Optimization of Multi-Layer Microperforated Systems for Absorption and Transmission Loss

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Optimization of multi-layer microperforated systems for absorption and transmission loss

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Purdue University
Introduction

- Microperforated Panel:
  Thin film with 100 microns scale holes

- Clean, light → one of alternative to fibrous sound absorbing material
Introduction

- Acoustic Properties are controlled by:
  1. Thickness of the panel
  2. Diameter of the hole
  3. Porosity
  4. Mass per unit area
  5. Air cavity depth

- By appropriate choice of these parameters, single panel can provide good acoustic performance in one or two octave band, but not in broader range.

  ➡️ Multiple-Layer Microperforated Panels are needed

- Multi-layer microperforated panels can make sound material, like functional absorber and barrier, lighter.
Introduction

- Functional absorber
  - Maximizing dissipation coefficient

- Barrier
  - Maximizing Transmission loss
Transfer Matrix Method

\[
\begin{bmatrix}
P_1 \\
u_1
\end{bmatrix} = \begin{bmatrix}
TM_{11} & TM_{12} \\
TM_{21} & TM_{22}
\end{bmatrix}_{\text{total}} \begin{bmatrix}
P_2 \\
u_2
\end{bmatrix} = [TM]_1 [TM]_2 [TM]_3 \cdots [TM]_n \begin{bmatrix}
P_2 \\
u_2
\end{bmatrix}
\]

\[
\Gamma = \frac{TM_{11}^{\text{total}} + TM_{12}^{\text{total}} (\cos \theta / \rho c) - (\rho c / \cos \theta)TM_{21}^{\text{total}} - TM_{22}^{\text{total}}}{TM_{11}^{\text{total}} + TM_{12}^{\text{total}} (\cos \theta / \rho c) + (\rho c / \cos \theta)TM_{21}^{\text{total}} + TM_{22}^{\text{total}}}
\]

\[
\tau = \frac{2e^{j\frac{\omega}{c}} \cos \theta L}{TM_{11}^{\text{total}} + TM_{12}^{\text{total}} (\cos \theta / \rho c) + (\rho c / \cos \theta)TM_{21}^{\text{total}} + TM_{22}^{\text{total}}}
\]

\[
\bar{\alpha}_d = \frac{\int_0^{\pi/2} \alpha_d(\theta) \sin(\theta) \cos(\theta) \, d\theta}{\int_0^{\pi/2} \sin(\theta) \cos(\theta) \, d\theta}
\]

\[
\bar{\tau} = \frac{\int_0^{\pi/2} \tau(\theta) \sin(\theta) \cos(\theta) \, d\theta}{\int_0^{\pi/2} \sin(\theta) \cos(\theta) \, d\theta}
\]
Transfer Matrix Method

\[ [P_1] = [TM_{11} \quad TM_{12}] \quad [P_2] = [TM]_1[TM]_2[TM]_3 \cdots [TM]_n [P_2] \]

\[ \Gamma = \frac{TM_{11}^{total} + TM_{12}^{total} (\cos \theta / \rho c) - (\rho c / \cos \theta) TM_{21}^{total} - TM_{22}^{total}}{TM_{11}^{total} + TM_{12}^{total} (\cos \theta / \rho c) + (\rho c / \cos \theta) TM_{21}^{total} + TM_{22}^{total}} \]

\[ \tau = \frac{2e^{j \frac{\omega c}{\rho} \cos \theta L}}{TM_{11}^{total} + TM_{12}^{total} (\cos \theta / \rho c) + (\rho c / \cos \theta) TM_{21}^{total} + TM_{22}^{total}} \]

\[ \bar{\alpha}_d = \frac{\int_0^{\pi/2} \alpha_d(\theta) \sin(\theta) \cos(\theta) \, d\theta}{\int_0^{\pi/2} \sin(\theta) \cos(\theta) \, d\theta} \quad \bar{\tau} = \frac{\int_0^{\pi/2} \tau(\theta) \sin(\theta) \cos(\theta) \, d\theta}{\int_0^{\pi/2} \sin(\theta) \cos(\theta) \, d\theta} \]
Transfer Matrix Method

\[ [TM]_{total} = [TM]_{mpp1}[TM]_{air1}[TM]_{mpp2}[TM]_{air2} \cdots [TM]_{mppN} \]

\[
[TM]_{air} = \begin{bmatrix}
\cos (\omega l/c) & j\rho csin(\omega l/c) \\
(j/\rho c)sin (\omega l)/c & \cos (\omega l/c)
\end{bmatrix}
\]

\[
[TM]_{mpp} = \begin{bmatrix}
1 & Z_{mpp} \\
0 & 1
\end{bmatrix}
\]
Microperforated panel

