivistic environment. Classrooms should try establishing a constructivistic environment with appropriately-designed scaffolding as a way to improve the self-explanation ability and discourse participation of students with LD, to help the teacher better understand students’ reasoning, and to promote learning outcome in problem-solving performance.
REFERENCES
REFERENCES


*Mathematical Thinking and Learning, 4*, 103-125.


APPENDICES
Appendix A  Sample EG Word Problems with Product Unknown, or the “Unit Rate” Unknown, or the “Number Of units” Unknown (Following the EG Problem Conceptual Model by Xin et al., 2008)

Instructions: Please read through each problem, write down all of the problem-solving processes, including math equation(s) in the space below the problems, and then explain your problem-solving process to me.

Problem 1

Gary made 41 buttons when running for class president. It takes 23 drops of glue to make each button. How many drops of glue did Gary use?

[Please write down ALL of your problem-solving processes, including math equation(s), and then explain your problem-solving process to me.]
Problem 2

Bobby found 7 boxes in the attic of his house. In each box there were an equal number of crystal drinking glasses. If there were 91 total glasses, how many glasses were there in each box?

[Please write down ALL of your problem-solving processes, including math equation(s), and then explain your problem-solving process to me.]
Problem 3

Sue wants to buy presents for all her friends. She has $153 and each present costs $9. How many presents can she buy?

[Please write down ALL of your problem-solving processes, including math equation(s), and then explain your problem-solving process to me.]
Appendix B  Fidelity Checklist

Instructions: Please check if the investigator has implemented the treatment components according to the different application contexts. Check “Yes” if the component is executed in the correct contexts. Check “No” if the investigator did not deliver the treatment component according to its application context.

<table>
<thead>
<tr>
<th>Scaffolding Components</th>
<th>Yes</th>
<th>No</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>The investigator provided general request in the form of “Do you want to try again to make it better?” when the participant provided an initial explanation which was incomplete, or relevant but not precise, or incorrect, or no response.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>The investigator requested for revision when, after the general request, the participant provided an explanation which was incorrect or no response, in the form of “This number means…and this number means…” (tells the participant the meaning of the two known numbers) “So do you want to revise your explanation?”</td>
<td></td>
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<td></td>
</tr>
<tr>
<td>The investigator requested for specification/clarification when, after the general request (or the general request and the request for revision), the participant provided an explanation which was relevant but not precise, in the form of repeating repairable part in the participant’s response and adding a wh-question word, or in the form of “And could you tell me more about why you did so?”</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>The investigator provided direct modeling of a self-explanation when the general request and the request for specification/clarification did not elicit satisfactory explanation from the participant.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>The investigator provided direct teaching when, after the general request and the request for revision, the participant provided an explanation which was incorrect, or when no response was given.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>In the direct teaching stage, the investigator followed the teaching script to teach how to solve the problem, and modeled how to explain the problem.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>The investigator asked the participant to repeat what has been taught after the direct modeling or direct teaching was given.</td>
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</tbody>
</table>
Appendix C  Modeling Self-Explanation

Problem 1
It takes a sewing machine 2 hours to finish sewing a dress. If there are 6 dresses to sew, how long will it take the sewing machine to finish sewing all the dresses?

\[ 2 \times 6 = 12 \]

Explanation: I did 2 times 6 is 12, so it takes 12 hours for the sewing machine to finish sewing all the dresses. The reason I did so is that the sewing machine takes 2 hours to sew one dress, another 2 hours to sew another dress, and another 2 hours for another dress. It is like many groups of 2 hours. There are 6 dresses, so it is like 6 groups of 2. To find the total number of hours, I used the number of hours for each dress, which is 2, times the number of dresses, which is 6.
Problem 2
It takes a sewing machine 2 hours to finish sewing a dress. If the sewing machine worked for 6 hours without stopping, how many dresses can it finish sewing?

\[ 6 \div 2 = 3 \]

Explanation: I did 6 divided by 2 is 3, so it can finish sewing 3 dresses. The reason I did so is that the sewing machine takes 2 hours to sew one dress, another 2 hours to sew another dress, and another 2 hours for another dress. It is like many groups of 2 hours. The total number of hours is 6, so it is like dividing 6 into groups of 2. Therefore, I did 6 divided by 2.
Problem 3
A sewing machine can sew a type of dress. The sewing machine worked for 6 hours without stopping and finished sewing 2 dresses. If it took the same amount of hours to sew each dress, how many hours did the sewing machine use to finish one dress?

\[6 \div 2 = 3\]

Explanation: I did 6 divided by 2 is 3, so it used 3 hours to finish each dress. The reason I did so is that the sewing machine worked for 6 hours in total and each dress took the same number of hours, which is 2. It is like distributing the 6 hours equally into 2 groups, so I did 6 divided by 2 to find out how many in each group.
Appendix D  Transfer Assessment

Instructions: Please read through each problem, write down all of the problem-solving processes, including math equation(s) in the space below the problems, and then explain your problem-solving process to me.

**Problem 1**

Phil and Marcy spent all day Saturday at the fair. Phil rode 3 rides each half hour and Marcy rode 2 rides each half hour. How many rides had Marcy ridden when Phil rode 24 rides?
Problem 2

A food cart on an airplane has 6 slots. Each slot holds 2 food trays. How many trays are in 8 food carts?
Problem 3

Gwen bought 4 dozen apples at the store. The apples were equally divided into 6 bags. How many apples were in each bag? (Hint: 1 dozen = 12)
Problem 4

Last month Steve earned $378 mowing lawns. He has 7 customers, and he mowed each lawn 3 times during the month. How much money does Steve charge to mow one lawn?
Appendix E  

Student Perception and Satisfaction Survey

Name: _____________  
Date: ________________

For each statement, please circle the choice that best describes your opinions and feelings.

