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THE SPATIAL AND TEMPORAL RESPONSES
OF FLUID-LOADED, LINE-DRIVEN PANEL STRUCTURES

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LITERATURE REVIEW

- FLUID-LOADED PLATES


- Procedure:
  1. Solve problem in the wavenumber-frequency domain
     \[ \tilde{V}_f(\gamma, \omega) = 1/(Z_a + Z_p) \]
  2. Inverse Transform to obtain spatial response
     \[ \tilde{V}_f(x, \omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \tilde{V}_f(\gamma, \omega) e^{i\gamma x} d\gamma \]
  3. Evaluate transform by: contour integration, numerical integration, or combination of the two

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PRESENT APPROACH

- Arbitrarily complicated layered structures:
  - not amenable to analytical approach

- Evaluate inversion integral numerically:
  - along steepest descent path, branch cuts, or real axis?

- Evaluate integral directly along real axis:
  - add damping to avoid singularities
  - perform numerical integration by using Fast Fourier Transform algorithm to evaluate the Inverse Discrete Fourier Transform

- Represent acoustic loading through surface normal impedance:
  - evaluate using simple plane wave theory

- Result:
  - efficient method for calculating response of damped structures
  - useful for design exercises, especially treatments involving porous layers

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PROBLEM CONSIDERED

Transverse Displacement of Line-Excited Panels Loaded by "Acoustic" Media: i.e., through a normal stress

i. Semi-Infinite Fluid:

ii. Layered Structure:

Note: Acoustical Medium may be fluid or limp porous material such as fiber glass

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iii Semi-Infinite Fluid-Loading With Line Constraints:
GOVERNING EQUATIONS - ACOUSTIC LOADING

- **Euler-Bernoulli Panel:**
  
  \[
  D \frac{\partial^4 w(x,t)}{\partial x^4} + m_s \frac{\partial^2 w(x,t)}{\partial t^2} = -p(x,0,t) + f(t) \delta(x)
  \]

- **Spatial and Temporal Transform:**
  
  \[(D\gamma^4 - \omega^2 m_s)\tilde{W}(\gamma, \omega) = \tilde{P}(\gamma, 0, \omega) + F\]

  or equivalently:

  \[\tilde{V}_f = \frac{1}{Z_a + Z_m}\]

  \[\tilde{V}(\gamma, \omega): \text{transformed panel velocity } (-i\omega\tilde{W}(\gamma, \omega))\]

  \[\tilde{V}_f(\gamma, \omega): \text{normalized panel velocity } (\tilde{V}(\gamma, \omega)/F)\]

  \[Z_a: \text{input impedance of acoustic space } (\tilde{P}(\gamma, 0, \omega)/\tilde{V}(\gamma, \omega))\]

  \[Z_m: \text{impedance of panel } (i[(D/\omega)\gamma^4 - \omega m_s])\]

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**IMPEDANCE CALCULATIONS**
*(Found by stepping through layers)*

1. **Across a panel**
   \[ Z_{in}^{n(-)} = Z_{in}^{n(+)} + Z_{m}^{n} \]

2. **Across a layer**
   \[ Z_{in}^{n-1(+) - Z_{cn}^{n-1}} = \frac{(Z_{in}^{n(-)} - iZ_{in}^{n(-)}\tan\Phi_{n-1})}{(Z_{cn}^{n-1} - iZ_{in}^{n(-)}\tan\Phi_{n-1})} \]
   **where:** \( \Phi_{n-1} = k_{(n-1)z}d_{n-1} \),

**VELOCITY OF THE EXTERIOR PANEL**

\[ \tilde{V}_{fN}(\gamma, \omega) = \tilde{V}_f(\gamma, \omega) \prod_{n=1}^{N-1} \frac{2 e^{i\Phi_n}}{\zeta_n + 1 + e^{i2\Phi_n}(1 - \zeta_n)} \]