- Guo Model

\[ R = \left( \text{Re} \left\{ \frac{j \omega t}{\sigma c} \left[ 1 - \frac{2}{k \sqrt{-j} J_1(k \sqrt{-j})} \right]^{-1} \right\} + \frac{\alpha 2R_s}{\sigma \rho c} \right) \times \rho c \]

\[ k = \frac{\sqrt{\omega \rho_0}}{4\eta} \quad R_s = \frac{\sqrt{2\omega \rho_0 \eta}}{2} \quad \alpha = 4 \quad \text{when sharp end} \]

- Previous work
  - adjusted \( \alpha \) by CFD calculation

\[ \alpha = (16.9 \frac{t}{d} + 152.8) f^{-0.5} \]
Microperforated panel

• Continuity and Force equilibrium

\[ v_y = (1 - \sigma)v_s + \sigma v_f \]

\[ P_1 - P_2 + (v_f - v_s)R \frac{\sigma^2}{1 - \sigma} = j\omega m v_s \]

\[ P_1 - P_2 + (v_f - v_s)R\sigma = \rho h_p j\omega v_f \]

\[ Z_{mpp} = \frac{R\sigma(1 - \sigma)(j\omega m - j\omega \rho(t + 2\delta)) + j\omega \rho(t + 2\delta)\{j\omega m(1 - \sigma) + R\sigma\}}{\sigma(1 - \sigma)(R + j\omega m) + (1 - \sigma)^2 \rho(t + 2\delta)j\omega + \sigma^2 R} \]
Optimization

• Assumption
  • Hole of the MPP is cylindrical and sharp edged.
  • Flexural stiffness of the panel can be ignored.
  • Only locally reaction case considered.
Optimization

• Constraints

<table>
<thead>
<tr>
<th></th>
<th>Minimum</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N$</td>
<td>2</td>
<td>20</td>
</tr>
<tr>
<td>$t$ [mm]</td>
<td>0.2</td>
<td>0.8</td>
</tr>
<tr>
<td>$d$ [mm]</td>
<td>0.1</td>
<td>0.3</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>0.01</td>
<td>0.2</td>
</tr>
<tr>
<td>$m$ [kg/m$^2$]</td>
<td>0.1</td>
<td>0.8</td>
</tr>
<tr>
<td>$l$ [m]</td>
<td>0.001</td>
<td>0.2</td>
</tr>
<tr>
<td>$M$ [kg/m$^2$]</td>
<td></td>
<td>3</td>
</tr>
<tr>
<td>$L$ [m]</td>
<td></td>
<td>0.5</td>
</tr>
</tbody>
</table>

• Genetic Algorithm was used for optimization
Optimization for absorption

- The number of panels

- For the error function, $1-\alpha_d$ was used in 500 to 4000 Hz.
Optimization for absorption

- Both direction
- Maximize averaged dissipation coefficient
Optimization for absorption

