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Effect of Rotation on the Natural Frequencies of Coupled Tire Structural-Acoustical Mode

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Effect of rotation on the natural frequencies of coupled tire structural-acoustical mode

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Advisor: J. Stuart Bolton

Aug 23rd–26th 2013
Tire-road noise can be really annoying!
Objective

Fully coupled model

Spinning tire

Frequency-split phenomenon
Previous work

Thompson (1995)

Sakata, Morimura (1990)

Kim, Bolton (2003)

Gunda, Gau, Dohrmann (2000)
Fully coupled tire-cavity case

- **Inviscid and incompressible**
- **2-dim**
- **Rotate about a fixed axis**
- **Tire-structure and air cavity move uniformly**
- **No energy radiates in radial direction**
- **Structural mode**
- **Plane mode**
- **higher modes**
Model description

<table>
<thead>
<tr>
<th>Configuration</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density</td>
<td>$\rho = 1200 \text{ kg/m}^3$</td>
</tr>
<tr>
<td>Radius</td>
<td>$R = 0.30 \text{ m}$</td>
</tr>
<tr>
<td>Inflation pressure</td>
<td>$P = 10300 \text{ Pa}$</td>
</tr>
<tr>
<td>Tension</td>
<td>$T = 1.03 \times 10^4 \text{ N/m}$</td>
</tr>
<tr>
<td>Channel height</td>
<td>$H = 0.1 \text{ m}$</td>
</tr>
</tbody>
</table>
Some dynamics

Linearized acoustic wave equation with uniform, inviscid flow is

$$\frac{\partial^2 p}{\partial t^2} = (c^2 - v^2) \frac{\partial^2 p}{\partial x^2} + c^2 \frac{\partial^2 p}{\partial y^2} - 2v \frac{\partial^2 p}{\partial x \partial t}$$

- $p$ - Sound pressure disturbance
- $c$ - Adiabatic speed of sound
- $v$ - Flow speed

While wave equation for a tensioned, moving string is

$$\rho_L \left( \frac{\partial^2 \xi}{\partial t^2} + 2v \frac{\partial^2 \xi}{\partial t \partial x} \right) + (\rho_L v^2 - T) \frac{d^2 \xi}{dx^2} = 0$$

- $\xi$ - Displacement disturbance in string
- $\rho_L$ - String density
- $T$ - Tension in string
1) The lower boundary

2) The upper boundary

i. Impermeability condition
\[ v_y = 0 \]

ii. Velocity matching condition
\[ v_y = \frac{\partial \xi}{\partial t} \]

iii. Force balance
\[ \sum f = p \]

iv. Periodic condition
End point 1 and 2 are the same.

Flow speed equals string translating speed

\[ k_x = k_s \]
Solving the model

Assume harmonic solutions for both air cavity and string:

\[ p = (A e^{-jk_y y} + B e^{+jk_y y}) e^{-jk_x x} e^{j\omega t} \]

\[ \xi = C e^{-jk_s x} e^{j\omega t} \]

Leads to characteristic equation:

\[ \omega^2 - 2v\omega k_x + (v^2 - \frac{T}{\rho_L}) k_x^2 = \frac{\rho_o \omega^2}{\rho_L k_y \tan(k_y H)} \]

The dispersion equation is no longer

\[ \frac{\omega^2}{c^2} = k^2 = k_x^2 + k_y^2 \quad \text{but} \quad \frac{\omega^2}{c^2} = (1 - \left(\frac{v}{c}\right)^2) k_x^2 + k_y^2 - 2 \frac{v}{c} \frac{\omega}{c} k_x \]

Solutions are close to singularities! Difficult to solve
Results

- Wave number plots
- Phase velocity plots
- Pressure distribution plots
- Natural frequencies
Structure mode

**Static and spinning case**

**Phase speed**

Graph showing Frequency [Hz] against Kx, with labels for real and imaginary components.
Spinning case, $v = 50 \text{ m/s}$

Phase speed
Structure mode

**Spinning case, \( v = 50 \text{ m/s} \)**

- **Phase speed**
  - Sound speed
  - String phase speed (in vacuo)

- **Frequency [Hz]**
  - **Real**
  - **Imaginary**

- **Phase speed [m/s]**
  - **Frequency [Hz]**
Spinning case, $v = 50 \text{ m/s}$

Pressure distribution (+)

Pressure distribution (-)
First acoustical mode

Static and spinning case

Phase speed

**Frequency [Hz]**

**Kx**

- **Real**
- **Imaginary**
First acoustical mode

Spinning case, \( v = 50 \text{ m/s} \)

<table>
<thead>
<tr>
<th>Frequency [Hz]</th>
<th>( K_x )</th>
<th>( \text{real} )</th>
<th>( \text{imaginary} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-40</td>
<td>real</td>
<td>imaginary</td>
</tr>
<tr>
<td>40</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Phase speed