**where:** \( \zeta_n = (Z_{m}^{n+1} + Z_{in}^{n+1(+)})/Z_{cn}^{n} \)
**GOVERNING EQUATIONS - LINE CONSTRAINTS**

- **Euler-Bernoulli Panel with Line Constraints:**

\[
\begin{align*}
D \frac{\partial^4 w(x,t)}{\partial x^4} + m_s \frac{\partial^2 w(x,t)}{\partial t^2} &= -p(x,0,t) + f(t) \delta(x) + F_l \delta(x-x_l) + M_l \delta'(x-x_l) \\
F_l &= -m_l \frac{\partial^2 w}{\partial t^2} \\
M_l &= -J_l \frac{\partial^2 \Theta}{\partial t^2} \\
\Theta &= \frac{\partial w}{\partial x}
\end{align*}
\]

- **mass per unit length of the constraint**
- **rotary inertia per unit length of the constraint**

- **Spatial and Temporal Transform:**

\[
\tilde{V}(\gamma, \omega) = [F(\omega) + (i \omega m_l V(x_l, \omega) - \omega \gamma J_l \dot{\Theta}(x_l, \omega)) e^{-i\gamma x_l}] / (Z_a + Z_m)
\]

- **velocity at the constraint**
- **angular velocity at the constraint**

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SOLUTION AT THE LINE CONSTRAINT

- **General Solution:**

\[ V(x, \omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \tilde{V}(\gamma, \omega) e^{i\gamma x} d\gamma \]

- **Solution at the Constraint for a Specified Frequency:**

\[ F(\omega) \int_{-\infty}^{\infty} e^{i\gamma x}/(Z_m+Z_a) d\gamma \]

\[ V(x_1, \omega) = \frac{2\pi - i\omega m_1 \int_{-\infty}^{\infty} \gamma/(Z_m+Z_a) d\gamma}{iF(\omega) \int_{-\infty}^{\infty} \gamma e^{i\gamma x}/(Z_m+Z_a) d\gamma} \]

\[ \dot{\Theta}(x_1, \omega) = \frac{2\pi + i\omega J_1 \int_{-\infty}^{\infty} \gamma^2/(Z_m+Z_a) d\gamma}{iF(\omega) \int_{-\infty}^{\infty} \gamma e^{i\gamma x}/(Z_m+Z_a) d\gamma} \]

**Note:** The response at the constraints must be found before the response at any arbitrary position can be evaluated.
**SOUND RADIATION**

- **Given:** $\tilde{V}_{f_N}(k_x, l, \omega)$

- **Calculate:** $P(k_x, l, \omega) = Z_{cn}^N \tilde{V}_{f_N}(k_x, l, \omega)$

- **Surface Normal Intensity:**
  \[
  I_n(x, \omega) = \frac{1}{2} \text{Re} \left\{ P(x, l, \omega)V_{f_N}^*(x, l, \omega) \right\}
  \]

- **Sound Power per Unit Width:**
  \[
  \bar{P}(\omega) = \frac{1}{4\pi} \int_{-k}^{k} Z_{cn}^N |\tilde{V}_{f_N}(k_x, \omega)|^2 dk_x
  \]

- **Also: Line Input Impedance of Panel 1:**
  \[
  Z_{1_i}(\omega) = \frac{1}{V_{f_1}}(0, \omega)
  \]
DISCRETIZATION - SPACE-FREQUENCY

- **CONTINUOUS**

\[
V_f(x, \omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \tilde{V}_f(\gamma, \omega) e^{i\omega x} d\gamma
\]

- **DISCRETE**

\[
V_f(k\Delta x, \omega) = \frac{1}{N_\gamma \Delta x} \sum_{l=0}^{N_\gamma-1} \tilde{V}_f(l\Delta \gamma, \omega) e^{i2\pi kl/N_\gamma}
\]

\[= \frac{1}{\Delta x} \text{IDFT} \left[ \tilde{V}_f(l\Delta \gamma, \omega) \right] \]

where: \( \Delta x = 2\pi/\gamma_s, \gamma_s = \text{spatial sampling frequency}, \)
\( \Delta \gamma = \gamma_s/N_\gamma = \text{spatial frequency resolution}, \)
\( N_\gamma = \text{transform length} \)

**Note:** IDFT can be evaluated using FAST FOURIER TRANSFORM algorithm
DISCRETIZATION - SPACE-TIME

- **CONTINUOUS**

\[ V_f(x, t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \tilde{V}_f(x, \omega)e^{-i\omega t} d\omega \]