1. I like explaining my thinking to other people, especially younger students.
   1=strongly disagree, 2=disagree, 3=neutral, 4=agree, 5=strongly agree

2. I find it hard to express how I think about the word problems, but the teacher’s prompts are very helpful.
   5=strongly agree, 4=agree, 3=neutral, 2=disagree, 1=strongly disagree

3. The teacher’s explanation did not really help my problem solving.
   5=strongly disagree, 4=disagree, 3=neutral, 2=agree, 1=strongly agree

4. I think that the teacher’s questions helped me to clarify my thinking.
   5=strongly agree, 4=agree, 3=neutral, 2=disagree, 1=strongly disagree

5. The strategy the teacher taught me is very helpful.
   1=strongly disagree, 2=disagree, 3=neutral, 4=agree, 5=strongly agree

6. This activity is a helpful experience to me.
   5=strongly agree, 4=agree, 3=neutral, 2=disagree, 1=strongly disagree
Appendix F  
Teaching Script for Direct Teaching Part of the Intervention (Adapted from Xin & Zhang, 2009)

The following script presents the essence of EG problem solving instruction in the context of solving problems with “number of units” unknown such as “It takes a sewing machine 8 hours to finish sewing a dress. If the sewing machine worked for 96 hours without stopping, how many dresses can it finish sewing?”

Teacher: This problem is about sewing dresses and the number of dresses finished in a given time. After we read the problem, we know that it is an equal group (EG) problem because it talks about equal number of dresses the sewing machine can sew each hour. It tells us the number of dresses the sewing machine can sew each hour, or the “unit rate” (i.e., 8); it also tells the total number of hours the sewing machine worked, or the product (i.e., 96); it asks for the number of dresses it can finish sewing, or the “number of units”. Let’s answer the three questions. For each of the three questions, find the relevant information or number in the problem and fill in the blanks accordingly in the equation.

First, which sentence tells about the “unit rate” or number of hours for sewing each dress? I read the problem and find that “it takes a sewing machine 8 hours to finish sewing a dress” tells the “unit rate”. (Writes “8” in the blank for “unit rate”.)

Second, which sentence or question tells about the “number of units”? Do we know how many dresses the sewing machine finished? No. Let’s use the question mark to represent the unknown quantity (Writes “?” in the blank for “number of units”).

Third, which sentence or question tells about the product? Do we know how many hours the sewing machine has worked in total? Yes, I find that it worked for 96 hours. (Writes 96 in the blank for product).

Now let’s look at the equation and read what it says. (Points to relevant parts of the equation.) Working for a total of 96 hours, the sewing machine finished question mark (unknown quantity) dresses with 8 hours for each dress. To solve for the unknown quantity or the question mark, we need to isolate the unknown quantity to one side of the equation. To have the question mark stay alone at one side of the equation, we can divide both sides of the equation by 8. By doing so, we get “8/8 × ? = 96/8,” (Writes this equation on the sheet the student is working on.) therefore, “? = 12”. (Writes this equation on the sheet the student is working on.) (Note. Students can also use mental math to solve for the unknown quantity if they can.) The complete answer to this question is that the sewing machine can finish 12 dresses.
Appendix G  Teaching Script for One Intervention Process Involving Direct Modeling

The following script presents one case of the intervention process with Amy in her first intervention session in the context of solving the problem “There are 22 ice cream bars in each box. How many ice cream bars would you have if you bought 37 boxes of ice cream bars?”

A (Amy): If you have a calculator, you will do either times or divide. I did times because you will have to do 22 times 37 equals 814. How I got it is you would do times or division instead of doing plus plus plus...or minus minus minus...done.
Researcher (R): Do you want to try again to make the explanation better?
A: No.
R: Could you tell me more why you did division?
A: Because then you don’t have to do plus or minus, and you don’t have to keep on doing it.
R: Here is how I would say. I did this because 22 is the number of ice cream bars in each box. It is like the number of items in each group. And 37 is the number of boxes we have. That is like the # of groups. In order to figure out the total number of ice cream bars, I time these together in order to find out the total number. Make sense to you? Would you please repeat that?
A: So 22 is the same number of how many ice cream bars on each box, and then 37…
R: And that means the number of…items in each group.
A: items in each group. And then 37 would mean how many…it would be 37 boxes of…filled with ice cream bars.
R: that also means the number of groups, right? So, then, in order to…
A: In order to do that, it would equal to 814.
R: In order to find out the total number of ice cream bars, would you please repeat that?
A: In order…
R: to find
A: Find how many are of ice cream bars…
R: We need to time these two together, right?
A: um-hum.
R: This is like 37 groups of 22. Would you please repeat?
A: This is kind of like, 27 sets of groups.
R: No. 37 groups, or sets of 22.
A: 37 sets…
R: of.
R: Great! Would you please repeat the whole process again?
A: That 22 would be how many cars in each box, and then 37 would be how much boxes there are filled with ice cream. And then, that’s it.
R: And then why we do division?
A: Because then it will be faster instead of doing plus, or minus.
R: Yes, because that’s like 37 groups of 22, right? Would you please repeat the last sentence?
A: It’s kind of like 37 groups of 22.
R: Great!
VITA
VITA

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