A Discrete Implementation of a Hankel Transform Technique for Predicting Multipole Sound Propagation over Plane Absorbing Surfaces

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A DISCRETE IMPLEMENTATION OF A
HANKEL TRANSFORM TECHNIQUE FOR PREDICTING
MULTIPOLE SOUND PROPAGATION OVER
PLANE ABSORBING SURFACES

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• **OBJECTIVE:**
  
  — accurate prediction of sound propagation from a higher order source over outdoor surfaces.

• **REQUIREMENTS:**
  
  — measurement of the reflection coefficients
  
  — appropriate sound propagation theory
INTRODUCTION

PREDICTION OF SOUND PROPAGATION

• PREVIOUS TECHNIQUES:
  — asymptotic approximation, limitation: accurate in a certain domain \((k_r r_2 > > 1 \text{ or } k_r r_2 < < 1)\)
  — one-dimensional Hankel transform technique for a monopole source

• PRESENT APPROACH:
  — apply the two-dimensional inverse Hankel transform

• RESULT:
  — theory capable of predicting multipole sound propagation over a finite impedance plane

• APPLICATION:
  — prediction of sound propagation from aerodynamic noise sources over outdoor surfaces
THEORY

- WAVE EQUATION
  - scalar Helmholtz equation
    \[ \nabla^2 p(r, \omega) + k^2 p(r, \omega) = 0 \]
    note: time dependence of \( e^{-i\omega t} \) is assumed here.
  - the sound pressure in cylindrical coordinates
    \[ p(r, \phi, z) = \sum_{n=0}^{\infty} \left[ Q_{1n}(r, z) \cos n\phi + Q_{2n}(r, z) \sin n\phi \right] \]
THEORY

- REPRESENTATION OF A SOUND FIELD BY USING TWO-DIMENSIONAL HANKEL TRANSFORM TECHNIQUE

  - the wavenumber spectrum in Cartesian coordinates:

    \[ \tilde{P}(k_x, k_y, z) = \frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} P(x, y, z) e^{-i(k_x x + k_y y)} \, dx \, dy \]

  - the wavenumber spectrum in cylindrical coordinates:

    \[ \tilde{P}(k_r, \psi, z) = \frac{1}{(2\pi)^2} \int_{0}^{\infty} \int_{0}^{2\pi} rP(r, \phi, z) e^{-ik_r r \cos(\phi - \psi)} \, d\phi \, dr \]

    where \( k_r \) is the radial wavenumber and \( \psi \) is the azimuth angle in the wavenumber domain.
— forward 2-D Hankel transform:

\[
\tilde{P}(k_r, \psi, z) = \sum_{n=0}^{\infty} \frac{(-i)^n}{2\pi} \left[ \tilde{Q}_1 n(k_r, z) \cos n\psi + \tilde{Q}_2 n(k_r, z) \sin n\psi \right]
\]

where

\[
\tilde{Q}_l n(k_r, z) = \int_0^\infty r Q_l n(r, z) J_n(k_r r) dk_r
\]

\[
l = 1, 2; \ n = 0, 1, 2, ...
\]

is the Hankel transform of nth-order and \( J_n \) is the Bessel function of order \( n \).

— inverse 2-D Hankel transform:

\[
P(r, \phi, z) = \sum_{n=0}^{\infty} \left[ Q_1 n(r, z) \cos n\phi + Q_2 n(r, z) \sin n\phi \right]
\]

where

\[
Q_l n(r, z) = \int_0^\infty k_r \tilde{Q}_l n(k_r, z) J_n(k_r r) dk_r
\]

\[
l = 1, 2; \ n = 0, 1, 2, ...
\]

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The Incident and Reflected Sound Fields

\[ \tilde{P}_r(k_r, \psi, 0) = R(k_r, \omega) \tilde{P}_i(k_r, \psi, 0) \]

- the wavenumber spectrum of the reflected pressure fields at \( z = 0 \)

\[ R(k_r, \omega) = \frac{z_n - \omega \rho/k_z}{z_n + \omega \rho/k_z} \]

- in which the plane wave reflection coefficient, \( R(k_r, \omega) \), may be calculated using

\[ \tilde{P}_r(k_r, \psi, z) = \tilde{P}_r(k_r, \psi, 0)e^{ik_z z} \]

- the wavenumber spectrum of the reflected field at any \( z \)-plane

\[ \tilde{P}(k_r, \psi, z) = \tilde{P}_i(k_r, \psi, z) + \tilde{P}_r(k_r, \psi, z) \]

- the total pressure in the wavenumber domain
PREDICTION OF SOUND FIELD

Know: Sources and Reflecting Surface

Objective: Prediction the Sound Field over a Finite Impedance Surface

Methodology:

- find $\tilde{P}_i(k_r, \psi, z)$ either analytically or numerically

- decompose $\tilde{P}_i(k_r, \psi, z)$ into $\tilde{Q}_{ln}(k_r, z)$’s

- multiply $\tilde{Q}_{ln}(k_r, z)$’s with $e^{i2k_zz}R_p(k_r)$ to obtain $\tilde{Q}_{ln}(k_r, z)$’s

- add $\tilde{Q}_{ln}(k_r, z)$’s and $\tilde{Q}_{ln}(k_r, z)$’s to give $\tilde{Q}_{ln}(k_r, z)$’s