- **DISCRETE**

\[ V_f(x_o, m\Delta t) = \frac{1}{N_t\Delta t} \sum_{n=0}^{N_t-1} \tilde{V}_f(x_o, n\Delta\omega)e^{-i2\pi mn/N_t} \]

\[ = \frac{1}{\Delta t} DFT \left[ \tilde{V}_f(x_o, n\Delta\omega) \right] \]

**where:** \( \Delta t = 2\pi/\omega_s, \) \( \omega_s = \text{temporal sampling frequency}, \)

\( \Delta\omega = \omega_s/N_t = \text{temporal frequency resolution}, \)

\( N_t = \text{transform length} \)

**Note:** DFT can be evaluated using **FAST FOURIER TRANSFORM** algorithm.
i. Spatial Truncation:
spatial response must decrease to small values
within spatial record length to ensure that wavenumber spectrum is sampled sufficiently quickly

ii. Spectral Truncation:
wavenumber spectrum must decrease to small values
within spectral record length to ensure that spatial response is sampled sufficiently quickly
FLUID-LOADED PLATE

- Fiet and Liu (JASA 78, 763-766)

- Present Approach
EXAMPLES PRESENTED

**EXAMPLE 1:**

1. Steel panel (12.7 mm) loaded by an acoustical half-space of water
2. Consider the velocity response at various distances from the force
3. Demonstrates radiation damping effects.

**EXAMPLE 2:**

1. Aluminum panels (0.762 mm and 0.254 mm) loaded by an acoustical half-space of air
2. Consider the pressures response 1.5 m from the force
3. Demonstrates the structure-borne and fluid-borne components of the signal
**EXAMPLE 3:**

1. Aluminum panel (0.762 mm) backed by a confined layer of air (0.0762m and 0.5 m)
2. Consider velocity and pressure responses at the surface of the panel
3. Demonstrates the reflections of the pressure field within the confined layer

**EXAMPLE 4:**

1. Aluminum panel (1.5 mm) in an acoustical half-space of air with a line constraint located 1.0 m from the force (constraint is 20.0 mm by 5 mm block of aluminum).
2. Consider the velocity, pressure and intensity on both sides of the constraint (responses found at 0.25 m and 1.75 m from the force).
3. Demonstrates the effect of the constraint on the transmitted structural response and the radiated acoustic field
EXAMPLE 1 - VELOCITY RESPONSE

Steel Panel - 12.7 mm
Fluid - water

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EXAMPLE 2 - PRESSURE RESPONSE

Aluminum Panel
Fluid - air

(a) Pressure vs. Time
(b) Frequency Response
(c) Pressure vs. Time
(d) Frequency Response

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EXAMPLE 3 - VELOCITY RESPONSE

Aluminum panel - 0.762 mm
Fluid - air
Response at $x = 0.5$ m

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EXAMPLE 3 - PRESSURE RESPONSE

Aluminum panel - 0.762 mm
Fluid - air
Response at $x = 0.5$ m

$l = 7.62$ mm

$l = 500.\text{ mm}$

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EXAMPLE 4 - VELOCITY RESPONSE

Aluminum panel - 1.5 mm thick
Fluid - air
Constraint at 1.0 m, 2.0 mm x 0.5 mm block
x1 = 0.25 m
x2 = 1.75 m

Response at x1

Response at x2
EXAMPLE 4 - INTENSITY RESPONSE

Aluminum panel - 1.5 mm thick
Fluid - air
Constraint at 1.0 m
Constraint - 2.0 mm x 0.5 mm beam
x1 = 0.25 m
x2 = 1.75 m

Response at x1

Response at x2
EXAMPLE 4 - REAL PART OF INTENSITY

Aluminum panel - 1.5 mm thick
Fluid - air
Constraint at 1.0 m
Constraint - 2.0 mm x 0.5 mm beam
EXAMPLE 4 - MAGNITUDE OF INTENSITY

Aluminum panel - 1.5 mm thick
Fluid - air
Constraint at 1.0 m
Constraint - 2.0 mm x 0.5 mm beam