<table>
<thead>
<tr>
<th>Panel</th>
<th>Thickness [mm]</th>
<th>Diameter [mm]</th>
<th>Porosity</th>
<th>Mass per unit area [kg/m²]</th>
<th>Distance to next panel [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.222</td>
<td>0.100</td>
<td>0.137</td>
<td>0.714</td>
<td>0.0001</td>
</tr>
<tr>
<td>2</td>
<td>0.200</td>
<td>0.300</td>
<td>0.089</td>
<td>0.100</td>
<td>0.0001</td>
</tr>
<tr>
<td>3</td>
<td>0.200</td>
<td>0.300</td>
<td>0.137</td>
<td>0.100</td>
<td>0.100</td>
</tr>
<tr>
<td>4</td>
<td>0.200</td>
<td>0.100</td>
<td>0.012</td>
<td>0.100</td>
<td>0.123</td>
</tr>
<tr>
<td>5</td>
<td>0.202</td>
<td>0.300</td>
<td>0.137</td>
<td>0.100</td>
<td>0.002</td>
</tr>
<tr>
<td>6</td>
<td>0.251</td>
<td>0.100</td>
<td>0.075</td>
<td>0.719</td>
<td>0.017</td>
</tr>
<tr>
<td>7</td>
<td>0.224</td>
<td>0.100</td>
<td>0.084</td>
<td>0.244</td>
<td>0.005</td>
</tr>
<tr>
<td>8</td>
<td>0.200</td>
<td>0.300</td>
<td>0.073</td>
<td>0.101</td>
<td>0.123</td>
</tr>
<tr>
<td>9</td>
<td>0.251</td>
<td>0.300</td>
<td>0.137</td>
<td>0.719</td>
<td>0.123</td>
</tr>
<tr>
<td>10</td>
<td>0.205</td>
<td>0.300</td>
<td>0.073</td>
<td>0.101</td>
<td>-</td>
</tr>
</tbody>
</table>
Optimization for absorption

- To compare optimized set, maximum resistance set was used.

- Maximum resistance set has same distance between panels, and same panel to make flow resistance maximum.

- The optimized set provides a much higher sound dissipation coefficient in the overall speech interference range.
Optimization for Transmission loss

- One direction
- Maximize transmission loss
## Optimization for Transmission loss

<table>
<thead>
<tr>
<th>Panel</th>
<th>Thickness [mm]</th>
<th>Diameter [mm]</th>
<th>Porosity</th>
<th>Mass per unit area [kg/m²]</th>
<th>Distance to next panel [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Panel 1</td>
<td>0.200</td>
<td>0.101</td>
<td>0.073</td>
<td>0.100</td>
<td>0.123</td>
</tr>
<tr>
<td>Panel 2</td>
<td>0.200</td>
<td>0.100</td>
<td>0.200</td>
<td>0.693</td>
<td>0.001</td>
</tr>
<tr>
<td>Panel 3</td>
<td>0.200</td>
<td>0.104</td>
<td>0.026</td>
<td>0.100</td>
<td>0.001</td>
</tr>
<tr>
<td>Panel 4</td>
<td>0.200</td>
<td>0.100</td>
<td>0.138</td>
<td>0.602</td>
<td>0.123</td>
</tr>
<tr>
<td>Panel 5</td>
<td>0.200</td>
<td>0.178</td>
<td>0.010</td>
<td>0.693</td>
<td>0.001</td>
</tr>
<tr>
<td>Panel 6</td>
<td>0.200</td>
<td>0.100</td>
<td>0.010</td>
<td>0.100</td>
<td>0.123</td>
</tr>
<tr>
<td>Panel 7</td>
<td>0.200</td>
<td>0.100</td>
<td>0.020</td>
<td>0.100</td>
<td>0.001</td>
</tr>
<tr>
<td>Panel 8</td>
<td>0.200</td>
<td>0.100</td>
<td>0.138</td>
<td>0.100</td>
<td>0.123</td>
</tr>
<tr>
<td>Panel 9</td>
<td>0.200</td>
<td>0.100</td>
<td>0.088</td>
<td>0.100</td>
<td>0.001</td>
</tr>
<tr>
<td>Panel 10</td>
<td>0.200</td>
<td>0.101</td>
<td>0.138</td>
<td>0.411</td>
<td>-</td>
</tr>
</tbody>
</table>
Optimization for Transmission loss

- To compare optimized set, maximum resistance set was used.

- Maximum resistance set has same distance between panels, and same panel to make flow resistance maximum.

- Improved in low frequency range, but need to make smooth
Conclusion

• An appropriate combination of microperforated panels can provide excellent performance for sound absorption.

• This suggests that a layered array of MPPs proper could be used to provide dissipation in a space

• Future work:
  • Make smooth for TL
  • Extend to the extended reaction case