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A MODEL FOR SOUND ABSORPTION BY AND SOUND TRANSMISSION THROUGH LIMP FIBROUS LAYERS

by

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INTRODUCTION

• A limp porous material model has been derived from Biot's theory by assuming zero stiffness of the frame.

• The simplified model yields a single second order wave equation governing the propagation of a single wave type.

• Experiments indicate that the predictions made using the present model more closely match experimental results related to limp fibrous materials than those made by using a rigid porous material model and give results equivalent to an elastic porous material model.
TRANSMISSION LOSS RESULTS

Measured parameters:
\[ \rho_b = 21.1 \text{Kg} / \text{m}^3 \]
\[ \sigma = 16660 \text{Rayls} / \text{m} \]
\[ l = 5.7 \text{cm} \]

Estimated parameters:
\[ N = 20N / \text{m}^2 \]
\[ \eta = 0.005 \]
\[ \vartheta = 0.3 \]
\[ q = 1.2 \]
\[ \phi = 0.98 \]
REVIEW OF EXISTING THEORIES

• RIGID
  - Rayleigh (1877, ‘Theory of sound’)
  - Monna (1938, Physica 5, 129-142)
  - Morse (1952, ‘Theoretical Acoustics’)
  - Attenborough (1985, JSV 99 (4), 521-544)

• LIMP
  - Wood (1941, ‘A textbook of sound’)
  - Beranek (1947, JASA 19, 556)
  - Kosten and Jansen (1957, Acustica 7, 372)
  - Geertsma and Smit (1957, Geophysics 26, 169-181)

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REVIEW OF EXISTING THEORIES

- ELASTIC
  - Zwikker and Kosten (1949, ‘Sound absorbing materials’)
  - Biot (1956, JASA 28, 168-191)
  - Allard (1993, ‘Propagation of sound in porous media’)

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OBJECTIVE OF THE PRESENT WORK

- To derive a limp porous material model (i.e., one that allows for motion of the solid phase) based on the Biot Theory.

- To show that the model gives the same result as the elastic porous theory for 'limp' fibrous materials.
ELASTIC POROUS MATERIAL MODEL

GOVERNING WAVE EQUATIONS (BIOT)

Solid: \[ Q \nabla^2 \varepsilon + \omega^2 \rho^*_s \varepsilon = -P \nabla^2 e_s - \omega^2 \rho^*_p \varepsilon \]

Fluid: \[ R \nabla^2 \varepsilon + \omega^2 \rho^*_p \varepsilon = -Q \nabla^2 e_s - \omega^2 \rho^*_p \varepsilon \]

where (from Gassman)

\[
P = \frac{(1-\phi) \left(1-\phi - \frac{K_b}{K_s} \right) K_s + \phi \frac{K_s}{K_f} K_b}{1-\phi - \frac{K_b}{K_s} + \phi \frac{K_s}{K_f}} + \frac{4}{3} N,
\]

\[
Q = \frac{\left(1-\phi - \frac{K_b}{K_s} \right) \phi K_s}{1-\phi - \frac{K_b}{K_s} + \phi \frac{K_s}{K_f}}
\]

and

\[
R = \frac{\phi^2 K_s}{1-\phi - \frac{K_b}{K_s} + \phi \frac{K_s}{K_f}}
\]
WAVENUMBERS IN ELASTIC MODEL

COMPRESSIONAL WAVES

\[ k_{1,2}^2 = \frac{\omega^2}{2(PR - Q^2)} \left[ P\rho_{22}^* + R\rho_{11}^* - 2Q\rho_{12}^* \pm \sqrt{\Delta} \right] \]

where,

\[ \Delta = \left( P\rho_{22}^* + R\rho_{11}^* - 2Q\rho_{12}^* \right)^2 - 4(PR - Q^2)(\rho_{11}^*\rho_{22}^* - \rho_{12}^*^2) \]

SHEAR WAVE

\[ k_t^2 = \frac{\omega^2}{N} \left( \frac{\rho_{11}^*\rho_{22}^* - \rho_{12}^*^2}{\rho_{22}^*} \right) \]
LIMP POROUS MATERIAL MODEL

- The frame bulk modulus is assumed to be zero. Therefore,
  \[ P = \frac{(1 - \phi)^2 K_s}{(1 - \phi) + \phi \frac{K_s}{K_f}} \]
  \[ Q = \frac{(1 - \phi)\phi K_s}{(1 - \phi) + \phi \frac{K_s}{K_f}} \]
  \[ R = \frac{\phi^2 K_s}{(1 - \phi) + \phi \frac{K_s}{K_f}} \]

- Applying these expressions within the governing equations of Biot’s theory, we obtain,
  \[(2\rho_{12}^* Q - \rho_{22}^* P - \rho_{11}^* R) \nabla^2 \varepsilon + \omega^2 (\rho_{12}^* - \rho_{11}^* \rho_{22}^*) \varepsilon = 0\]

and
  \[ e_s = \frac{(\rho_{12}^* Q - \rho_{22}^* P)}{(\rho_{12}^* P - \rho_{11}^* Q)} \varepsilon = \frac{1}{b} \varepsilon. \]
\[ k_p^2 = \frac{\omega^2 \left( \rho_{11}^* \rho_{22}^* - \rho_{12}^* \right)}{\left( \rho_{22}^* P + \rho_{11}^* R - 2 \rho_{12}^* Q \right)} \]

- Single dilatational wave propagates through both phases of limp porous material.
\[ \varepsilon = \nabla U_y = e^{-ik_x x} \left( C_1 e^{-ik_y y} + C_2 e^{ik_y y} \right) \]

\[ e_s = \nabla u_y = \frac{1}{b} e^{-ik_x x} \left( C_1 e^{-ik_y y} + C_2 e^{ik_y y} \right) \]

**Fluid stress:** \[ s = R \varepsilon + Q e_s \]

**Solid stress:** \[ \sigma_y = Pe_s + Q \varepsilon \]
**BOUNDARY CONDITIONS**

\[ y=0 \]
\[-\phi p = s \]
\[-(1-\phi)p = \sigma_y \]

\[ y=d \]
\[ u_y = 0 \]
\[ U_y = 0 \]

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NORMAL SPECIFIC IMPEDANCE

\[
Z_n = \frac{1}{\rho_0 c} \left( \frac{p}{v_y} \right)_{y=0} = -\frac{i(Rb + Q)k \cot kd}{\rho_0 c \phi \omega [1 - \phi (1 - b)]}
\]

where,

\[
v_y = i\omega (1 - \phi) u_y + i\omega \phi U_y
\]
COMPARISON WITH EXPERIMENT

TRANSMISSION LOSS COMPARISON

![Graph showing transmission loss comparison between different models.]

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COMPARISON WITH ELASTIC MODEL

TRANSMISSION LOSS

ABSORPTION COEFFICIENT

- How limp is 'Limp'?

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CONCLUSIONS

A theoretical model for limp fibrous materials has been constructed using Biot’s theory. The new model has the following advantages over the two prevalent models:

- More accurate predictions than the rigid porous material model.
- Numerically robust compared to the elastic porous material model.
- Computationally faster compared to the elastic porous material model.