Sound speed

Frequency [Hz]
First acoustical mode

Spinning case, $v = 50 \text{ m/s}$

<table>
<thead>
<tr>
<th>Frequency [Hz]</th>
<th>$k_x$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2000</td>
<td></td>
</tr>
<tr>
<td>1800</td>
<td></td>
</tr>
<tr>
<td>1600</td>
<td></td>
</tr>
<tr>
<td>1400</td>
<td></td>
</tr>
<tr>
<td>1200</td>
<td></td>
</tr>
<tr>
<td>1000</td>
<td></td>
</tr>
<tr>
<td>800</td>
<td></td>
</tr>
<tr>
<td>600</td>
<td></td>
</tr>
<tr>
<td>400</td>
<td></td>
</tr>
<tr>
<td>200</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

Phase speed

Sound speed

Real

Imaginary
Spinning case, \( v = 50 \text{ m/s} \)
Second acoustical mode

Static and spinning case

Phase speed

![Graph showing frequency vs. Kx with real and imaginary components marked]
Second acoustical mode

**Spinning case, \( v = 50 \text{ m/s} \)**

- **Frequency [Hz]**
- **Kx**
- **Phase speed**

![Graph showing phase speed](image)
Second acoustical mode

**Spinning case, \( v = 50 \text{ m/s} \)**

**Phase speed**

![Graph showing frequency and phase speed](graph.png)

- **Frequency [Hz]**
  - **Kx**
    - **Real**
    - **Imaginary**

- **Phase speed [m/s]**
  - **Sound speed**
  - **Phase speed**

- **Graphs**
  - Frequency vs. Kx
  - Phase speed vs. Frequency
Spinning case, $v = 50 \text{ m/s}$

Pressure distribution (+) | Pressure distribution (-)
### Third acoustical mode

<table>
<thead>
<tr>
<th>Frequency [Hz]</th>
<th>Kx</th>
<th>Real</th>
<th>Imaginary</th>
</tr>
</thead>
<tbody>
<tr>
<td>6000</td>
<td>0</td>
<td>50</td>
<td>0</td>
</tr>
<tr>
<td>5500</td>
<td>50</td>
<td>45</td>
<td>5</td>
</tr>
<tr>
<td>5000</td>
<td>100</td>
<td>40</td>
<td>10</td>
</tr>
<tr>
<td>4500</td>
<td>-50</td>
<td>35</td>
<td>-5</td>
</tr>
<tr>
<td>4000</td>
<td>0</td>
<td>30</td>
<td>0</td>
</tr>
<tr>
<td>3500</td>
<td>-50</td>
<td>25</td>
<td>-5</td>
</tr>
<tr>
<td>3000</td>
<td>100</td>
<td>20</td>
<td>10</td>
</tr>
<tr>
<td>2500</td>
<td>0</td>
<td>15</td>
<td>0</td>
</tr>
<tr>
<td>2000</td>
<td>-50</td>
<td>10</td>
<td>-5</td>
</tr>
<tr>
<td>1500</td>
<td>50</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>1000</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>500</td>
<td>-50</td>
<td>-5</td>
<td>-5</td>
</tr>
<tr>
<td>0</td>
<td>50</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

**Phase speed**

**Static and spinning case**
Third acoustical mode

**Spinning case with v = 50 m/s**

**Phase speed**

- **Frequency [Hz]**
- **Kx**
- **real**
- **imaginary**
Third acoustical mode

**Spinning case with \( v = 50 \text{ m/s} \)**

**Phase speed**

- **Frequency [Hz]**
- **Real**
- **Imaginary**
- **Sound speed**
- **Phase speed**
Spinning case, \( v = 50 \text{ m/s} \)

Pressure distribution (+)  
Pressure distribution (-)
### Natural frequencies when ν = 50 m/s

<table>
<thead>
<tr>
<th>Structure mode</th>
<th>Frequency (+) [Hz]</th>
<th>Frequency (-) [Hz]</th>
<th>Frequency (static) [Hz]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mode 1</td>
<td>299.80</td>
<td>222.12</td>
<td>253.87</td>
</tr>
<tr>
<td>Mode 2</td>
<td>1735.87</td>
<td>1684.59</td>
<td>1709.10</td>
</tr>
<tr>
<td>Mode 3</td>
<td>3431.11</td>
<td>3363.12</td>
<td>3404.70</td>
</tr>
</tbody>
</table>

\[ \phi = kl = n \cdot 2\pi \]

Phase-closure principle is applied here to obtain the results!
Frequency-split phenomenon

First acoustical mode

Structure mode
Conclusion

- Rotation induces frequency-split phenomenon.

- In spinning case, the phase velocity of each mode is approximately the associated static phase velocity plus/minus flow velocity.

- The radiation efficiency varies for positive-going wave and for the negative-going wave.

- The plane wave mode is significantly influenced by the coupling effect while higher modes are less influenced.
End

Thank you!