- inverse transform $\tilde{Q}_{ln}(k_r, z)$’s to give $Q_{ln}(r, z)$’s

- synthesize $Q_{ln}(r, z)$’s to give $P(r, \psi, z)$

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FAST HANKEL TRANSFORM

Summary:

— FHT is not as accurate as DHT but is faster with the same numerical parameters

— performance ($|E|^2 \times T_{cpu}$) of the FHT is generally superior to that of the DHT

— most efficient to compute higher order azimuthal harmonic components (shifted sources)

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THEORY

THE SOUND FIELD OF A MONOPOLE

— the direct sound pressure from the monopole in the space domain

\[ P_i(r, \phi, z) = \frac{A_m}{R} e^{ikR} \]

— the wavenumber domain spectrum of the monopole is

\[ \tilde{P}_i(k_r, \psi, z) = \frac{A_m}{2\pi k_z} e^{ik_z(z_s - z)} \]

where \( k_z = \sqrt{k^2 - k_r^2} \)
THE SOUND FIELD OF AN ARBITRARILY ORIENTED DIPOLE

the direct sound pressure from the inclined dipole in the space domain

\[ P_i(r, \phi, z) = A_d \frac{(1 - ikR)}{R^3} e^{ikR} \left[ (z_s - z) \cos \gamma + r \sin \gamma \cos (\phi - \phi_0) \right] \]

the wavenumber domain spectrum of the inclined dipole

\[ \tilde{P}_i(k_r, \psi, z) = \frac{A_d}{(2\pi k_z)} \left[ k_z \cos \gamma + k_r \sin \gamma \cos (\psi - \phi_0) \right] e^{ik_z(z_s - z)} \]
MODEL OF SURFACE

- **assumption:**
  the test carpet is a finite-depth layer of an extended reaction Delany and Bazley-type material

- **the characteristic impedance** $z_{o1}$
  
  \[ z_{o1} = \rho c [1 + 0.0571(\rho f/\sigma)^{-0.754} + i0.087(\rho f/\sigma)^{-0.732}] \]
  
  *where* $\sigma$ *is the flow resistivity of the material in MKS Rayls/m*

- **the complex wavenumber** $k_1$
  
  \[ k_1 = k [1 + 0.0978(\rho f/\sigma)^{-0.70} + i0.189(\rho f/\sigma)^{-0.595}] \]

- **the surface normal impedance**
  
  \[ z_n = \frac{iz_{o1}}{\cos\theta_1} \cot(k_1 l \cos\theta_1) \]
  
  *where* $l$ *is the layer depth and* $\cos\theta_1 = \sqrt{1 - (k/k_1)^2 \sin^2 \theta}$
EXAMPLE: VERTICAL DIPOLE

\[ 20 \log_{10} \left[ \frac{P_{d1}}{P_{d1}} + P_{dr} \right] \text{ [dB]} \]

\[ f = 4000 \text{ Hz} \quad z_s = 0.1 \text{ m} \quad z = 0.025 \text{ m} \]

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EXAMPLE: INCLINED DIPOLE

\[ f = 4000 \text{ Hz} \quad \gamma_d = 30^\circ \quad \phi_d = 90^\circ \quad z_s = 0.1 \text{ m} \quad z = 0.025 \text{ m} \]

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PROPAGATION MEASUREMENTS

Geometry

1.2 m window size
0.002m
0.1m
0.01m spatial sample interval
0.025m
0.012m
Two measuring lines

Instrumentation

Power Amplifier

Schroeder-phased
Signal
D/A

MASSCOMP
Computer

A/D
Data Acquisition,
Signal Enhancement
and FFT

Unbaffled Loudspeaker

1/4" Microphone
B&K 4125
Preamplifier B&K 2642

Amplifier
B&K 2810

WaveTek Dual Hi/Lo
Filter 852

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PROPAGATION MEASUREMENTS

$f = 1953 \text{ Hz, } z = 0.025 \text{ m}$

![Graph showing sound pressure level vs. r for $f = 1953 \text{ Hz, } z = 0.025 \text{ m}$]

$f = 1953 \text{ Hz, } z = 0.012 \text{ m}$

![Graph showing sound pressure level vs. r for $f = 1953 \text{ Hz, } z = 0.012 \text{ m}$]
PROPAGATION MEASUREMENTS

Optimal Monopole Source Strength

\[ 20 \log_{10}(A_m/A_d) \text{ [dB]} \]

\[ 2(\angle(A_m/A_d)) \text{ [degrees]} \]

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PROPAGATION MEASUREMENTS

\[ f = 3613 \text{ Hz}, \ z = 0.025 \text{ m} \]

\[ f = 3613 \text{ Hz}, \ z = 0.012 \text{ m} \]

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PROPAGATION MEASUREMENTS

\[ z = 0.012 \text{ m} \]

**Measurement**

**Theory**

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PROPAGATION MEASUREMENTS

\[ z = 0.025 \, \text{m} \]

Measurement

Theory

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CONCLUSIONS

- The two-dimensional Discrete Hankel Transform approach presented here offers an effective means for predicting the sound radiation from multipole sources over plane absorbing surfaces.

- The approach may be extended to arbitrarily directional sources if those sources can be modeled as a superposition of multipoles.

- The performance of the two-dimensional Discrete Hankel Transform prediction procedure can be enhanced by the use of a Fast Hankel Transform algorithm.

- The procedure outlined here must be supplemented by a farfield prediction to allow predictions at arbitrarily large distances from the source.

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